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The Compression Behaviour of an Oleo

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Preamble

This document examines the dynamic compression behaviour of an oleo. This is modelled by a linear spring and damper in parallel.

Figure 1 – Airbus A340 – 600 at Farnborough 2002

Nomenclature

Variable	Description
x	Deflection
x_s	Static Deflection
M	Mass
g	Gravity
t	Time
k	Spring Rate
C	Damping Rate
ζ	Critical Damping Factor
V_0	Vertical Touchdown Velocity
T	Kinetic Energy
U	Potential Energy
ξ	Damping Potential
δW	Virtual Work Done
Q_x	External Work Coefficient
n	Ratio of Energy Absorptions

Basic Analysis

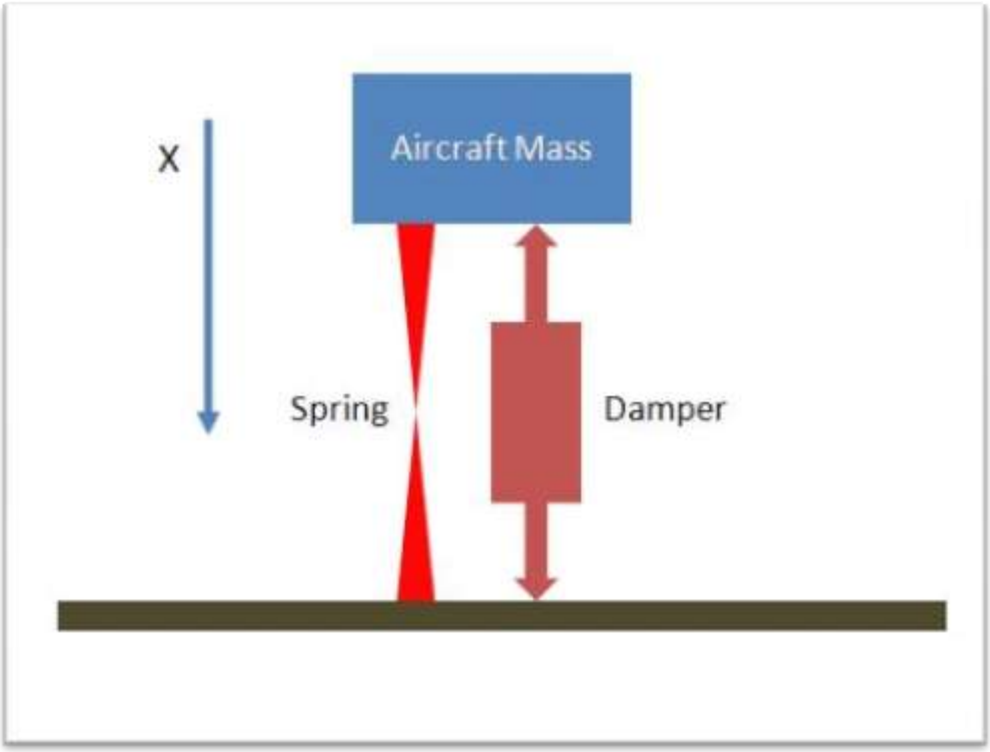

Figure 2

The static deflection is given by:

$$x_s = \frac{Mg}{k} \tag{1.}$$

From which the spring rate is given by:

$$k = \frac{Mg}{x_s} \tag{2.}$$

The natural circular frequency is given by:

$$\omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{g}{x_s}} \tag{3.}$$

Application of Lagrange's Method

The kinetic energy is given by:

$$T = \frac{1}{2}M\dot{x}^2 \tag{4.}$$

The potential energy in the spring is given by:

$$U = \frac{1}{2}kx^2 \tag{5.}$$

The damping potential is given by:

$$\xi = \frac{1}{2}C\dot{x}^2 \tag{6.}$$

The virtual work done by the external forces is given by:

$$\delta W = Mg \cdot \delta x \tag{7.}$$

The external force contribution to the RHS is given by:

$$Q_x = Mg \tag{8.}$$

Lagrange's equation is:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial T}{\partial x} + \frac{\partial U}{\partial x} + \frac{\partial \xi}{\partial \dot{x}} = Q_x \qquad (9.)$$

From which we obtain:

$$\begin{aligned} \frac{\partial T}{\partial \dot{x}} &= m\dot{x} \\ \frac{\partial T}{\partial x} &= 0 \\ \frac{\partial U}{\partial x} &= kx \\ \frac{\partial \xi}{\partial \dot{x}} &= C\dot{x} \\ Q_x &= Mg \end{aligned} \qquad (10.)$$

Assembling the Lagrange Equation:

$$M\ddot{x} + C\dot{x} + kx = Mg \qquad (11.)$$

We use the following definitions:

$$\begin{aligned} k &= M\omega^2 \\ C &= 2M\omega\zeta \end{aligned} \qquad (12.)$$

