

# **Aerodynamics & Flight Mechanics Research Group**

## **The Response of a Vibration Absorber or Isolator**

S. J. Newman

Technical Report AFM-11/23

January 2011

UNIVERSITY OF SOUTHAMPTON

SCHOOL OF ENGINEERING SCIENCES

AERODYNAMICS AND FLIGHT MECHANICS RESEARCH GROUP

**The Response of a Vibration Absorber or Isolator**

by

**S. J. Newman**

AFM Report No. AFM 11/23

January 2011

© School of Engineering Sciences, Aerodynamics and Flight Mechanics Research Group



## COPYRIGHT NOTICE

(c) SES University of Southampton All rights reserved.

SES authorises you to view and download this document for your personal, non-commercial use. This authorization is not a transfer of title in the document and copies of the document and is subject to the following restrictions: 1) you must retain, on all copies of the document downloaded, all copyright and other proprietary notices contained in the Materials; 2) you may not modify the document in any way or reproduce or publicly display, perform, or distribute or otherwise use it for any public or commercial purpose; and 3) you must not transfer the document to any other person unless you give them notice of, and they agree to accept, the obligations arising under these terms and conditions of use. This document, is protected by worldwide copyright laws and treaty provisions.



# Preamble

This document examines a system of masses, springs and dampers placed in line on a frictionless table. From these equations, the responses of a generic vibration absorber or isolator can be derived.

The generic system is shown in Figure 1

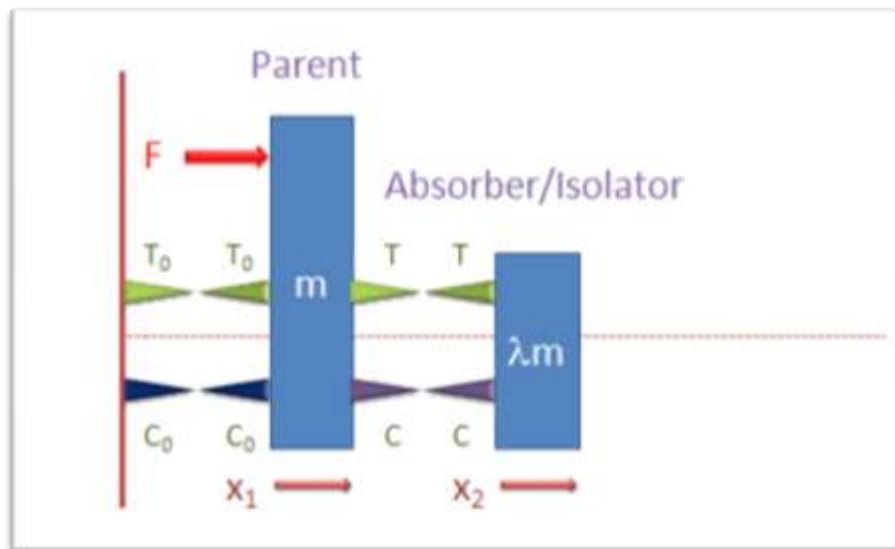


Figure 1

## Basic Analysis

The parent – earth spring tension is given by:

$$T_0 = k_0 x_1 \quad (1.)$$

The parent – absorber spring tension is given by:

$$T = k(x_2 - x_1) \quad (2.)$$



The parent – earth damper force is given by:

$$C_0 = c_0 \dot{x}_1 \quad (3.)$$

The parent – absorber spring tension is given by:

$$C = c(\dot{x}_2 - \dot{x}_1) \quad (4.)$$

The equation of motion for the parent mass is then:

$$F + T - T_0 + C - C_0 = m\ddot{x}_1 \quad (5.)$$

and for the abs/iso mass:

$$-C - T = \lambda m\ddot{x}_2 \quad (6.)$$

From which we obtain:

and:

$$\lambda m\ddot{x}_2 + k(x_1 - x_2) + c(\dot{x}_2 - \dot{x}_1) = 0 \quad (7.)$$

If we make the following substitutions:

$$\begin{bmatrix} F \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ X_1 \\ X_2 \end{bmatrix} e^{i\omega t} \quad (8.)$$

