

Metastability in spin-polarized Fermi gases and quasiparticle decays

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2011 New J. Phys. 13 055011

(<http://iopscience.iop.org/1367-2630/13/5/055011>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 152.78.41.124

The article was downloaded on 12/12/2011 at 16:19

Please note that [terms and conditions apply](#).

Metastability in spin-polarized Fermi gases and quasiparticle decays

**K Sadeghzadeh^{1,7}, G M Bruun², C Lobo^{1,3}, P Massignan^{4,5}
and A Recati⁶**

¹ Department of Physics, Cavendish Laboratory, JJ Thomson Avenue, Cambridge, CB3 0HE, UK

² Department of Physics and Astronomy, Aarhus University, Ny Munkegade 120, DK-8000 Aarhus C, Denmark

³ School of Mathematics, University of Southampton, Highfield, Southampton, SO17 1BJ, UK

⁴ Física Teòrica: Informació i Processos Quàntics, Universitat Autònoma de Barcelona, 08193 Bellaterra, Spain

⁵ ICFO—Institut de Ciències Fotòniques—Mediterranean Technology Park, 08860 Castelldefels (Barcelona), Spain

⁶ INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, via Sommarive 14, I-38123 Povo, Italy

E-mail: ks539@cam.ac.uk

New Journal of Physics **13** (2011) 055011 (13pp)

Received 1 December 2010

Published 24 May 2011

Online at <http://www.njp.org/>

doi:10.1088/1367-2630/13/5/055011

Abstract. The metastability associated with the first-order transition from the normal to the superfluid phase is investigated in the phase diagram of two-component polarized Fermi gases. We begin by detailing the dominant decay processes of single quasiparticles, determining the momentum thresholds of each process and calculating their rates. This understanding is then applied to a Fermi sea of polarons, and we predict a region of metastability for the normal partially polarized phase. We propose experiments to observe the threshold of the metastable region, the interaction strength at which the quasiparticle ground state changes character, and the decay rate of polarons.

⁷ Author to whom any correspondence should be addressed.

Contents

1. Introduction	2
2. Background	3
3. Decay processes and thresholds	4
3.1. Polaron decay	4
3.2. Molecule decay	6
4. Metastability of polaron gas	6
5. Single quasiparticle decay rates and experimental observability	8
5.1. Decay rate calculation	8
5.2. Decay rate experiment	10
6. Conclusions	11
Acknowledgments	12
References	12

1. Introduction

The experimental realization of spin-polarized ultracold Fermi gases has initiated a variety of new physics. Of particular interest is the understanding of strongly interacting two-component Fermi gases at zero temperature. Recent experiments have observed a rich phase diagram and located a first-order transition from the normal to the superfluid phase. Phase separation has been observed by the authors of [1–6], who showed that the trapped Fermi gas arranges itself in a shell structure. A precise determination of the equation of state was made by the authors of [7–9]. Furthermore, direct observations of the quasiparticle parameters are presented in [6, 9, 10]. Two theoretical approaches have been used to throw light on this intriguing problem: the study of a single atom immersed in an ideal Fermi gas of atoms in a different spin state, and a Monte Carlo calculation at finite polarization. The Monte Carlo approach revealed the theoretical phase diagram of a homogeneous Fermi gas, as a function of polarization $P = (N_{\uparrow} - N_{\downarrow}) / (N_{\uparrow} + N_{\downarrow})$ and interaction strength [11], mapping out a phase separation of the superfluid and normal phases⁸. The single impurity approach examines quasiparticles used as building blocks to describe these phases. In the strongly imbalanced limit, a single \downarrow fermion immersed in a Fermi sea of \uparrow fermions, the spin impurity atom becomes either a fermionic (polaron) or a bosonic (molecule) quasiparticle. Complementary wave functions for each quasiparticle ([14, 15] and [16–18], respectively) provide ground state energies and effective masses. Previous studies showed that the critical interaction strength at which the ground state of a single impurity at zero momentum switches from the fermionic to the bosonic branch [19] occurs at $1/k_{F\uparrow}a_c \sim 0.88$, with $k_{F\uparrow} = (6\pi^2n_{\uparrow})^{1/3}$ the Fermi momentum of a non-interacting Fermi sea of \uparrow atoms with density n_{\uparrow} , and a the scattering length parametrizing the interaction strength between \uparrow and \downarrow atoms. This value is higher than the interaction strength $1/k_{F\uparrow}a \sim 0.73$ at which a superfluid phase was found to emerge in the limit of full polarization [11]. In this paper, we will assume

⁸ There is also the possibility, discussed by Kohn and Luttinger, that the zero-temperature ground state of the partially polarized Fermi gas is unstable to superfluid pairing in higher-angular-momentum channels, which would lead to new regions of superfluidity, as discussed recently in [12, 13]. The inclusion of such a scenario is, anyway, beyond the scope of this paper.

that this is indeed the case, i.e. that the normal–superfluid transition appears at a lower value of $1/k_{F\uparrow}a$ than does the crossing of the single polaron and molecule branches. The interaction strength at which the normal–superfluid transition occurs is derived from a thermodynamic calculation, in which the energies of a normal partially polarized phase and a phase-separated state of a polarized superfluid and a fully polarized normal gas are compared. Hence, the value $1/k_{F\uparrow}a \sim 0.73$ includes the effects of molecule–molecule interactions as well as the energy cost of phase separation. Since phase separation occurs before a single quasiparticle changes from fermionic to bosonic character, no measurements made in the ground state will allow us to study the transition from the polaron to the molecule in the homogeneous phase. As we will show later, the use of metastable and out of equilibrium processes helps overcome this problem.