Solution of Equation of Motion

Particular Integral:

$$\frac{Mg}{k} = x_s \tag{13.}$$

Complementary Function:

$$Ae^{\lambda_1 t} + Be^{\lambda_2 t} \tag{14.}$$

Where:

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} \dfrac{-C + \sqrt{C^2 - 4kM}}{2M} \\ \dfrac{-C - \sqrt{C^2 - 4kM}}{2M} \end{bmatrix} \tag{15.}$$

Defining:

$$\begin{aligned} k &= M\omega^2 \\ C &= 2\zeta M\omega \end{aligned} \tag{16.}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} \omega\left\{-\zeta + \sqrt{\zeta^2 - 1}\right\} \\ \omega\left\{-\zeta - \sqrt{\zeta^2 - 1}\right\} \end{bmatrix} \tag{17.}$$

The boundary conditions are:

$$\begin{aligned} t &= 0 \\ x &= 0 \\ \dot{x} &= V_0 \end{aligned} \quad (18.)$$

General Solution

$\zeta>1$

$$x = x_s + Ae^{\lambda_1 t} + Be^{\lambda_2 t} \quad (19.)$$

$$\dot{x} = A\lambda_1 e^{\lambda_1 t} + B\lambda_2 e^{\lambda_2 t} \quad (20.)$$

Applying the boundary conditions gives:

$$\begin{aligned} A + B &= -x_s \\ A\lambda_1 + B\lambda_2 &= V_0 \end{aligned} \quad (21.)$$

From which:

$$\begin{aligned} A &= \frac{-(\lambda_2 x_s + V_0)}{(\lambda_2 - \lambda_1)} \\ B &= \frac{(\lambda_1 x_s + V_0)}{(\lambda_2 - \lambda_1)} \end{aligned} \quad (22.)$$

ζ=1

$$x = x_s + (A + Bt)e^{-\omega t} \tag{23.}$$

$$\dot{x} = -\omega(A + Bt)e^{-\omega t} + Be^{-\omega t} \tag{24.}$$

Applying the boundary conditions gives:

$$\begin{gathered} A + x_s = 0 \\ -\omega A + B = V_0 \end{gathered} \tag{25.}$$

From which:

$$\begin{gathered} A = -x_s \\ B = V_0 - \omega x_s \end{gathered} \tag{26.}$$

ζ<1

$$\begin{aligned} x = x_s + e^{-\zeta\omega t} \Big(A \cos\left\{\omega t\sqrt{1-\zeta^2}\right\} \\ + B \sin\left\{\omega t\sqrt{1-\zeta^2}\right\}\Big) \end{aligned} \tag{27.}$$

$$\begin{aligned} \dot{x} = -\zeta\omega e^{-\zeta\omega t} \cdot \Big(A\cos\left\{\omega t\sqrt{1-\zeta^2}\right\} + B\sin\left\{\omega t\sqrt{1-\zeta^2}\right\}\Big) \\ + \omega\sqrt{1-\zeta^2} \cdot e^{-\zeta\omega t}\Big(-A\sin\left\{\omega t\sqrt{1-\zeta^2}\right\} \\ + B\cos\left\{\omega t\sqrt{1-\zeta^2}\right\}\Big) \end{aligned} \tag{28.}$$

Applying the boundary conditions gives:

$$A + x_s = 0$$
$$-\omega\zeta A + B\omega\sqrt{1-\zeta^2} = V_0 \tag{29.}$$

From which:

$$A = -x_s$$
$$B = \frac{V_0 - \zeta\omega x_s}{\omega\sqrt{1-\zeta^2}} \tag{30.}$$

Energy Absorption

At the moment of touchdown the energy is the potential energy of the aircraft together with its vertical kinetic energy – viz:

$$\frac{1}{2}MV_0^2 + Mgx_s \tag{31.}$$

When the aircraft reaches its final steady state the energy is the potential energy in the spring viz:

$$\frac{1}{2}kx_s^2 = \frac{1}{2}\frac{Mg}{x_s}x_s^2 = \frac{1}{2}Mgx_s \tag{32.}$$

From which we conclude that the energy absorbed by the damper is given by:

$$\frac{1}{2}\{MV_0^2 + Mgx_s\} \tag{33.}$$

Whence the ratio of the energy absorbed by the damper and spring is given by:

$$\begin{aligned}\frac{Damper\ Energy}{Spring\ Energy} &= \frac{\frac{1}{2}\{MV_0^2 + Mgx_s\}}{\frac{1}{2}Mgx_s} \\ &= 1 + \frac{V_0^2}{gx_s} = n\end{aligned} \tag{34.}$$