Substituting (9) into (5) & (7) gives:

$$\begin{aligned} -\omega^2 mX_1 + (k + k_0)X_1 - kX_2 \\ -i\omega c(X_2 - X_1) + i\omega c_0 X_1 = F_0 \end{aligned} \quad (9.)$$

$$-\omega^2 \lambda mX_2 + k(X_1 - X_2) + i\omega c(X_2 - X_1) = 0 \quad (10.)$$

$$\begin{aligned} (k + k_0 - m\omega^2 + i\omega c + i\omega c_0)X_1 \\ + X_2(-k - i\omega c) = F_0 \end{aligned} \quad (11.)$$



$$(-k - i\omega c)X_1 + (k - \lambda m\omega^2 + i\omega c)X_2 = 0 \quad (12.)$$

$$\begin{aligned} k &= \lambda m\omega_2^2 \\ k_0 &= m\omega_0^2 \\ c &= 2\zeta m\omega_2 \\ c_0 &= 2\zeta_0 m\omega_0 \end{aligned} \quad (13.)$$

$$A = \begin{bmatrix} \lambda\omega_2^2 + \omega_0^2 - \omega^2 + i2\zeta\omega\omega_2 + i2\zeta_0\omega\omega_0 & -\lambda\omega_2^2 - i2\zeta\omega\omega_2 \\ -\lambda\omega_2^2 - i2\zeta\omega\omega_2 & \lambda\omega_2^2 - \lambda\omega^2 + i2\zeta\omega\omega_2 \end{bmatrix} \quad (14.)$$

$$A \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ m \\ 0 \end{bmatrix} \quad (15.)$$

$$\begin{aligned} \bar{\omega} &= \frac{\omega}{\omega_0} \\ \bar{\omega}_2 &= \frac{\omega_2}{\omega_0} \end{aligned} \quad (16.)$$

$$A' = \frac{A}{\omega_0^2} = \begin{bmatrix} \lambda\bar{\omega}_2^2 + 1 - \bar{\omega}^2 + i2\zeta\bar{\omega}\bar{\omega}_2 + i2\zeta_0\bar{\omega} & -\lambda\bar{\omega}_2^2 - i2\zeta\bar{\omega}\bar{\omega}_2 \\ -\lambda\bar{\omega}_2^2 - i2\zeta\bar{\omega}\bar{\omega}_2 & \lambda\bar{\omega}_2^2 - \lambda\bar{\omega}^2 + i2\zeta\bar{\omega}\bar{\omega}_2 \end{bmatrix} \quad (17.)$$



This standard result gives SHM with circular frequency given by:

$$A' \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \frac{F_0}{m\omega_0^2} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{F_0}{k} \\ 0 \end{bmatrix} = \begin{bmatrix} X_0 \\ 0 \end{bmatrix} \quad (18.)$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = A'^{-1} \begin{bmatrix} X_0 \\ 0 \end{bmatrix} \quad (19.)$$



# MATLAB File

```

%
% Response of Vibration Absorber or Isolator
%
% SJN 01/03/08
%
%-----
clear
%-----
lambda=1; % Ratio of Absorber Mass to Parent Mass
zeta0=2.5; % Damping Coefficient for Parent - Earth
zeta=0; % Damping Coefficient for Parent - Absorber
om2bar=2; % Natural Frequency Ratio - Absorber to Parent
%-----
ommax=3; % Maximum Frequency Ratio - Forcing to Parent
%-----
ombar=linspace(0,ommax,1000);
ombar2=ombar.^2;
t0=i*2*zeta0*ombar;
t1=i*2*zeta*ombar*om2bar;
t2=lambda*om2bar^2;
det=(t2+1-ombar2+t1+t0).*(t2-lambda*ombar2+t1)-(t2+t1).*(t2+t1);
%-----
X1=(t2+t1-lambda*ombar2)./det;
X2=(t2-ombar2+t1+t0)./det;
parent=abs(X1);
phaseparent=57.296*angle(X1);
absorber=abs(X2);
phaseabsorber=57.296*angle(X1);
%-----
% Response Amplitude Plot
%-----
clf
semilogy(ombar,parent,'b');
hold on
semilogy(ombar,absorber,'r');
grid on
legend('Parent','Absorber');
xlabel('Forcing Frequency / Parent Natural Frequency');
ylabel('Response Amplitude');
title(['Response Amplitude - \lambda < ',num2str(lambda),' > \omega_2 / \omega_0 < ',num2str(om2bar),' > \zeta < ',num2str(zeta),' > \zeta_0 < ',num2str(zeta0),' >']);
%-----
figure
%-----
% Phase Plot
%-----
clf
plot(ombar,phaseparent,'b');
hold on
plot(ombar,phaseabsorber,'r');
grid on
legend('Parent','Absorber');
set(gca,'YTick',[-180 -120 -60 0 60 120 180]);
axis([0 ommax -180 180]);
xlabel('Forcing Frequency / Parent Natural Frequency');

```





```
YLabel('Phase Lead over Forcing');  
title(['Phase Lead - \lambda < ', num2str(lambda), ' > \omega_2 / \omega_0 <  
' , num2str(om2bar), ' > \zeta < ', num2str(zeta), ' > \zeta_0 <  
' , num2str(zeta0), ' >']);  
%-----
```

