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The Response of a Vibration Absorber or Isolator

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Preamble

This document examines a system of masses, springs and dampers placed in line on a frictionless table. From these equations, the responses of a generic vibration absorber or isolator can be derived.

The generic system is shown in Figure 1

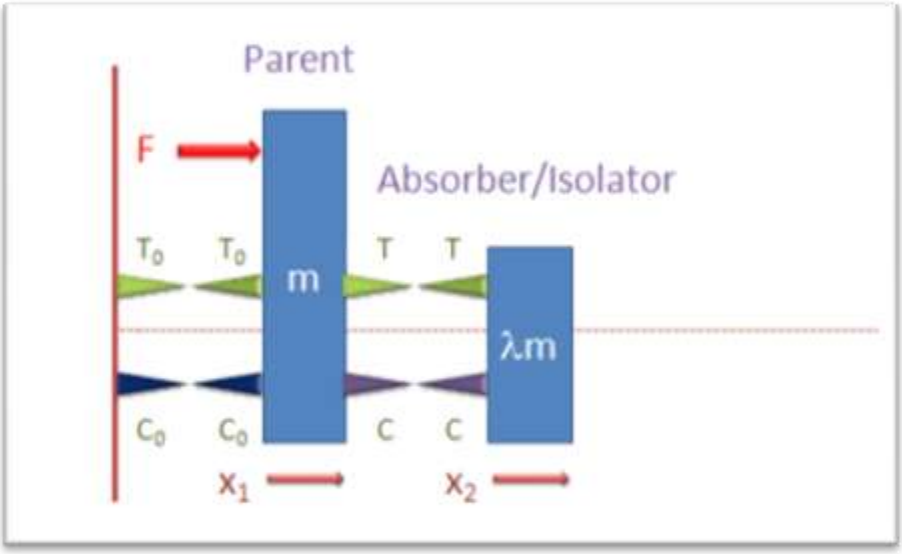

Figure 1

Basic Analysis

The parent – earth spring tension is given by:

$$T_0 = k_0 x_1 \tag{1.}$$

The parent – absorber spring tension is given by:

$$T = k(x_2 - x_1) \tag{2.}$$

The parent – earth damper force is given by:

$$C_0 = c_0 \dot{x}_1 \qquad (3.)$$

The parent – absorber spring tension is given by:

$$C = c(\dot{x}_2 - \dot{x}_1) \qquad (4.)$$

The equation of motion for the parent mass is then:

$$F + T - T_0 + C - C_0 = m\ddot{x}_1 \qquad (5.)$$

and for the abs/iso mass:

$$-C - T = \lambda m \ddot{x}_2 \qquad (6.)$$

From which we obtain:

and:

$$\lambda m \ddot{x}_2 + k(x_1 - x_2) + c(\dot{x}_2 - \dot{x}_1) = 0 \qquad (7.)$$

If we make the following substitutions:

$$\begin{bmatrix} F \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ X_1 \\ X_2 \end{bmatrix} e^{i\omega t} \qquad (8.)$$

Substituting (9) into (5) & (7) gives:

$$\begin{gathered} -\omega^2 m X_1 + (k + k_0) X_1 - k X_2 \\ -i\omega c(X_2 - X_1) + i\omega c_0 X_1 = F_0 \end{gathered} \qquad (9.)$$

$$-\omega^2 \lambda m X_2 + k(X_1 - X_2) + i\omega c(X_2 - X_1) = 0 \qquad (10.)$$

$$\begin{gathered} (k + k_0 - m\omega^2 + i\omega c + i\omega c_0) X_1 \\ + X_2(-k - i\omega c) = F_0 \end{gathered} \qquad (11.)$$

$$(-k - i\omega c)X_1 + (k - \lambda m\omega^2 + i\omega c)X_2 = 0 \quad (12.)$$

$$\begin{aligned} k &= \lambda m\omega_2^2 \\ k_0 &= m\omega_0^2 \\ c &= 2\zeta m\omega_2 \\ c_0 &= 2\zeta_0 m\omega_0 \end{aligned} \quad (13.)$$