From which the fraction of the total is given by:

$$Damper\ Energy\ Fraction = \frac{n}{1+n} \tag{35.}$$

$$Spring\ Energy\ Fraction = \frac{1}{1+n} \tag{36.}$$

Example

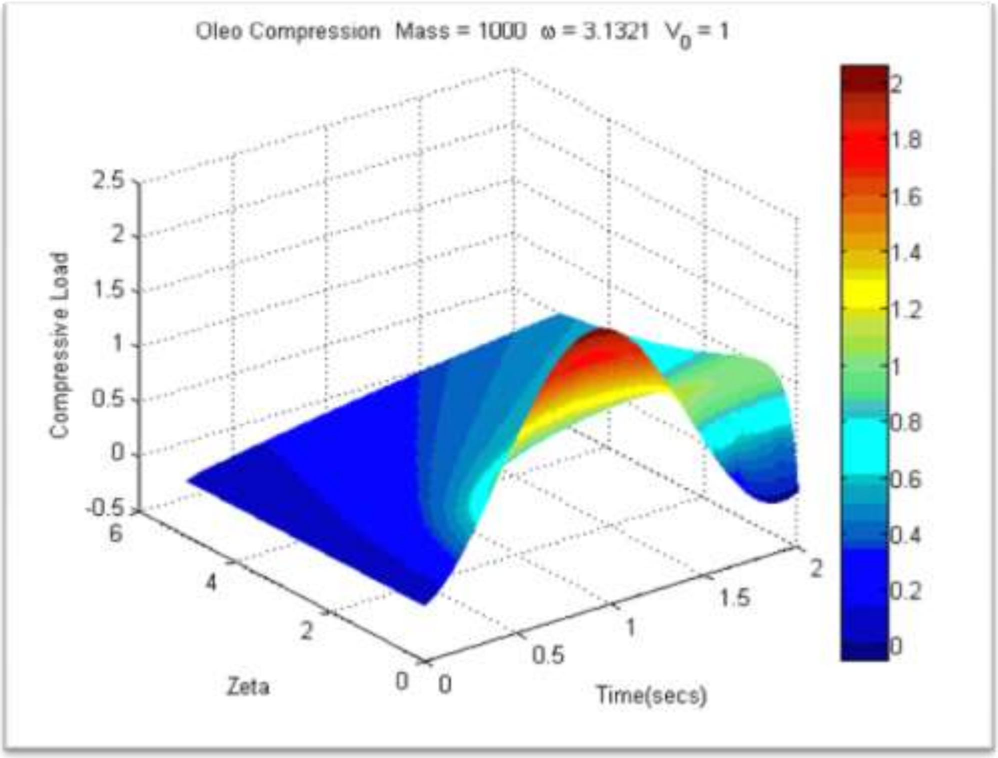

Figure 3

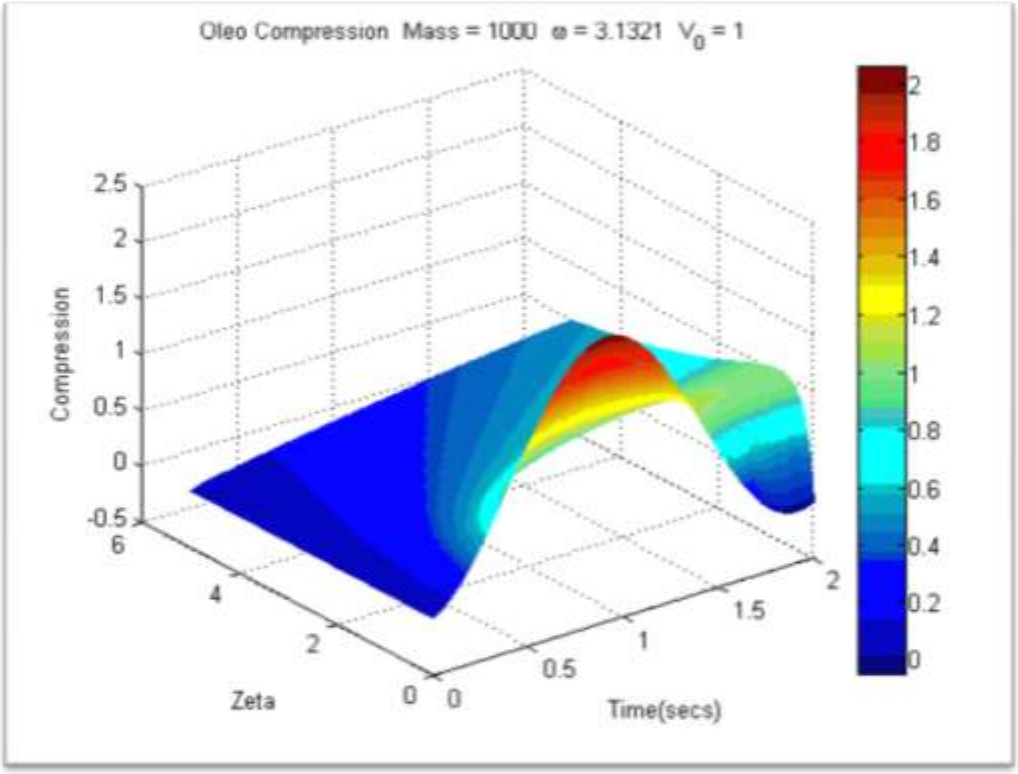
Oleo Compression Mass = 1000 ω = 3.1321 V_0 = 1
Compression
Zeta
Time(secs)
2.5
2
1.5
1
0.5
0
-0.5
6
4
2
0
0
0.5
1
1.5
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0