Firstly, we determine the momentum thresholds for the decay of single impurities with finite momentum. We then consider the metastability associated with the normal to superfluid first-order phase transition in the thermodynamic phase diagram. Indeed, by applying our understanding of single quasiparticle decay, we predict that there is a region where a finite concentration of quasiparticles is metastable. We denote such a phase as a Fermi sea of polarons. Experimentally, this corresponds to a spin-imbalanced configuration with a finite number of impurities. We propose the use of a Fermi sea of polarons as an experimental probe for determining the momentum threshold of the quasiparticle decay. As this threshold goes to zero at $1/k_{F\uparrow}a_c$, we can observe this point for the first time. Beyond this threshold the decay of polarons into molecules may lead to a mixture of molecules and polarons, which suggests a possible way of measuring the molecule–polaron scattering length. In this way, we hope to open up a new regime of metastable physics in Fermi gases for experimental exploration. Finally, we calculate the decay rates for each process, within the key regions of interaction strength and momenta, to determine the fate of the quasiparticles. The presence of a Fermi sea of polarons would again be instrumental in measuring the various decay rates.

2. Background

Our starting point is an understanding of the single quasiparticle ground state as a function of $1/k_{F\uparrow}a$ going from unitarity ($1/k_{F\uparrow}a \rightarrow 0$) to the ‘Bose–Einstein condensate (BEC)’ limit ($1/k_{F\uparrow}a \rightarrow +\infty$) (see figure 1). At the critical value $1/k_{F\uparrow}(a)_c \sim 0.88$, the zero-momentum energies of the polaron and the molecule cross. For smaller values of $1/k_{F\uparrow}a$ the polaron is the ground state of the system, and for larger values the molecule is the ground state. In figure 1 and throughout this paper, we use the polaron energy calculated using the wave function proposed by Chevy [14]. The energy in the Bardeen–Cooper–Schrieffer (BCS) limit tends to $E_{\text{Pol}} = 4\pi a n_{\uparrow}/m$, the mean-field interaction energy, and that in the BEC limit tends to $E_{\text{Pol}} = -\frac{1}{ma^2} - \frac{\epsilon_{F\uparrow}}{2}$. Here and in the following, we take $\hbar = 1$. The effective mass m_{Pol}^* of a polaron obtained from [18] becomes the bare mass in the BCS limit and diverges at $1/k_{F\uparrow}a \sim 1.17$. For the molecule, the energies and effective masses are obtained from [16–18]. In the BEC limit the energy tends to $E_{\text{Mol}} = -\frac{1}{ma^2} - \epsilon_{F\uparrow}$. The effective mass equals the bare molecule mass $m_{\text{Mol}}^* = 2m$ in that limit; while approaching unitarity it grows; it diverges at $1/k_{F\uparrow}a \sim 0.55$. These masses and energies for both polarons and molecules have been found to be very accurate in comparison with quantum Monte Carlo calculations [11, 19].

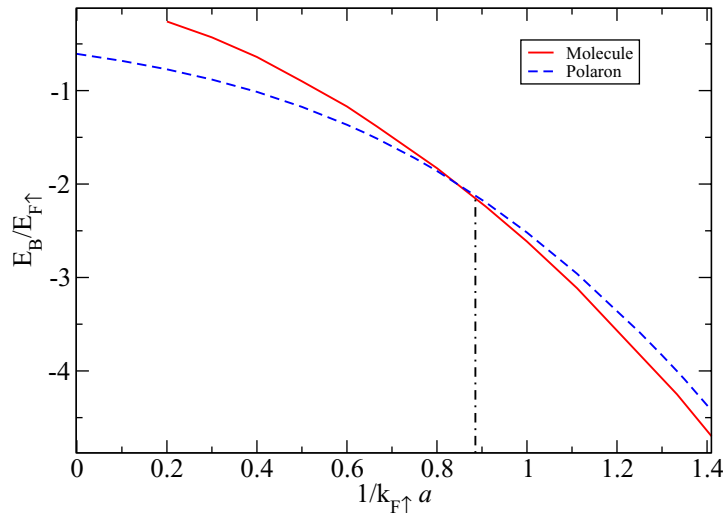


Figure 1. Molecule and polaron zero momentum energies as a function of interaction strength on the ‘BEC’ side. Red line: molecule. Blue dashed line: polaron. The dot-dashed line marks the critical interaction strength $1/(k_{F\uparrow}a)_c \sim 0.88$. To the left of it, the polaron energy is lower, and to the right it is the molecule that becomes the ground state.

3. Decay processes and thresholds

We begin by considering the decay that the polarons and molecules can undergo when they have a non-zero momentum. The decay process of a quasiparticle has to satisfy energy and momentum conservation. This is generally possible only when the initial quasiparticle momentum p is above a threshold momentum P_{Th} .

3.1. Polaron decay

For the polaron at zero temperature, the decay processes we consider are

- A. polaron \rightarrow molecule + hole,
- B. polaron \rightarrow molecule + particle + 2 holes.

