A comparison of two methods of estimating propensity scores after multiple imputation - online supplement

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In the online supplement we present details of additional simulations and results that have been referred to in the main text of the paper. In Appendix 1 we present results of the treatment effect estimates from the Across and Within methods in the simulations of Section 3 when matching is performed with replacement. In Appendix 2 we investigate different applications of propensity scores to estimate treatment effects in the simulations of Section 3. In Appendix 3 we present the results of a logistic regression of treatment assignment on the covariates for the 1306 complete cases in the National Longitudinal Survey of Youth (NLSY) data before introduction of missing values.

Appendix 1 - Matching with replacement

We present results of the treatment effect estimates from the Across and Within methods in the simulations of Section 3 when matching is performed with replacement. We see similar properties in the bias and variance profiles to those in Section 3. The biases in both the Across and Within estimates are small when treatment only depends on x_1. When treatment depends on x_2 the Across biases are consistently smaller than the Within biases. The variances of the Within estimates tend to decrease with m, and are always smaller than the corresponding variances of the Across estimates, which do not decrease with m.

		Across			Within	
m	Pt. Est.	Variance	MSE	Pt. Est.	Variance	MSE
Only control units missing $\mathbf{x}_2$						
5	-0.004	0.097	0.097	0.033	0.053	0.054
10	0.002	0.100	0.100	0.028	0.048	0.049
15	0.010	0.091	0.091	0.030	0.045	0.046
20	0.011	0.097	0.097	0.030	0.045	0.046
50	-0.001	0.104	0.104	0.028	0.043	0.044
Treatment and control units missing $\mathbf{x}_2$						
5	-0.026	0.101	0.102	0.024	0.056	0.057
10	-0.019	0.103	0.103	0.023	0.051	0.051
15	-0.020	0.099	0.099	0.026	0.049	0.050
20	-0.020	0.098	0.098	0.026	0.047	0.048
50	-0.028	0.102	0.103	0.024	0.045	0.046

Table 1: Treatment effect estimates from the Across and Within approaches in the simulation design where treatment assignment depends only on $\mathbf{x}_1$. The average treatment effect estimates based on the covariates before introduction of missing data is 0.0213.

m	Pt. Est.	Across Variance	MSE	Pt. Est.	Within Variance	MSE
Only control units missing $\mathbf{x}_2$						
5	0.444	0.114	0.311	0.803	0.050	0.695
10	0.408	0.111	0.277	0.809	0.043	0.697
15	0.414	0.116	0.288	0.810	0.040	0.696
20	0.411	0.115	0.283	0.811	0.038	0.696
50	0.419	0.127	0.303	0.811	0.035	0.693
Treatment and control units missing $\mathbf{x}_2$						
5	0.213	0.127	0.172	0.825	0.062	0.742
10	0.133	0.128	0.146	0.827	0.047	0.731
15	0.107	0.123	0.134	0.828	0.043	0.729
20	0.093	0.126	0.135	0.828	0.042	0.729
50	0.080	0.132	0.139	0.827	0.038	0.722

Table 2: Treatment effect estimates from the Across and Within approaches in the simulation design where treatment assignment depends only on $\mathbf{x}_2$. The average treatment effect estimates based on the covariates before introduction of missing data is 0.0187.

m	Pt. Est.	Across Variance	MSE	Pt. Est.	Within Variance	MSE
Only control units missing $\mathbf{x}_2$						
5	0.296	0.074	0.162	0.532	0.044	0.326
10	0.258	0.076	0.142	0.534	0.038	0.323
15	0.241	0.071	0.129	0.533	0.037	0.321
20	0.227	0.074	0.126	0.532	0.036	0.319
50	0.228	0.073	0.125	0.533	0.034	0.319
Treatment and control units missing $\mathbf{x}_2$						
5	0.209	0.105	0.149	0.534	0.049	0.334
10	0.163	0.102	0.129	0.535	0.042	0.328
15	0.132	0.106	0.123	0.536	0.040	0.328
20	0.124	0.101	0.116	0.534	0.039	0.324
50	0.114	0.105	0.118	0.533	0.037	0.321

Table 3: Treatment effect estimates from the Across and Within approaches in the simulation design where treatment assignment depends equally on $\mathbf{x}_1$ and $\mathbf{x}_2$. The average treatment effect estimates based on the covariates before introduction of missing data is 0.0172.

