The Process of Redesigning the Geometry Curriculum: The Case of the Mathematical Association in England in the Early Twentieth Century

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Abstract

This paper examines a key period of change in geometry teaching in England. Our focus is the character and nature of the recommendations of the 1902 geometry report of the UK Mathematical Association. We analyse historical documents of the Mathematical Association using a theoretical framework informed by work in the sociology of education. Our analysis shows that the character and recommendations of the Mathematical Association report were influenced by various factors including: that Mathematical Association members at the time still respected the traditional Euclidean approach to geometry as a basis for school geometry; that the academic and "power" resources available to the Mathematical Association at the time were not sufficient to enable a complete change from the traditional approach; that a lack of consensus between the various members of the Mathematical Association prevented a more radical proposal; and that the general climate in schools at that time was not prepared for far-reaching changes to the teaching of geometry. These findings accord with other research on educational reform which indicates that curriculum change processes are invariably complex and often subject to much politicking.

Introduction

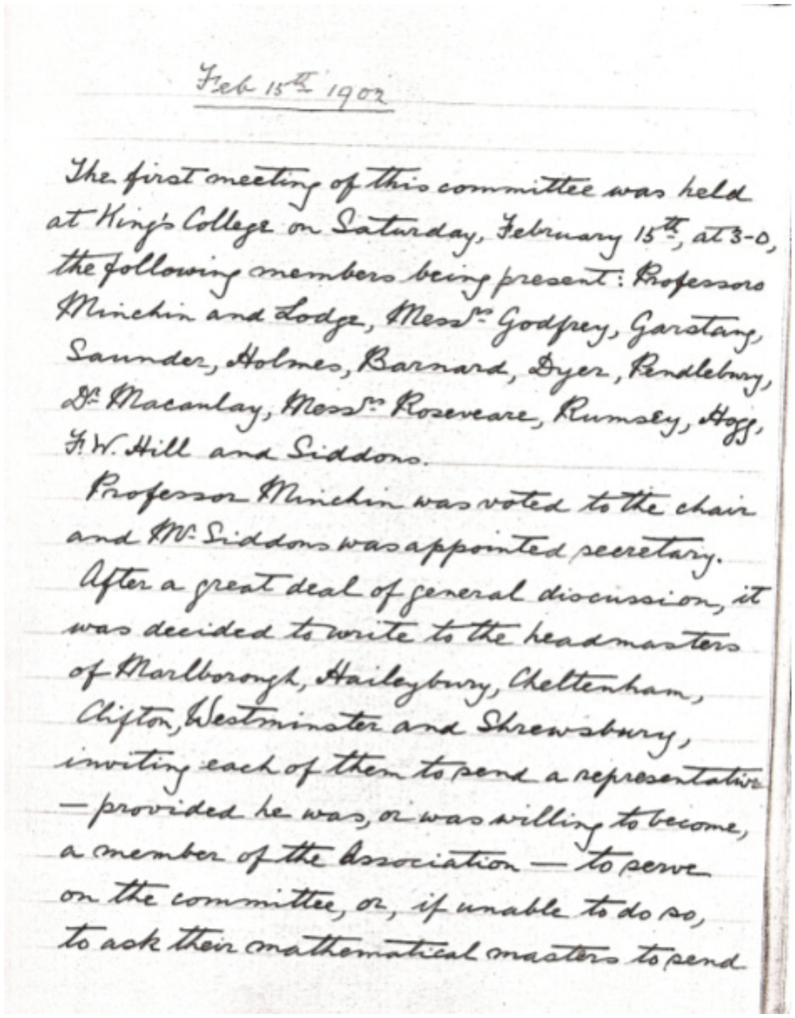
In the history of mathematics education in secondary schools in England, one of the major events occurred in and around 1900; it came to be known not only in

England but also internationally as the *Perry movement* (Price, 1986; 2001, p. 217; 2003, p. 465). The main argument at that time concerned the use of geometry textbooks based on a strict following of Euclid's *Elements*—such as those edited by Potts (1845) or Todhunter (1862). The Mathematical Association (MA), originally founded in the UK in 1871 as the Association for the Improvement of Geometrical Teaching (AIGT), acted for the reform of the teaching of geometry, and published an important geometry report in 1902 (MA, 1902b). This report can be considered a key document in the history of the teaching of geometry given its prominence in various articles and studies (see, for example, Howson, 1982, p. 149; Price, 1994, p. 56).