A finite momentum polaron also has a finite relaxation time as described in [20], which schematically reads

- C. polaron(p) \rightarrow polaron($p' < p$) + particle + hole

The three-body decay of a polaron into a molecule, two holes and a particle has been considered in [21] in the special case $\mathbf{p} = 0$. Generalizing this calculation to non-zero momentum, we find that process B can occur at any momentum when $\Delta E = E_{Pol} - E_{Mol} > 0$, as expected, and only for

$$p > P_{Th}^B = \sqrt{-2m_{Pol}^* \Delta E}, \quad (1)$$

when $\Delta E < 0$. Here, E_{Pol} and E_{Mol} are the energies of a zero-momentum polaron and molecule at the given value of interaction strength.

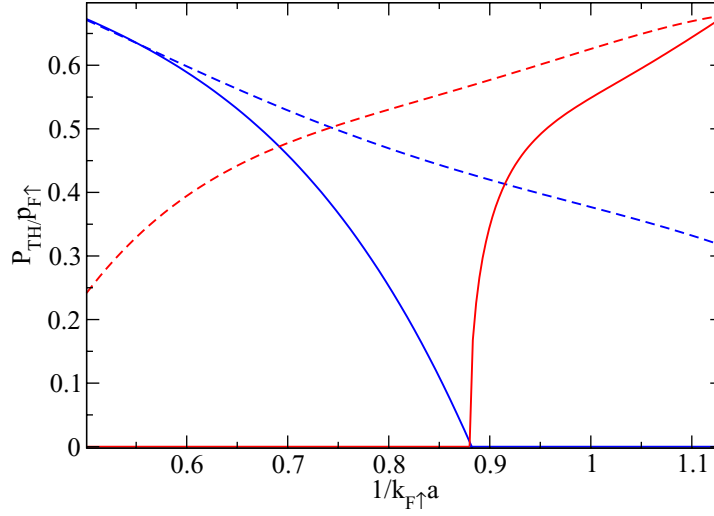


Figure 2. Momentum thresholds for various decay processes for both polarons and molecules about the critical point. Blue solid line: process B, *polaron* \rightarrow *molecule* + 2 *holes* + *particle*; blue dashed line: process A, *polaron* \rightarrow *molecule* + *hole*; red solid line: process \tilde{B} , *molecule* \rightarrow *polaron* + 2 *particles* + *hole*; red dashed line: process \tilde{A} , *molecule* \rightarrow *polaron* + *particle*. The ground state of the system changes from being a polaron to a molecule at $1/k_{F\uparrow}a \sim 0.88$, beyond which even a polaron at $p = 0$ can decay via process B; otherwise, only polarons at $p > 0$ are unstable in this way. Process A is relevant at higher polaron momentum and generally results in a finite-momentum molecule even on the threshold. The process A and B thresholds meet when the molecule's (polaron's) effective mass diverges at $1/k_{F\uparrow}a \sim 0.55$ (1.17). Note that for polarons at large $p \sim p_{F\uparrow}$, momentum relaxation processes (process C) are very strong, so that such a polaron is no longer a well-defined quasiparticle.

For process A, the two-body decay of a polaron with momentum \mathbf{p} into a molecule and a hole, the conservation of energy and momentum requires the following equality to be verified,

$$E_{\text{Pol}} + \frac{p^2}{2m^*} + \xi_q = E_{\text{Mol}} + \frac{(\mathbf{p} + \mathbf{q})^2}{2M^*}, \quad (2)$$

where $\xi_q = \frac{q^2}{2m} - \epsilon_{F\uparrow}$ is the kinetic energy of a majority particle measured with respect to the Fermi surface. The minimum momentum P_{Th}^A at which process A is allowed is found by setting the hole at the Fermi surface, and by taking its momentum to be anti-parallel to the one of the polaron,

$$\frac{P_{\text{Th}}^A{}^2}{2m_{\text{Pol}}^*} + E_{\text{Pol}} = \frac{(P_{\text{Th}}^A - p_F)^2}{2m_{\text{Mol}}^*} + E_{\text{Mol}}. \quad (3)$$

Processes including more particle-hole pairs (e.g. *molecule* + 3 *holes* + 2 *particles*) have the same energy threshold as process B, but lower rates, so we do not consider them here.

Figure 2 shows our results for the polaron decay thresholds, as given by (1) and (3). In the region below $1/k_{F\uparrow}a_c$, where the zero-momentum polaron is the ground state of the system, a

polaron at finite momentum can nevertheless be unstable to decay processes A, B and C. The solid blue line gives the momentum threshold P_{Th}^{B} for the polaron to decay via process B. On this line, decay results in a zero-momentum molecule for $1/k_{\text{F}\uparrow}a < 1/k_{\text{F}\uparrow}a_{\text{c}}$. Finite momentum molecules also result from process B above the solid blue line. The blue dashed line gives the momentum threshold P_{Th}^{A} for process A, which generally creates a molecule at finite momentum even on the threshold. Decay processes including more particle–hole pairs all set in at momenta above the solid blue line, i.e. for $p > P_{\text{Th}}^{\text{B}}$. For $1/k_{\text{F}\uparrow}a > 1/k_{\text{F}\uparrow}a_{\text{c}}$, a polaron at zero momentum that is unstable to process B decays into a molecule at finite momentum, whereas process A continues to affect only higher-momentum polarons. Polarons with any finite momentum are unstable to momentum relaxation (process C) on both sides of $1/k_{\text{F}\uparrow}a_{\text{c}}$. The momentum thresholds for processes A and B become equal where the molecule’s effective mass diverges ($1/k_{\text{F}\uparrow}a \sim 0.55$).