Appendix 2 - Alterative uses of propensity scores

We also investigated different applications of propensity scores to estimate treatment effects for the scenarios described in Section 3. These were: (1) Inverse weighting of the propensity scores[1,2], (2) Regression estimates of the treatment effect including the propensity score as a covariate[3] , and (3) Sub-classification based on the propensity score[1,4]. In particular, we were interested in observing whether similar properties of the Across and Within methods would also be apparent in these uses of the propensity score.

Inverse weighting

To estimate treatment effects here, we first weight each unit's outcome, where the unit's weight is determined by its propensity score[2]. The treatment effect is then estimated by the difference between the average of the weighted outcomes in the treatment group and the weighted outcomes in the control group. Using the notation in Section 2, we can compute treatment effect estimates using the Across and Within methods in the following way. We compute the Across treatment effect estimate by:

$$\hat{\tau}^{A,m} = \frac{1}{n}\sum_{i=1}^{n}\left\{Y_iT_i - \frac{Y_i e^{A,m}(\boldsymbol{x}_i)}{1-e^{A,m}(\boldsymbol{x}_i)}(1-T_i)\right\}, \tag{1}$$

and we compute the Within treatment effect estimate by:

$$\hat{\tau}^{W,m} = \frac{1}{m}\left[\sum_{k=1}^{m}\frac{1}{n}\sum_{i=1}^{n}\left\{Y_iT_i - \frac{Y_i e(\boldsymbol{x}_{i,\text{com}}^{(k)})}{1-e(\boldsymbol{x}_{i,\text{com}}^{(k)})}(1-T_i)\right\}\right], \tag{2}$$

As with the simulations in Section 3, when treatment only depends on $\mathbf{x}_1$ both the Across and Within biases are relatively small. When treatment depends only on $\mathbf{x}_2$ or equally on $\mathbf{x}_1$ and $\mathbf{x}_2$, the Within method has consistently lower biases than the Across method for all values of m. The variances of the Within estimates tend to be smaller than the variance of the Across estimates, and they tend to decrease as m increases. No such trend is evident in the variances of the Across estimates.

	Across			Within		
m	Pt. Est.	Variance	MSE	Pt. Est.	Variance	MSE
Only control units missing $\mathbf{x}_2$						
5	0.006	0.002	0.002	0.001	0.002	0.002
10	0.007	0.002	0.002	0.001	0.002	0.002
15	0.007	0.002	0.002	0.001	0.002	0.002
20	0.007	0.002	0.002	0.001	0.002	0.002
50	0.007	0.002	0.002	0.001	0.002	0.002
Treatment and control units missing $\mathbf{x}_2$						
5	0.009	0.002	0.003	0.000	0.002	0.002
10	0.010	0.002	0.003	0.000	0.002	0.002
15	0.010	0.002	0.003	0.000	0.002	0.002
20	0.010	0.002	0.003	0.000	0.002	0.002
50	0.010	0.002	0.003	0.000	0.002	0.002

Table 4: Treatment effect estimates from the Across and Within approaches in the simulation design where treatment assignment depends only on $\mathbf{x}_1$. The average treatment effect estimates based on the covariates before introduction of missing data is -0.0009.

	Across			Within		
m	Pt. Est.	Variance	MSE	Pt. Est.	Variance	MSE
Only control units missing $\mathbf{x}_2$						
5	0.172	0.003	0.032	0.061	0.001	0.005
10	0.181	0.003	0.036	0.061	0.001	0.005
15	0.185	0.003	0.037	0.061	0.001	0.005
20	0.186	0.003	0.038	0.061	0.001	0.005
50	0.189	0.003	0.039	0.061	0.001	0.005
Treatment and control units missing $\mathbf{x}_2$						
5	0.174	0.003	0.033	0.065	0.001	0.006
10	0.184	0.003	0.037	0.065	0.001	0.005
15	0.187	0.003	0.038	0.065	0.001	0.005
20	0.189	0.003	0.039	0.065	0.001	0.005
50	0.192	0.003	0.040	0.065	0.001	0.005