Figure 4

MATLAB File

```
%
%   Oleo Drop Test
%
%   SJN 20/04/08
%
clear
colordef white
%-----------------------------------------------------------------------
xs=1;
%z=[0 .1 .2 .5 1 2 5 10];
z=0:.1:5;
M=1000.;

v0=1.;

nt=100;
tmax=2;
%-----------------------------------------------------------------------
[drivel,nz]=size(z);

grav=9.81;
omg=sqrt(grav/xs);
k=xs/(M*grav);
cdamp=2*M*omg*z;

t=linspace(0,tmax,nt);

xplt=[];
loadplt=[];
landloadplt=[];

[T,Z]=meshgrid(t,z);
%-----------------------------------------------------------------------
for iz=1:nz
    zeta=z(iz);

    if zeta>1;
        l1=omg*(-zeta+sqrt(zeta^2-1));
        l2=omg*(-zeta-sqrt(zeta^2-1));
        A=-(l2*xs+v0)/(l2-l1);
        B=(l1*xs+v0)/(l2-l1);
        x=xs+A*exp(l1*t)+B*exp(l2*t);
        xd=l1*A*exp(l1*t)+l2*B*exp(l2*t);
        xdd=l1^2*A*exp(l1*t)+l2^2*B*exp(l2*t);
    elseif zeta==1
        A=-xs;
        B=v0-omg*xs;
        x=xs+(A+B*t).*exp(-omg*t);
        xd=-omg*(A+B*t).*exp(-omg*t)+B*exp(-omg*t);
        xdd=-omg*exp(-omg*t).*(2*B-omg*A+B*t);
    else
        A=-xs;
        B=(-zeta*omg*xs+v0)/(omg*sqrt(1-zeta^2));
        omg1=omg*sqrt(1-zeta^2);
```

```
        cosomg1t=cos(omg1*t);
        sinomg1t=sin(omg1*t);
        x=xs+exp(-zeta*omg*t).*(A*cosomg1t+B*sinomg1t);
        coef1=zeta^2;
        coef2=2*zeta*sqrt(1-zeta^2);
        coef3=1-zeta^2;
        xd=-zeta*omg*exp(-zeta*omg*t).*(A*cosomg1t+B*sinomg1t)+omg1*exp(-
zeta*omg*t).*(-A*sinomg1t+B*cosomg1t);
        xdd=omg^2*exp(-zeta*omg*t).*((coef1*cosomg1t+B*sinomg1t)-coef2*(-
A*sinomg1t+B*cosomg1t)-coef3*(A*cosomg1t+B*sinomg1t));
    end
%------------------------------------------------------------------------
    load=-M*xdd;
    cdamp=2*M*omg*zeta;
    landload=cdamp*xd;
    xplt=[xplt;x];
    loadplt=[loadplt;load];
    landloadplt=[landloadplt;landload];
end
%------------------------------------------------------------------------
%   Deflection Plot
surf(T,Z,xplt);
shading interp;
grid on
colorbar
xlabel('Time(secs)');
ylabel('Zeta');
zlabel('Compression');
title(['Oleo Compression  Mass = ',num2str(M),'  \omega = ',num2str(omg),'
V_0 = ',num2str(v0)]);
%------------------------------------------------------------------------
%   Load Plot
figure
surf(T,Z,xplt);
shading interp;
grid on
colorbar
xlabel('Time(secs)');
ylabel('Zeta');
zlabel('Compressive Load');
title(['Oleo Compression  Mass = ',num2str(M),'  \omega = ',num2str(omg),'
V_0 = ',num2str(v0)]);
%------------------------------------------------------------------------
%   Load v Compression Plot
figure
maxcomp=[];
for ic=1:nz
    maxcomp=[maxcomp max(xplt(ic,:))];
end
maxload=landloadplt(:,1);

plot(maxcomp,maxload);
grid on
xlabel('Maximum Compression');
ylabel('Landing Load');
title(['Oleo Compression  Mass = ',num2str(M),'  \omega = ',num2str(omg),'
V_0 = ',num2str(v0)]);
```