3.2. Molecule decay

The stability of a molecule is calculated in a similar way. The decay channels we consider are:

- \tilde{A} . molecule \rightarrow polaron + particle
- \tilde{B} . molecule \rightarrow polaron + 2 particles + hole
- \tilde{C} . molecule(p) \rightarrow molecule ($p' < p$) + hole + particle

For $1/k_{\text{F}a} < 1/k_{\text{F}\uparrow}a_{\text{c}}$, a zero-momentum molecule decays via process \tilde{B} into a polaron with finite momentum, and via process \tilde{A} at higher momentum. For $1/k_{\text{F}a} > 1/k_{\text{F}\uparrow}a_{\text{c}}$, process \tilde{B} precedes process \tilde{A} with increasing momentum until where the polaron’s effective mass diverges. In analogy to the polaron decay considered above, the momentum thresholds for the molecule are determined by energy and momentum conservation. They are

$$\frac{P_{\text{Th}}^{\tilde{A}2}}{2m_{\text{Mol}}^*} + E_{\text{Mol}} = \frac{(P_{\text{Th}}^{\tilde{A}} - p_{\text{F}})^2}{2m_{\text{Pol}}^*} + E_{\text{Pol}}, \quad (4)$$

$$P_{\text{Th}}^{\tilde{B}} = \sqrt{2m_{\text{Mol}}^* \Delta E} \quad \text{for} \quad \frac{1}{k_{\text{F}\uparrow}a} > \frac{1}{k_{\text{F}\uparrow}a_{\text{c}}}, \quad (5)$$

where $P_{\text{Th}}^{\tilde{B}} = 0$ for $1/k_{\text{F}\uparrow}a \leq 1/k_{\text{F}\uparrow}a_{\text{c}}$. Again, the threshold momentum for process \tilde{A} is found when the majority particle is on the Fermi surface.

Our results for the threshold momenta for polaron and molecule decay, as given by (1), (3), (4) and (5), are shown in figure 2.

4. Metastability of polaron gas

So far, we have analysed the decay processes of single quasiparticles. The single quasiparticle problem is a limiting case of the spin imbalanced Fermi gas. The ground state phase diagram of a spin imbalanced Fermi gas, with polarization versus interaction strength, has been calculated in [11] using a Monte Carlo approach. In this calculation, the \downarrow atom concentration is kept finite for a macroscopic system even in the $P \rightarrow 1$ limit. We now examine how the stability analysis above is connected to this equilibrium phase diagram.

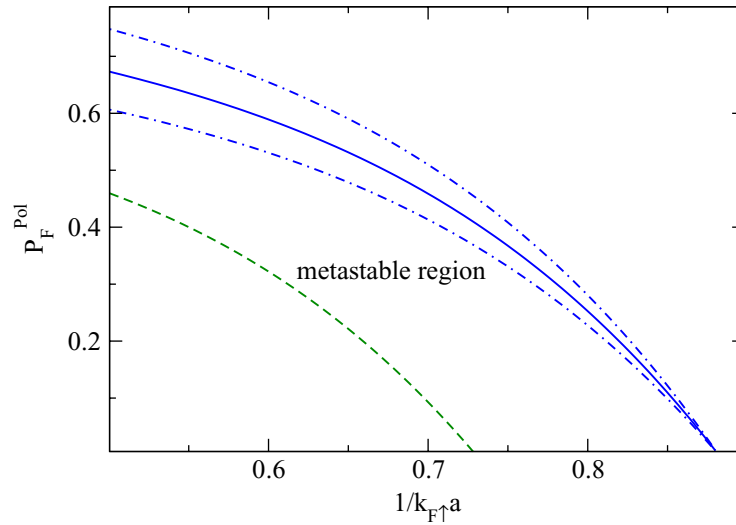


Figure 3. The stability of a Fermi sea of polarons: the dashed green line shows the first-order transition from normal to phases including a superfluid (figure 4 of [11]). The solid blue line is the Fermi momentum of a Fermi sea of polarons equal to the momentum threshold for process B ($p_F^{\text{Pol}} = P_{\text{Th}}^{\text{B}}$). The dot-dashed blue lines show how the solid blue line might be shifted due to molecule–polaron interactions.

Our particular consideration is the normal to superfluid first-order phase transition predicted by the Monte Carlo calculations [11]. This transition from a partially polarized normal phase to a state with separated superfluid and normal phases is represented by the dashed green line in figure 3. To make this line in figure 3, we have assumed that the polarons form a weakly interacting Fermi sea, so that we can convert the critical density at which phase separation occurs, into a critical Fermi momentum $p_F^{\text{Pol}}(1/k_{F\uparrow}a)$ for the polarons. In the phase-separated state, the Monte Carlo calculation includes the interactions between molecules in the condensate. In contrast, in the single \downarrow atom calculation, there is at most one molecule and therefore no condensate.

If we assume that the relevant processes in which the polarons can be converted into molecules, within experimentally realistic timescales, are only the ones considered in section 3.1 and that these single impurity processes can be used to analyse the stability of the Fermi sea of polarons (neglecting for instance multiple polaron decay), this gives rise to a region of *metastability* in the phase diagram. If a Fermi sea of polarons prepared in the ground state (below the green dashed line) is adiabatically taken above the dashed green line by increasing $1/k_{F\uparrow}a$, we expect it to persist as a metastable state. Even though the density of the \downarrow atoms is so high that the true ground state is a phase-separated state, the state is stable since there are no polarons with a large enough momentum to decay to a molecule via process B. The decay of polarons to molecules sets in only when $1/k_{F\uparrow}a$ is increased further so that the Fermi momentum of the polarons is larger than the momentum threshold for process B ($p_F^{\text{Pol}} = P_{\text{Th}}^{\text{B}}$); we ignore here molecule–polaron interactions (see below). This momentum is given by the solid blue line in figure 3. At this point, the highest-momentum polarons will decay into molecules.