$$A = \begin{bmatrix} \lambda\omega_2^2 + \omega_0^2 - \omega^2 + i2\zeta\omega\omega_2 + i2\zeta_0\omega\omega_0 & -\lambda\omega_2^2 - i2\zeta\omega\omega_2 \\ -\lambda\omega_2^2 - i2\zeta\omega\omega_2 & \lambda\omega_2^2 - \lambda\omega^2 + i2\zeta\omega\omega_2 \end{bmatrix} \quad (14.)$$

$$A\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \frac{F_0}{m} \\ 0 \end{bmatrix} \quad (15.)$$

$$\begin{aligned} \varpi &= \frac{\omega}{\omega_0} \\ \varpi_2 &= \frac{\omega_2}{\omega_0} \end{aligned} \quad (16.)$$

$$A' = \frac{A}{\omega_0^2} = \begin{bmatrix} \lambda\varpi_2^2 + 1 - \varpi^2 + i2\zeta\varpi\varpi_2 + i2\zeta_0\varpi & -\lambda\varpi_2^2 - i2\zeta\varpi\varpi_2 \\ -\lambda\varpi_2^2 - i2\zeta\varpi\varpi_2 & \lambda\varpi_2^2 - \lambda\varpi^2 + i2\zeta\varpi\varpi_2 \end{bmatrix} \quad (17.)$$

This standard result gives SHM with circular frequency given by:

$$A^{'}\begin{bmatrix}X_1\\X_2\end{bmatrix}=\begin{bmatrix}\dfrac{F_0}{m\omega_0^2}\\0\end{bmatrix}=\begin{bmatrix}\dfrac{F_0}{k}\\0\end{bmatrix}=\begin{bmatrix}X_0\\0\end{bmatrix} \tag{18.}$$

$$\begin{bmatrix}X_1\\X_2\end{bmatrix}=A^{'\,-1}\begin{bmatrix}X_0\\0\end{bmatrix} \tag{19.}$$

MATLAB File

```
%
%   Response of Vibration Absorber or Isolator
%
%   SJN 01/03/08
%
%------------------------------------------------------
clear
%------------------------------------------------------
lambda=1;    % Ratio of Absorber Mass to Parent Mass
zeta0=2.5;     % Damping Coefficient for Parent - Earth
zeta=0;      % Damping Coefficient for Parent - Absorber
om2bar=2;    % Natural Frequency Ratio - Absorber to Parent
%------------------------------------------------------
ommax=3;     % Maximum Frequency Ratio - Forcing to Parent
%------------------------------------------------------
ombar=linspace(0,ommax,1000);
ombar2=ombar.^2;
t0=i*2*zeta0*ombar;
t1=i*2*zeta*ombar*om2bar;
t2=lambda*om2bar^2;
det=(t2+1-ombar2+t1+t0).*(t2-lambda*ombar2+t1)-(t2+t1).*(t2+t1);
%------------------------------------------------------
X1=(t2+t1-lambda*ombar2)./det;
X2=(t2-ombar2+t1+t0)./det;
parent=abs(X1);
phaseparent=57.296*angle(X1);
absorber=abs(X2);
phaseabsorber=57.296*angle(X1);
%------------------------------------------------------
%   Response Amplitude Plot
%------------------------------------------------------
clf
semilogy(ombar,parent,'b');
hold on
semilogy(ombar,absorber,'r');
grid on
legend('Parent','Absorber');
XLabel('Forcing Frequency / Parent Natural Frequency');
YLabel('Response Amplitude');
title(['Response Amplitude - \lambda < ',num2str(lambda),' > \omega_2 /
\omega_0 < ',num2str(om2bar),' > \zeta < ',num2str(zeta),' > \zeta_0 <
',num2str(zeta0),' >']);
%------------------------------------------------------
figure
%------------------------------------------------------
%   Phase Plot
%------------------------------------------------------
clf
plot(ombar,phaseparent,'b');
hold on
plot(ombar,phaseabsorber,'r');
grid on
legend('Parent','Absorber');
set(gca,'YTick',[-180 -120 -60 0 60 120 180]);
axis([0 ommax -180 180]);
XLabel('Forcing Frequency / Parent Natural Frequency');
```

```
YLabel('Phase Lead over Forcing');
title(['Phase Lead - \lambda < ',num2str(lambda),' > \omega_2 / \omega_0 <
',num2str(om2bar),' > \zeta < ',num2str(zeta),' > \zeta_0 <
',num2str(zeta0),' >']);
%-------------------------------------------------------
```