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Continuous quantum measurement of a light-matter system

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Continuous measurements on correlated quantum systems, in addition to providing information on the state vector of the system in question, induce evolution in the unmeasured degrees of freedom conditioned on the measurement outcome. However, experimentally accessing these nontrivial regimes requires high-efficiency measurements over time scales much longer than the temporal resolution of the measurement apparatus. We report the observation of such a continuous conditioned evolution in the state of a light-collective atomic excitation system undergoing photoelectric measurement.

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Continuous measurements provide information on the conditional evolution of the state of a quantum system (see, e.g, Refs. [1,2]), but examples of such measurements are rare in the literature. It has been observed that photoelectric detection of a field emitted from an optical cavity causes abrupt changes in the state of the intracavity field [3–5]. The quantum theory of continuous measurements also predicts dissipative conditional evolution during periods in which no detection events are registered. However, for such a conditioned evolution to be observed, the detection apparatus must have high efficiency. Otherwise, a lack of detection cannot be interpreted as the absence of a field excitation, and, hence, one obtains little information about the underlying state. Therefore, observation of the *complete* conditional dynamics is difficult. Conditional dynamics has been observed, however, in a superconducting qubit system [6].

The goal is to continuously monitor, with high efficiency and temporal resolution, a system with significant quantum correlations. It is often difficult to meet this challenge in experimentally accessible systems. For example, parametric down-conversion has been a successful paradigm for a variety of investigations involving correlated quantum states [7]. It does not, however, readily lend itself to continuous quantum measurement. To understand this, consider a laser pulse incident on a nonlinear crystal. The nonlinear interaction yields a state for the down-converted fields of the form $|\psi\rangle \sim |\mathrm{vac}\rangle + \chi \int dt dt' f(t,t')\hat{\psi}_s^\dagger(t)\hat{\psi}_i^\dagger(t')|\mathrm{vac}\rangle + O(\chi^2)$, where the interaction strength $\chi \ll 1$, $\hat{\psi}_s^\dagger(t)$ and $\hat{\psi}_i^\dagger(t')$ are continuous boson operators for the *signal* and *idler* fields, respectively, and the two-photon amplitude $f(t,t')$ is, in general, a nonseparable function of t and t'. This nonseparability is often referred to as *time-frequency entanglement* and has an associated time scale much shorter than photodetector resolution times [8,9]. The degree of spectral filtering required would result in an unacceptably low detection efficiency, which as we note below, makes conditional evolution unobservable.

A cold atomic ensemble can provide a system analogous to parametric amplification while ensuring that the joint *signal-idler* amplitude remains separable. In this Rapid Communiation, we consider an ensemble of $N \gg 1$ atoms in a Λ level configuration with ground levels $|b\rangle$ and $|a\rangle$ and excited level $|c\rangle$ initially prepared in level $|b\rangle$. A weak *write* pulse nearly resonant on the $|b\rangle \leftrightarrow |c\rangle$ transition impinges on the ensemble. This *write* field induces Raman scattering of *signal* photons nearly resonant on the $|a\rangle \leftrightarrow |c\rangle$ transition with a temporal envelope identical to that of the *write* pulse [10]. Independently of the time at which a *signal* photon might be emitted, the Raman scattering imprints an *idler* excitation onto a unique spatial spin-wave mode [10]. After the *write* process is complete, the state of the *idler* spin wave can then be mapped to an *idler* field mode through application of a *read* field resonant on the $|a\rangle \leftrightarrow |c\rangle$ transition. The scattering dynamics are thus equivalent to those of a two-mode parametric amplifier, i.e., the *signal-idler* amplitude remains separable. Furthermore, by extending the *write* process over a long period (0.8 μs), we are able to ensure that the photodetectors employed have a temporal resolution much shorter than the emitted *signal* field.

A remaining challenge here is to achieve the required high detection efficiency. By optimizing the *write/read* and *signal/idler* spatial modes, we obtain a measured *signal* detection efficiency of 0.17, which may be compared with 0.08 in Ref. [11]. Ideally, the retrieval efficiency should be independent of storage time over the duration of an experimental trial. To achieve this we must compensate ambient magnetic fields which induce dephasing of the spin waves due to Larmor precession. Spin wave lifetimes of several milliseconds have been demonstrated by employing magnetically insensitive coherences and optical pumping [12,13]. This technique cannot be used here, however, since it would involve the application of a bias field resulting in oscillations of the retrieval efficiency as a function of storage time [12]. We therefore obtain sufficiently long memory lifetimes (25 μs) by minimizing the ambient magnetic fields.