This motivates the following proposal to use a Fermi sea of polarons as an experimental probe for exploring the metastable region and observe molecule formation through process B. Due to the Pauli exclusion principle, the momentum relaxation process (C) is suppressed, allowing for an analysis of the other decay processes. Let us suppose for the moment that the molecules into which polarons can decay do not interact with the polarons. Then, to determine the shape of P_{Th}^{B} , we can prepare a polaron Fermi sea with a Fermi momentum smaller than the dashed green line in figure 3 at unitarity, for example. Then we increase $1/k_{\text{F}\uparrow}a$ adiabatically. The system will cross the first-order phase transition line, but since it is metastable (as discussed above), it will remain a Fermi sea of polarons rather than form a condensate of molecules. As we continue beyond the phase transition, the Fermi momentum will become equal to the momentum threshold for process B ($p_{\text{F}}^{\text{Pol}} = P_{\text{Th}}^{\text{B}}$). Increasing $1/k_{\text{F}\uparrow}a$ beyond this point will now lead to the decay of polarons at $p_{\text{F}}^{\text{Pol}}$. The polarons on the Fermi surface decay into molecules. The appearance of these molecules can then be detected experimentally as the tell-tale sign of the solid blue line P_{Th}^{B} . The experiment can be repeated using an appropriate $p_{\text{F}}^{\text{Pol}}$ to find P_{Th}^{B} at different interaction strengths, making sure that only a small number of molecules are created each time (so as to be able to ignore effects beyond the threshold, e.g. molecule–molecule interactions) but a large enough number are observable. The size of the polaron Fermi sea needed will decrease as P_{Th}^{B} decreases, so that the value $1/k_{\text{F}\uparrow}a_c$ can be found in the limit of a single polaron.

If we now take into account molecule–polaron interactions, the value of $p_{\text{F}}^{\text{Pol}}$ at which molecules are first formed will change. If we consider one molecule in the final state, the (unknown) molecule–polaron interaction changes the energy by $\Delta E = g_{\text{PM}}n_{\text{Pol}}$ (assuming a mean-field approximation). This will lead to a positive or negative shift in the threshold curve of the production of molecules (see dot-dashed blue lines in figure 3). This shift tends to zero at $1/k_{\text{F}\uparrow}a_c$ where $n_{\text{Pol}} \rightarrow 0$ and where the threshold curve meets P_{Th}^{B} . One could in principle use the difference between the experimentally observed curve and P_{Th}^{B} (which is known theoretically) to determine this shift and so the molecule–polaron scattering length. Note that we have ignored the effects of the polaron–polaron interaction, which are known to be small.

We have therefore established that the region between the dashed green and dot-dashed blue lines in figure 3 represents a metastable phase consisting of a Fermi sea of polarons. Such a metastable phase is characteristic of a first-order phase transition. It is important to note that the state may well be metastable beyond the dot-dashed blue line since an analysis of its stability in that region would require us to take into account the presence of a finite quantity of molecules. We also raise the intriguing possibility of a final state containing the remaining Fermi sea of polarons coexisting with a condensate of molecules, within a background of \uparrow particles.

5. Single quasiparticle decay rates and experimental observability

5.1. Decay rate calculation

The phase diagram is animated by considering the rates of each decay process. The decay rates of zero-momentum polarons and molecules via process B are presented in [21]. The decay rate of a polaron with momentum p through process A is given in terms of the imaginary part of the on-shell polaron self-energy $\Sigma_{\text{Pol}}^{\text{A}}$ shown in figure 4(a), i.e. $\Gamma_{\text{Pol}}^{\text{A}}(p) = -Z_{\text{Pol}} \text{Im} \Sigma_{\text{Pol}}^{\text{A}}(p, E_{\text{Pol}} + \frac{p^2}{2m_{\text{Pol}}^*})$, where the quasiparticle residue Z_{P} has been explicitly included in the decay rate [22]

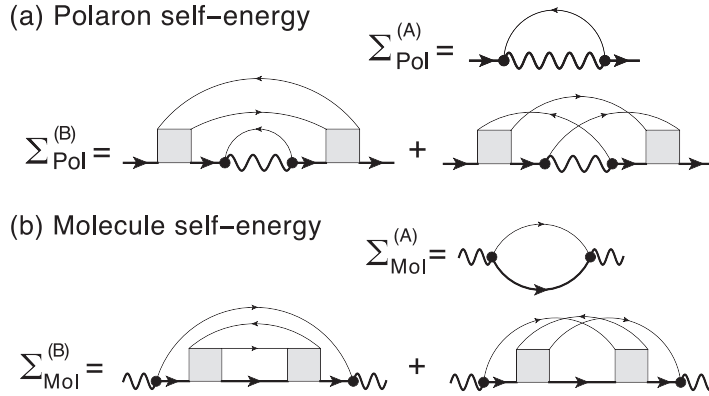


Figure 4. Diagrams showing the decay processes A and B for (a) polarons and (b) molecules. The thin lines represent majority atoms, wavy lines a molecule and thick lines a polaron. The polaron–molecule matrix element g is represented by a thick dot and the square represents off-resonant scattering between a polaron and the majority atoms.