Table 5: Treatment effect estimates from the Across and Within approaches in the simulation design where treatment assignment depends only on $\mathbf{x}_2$. The average treatment effect estimates based on the covariates before introduction of missing data is -0.0015.

m	Pt. Est.	Across Variance	MSE	Pt. Est.	Within Variance	MSE
Only control units missing $\mathbf{x}_2$						
5	0.089	0.002	0.010	0.042	0.001	0.003
10	0.093	0.002	0.011	0.041	0.001	0.003
15	0.095	0.002	0.011	0.041	0.001	0.003
20	0.096	0.002	0.011	0.041	0.001	0.003
50	0.098	0.002	0.012	0.041	0.001	0.003
Treatment and control units missing $\mathbf{x}_2$						
5	0.091	0.002	0.010	0.042	0.001	0.003
10	0.096	0.002	0.011	0.042	0.001	0.003
15	0.098	0.002	0.012	0.042	0.001	0.003
20	0.099	0.002	0.012	0.042	0.001	0.003
50	0.101	0.002	0.012	0.042	0.001	0.003

Table 6: Treatment effect estimates from the Across and Within approaches in the simulation design where treatment assignment depends equally on $\mathbf{x}_1$ and $\mathbf{x}_2$. The average treatment effect estimates based on the covariates before introduction of missing data is -0.0004.

Regression including the propensity score as a covariate

To estimate treatment effects, we regress the outcome variable on the treatment indicator, the propensity score, and other covariates. For the Across method we estimate the treatment effect $\hat{\tau}^{A,m} = \sum_{i=1}^{m} \frac{\hat{\tau}^{A,k}}{m}$, where $\hat{\tau}^{A,k}$ is the estimated regression coefficient from

$$E(Y_i) = \alpha^A + \beta^A e^{A,m}(\boldsymbol{x}_i) + \boldsymbol{\gamma}^{A'} \boldsymbol{x}_{i,\text{com}}^{(k)} + \hat{\tau}^{A,k} T_i. \tag{3}$$

For the within method, we estimate the treatment effect $\hat{\tau}^{W,m} = \sum_{i=1}^{m} \frac{\hat{\tau}^{W,k}}{m}$, where $\hat{\tau}^{W,k}$ is the estimated regression coefficient from

$$E(Y_i) = \alpha^W + \beta^W e(\boldsymbol{x}_{i,\text{com}}^{(k)}) + \boldsymbol{\gamma}^{W'} \boldsymbol{x}_{i,\text{com}}^{(k)} + \hat{\tau}^{W,k} T_i \ , \tag{4}$$

where $i = 1, \ldots, n$. The Across method includes a common set of propensity scores in each regression model for each imputed $\boldsymbol{x}_{i,\text{com}}^{(k)}$. In contrast, the Within method uses the propensity scores estimated with $\boldsymbol{x}_{i,\text{com}}^{(k)}$.

As with the simulations in Section 3, when treatment only depends on $\mathbf{x}_1$ the biases for the Across and Within estimates are relatively small. When treatment depends only on $\mathbf{x}_2$ or equally on $\mathbf{x}_1$ and $\mathbf{x}_2$, the Across method has consistently lower biases than the Within method for all values of m. In these cases, the Across biases decrease as m increases, and no such decrease is evident with the Within biases. The variance of the Within estimates tend to decrease with m, and no such trend is evident in the variance of the Across estimates.

m	Across Pt. Est.	Across Variance	Across MSE	Within Pt. Est.	Within Variance	Within MSE
Only control units missing $\mathbf{x}_2$						
5	0.004	0.026	0.026	0.005	0.028	0.029
10	0.005	0.025	0.025	0.006	0.028	0.028
15	0.005	0.025	0.025	0.006	0.027	0.028
20	0.006	0.024	0.025	0.006	0.027	0.027
50	0.006	0.024	0.024	0.007	0.027	0.027
Treatment and control units missing $\mathbf{x}_2$						
5	0.004	0.040	0.040	0.005	0.035	0.035
10	0.004	0.040	0.040	0.005	0.035	0.035
15	0.004	0.040	0.040	0.005	0.035	0.035
20	0.003	0.040	0.040	0.005	0.034	0.034
50	0.002	0.040	0.040	0.004	0.034	0.034