Despite its prominence, some of the commentary on the MA report has suggested that it was rather conservative and quite cautious; that it favoured tradition and proposed only gradual change. For example, Godfrey, a prominent member of the MA at the time of the report, stated later that "the M.A. published a report on Geometry teaching; a conservative report, as it was considered impracticable to secure the abolition of the [Euclidean] sequence" (Godfrey, 1920, p. 20). Much later a UK Government report commented that "this body [the Teaching Committee of the Mathematical Association] despaired of abolishing Euclid as an examination textbook and concentrated on less sweeping changes" (DES, 1958, p. 9).

The goal of this paper is to give a comprehensive account for *why* the MA report of 1902 can be seen as a modest reform, something which has yet to be addressed in historical studies. Our focus is on why this report was quite "modest," as compared to what was proposed for the improvement in the teaching of geometry in 1901–1902. To achieve our goal, we employ an historical case-study approach. Our approach is to analyse historical documents that record the discussions leading up to the MA report of 1902, including the unpublished book of minutes of the Teaching Committee of the MA (stored in the MA archive at Leicester, UK); see Figure 1 for an example entry.

While our focus is on mathematics education in England, we consider analysing curriculum changes as important within international contexts because examining such changes provides useful insights into changes in policies that may be compared to changes in mathematics education in other countries.



Feb 15th 1902

The first meeting of this committee was held at King's College on Saturday, February 15th, at 3-0, the following members being present: Professors Minchin and Lodge, Messrs Godfrey, Garstang, Saunder, Holmes, Barnard, Dyer, Pendlebury, Dr Macaulay, Messrs Roseveare, Rumsey, Hogg, F.W. Hill and Siddons.

Professor Minchin was voted to the chair and Mr Siddons was appointed secretary.

After a great deal of general discussion, it was decided to write to the headmasters of Marlborough, Haileybury, Cheltenham, Clifton, Westminster and Shrewsbury, inviting each of them to send a representative —provided he was, or was willing to become, a member of the Association — to serve on the committee, or, if unable to do so, to ask their mathematical masters to send

Figure 1. MA book of minutes, 1902 (written by Minchin)

Secondary education and the teaching of geometry in the late nineteenth century in England

The ongoing development of the teaching of mathematics is always accompanied by moves to design and redesign syllabi, to adjust the content of textbooks, and others. Such changes may be partially a result of progress in teaching and learning theories, teaching methods, technology, social demands, and so on. In this paper, we refer to this process of change as "reform." Marsh (1997, p. 211) states "Proposals for curriculum reform come from various sources including: teachers, teacher unions, policy-makers, academics, politicians, media and pressure groups." Fullan (1993, p. 19) has characterised such change processes in education as "uncontrollably complex," and the case of English mathematics education is by no means an exception. In this section, we provide a brief overview of secondary school education at the turn of the twentieth century in England, together with a short account of the teaching of geometry at that time. We do this in order to give appropriate background to the issues which were being discussed by the Teaching Committee of the Mathematical

Association during the drawing up of the 1902 geometry report. As we show in what follows, the major force for reform was educational, i.e., it was mainly from mathematicians, mathematics educators, and teachers who fundamentally wanted to improve the quality of teaching of geometry.

Whereas the 1870 Education Act established a national system of primary education up to the age of 13 in England (see Price, 1994, p. 15), opportunities for secondary education in England in the late nineteenth century were of varying quality. In 1894 a Royal Commission on Secondary Education was given the task of considering "the best methods of establishing a well-organised system of Secondary Education in England" (Barnard, 1961, p. 204). The report of the commission, published in 1895, recommended a national system of secondary schools and this was enacted from 1902 (ibid., pp. 204–211). The reason for the Commission, and for Government action in 1902, was that secondary education in England in the late nineteenth century was in complete muddle. In brief, there existed a range of secondary schools that can be grouped as "public schools," "grammar schools," and "private schools." The public schools (nine in total in the entire country) were, in fact, anything but public. As Howson (2010) explains, these so-called public schools had been established some hundreds of years previously (Winchester College, for example, in 1382; Eton in 1440) with provisions made for poor scholars—hence the name "public school"—but, by the beginning of the nineteenth century, they had become schools for the children of the rich. The grammar schools (an example being King Edward's School in Birmingham) were endowed in some way—this could be by the Church or by a trade guild or by one or more wealthy individuals. Private schools were just that: schools run by private individuals, usually for profit.