since it is of the order of $1/2$ in the interesting region. At zero temperature, one obtains

$$\Gamma_{\text{Pol}}^{(A)}(p) = \pi Z_{\text{Pol}} Z_{\text{Mol}} g^2 \int_{q < k_{F\uparrow}} \delta \left(\Delta E + \frac{p^2}{2m_{\text{Pol}}^*} + \xi_q - \frac{(p+q)^2}{2m_{\text{Mol}}^*} \right), \quad (6)$$

which is a simple Golden rule expression. Here, $g = -\sqrt{2\pi/m_r^2 a}$ is the atom–molecule coupling in vacuum [21], and m_r is the atom–molecule reduced mass. To derive this equation, we have used a pole expansion of the molecule propagator with quasiparticle residue Z_{Mol} . The decay rate may be calculated analytically. Assuming for simplicity $m_{\text{Mol}}^* = 2m_{\uparrow}$ and $m_{\text{Pol}}^* = m_{\uparrow}$, we obtain

$$\Gamma_{\text{Pol}}^A(p) = Z_{\text{Pol}} Z_{\text{Mol}} \epsilon_{F\uparrow} \frac{4}{k_{F\uparrow} a} \frac{(p - P_{\text{Th}}^A)(P_{\text{Th}}^A + 2k_{F\uparrow} - p)}{pk_{F\uparrow}}, \quad (7)$$

for $P_{\text{Th}}^A < p < P_{\text{Th}}^A + 2k_{F\uparrow}$ and zero otherwise. The momentum threshold for this process is $P_{\text{Th}}^A = k_{F\uparrow}(\sqrt{2 - 2\Delta E/\epsilon_{F\uparrow}} - 1)$. Note that $P_{\text{Th}}^A > 0$ even when $\Delta E > 0$ and that one has $\frac{\Delta E}{\epsilon_{F\uparrow}} < 0.5$ for all scattering lengths [16, 19].

The decay rate of a molecule with momentum p via process A, the creation of a polaron and a majority particle, is given in terms of the imaginary part of the molecule self-energy Σ_{Mol}^A depicted in figure 4(b). A calculation analogous to the polaron case considered above yields

$$\Gamma_{\text{Mol}}^A(p) = Z_{\text{Mol}} Z_{\text{Pol}} \epsilon_{F\uparrow} \frac{2}{k_{F\uparrow} a} \frac{(p - \tilde{P}_{\text{Th}}^A)(p - \tilde{P}_{\text{Th}}^A + 4k_{F\uparrow})}{pk_{F\uparrow}}. \quad (8)$$

The threshold momentum for this process is $\tilde{P}_{\text{Th}}^A = k_{F\uparrow}(2 - \sqrt{2 - 2\Delta E/\epsilon_{F\uparrow}})$ with $\Gamma_{\text{Mol}}^A = 0$ for $p < \tilde{P}_{\text{Th}}^A$. Note that in this case, as opposed to the polaron case, there is no maximum momentum for the molecule above which there is no decay via process A. This is because for large momentum, the molecule can always dispose its energy and momentum to a majority particle, whereas the polaron has to dispose it into a majority hole within the Fermi sea.

The corresponding self-energy for process B, $\Sigma_{\text{Pol}}^{\text{B}}$, is shown in figure 4(a). Generalizing the calculations in [21] to non-zero momentum p yields

$$\Gamma_{\text{Pol}}^{\text{B}}(p) \propto Z_{\text{Pol}} Z_{\text{Mol}} \epsilon_{\text{F}\uparrow} \left[\frac{(\Delta E + (p^2/2m_{\text{Pol}}^*))}{\epsilon_{\text{F}\uparrow}} \right]^{9/2}. \quad (9)$$

The analysis in [21] has shown that the proportionality constant in this expression is of the order of unity.

Likewise, the decay rate of a molecule via process B can be obtained by generalizing the calculations in [21] to a finite momentum p . We obtain

$$\Gamma_{\text{Mol}}^{\text{B}}(p) \propto Z_{\text{Mol}} Z_{\text{Pol}} \epsilon_{\text{F}\uparrow} \left[\frac{(-\Delta E + (p^2/2m_{\text{Mol}}^*))}{\epsilon_{\text{F}\uparrow}} \right]^{9/2}, \quad (10)$$

where the constant of proportionality is, again, of the order of unity. P_{Th}^{B} ($P_{\text{Th}}^{\tilde{\text{B}}}$) is the point at which $\Gamma_{\text{Pol}}^{\text{B}}$ ($\Gamma_{\text{Mol}}^{\text{B}}$) is equal to zero.

For a finite momentum, the polaron can scatter off the majority particles, giving rise to momentum relaxation with a rate $1/\tau_{\text{Pol}}$ (process C). The high velocity regime $k_{\text{F}\downarrow} \ll m_{\downarrow}^* v \ll k_{\text{F}\uparrow}$ analysed in [20] determines the rate of relaxation of a single impurity. If we consider only a single impurity, spin statistics become redundant, allowing the same calculation to be used for the molecule. The momentum relaxation rate, at $T = 0$ for the molecule or polaron, is then

$$\frac{1}{\tau_{\text{Mol/Pol}}} = \frac{2\pi}{35} |\gamma|^2 \frac{m_{\text{Mol/Pol}}^{*3} v^4}{k_{\text{F}\uparrow}^2}, \quad (11)$$