Table 7: Treatment effect estimates from the Across and Within approaches in the simulation design where treatment assignment depends only on $\mathbf{x}_1$. The average treatment effect estimates based on the covariates before introduction of missing data is 0.0011.

m	Across Pt. Est.	Across Variance	Across MSE	Within Pt. Est.	Within Variance	Within MSE
Only control units missing $\mathbf{x}_2$						
5	0.171	0.022	0.051	0.624	0.022	0.411
10	0.081	0.021	0.028	0.623	0.022	0.410
15	0.049	0.021	0.024	0.624	0.021	0.411
20	0.033	0.021	0.022	0.624	0.021	0.411
50	0.001	0.021	0.021	0.625	0.021	0.411
Treatment and control units missing $\mathbf{x}_2$						
5	0.199	0.040	0.080	0.718	0.032	0.548
10	0.084	0.039	0.046	0.720	0.030	0.549
15	0.039	0.040	0.041	0.720	0.030	0.548
20	0.015	0.040	0.040	0.719	0.030	0.548
50	-0.029	0.040	0.040	0.719	0.029	0.546

Table 8: Treatment effect estimates from the Across and Within approaches in the simulation design where treatment assignment depends only on $\mathbf{x}_2$. The average treatment effect estimates based on the covariates before introduction of missing data is -0.0011.

	Across			Within		
m	Pt. Est.	Variance	MSE	Pt. Est.	Variance	MSE
Only control units missing $\mathbf{x}_2$						
5	0.151	0.024	0.047	0.396	0.024	0.180
10	0.107	0.024	0.035	0.396	0.023	0.180
15	0.091	0.024	0.032	0.396	0.023	0.180
20	0.082	0.024	0.031	0.396	0.023	0.180
50	0.067	0.024	0.029	0.396	0.023	0.180
Treatment and control units missing $\mathbf{x}_2$						
5	0.172	0.041	0.070	0.439	0.031	0.224
10	0.120	0.043	0.057	0.439	0.030	0.223
15	0.101	0.044	0.055	0.439	0.030	0.223
20	0.090	0.045	0.053	0.439	0.030	0.223
50	0.071	0.046	0.051	0.439	0.030	0.223

Table 9: Treatment effect estimates from the Across and Within approaches in the simulation design where treatment assignment depends equally on $\mathbf{x}_1$ and $\mathbf{x}_2$. The average treatment effect estimates based on the covariates before introduction of missing data is -0.0017.

Sub-classification on the propensity score

In this approach, we group units with similar values of propensity scores, and estimate the treatment effect within each of these groups. To estimate the treatment effect, we compute a weighted average of these estimates, where the weight assigned to each group is equal to the proportion of treated units in that group[2].

To form the groups, we use the sample quintiles of the propensity scores. For $j = 1, \ldots, 5$, let q_j be the value such that approximately $j/5$ units have $e(\boldsymbol{x}_i)$ satisfying $q_{j-1} < e(\boldsymbol{x}_i) \leq q_j$, where $q_0 = 0$ and $q_5 = 1$. Let $Q_j = (q_{j-1}, q_j]$ for each j. For $j = 1, \ldots, 5$, let $n_j = \sum_{i=1}^{n} I(e(\boldsymbol{x}_i) \in Q_j)$ be the number of units in group j, and let $n_{jT} = \sum_{i=1}^{n} I(e(\boldsymbol{x}_i) \in Q_j)T_i$ be the number of treated units in group j; here, $I(\cdot)$ is the indicator function. We have

$$\hat{\tau} = \sum_{j=1}^{5} \frac{n_{jT}}{n_T} \left\{ \frac{\sum_{i=1}^{n} Y_i T_i I(e(\boldsymbol{x}_i) \in Q_j)}{n_{jT}} - \frac{\sum_{i=1}^{n} Y_i (1 - T_i) I(e(\boldsymbol{x}_i) \in Q_j)}{n_j - n_{jT}} \right\} \tag{5}$$

For the Across method, we determine each q_j and n_j using $e^{A,m}(\boldsymbol{x}_i)$, where $i =$