We consider a source of correlated *signal* and *idler* fields with the assumption that the two-photon amplitude is separable: $f(t,t') = \varphi_s(t)\varphi_i(t')$. We define the single-mode *signal* and *idler* annihilation operators $\hat{a}_{s,i} \equiv \int_{-\infty}^{\infty} dt \varphi_{s,i}^*(t)\hat{\psi}_{s,i}(t)$. The state produced by the effective two-mode parametric amplification process of the *write-read* process is given by [7]

$$|\Psi(\chi)\rangle = \frac{1}{\cosh(\chi)} \sum_{n=0}^{2} \tanh^n \chi \frac{\hat{a}_s^{\dagger n}\hat{a}_i^{\dagger n}}{n!} |vac\rangle + O(\chi^3), \quad (1)$$

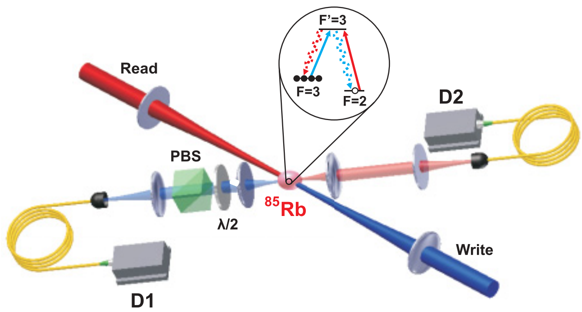

FIG. 1. (Color online) Schematic of experimental setup with the inset showing the atomic level scheme. Here, $\lambda/2$ and PBS are half-wave plate and polarizing beam-splitter for the *signal* field, respectively, and D1 and D2 are single-photon detectors. See text for a discussion.

where χ is the interaction parameter, and the higher-order terms $O(\chi^3)$ account for multiple *signal* mode excitations and emission into other temporal modes arriving at the detector. The *signal* field undergoes a continuous photon-counting measurement as it impinges on detector $D1$ with efficiency ϵ_s (see Fig. 1). We model the photodetector as one of unit efficiency preceded by a beam splitter with intensity transmittance ϵ_s. In the absence of a detection event in the interval $(-\infty, t_c)$, the vector $|\Psi(\chi)\rangle$ is projected into the subspace in which no photons arrive at the hypothetical unit efficiency detector before the conditioning time t_c. We calculate the effects of the conditioning by applying the projection operator: $\exp(-\int_{-\infty}^{t_c} dt \hat{\psi}_{D1}^\dagger(t)\hat{\psi}_{D1}(t))$: to $|\Psi(\chi)\rangle$. Here, $\hat{\psi}_{D1}(t) \equiv \sqrt{\epsilon_s}\hat{\psi}_s(t) + \sqrt{1-\epsilon_s}\hat{\xi}_1(t)$ and $\hat{\xi}_1(t)$ is a bosonic noise operator associated with detector $D1$. This projection thus yields the conditioned state

$$\sqrt{\pi_0(t_c)}\,|\Psi_c(t_c)\rangle = O(\chi^3) + \sum_{n=0}^{\infty} \frac{[\sqrt{1-\epsilon_s\mu(t_c)}\tanh(\chi)]^n}{\cosh\chi} \frac{\hat{c}^{\dagger n}(t_c)}{\sqrt{n!}} \frac{\hat{a}_i^{\dagger n}}{\sqrt{n!}} |\text{vac}\rangle\,, \tag{2}$$

where the increasing function $\mu(t_c) \equiv \int_{-\infty}^{t_c} dt |\varphi_s(t)|^2$, with $\mu(\infty) = 1$. Provided the detection efficiency ϵ_s is not negligibly small, conditioning effectively results in a state with a reduced interaction parameter, where the *signal* mode $\hat{a}_s$ has been replaced by the conditioned bosonic mode

$$\hat{c}(t_c) = -\sqrt{\frac{\mu(t_c)(1-\epsilon_s)}{1-\epsilon_s\mu(t_c)}}\hat{V}(t_c) + \sqrt{\frac{1-\mu(t_c)}{1-\epsilon_s\mu(t_c)}}\hat{\alpha}(t_c), \tag{3}$$

in which the *prior signal* field, that arriving before t_c, is replaced by the vacuum noise field $\hat{V}(t_c)$, and the *postsignal* field, arriving after t_c, is defined by $\hat{\alpha}(t_c) = \int_{t_c}^{\infty} dt \varphi_s^*(t)\hat{\psi}_s(t)/\sqrt{1-\mu(t_c)}$.