where γ is determined by the scattering amplitude $U = \frac{\partial \mu_{\text{Mol/Pol}}}{\partial n_{\uparrow}} = \frac{2\pi^2}{m_{\uparrow} k_{\text{F}\uparrow}} \gamma$. At unitarity, we have $\mu_{\text{Pol}} = -\alpha \epsilon_{\text{F}\uparrow}$ with $\alpha \simeq 0.6$ [19]. One can also use $\mu_{\text{Mol}} = -\frac{1}{ma^2} + \frac{3\pi\tilde{a}}{m} n_{\uparrow}$, valid for $\frac{1}{k_{\text{F}\uparrow} a} \gtrsim 0.7$ [19] (where $\tilde{a} \simeq 1.18a$), to find the molecule momentum relaxation rate in terms of the interaction strength,

$$\frac{1}{\tau_{\text{Mol}}} = \frac{9}{70\pi} (k_{\text{F}\uparrow} \tilde{a})^2 \frac{m_{\text{Mol}}^{*3} v^4}{k_{\text{F}\uparrow}^2}. \quad (12)$$

The high power of the velocity v and the effective mass m^* indicates that impurities at large momenta $p \sim p_{\text{F}\uparrow}$ are no longer well-defined quasiparticles.

In figure 5, we plot the decay rate of polarons and molecules via processes A and B as a function of momentum p for various ΔE . We see that once $p > P_{\text{Th}}^{\text{A}}$ and process A sets in, it quickly dominates B. In fact, once active, process A dominates process B by several orders of magnitude since the ratio is, to lowest order, $\frac{\Gamma_{\text{A}}}{\Gamma_{\text{B}}} \propto \left(\frac{p - P_{\text{Th}}^{\text{A}}}{p_{\text{F}\uparrow}} \right) \left(\frac{p_{\text{F}\uparrow}}{P_{\text{Th}}^{\text{A}}} \right)^{10}$. This is expected since process A is a two-body process, whereas process B is a three-body process. On the other hand, for $p < P_{\text{Th}}^{\text{A}}$ process B of course dominates, as process A is not allowed. However, since the typical timescale for process B is very long, of the order of 10–100 ms, the momentum relaxation of the polaron via process C is in general much faster. This justifies our proposed experiment to determine P_{Th}^{B} using a Fermi sea of polarons in equilibrium, in order to suppress process C.

5.2. Decay rate experiment

The polaron Fermi sea can also be used to measure the rate of process B. The initial state is a polaron Fermi sea prepared in the ground state (i.e. below the dashed green line in figure 3). We

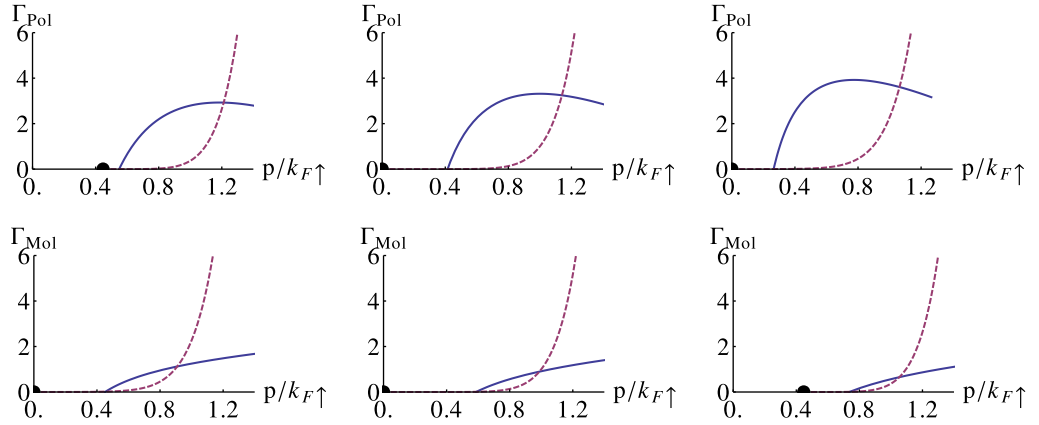


Figure 5. Decay rates of finite momentum polarons (top) and molecules (bottom) for processes A (continuous line) and B (dashed line). From left to right, we have $\Delta E/\epsilon_{F\uparrow} = -0.2, 0$ and 0.2 . Decay rates are in units of $(Z_{\text{Pol}}Z_{\text{mol}}\epsilon_{F\uparrow})$. The thick dot indicates the threshold momentum for process B, P_{Th}^{B} . Generally, process B dominates over A for $p \gtrsim k_{F\uparrow}$.

then instantaneously increase the interaction strength so that some fraction of the polarons have a momentum $p > P_{\text{Th}}^{\text{B}}$ and hence are then susceptible to decay via process B. Alternatively, by increasing the interaction strength to above $1/k_{F\uparrow}a_c$, every polaron has a momentum $p > P_{\text{Th}}^{\text{B}}$, and so the whole Fermi sea is susceptible to decay via process B. The molecules resulting from this decay are unable to decay via process \tilde{A} or \tilde{B} ; however, they lose momentum via process \tilde{C} . The resultant state is therefore expected to be a condensate of molecules. A measurement of the initial growth rate of the number of molecules or the loss rate of polarons determines the rate of process B for polarons, averaged over the momenta of the polarons decaying, at a given $1/k_{F\uparrow}a$.

Similarly, the rate of process A can be measured, with some fraction of the polarons having a momentum $p > P_{\text{Th}}^{\text{A}}$. In this case, the Fermi sea of polarons will decay via both processes A and B. The two processes can be distinguished, since process A has a significantly higher rate and typically results in molecules at finite momentum.