$1, \dot{;} n$. Using superscript A, m notation to distinguish the Across method, we have a treatment effect estimate of

$$\hat{\tau}^{A,m} = \sum_{j=1}^{5} \frac{n_{jT}^{A,m}}{n_T} \left\{ \frac{\sum_{i=1}^{n} Y_i T_i I(e(\boldsymbol{x}_i) \in Q_j^{A,m})}{n_{jT}^{A,m}} - \frac{\sum_{i=1}^{n} Y_i (1-T_i) I(e(\boldsymbol{x}_i) \in Q_j^{A,m})}{n_j^{A,m} - n_{jT}^{A,m}} \right\}. \tag{6}$$

For the Within method, we create quintiles and apply (17) in each completed dataset. Using superscript W, k notation to distinguish quantities for the kth completed dataset for the Within method, we have a treatment effect estimate of $e^{A,m}(\boldsymbol{x}_i)$, where $i = 1, \dot{;} n$.

$$\hat{\tau}^{W,k} = \sum_{j=1}^{5} \frac{n_{jT}^{W,k}}{n_T} \left\{ \frac{\sum_{i=1}^{n} Y_i T_i I(e(\boldsymbol{x}_{i,\mathrm{com}}^{(k)}) \in Q_j^{W,k})}{n_{jT}^{W,k}} - \frac{\sum_{i=1}^{n} Y_i (1-T_i) I(e(\boldsymbol{x}_{i,\mathrm{com}}^{(k)}) \in Q_j^{W,k})}{n_j^{W,k} - n_{jT}^{W,k}} \right\}. \tag{7}$$

As with the simulations in Section 3, when treatment only depends on $\mathbf{x}_1$ the biases in the Across and Within methods are relatively small. When treatment depends only on $\mathbf{x}_2$ or equally on $\mathbf{x}_1$ and $\mathbf{x}_2$, the Across method has consistently lower biases than the Within method for all values of m. The Across biases decrease as m increases, and no such decrease is evident with the Within biases. The variances of the Within estimates tend to decrease with m, and no such trend is evident in the variance of the Across estimates.

		Across			Within	
m	Pt. Est.	Variance	MSE	Pt. Est.	Variance	MSE
Only control units missing $\mathbf{x}_2$						
5	0.539	0.058	0.349	0.547	0.056	0.356
10	0.538	0.057	0.347	0.548	0.056	0.356
15	0.539	0.057	0.347	0.548	0.055	0.355
20	0.538	0.057	0.346	0.548	0.055	0.355
50	0.537	0.057	0.346	0.548	0.055	0.356
Treatment and control units missing $\mathbf{x}_2$						
5	0.523	0.063	0.336	0.548	0.059	0.359
10	0.521	0.062	0.333	0.547	0.058	0.358
15	0.520	0.062	0.333	0.548	0.058	0.358
20	0.519	0.061	0.330	0.547	0.058	0.358
50	0.518	0.061	0.330	0.547	0.058	0.357

Table 10: Treatment effect estimates from the Across and Within approaches in the simulation design where treatment assignment depends only on $\mathbf{x}_1$. The average treatment effect estimates based on the covariates before introduction of missing data is 0.5377.

		Across			Within	
m	Pt. Est.	Variance	MSE	Pt. Est.	Variance	MSE
Only control units missing $\mathbf{x}_2$						
5	0.969	0.049	0.988	1.101	0.051	1.262
10	0.922	0.051	0.900	1.100	0.050	1.261
15	0.899	0.049	0.857	1.101	0.050	1.263
20	0.884	0.050	0.832	1.101	0.050	1.263
50	0.854	0.050	0.780	1.101	0.050	1.262
Treatment and control units missing $\mathbf{x}_2$						
5	0.710	0.060	0.564	1.120	0.052	1.307
10	0.591	0.061	0.411	1.122	0.051	1.309
15	0.540	0.063	0.355	1.122	0.051	1.309
20	0.509	0.064	0.323	1.121	0.051	1.308
50	0.444	0.067	0.264	1.121	0.050	1.307

Table 11: Treatment effect estimates from the Across and Within approaches in the simulation design where treatment assignment depends only on $\mathbf{x}_2$. The average treatment effect estimates based on the covariates before introduction of missing data is 0.5339.

