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Projectile with Drag

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SCHOOL OF ENGINEERING SCIENCES

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by

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Calculation of Trajectory of a Brick with a Drag Force

The trajectory of a brick with the inclusion of a drag force is modelled. The integration of the resulting equations of motion is achieved using a time marching numerical method. The acceleration components are assumed constant during each time interval.

GLOSSARY OF TERMS

The terminology used in the analysis is presented in the following glossary.

Horizontal Displacement (+ve downrange)	x
Vertical Displacement (+ve upward)	y
Horizontal Velocity	u
Vertical Velocity	v
Horizontal Acceleration	a_x
Vertical Acceleration	a_y
Start Height	h_{ST}
Start Horizontal Velocity	V_{ST}
Drag Force	D
Flat Plate Drag Area (C_D=1)	A
Mass	m
Gravity	g
Air Density	ρ
Flight Path Inclination Angle	θ

Derivation of Equations of Motion

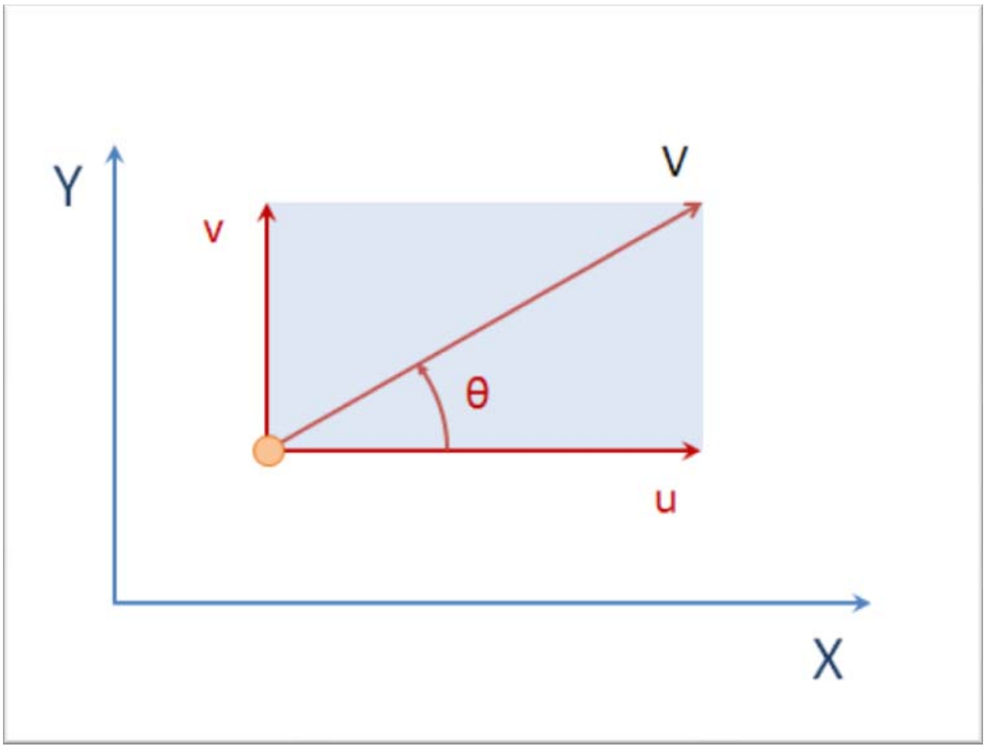

Figure 1 – Coordinate System

The drag force on the vehicle is given by:

$$\begin{aligned} D &= \frac{1}{2}\rho V^2 \cdot A \\ &= \frac{1}{2}\rho\left(u^2 + v^2\right)\cdot A \end{aligned} \qquad (1)$$

The flight path inclination and associated trigonometric identities are given by:

$$\begin{aligned}\tan\theta &= \frac{v}{u} \\ \sin\theta &= \frac{v}{V} \\ \cos\theta &= \frac{u}{V}\end{aligned} \tag{2}$$

Applying Newton's Second Law gives:

$$\begin{aligned} ma_x &= -D\cos\theta \\ &= -\frac{1}{2}\rho V u A \\ ma_y &= -D\sin\theta - mg \\ &= -\frac{1}{2}\rho V u A - mg \end{aligned} \tag{3}$$

Solution of the Equations of Motion

If a time increment of Δt is applied *with the acceleration values being held constant* we have the following expressions giving the displacement increments:

$$\Delta x = u\Delta t + \frac{1}{2}a_x\left(\Delta t\right)^2$$
$$\Delta y = v\Delta t + \frac{1}{2}a_y\left(\Delta t\right)^2 \tag{4}$$

And the corresponding velocity increments are:

$$\Delta u = a_x\Delta t$$
$$\Delta v = a_y\Delta t \tag{5}$$

Finally, the height of the vehicle is given by:

$$h = h_{ST} + y \tag{6}$$

Flat Plate Drag Area

In order to gain an estimate of the drag of the aircraft, three situations are considered:

1. No drag – zero flat plate drag area
2. Flight in conventional fuselage attitude – flat plate drag area is projected frontal area, assumed to be circular, based on equivalent circle radius
3. Flight in a high nose up attitude – flat plate drag area is projected frontal area, assumed to be elliptic, based on length and width of fuselage as major and minor axes

The dimensions are obtained by scaling and have the following values:

Front Circle Radius	1.66m
Fuselage Length (Belly)	10.65m
Fuselage Width (Belly)	3.3m

From which the flat plate drag area values are:

1	0
2	8.66m^2
3	27.9m^2

Calculation Results

If we take the initial conditions to be 61.1m/s (220kph) horizontally at 300m height:

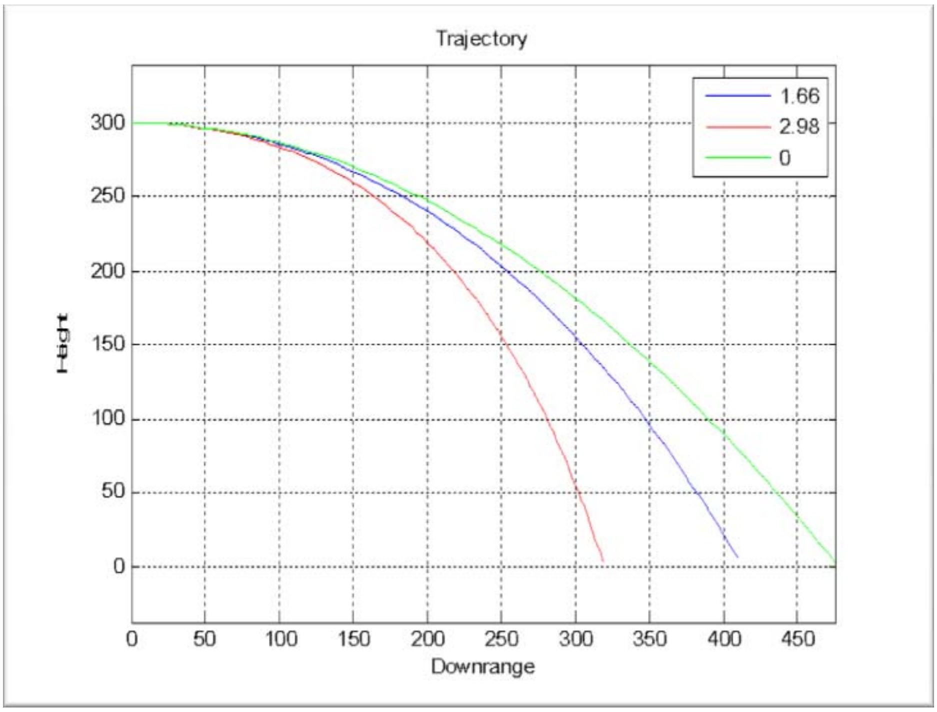

Figure 2

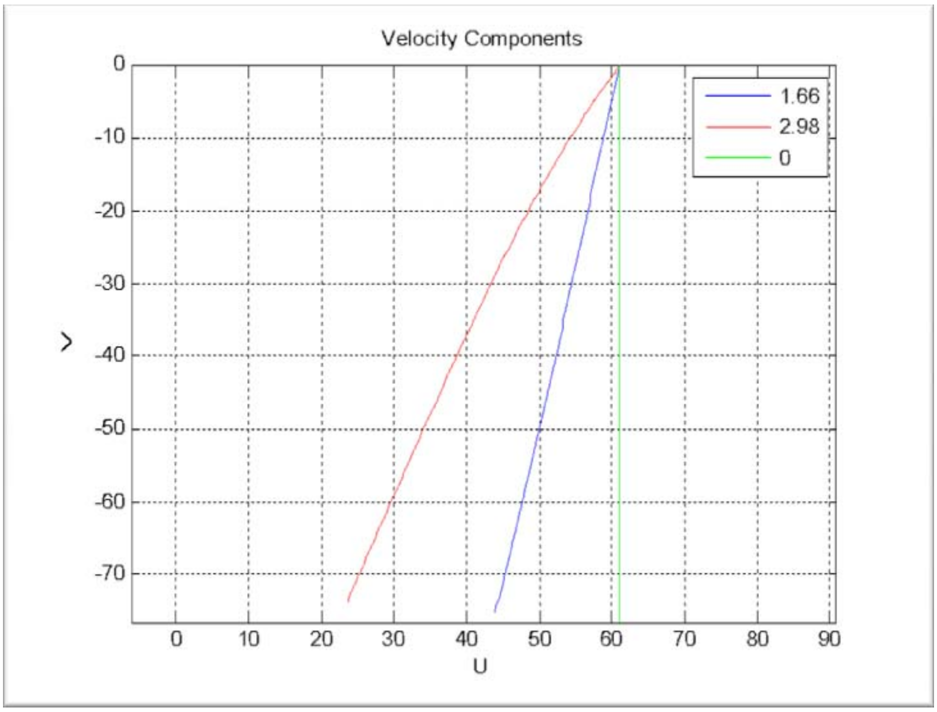

Figure 3

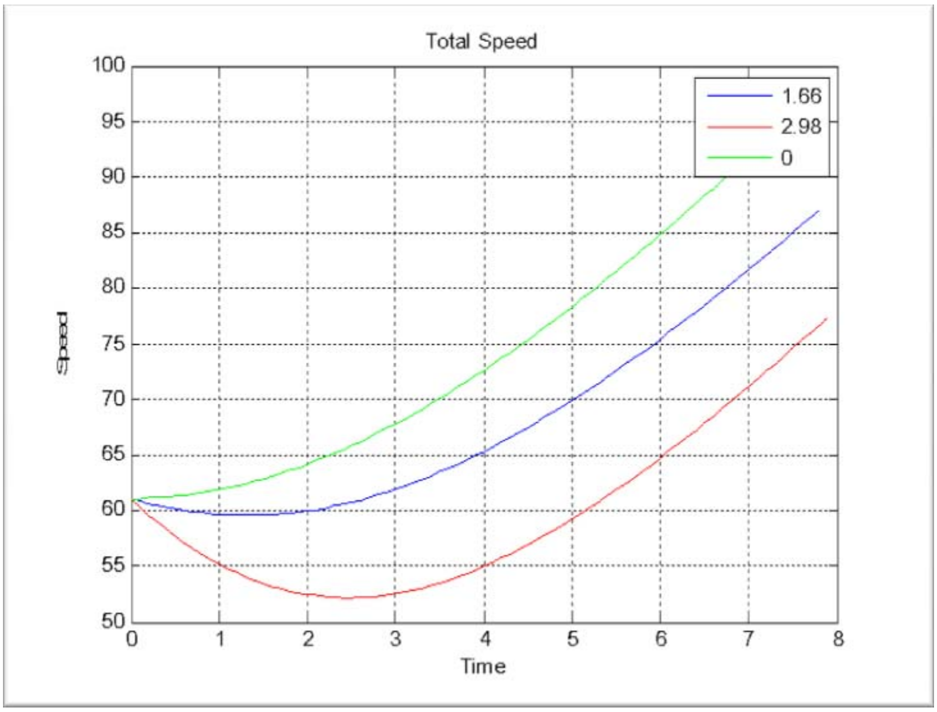

Figure 4

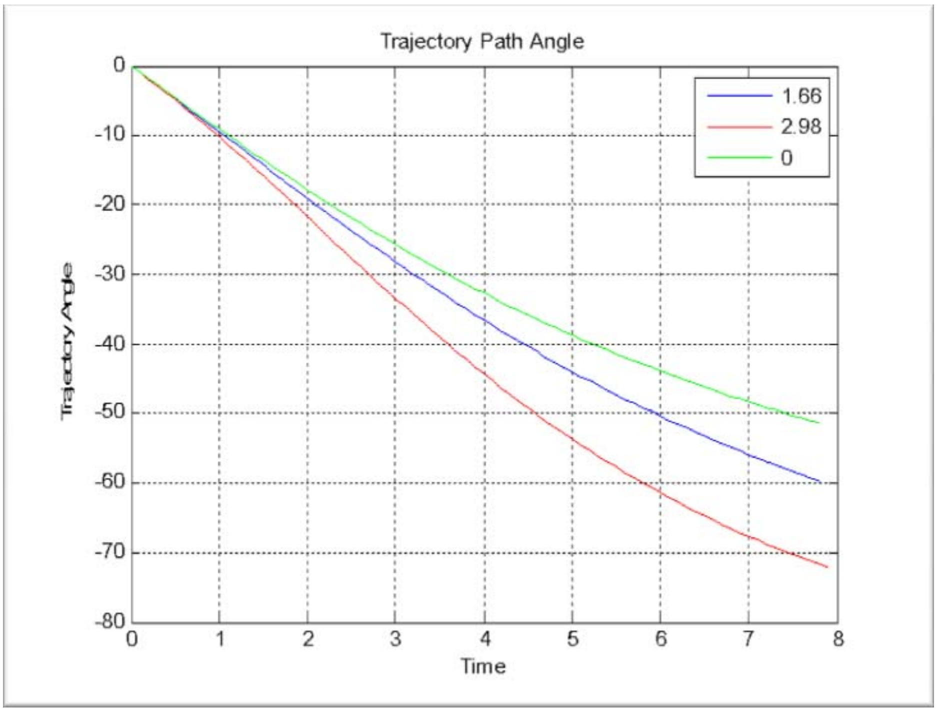
Trajectory Path Angle
1.66
2.98
0
Trajectory Angle
0
-10
-20
-30
-40
-50
-60
-70
-80
0 1 2 3 4 5 6 7 8
Time

Figure 5