In terms of the teaching of geometry in these schools, Howson (1984) explains how Euclid[1] occupied a dominant position, especially the first six books on plane geometry. The dominant value was training students' ability in logical reasoning (see Howson, 1982, p. 131), such that "every Gentleman should know Greek thought" (Griffiths, 1998, p. 195). Equally important was the aim to prepare students for the entrance examinations for Oxford or Cambridge. Todhunter, a Cambridge mathematician and prominent textbook writer for secondary schools (see Barrow-Green, 2001), wrote that "In England the text-book of Geometry consists of the Elements of Euclid; for nearly every official programme of instruction or examination explicitly includes some portion of this work" (Todhunter, 1862, p. vii). In such teaching, the logical and deductive of Euclidean geometry was stressed, while practical approaches, measurement, and calculations were notable by their absence.

Yet, in nineteenth-century England, alternative educational opportunities were also being developed. For example, the "Great Exhibition" of 1851 led to the establishment in 1853 of the Department of Science and Art (DSA) to promote scientific and technical education. The DSA provided financial support for small grammar schools to adopt technical and scientific curricula and to provide evening classes for artisans (Howson, 1982, pp. 145–146; Price, 1994, p. 14). While this was happening, it should be noted that despite the extension of schooling to girls (following the recommendations of a commission in 1869), it was not until the late nineteenth century that mathematics became firmly established in the curriculum of girls' schools. Even then, mathematics was not regarded by society as a subject really suitable for girls (see Harris, 1997, particularly chapters 3 and 4; Howson, 1982, p. 177).

While textbooks were based strictly on Euclid's *Elements*, the direct teaching of strict Euclidean-style geometry in secondary schools at that time was not altogether successful. For example, the Report of the Schools Inquiry Commission in 1868 (reported in Jackson, 1924), summarised some of the causes of the difficulties in teaching geometry in a strict Euclidean-style as follows: the lack of an introductory course; the ban on hypothetical constructions [using geometrical constructions such as an angle bisector without showing how to draw them with only a compass and ruler][2]; the treatment of parallels; and the treatment of incommensurable magnitude in the Book V[3] (Jackson, 1924, pp. 36–37). In 1871, the Association for the Improvement of Geometrical Teaching (AIGT) was founded by UK University mathematicians, together with teachers from prominent schools, as a means of pressing for improvements in the teaching of geometry. In 1875, the AIGT's *Syllabus of Plane Geometry* was published. This was approved by the British Association for the Advancement of Science (the BAAS) in 1876 (see AIGT, 1877, p. 11). In 1877, the AIGT circulated their syllabus to Examination Boards at universities including Oxford, Cambridge, London, Durham, as well as the relevant Government department, the DSA (see AIGT, 1878, pp. 18–21). However, it transpired that the English universities, in particular the University of Cambridge, were not in favour of the AIGT syllabus. The University of Durham, for example, reported that it could "do nothing until it saw a textbook based upon the syllabus" (see Brock, 1975, p. 28). Such comments from the English universities may have made the AIGT change its activities and begin working on the publication of a textbook of its own.

In 1884 and 1886, the AIGT edited and published a geometry textbook entitled *The Elements of Plane Geometry* which included proofs of the theorems contained in the 1875 syllabus. Reflecting the issues in the teaching of geometry at that time, neither introductory stages with practical geometry, nor the use of algebraic approaches, were included. In this sense, this textbook can be seen as very modest reform. Howson reports that the AIGT textbook retained Euclid's overall sequence but "rearranged theorems within allied groups and supplied new proofs." Even so, Howson observes that the book was "without doubt, one of the dreariest books the present author has ever seen" (Howson, 1973, p. 158). For the members of the AIGT, reform on the teaching of geometry meant rewriting and adding definitions and axioms, and sorting out logical relationships between Euclid's propositions. In fact, their views on the teaching of geometry were still as a training of students' logical way of thinking. The following extract tells us what members considered the teaching of geometry:

> Your Secretaries … issued a circular in February last, asking for the definite views of Members on the following questions:

> (1) Is it to be held a part of the work of the Association to consider the wants of those who require instruction in Geometry for technical purpose, or is the Association to confine its attention to strictly scientific Geometry?
> (2) Is the Association eventually to bring out a Geometrical text-book, stamped with its authority?
> To the first query eleven Members replied, ten of whom were against the Association dealing with Geometry for practical purposes—for the present at least—one only was in favour of such as course. …
>
> (AIGT, 1873, p. 11)