	Across			Within		
m	Pt. Est.	Variance	MSE	Pt. Est.	Variance	MSE
Only control units missing $\mathbf{x}_2$						
5	0.841	0.052	0.760	0.870	0.053	0.810
10	0.833	0.051	0.745	0.870	0.053	0.810
15	0.829	0.052	0.740	0.871	0.053	0.811
20	0.827	0.051	0.735	0.871	0.053	0.812
50	0.825	0.052	0.733	0.871	0.053	0.811
Treatment and control units missing $\mathbf{x}_2$						
5	0.730	0.059	0.592	0.877	0.055	0.824
10	0.707	0.060	0.560	0.877	0.054	0.823
15	0.697	0.060	0.546	0.878	0.054	0.824
20	0.691	0.060	0.538	0.877	0.054	0.823
50	0.681	0.061	0.525	0.878	0.053	0.824

Table 12: Treatment effect estimates from the Across and Within approaches in the simulation design where treatment assignment depends equally on $\mathbf{x}_1$ and $\mathbf{x}_2$. The average treatment effect estimates based on the covariates before introduction of missing data is 0.5302.

Appendix 3: Results of Propensity Score Regression in NLSY data

Table 13 displays the results from a logistic regression of treatment assignment indicator on the covariates for the 1306 complete cases in the NLSY data before introduction of missing values. Also included is the proportion of missing cases for each variable. The variables with largest fractions of missing data, namely family income and weeks mother worked before birth, are not among the strongest predictors of treatment assignment, which include mother's highest attained level of education and mother's AFQT score.

Coefficient	Est	SE	95% CI	miss
Intercept	-3.98	1.15	(-6.24, -1.73)	0.00
Mother's race - Black	1.62	2.19	(-2.67, 5.92)	0.01
Mother's race - Asian	1.70	0.96	(-0.17, 3.58)	0.01
Mother's race - White	0.49	0.47	(-0.43, 1.41)	0.01
Mother's race - Hawaiin/PI/American Indian	0.76	0.55	(-0.32, 1.84)	0.01
Mother's race - Other	0.22	0.54	(-0.83, 1.27)	0.01
Child's race - Black	-2.22	2.18	(-6.49, 2.06)	0.00
Child's race - Not Black and Not Hispanic	-0.40	0.42	(-1.23, 0.42)	0.00
Child's sex - female	-0.23	0.16	(-0.55, 0.09)	0.00
Presence of spouse/partner at birth - 2	-0.88	0.50	(-1.86, 0.10)	0.02
Presence of spouse/partner at birth - 3	-0.33	0.33	(-0.97, 0.32)	0.02
Grandparents in HH 1 YR before birth - Yes	-0.13	0.36	(-0.82, 0.57)	0.02
Wks mother worked in YR before birth - 1-48 wks	-0.02	0.26	(-0.54, 0.49)	0.24
Wks mother worked in YR before birth - 49-51 wks	-0.27	0.31	(-0.89, 0.34)	0.24
Wks mother worked in YR before birth - 52 wks	-0.46	0.29	(-1.02, 0.10)	0.24
Weeks preterm - 1-4 weeks	0.06	0.22	(-0.37, 0.48)	0.03
Weeks preterm - >5 weeks	-0.93	0.82	(-2.55, 0.68)	0.03
Sq. root(mother's age - mother's age in 1979)	-0.07	0.12	(-0.31, 0.17)	0.00
Sq. root of mother's AFQT score	0.15	0.06	(0.04, 0.27)	0.03
Child's birth weight	-0.00	0.01	(-0.01, 0.01)	0.01
Log of number of days child spent in hospital	-0.67	0.39	(-1.44, 0.09)	0.04
Log of number of days mother spent in hospital	0.07	0.42	(-0.75, 0.89)	0.05
Mother's attained education	0.20	0.05	(0.10, 0.30)	0.02
Log of family income	0.02	0.10	(-0.16, 0.21)	0.22

Table 13: Summaries of the cofficients in the logistic regression to estimate treatment (estimated from the complete cases)

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