Taking the norm of Eq. (2), we find the probability for no photoelectric event registered before t_c

$$\pi_0(t_c) = \frac{1}{1+\epsilon_s\mu(t_c)\sinh^2\chi} + O(\chi^3). \tag{4}$$

Conditioned on the absence of a *signal* event in the interval $(-\infty, t_c)$, we can calculate the probability of registering at least one photoelectric detection event in the *idler* channel by taking the expectation value of the projection operator $\hat{P}_i \equiv: 1 - \exp(-\hat{d}_i^\dagger \hat{d}_i)$: with respect to the conditioned state of Eq. (2), where $\hat{d}_i = \sqrt{\epsilon_i}\hat{a}_i + \sqrt{1-\epsilon_i}\hat{\xi}_i$, ϵ_i is the efficiency of the *idler* detector, and $\hat{\xi}_i$ is an *idler* noise operator. The second term of the operator $\hat{P}_i$ projects onto the subspace in which no *idler* photons are detected. Similarly, one finds the probability of detecting a *signal* between times t_1 and t_2, $p_{s|0}(t_1, t_2; t_c)$, corresponding to the expectation value of $\hat{P}_s(t_1, t_2) =: 1 - \exp(-\int_{t_1}^{t_2} dt \hat{\psi}_{D1}^\dagger(t)\hat{\psi}_{D1}(t))$:. Finally, the joint conditioned probability that a photoelectric event is registered at both the *idler* detector (at any time during the trial) and the *signal* detector between times t_1 and t_2 corresponds to the expectation value of $\hat{P}_i\hat{P}_s(t_1, t_2)$. Dark counts on the *signal* and *idler* channels can be modelled by taking the noise fields associated with $\hat{\xi}_s(t)$ and $\hat{\xi}_i$ to be in coherent states; the dark count rates would be given by $B_i = \langle\hat{\xi}_i^\dagger\hat{\xi}_i\rangle$ and $B_s(t_1, t_2) = \int_{t_1}^{t_2} dt \langle\hat{\xi}_s^\dagger(t)\hat{\xi}_s(t)\rangle$. In the weak excitation limit where $B_i \sim B_s(t_1, t_2) \sim \sinh^2\chi \ll 1$, these conditioned detection probabilities are, to first order in $\sinh^2\chi$,

$$p_{s|0}(t_1, t_2; t_c) \approx \epsilon_s [\mu(t_2) - \mu(t_1)] \sinh^2\chi + B_s(t_1, t_2), \tag{5a}$$

$$p_{i|0}(t_c) \approx \epsilon_i [1 - \epsilon_s\mu(t_c)] \sinh^2\chi + B_i, \tag{5b}$$

$$p_{si|0}(t_1, t_2; t_c) \approx \epsilon_s\epsilon_i [\mu(t_2) - \mu(t_1)] \sinh^2\chi. \tag{5c}$$

As intuitively expected, the conditioned *idler* probability, $p_{i|0}(t_c)$, becomes progressively smaller as the conditioning interval increases. By contrast, the probabilities $p_{s|0}(t_1, t_2; t_c)$ and $p_{si|0}(t_1, t_2; t_c)$ are proportional to the detection window $\mu(t_2) - \mu(t_1)$ and are identical to the corresponding unconditioned probabilities. The conditioning only manifests itself in $p_{s|0}(t_1, t_2; t_c)$ and $p_{si|0}(t_1, t_2; t_c)$ through the requirement that the detection window occurs after t_c and $\mu(t_2) - \mu(t_1) \leqslant 1 - \mu(t_c)$. From Eqs. (5) we can also determine the unconditioned, integrated probabilities $(t_c, t_1 \to -\infty)$, $(t_2 \to \infty)$ [11]:

$$p_s \approx \epsilon_s \sinh^2\chi + B_s, \tag{6a}$$

$$p_i \approx \epsilon_i \sinh^2\chi + B_i, \tag{6b}$$

$$p_{si} \approx \epsilon_s\epsilon_i \sinh^2\chi. \tag{6c}$$

From these detection probabilities, one can define the experimentally measurable effective *signal* efficiency as

$$\epsilon_s' \equiv p_{si}/p_i = \epsilon_s\left(1 - \frac{B_i}{\epsilon_i\sinh^2\chi + B_i}\right), \tag{7}$$

which is reduced by background *idler* counts B_i. Inspection of Eqs. (5) and (7) reveals that this efficiency can also be obtained through the conditioned *idler* detection probability as

$$\epsilon_s' = -\frac{1}{p_i}\frac{dp_{i|0}(t_c)}{d\mu}. \tag{8}$$

This correspondence between the predictions of conditioned and unconditioned quantum dynamics provides a quantitative measure to experimentally test the dynamics of conditional quantum measurement.

To implement such a test, we prepare an optically thick atomic cloud of ^{85}Rb by switching on a magneto-optical trap (MOT) for a period of 14 ms (Fig. 1). The atomic ground levels

$\{|a\rangle; |b\rangle\}$ correspond to the $5S_{1/2}$, $F_{a,b} = \{2, 3\}$ hyperfine levels, while the excited level $|c\rangle$ represents the $5P_{1/2}$, $F_c = 3$ level of the D_1 line at 795 nm. After switching off the MOT fields, the experimental sequence begins with all of the atoms pumped into level $|b\rangle$ by sequentially switching off first the trapping light followed 10 μs later by the repumping light. The quadrupole magnetic field of the MOT is extinguished for the 2.5 ms duration of the measurement sequence. Compensation of the ambient magnetic field is provided by three pairs of Helmholtz coils.

The measurement sequence consists of 1666 cycles of duration 1.5 μs. The cycle begins when a weak, approximately square, linearly polarized *write* laser pulse, tuned to the $|b\rangle \leftrightarrow |c\rangle$ transition, illuminates the ensemble for $T = 0.8$ μs; that is, in each cycle $\mu(t_c = 0) = 0$ and $\mu(T) = 1$. The light pulse generates an orthogonally polarized *signal* field by spontaneous Raman scattering on the $|c\rangle \leftrightarrow |a\rangle$ transition together with spin wave excitation of the atomic medium associated with the $|b\rangle \leftrightarrow |a\rangle$ hyperfine coherence [14]. After a 200 ns delay, a 200 ns long *read* pulse, tuned to the $|a\rangle \leftrightarrow |c\rangle$ transition, illuminates the atoms. This *read* field, with power 170 μW and linear polarization orthogonal to that of the *write* pulse, converts the atomic spin excitation into an orthogonally polarized *idler* field, which is emitted on the $|c\rangle \leftrightarrow |b\rangle$ transition. Both the *write-read* and *signal-idler* pairs of mode-matched fields are counterpropagating, with Gaussian waists of 400 μm for the former and 130 μm for the latter. The *signal* and *idler* fields are measured by single photon detectors D1 and D2, respectively.

The photoelectric detection events for the *signal* and *idler* fields are measured and recorded with 2 ns time resolution, allowing conditioned and unconditioned detection probabilities to be determined. The unconditioned detection probability for the *idler* field is defined by the ratio of the number of cycles, N_i, with at least one photoelectric detection event recorded to the total number of cycles: $P_i \equiv N_i/N_T$. The conditioned probability is determined similarly, except that all cycles in which a *signal* photoelectric event has been recorded prior to time t_c are omitted, hence $P_{i|0}(t_c) \equiv N_{i|0}(t_c)/N_T$. In order to test the predictions of the conditional quantum theory, we measure both the unconditioned *signal-idler* coincidence probability, $P_{si} \equiv N_{si}/N_T$, and the gradient of the conditional *idler* detection probability, $D_{i|0}(t_c) \equiv -dP_{i|0}/d\mu(t_c)$, as a function of $\mu(t_c)$, which according to Eq. (8), must be equal to $\epsilon_s' P_i$. By varying ϵ_s using the half-wave plate placed in the *signal* beam path before the polarizer (Fig. 1), we measure a set of values for P_{si}. For each plate setting, we construct $D_{i|0}(t_c)$