6. Conclusions

In this paper, we have shown how quasiparticles in a two-component Fermi gas behave at finite momenta in the limit of extreme imbalance. By considering energy and momentum conservation of single impurities, we determined the momentum thresholds beyond which quasiparticles are susceptible to the most significant decay processes, and we calculated the rates of each.

We have identified a region of metastability for the partially polarized normal phase about the normal to superfluid first-order transition, and we have shown how a Fermi sea of polarons can be used as an experimental probe to observe the boundary of this region. Moreover, we suggested an experiment able to determine the location of the polaron–molecule transition and to measure the decay rate of processes involving one and two particle–hole pairs.

The experiments we propose would yield the observation of a novel state, a mixture of molecules and polarons, and a measure of the unknown molecule–polaron interaction strength.

Acknowledgments

KS and CL acknowledge support from the EPSRC through Advanced Fellowship EP/E053033/1. PM acknowledges support from the ESF/MEC project FERMIX (FIS2007-29996-E), the Spanish MEC project FIS2008-01236 and the Catalan project 2009-SGR-985. AR acknowledges support from the ESF/MEC project FERMIX and the ERC through a QGBE grant (FIS2007-29996-E).

References

- [1] Zwierlein M W, Schirotzek A, Schunck C H and Ketterle W 2006 Fermionic superfluidity with imbalanced spin populations *Science* **311** 492–6
- [2] Partridge G B, Li W, Kamar R I, Liao Y-A and Hulet R G 2006 Pairing and phase separation in a polarized Fermi gas *Science* **311** 503–5
- [3] Shin Y, Zwierlein M W, Schunck C H, Schirotzek A and Ketterle W 2006 Observation of phase separation in a strongly interacting imbalanced Fermi gas *Phys. Rev. Lett.* **97** 030401
- [4] Partridge G B, Wenhui Li, Liao Y A, Hulet R G, Haque M and Stoof H T C 2006 Deformation of a trapped Fermi gas with unequal spin populations *Phys. Rev. Lett.* **97** 190407
- [5] Shin Y-I, Schunck C H, Schirotzek A and Ketterle W 2008 Phase diagram of a two-component Fermi gas with resonant interactions *Nature* **451** 689
- [6] Schirotzek A and Wu C-H 2009 Sommer A and Zwierlein M W Observation of Fermi polarons in a tunable Fermi liquid of ultracold atoms *Phys. Rev. Lett.* **102** 230402
- [7] Nascimbene S, Navon N, Jiang K J, Chevy F and Salomon C 2010 Exploring the thermodynamics of a universal Fermi gas *Nature* **463** 1057
- [8] Navon N, Nascimbene S, Chevy F and Salomon C 2010 The equation of state of a low-temperature Fermi gas with tunable interactions *Science* **328** 729–32
- [9] Nascimbène S, Navon N, Pilati S, Chevy F, Giorgini S, Georges A and Salomon C 2010 A new Fermi liquid: the normal phase of a strongly interacting gas of cold atoms arXiv:1012.4664
- [10] Nascimbène S, Navon N, Jiang K J, Tarruell L, Teichmann M, McKeever J, Chevy F and Salomon C 2009 Collective oscillations of an imbalanced Fermi gas: axial compression modes and polaron effective mass *Phys. Rev. Lett.* **103** 170402
- [11] Pilati S and Giorgini S 2008 Phase separation in a polarized Fermi gas at zero temperature *Phys. Rev. Lett.* **100** 030401
- [12] Bulgac A, McNeil Forbes M and Schwenk A 2006 Induced p-wave superfluidity in asymmetric Fermi gases *Phys. Rev. Lett.* **97** 020402
- [13] Patton K R and Sheehy D E 2010 Induced p-wave superfluidity in strongly interacting imbalanced Fermi gases arXiv:1011.4536
- [14] Chevy F 2006 Universal phase diagram of a strongly interacting Fermi gas with unbalanced spin populations *Phys. Rev. A* **74** 063628
- [15] Combescot R, Recati A, Lobo C and Chevy F 2007 Normal state of highly polarized Fermi gases: simple many-body approaches *Phys. Rev. Lett.* **98** 180402
- [16] Punk M, Dumitrescu P T and Zwerger W 2009 Polaron-to-molecule transition in a strongly imbalanced Fermi gas *Phys. Rev. A* **80** 053605
- [17] Mora C and Chevy F 2009 Ground state of a tightly bound composite dimer immersed in a Fermi sea *Phys. Rev. A* **80** 033607
- [18] Combescot R, Giraud S and Leyronas X 2009 Analytical theory of the dressed bound state in highly polarized fermi gases *Europhys. Lett.* **88** 60007

- [19] Prokof'ev N and Svistunov B 2008 Fermi-polaron problem: diagrammatic Monte Carlo method for divergent sign-alternating series *Phys. Rev. B* **77** 020408
- [20] Bruun G M, Recati A, Pethick C J, Smith H and Stringari S 2008 Collisional properties of a polarized Fermi gas with resonant interactions *Phys. Rev. Lett.* **100** 240406
- [21] Bruun G M and Massignan P 2010 Decay of polarons and molecules in a strongly polarized Fermi gas *Phys. Rev. Lett.* **105** 020403
- [22] Fetter A L and Walecka J D 2003 *Quantum Theory of Many-Particle Systems* (New York: Dover)