$$D_{i|0}(t_c) \simeq \frac{P_{i|0}(t_c) - P_{i|0}(t_c + \Delta t)}{\mu(t_c + \Delta t) - \mu(t_c)},$$

where Δt is a sufficiently small time interval that the determined $D_{i|0}(t_c)$ does not depend on its value. The results are presented in Fig. 2 and show very good agreement between the conditional measurement data, $D_{i|0}(t_c)/P_i$, and the unconditional data, P_{si}/P_i, which Eqs. (7) and (8) predict should be equal. The error bars on conditional data are based on the statistics of the photoelectric counting events, while statistical errors on the unconditioned data are negligible.

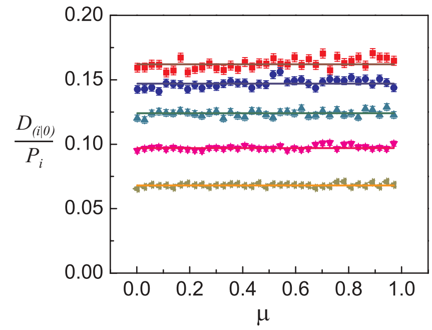

FIG. 2. (Color online) Experimental data for $D_{i|0}(t_c)/P_i$ (discrete data) and P_{si}/P_i (solid lines) vs. *signal* field gate function $\mu(t_c)$ for five different settings of the half-wave plate (Fig. 1). According to conditional quantum theory, these should be equal, Eqs. (7) and (8). Data shown from top to bottom correspond to decreasing measured *signal* efficiencies. The data acquisition time was 5 h.

At high measured efficiencies, there is a small systematic discrepancy in excess of the statistical errors that requires a careful examination in order to exclude spurious effects which could potentially mimic the predictions of conditional quantum theory. The two most relevant processes are (1) the residual effects of Raman scattering of the *write* pulse and (2) the Larmor precession of the hyperfine coherences in the uncompensated magnetic field. Although Raman scattering is responsible for the creation of the *signal* photon and the accompanying spin excitation, excessive scattering into undetected field modes can slightly deplete the population of level $|b\rangle$ during the *write* process and therefore reduce the efficiency with which spin waves generated early in the *write* process are retrieved. We can model this effect using the time-dependent retrieval efficiency $\eta(1-\mu) = \epsilon_i \exp[-\alpha(1-\mu)]$, where $1-\mu$ is proportional to the time a spin wave is exposed to the deleterious effects of the *write* beam. This would manifest itself in an increasing time-dependent correction to ϵ_s' in Eq. (8) and as a reduced joint *signal-idler* coincidence detection probability at small values of μ. Based on this picture, we can model $p_{si}(\mu, \mu + \Delta\mu)/\Delta\mu \sim p_s \exp(-\alpha(1-\mu))$, where α is proportional to the *write* pulse energy, as shown by the data in Fig. 3. As can be seen from Fig. 3, for the 0.1 μW power at which data in Fig. 2 were taken, these effects are not significant. Similarly, the Larmor precession reduces p_{si} for small values

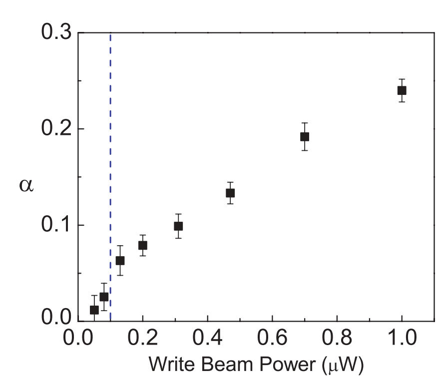

FIG. 3. (Color online) Coefficient α as the function of *write* beam power (see text). The vertical dashed line shows the value of *write* beam power at which the data in Fig. 2 were taken.

of $\mu(t_c)$, according to $p_{si} \sim \exp(-\gamma(1-\mu)^2)$ [15,16]. For the measured coherence time, $\tau_c \approx 25$ μs, and maximum *write-read* delay, $T_t \sim 1$ μs, the expected decoherence effect is negligible ($\gamma = (T_t/\tau_c)^2 \approx 0.004$).

In summary, we have observed conditional dynamics of a correlated atomic spin wave-light system. This work was supported by the National Science Foundation and the Air Force Office of Scientific Research.

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