In 1887, the AIGT sent this textbook to both Oxford and Cambridge Universities. Again, the universities only agreed that "proofs other than Euclid could be used, providing the Euclidean order was not violated" (Siddons, 1936, p. 18). In this way, the answer from the English universities was very modest (note that the comments from Cambridge and Oxford are provided at the end of the part I of the MA's *The Elements of Plane Geometry*, 1903 edition). As such, and as has already been pointed out by Brock and by Price, the efforts of the AIGT failed to make radical changes to the teaching of geometry (see, for instance, Brock, 1975, p. 29). By 1897 the AIGT had changed its name to the Mathematical Association to reflect its wider ambitions.

Perry's address in 1901 and the geometry reform by the MA

Reform of mathematics teaching in the early twentieth century in England was prompted by J. Perry, Professor of Engineering at the Royal College of Science, with his talk entitled "The Teaching of Mathematics" given at the British Association for the Advancement of Science (BAAS) meeting in Glasgow on September 13, 1901 (see Perry, 1902). In his speech, Perry took an engineer's point of view and roundly denounced the then teaching of mathematics in England. With regard to the teaching of geometry, Perry questioned the educational value of Euclidean geometry for all students, and emphasised the importance of using experimental tasks in the early stages of secondary education (ibid., pp. 158–181), a stance that was much more "radical" than that advocated by the nineteenth-century reformers. Following Perry's speech, opinions from various people from inside and outside mathematics were voiced and argued: examples include the debate by the BAAS (1901), a letter from a group of teachers from various prominent schools (later known as "the letter of the 22 schoolmasters"; see Godfrey et al., 1902), and the Annual Meeting of the Mathematical Association in 1902. In general, although these people did agree with Perry that the teaching of geometry needed significant reform, they considered that Perry's proposals were unrealistic to achieve in secondary schools. In 1901, Godfrey (1901, p. 107), of Winchester College and a member of the MA, introduced a syllabus which he said "may be described as a compromise; but we hope that Professor Perry, in an indulgent mood, would not condemn it utterly." In this syllabus, which mainly comprised the contents from Euclid's *Elements*, experimental tasks were included in the early stages in geometry. Siddons, of Harrow School and also a member of MA, stated that

Perry's proposal "may be admirably adapted to the wants of training colleges, but seems quite impracticable for public schools" (Siddons, 1901, p. 108).

Given this background, we now focus on the Annual Meeting of the MA in 1902, which was, according to Siddons (1952, pp. 153–155), one of the "main causes of the appointment of the first Teaching Committee" of the MA. The MA, numbering about 300 members at that time (Price, 1994, p. 64) was described by Godfrey as having "awoke as one out of sleep, thanks to Perry" (Godfrey, 1906, p. 76). The 1902 Annual Meeting of the MA was held at King's College, London, on Saturday, January 18 (MA, 1902a, pp. 129–143). At this meeting, first, the chair (Minchin, Royal Indian Engineering College) declared the object of the meeting to be the reform of geometry teaching (ibid., p. 129). Lodge then read his paper entitled *Reform in the Teaching of Mathematics* in which he pointed out the problems of the teaching of geometry as well as giving his suggestions for improvement. Lodge, in his paper, identified that the main problem was caused by "a fixed ancient model" based on the teaching of traditional Euclidean-style geometry (ibid.). He then outlined his suggestions for the reform the teaching of geometry, referring to French textbooks. Lodge's suggestions included: the introduction of practical work, the rearrangement of the order of theorems in Euclid, the teaching of proportions, the introduction of algebra, and so on (ibid., pp. 130–131). Following this, the other MA members at the meeting reacted to Lodge's suggestions.

Most of the members seemed to recognise that the traditional style of geometry teaching was the main cause of the problem. In particular, the members considered the greatest problem to be the strict allegiance to the order of theorems in Euclid, primarily because their only teaching method was to expect students to memorise the particular order. To overcome this problem, first, an introductory course was suggested—comprising practical work in the early stages, with the idea that this would enable students to grasp important geometrical facts. Secondly, the members considered that a rearrangement of theorems was necessary. Lodge introduced some ideas and Hill also suggested that the order be rearranged in "a more natural order." Godfrey briefly stated his idea of the rearrangement of Euclid's *Elements* such that "[Euclid] Book II [areas] taken after [Euclid] Book III [properties of circles]" (MA, 1902a, p. 140). A reason for this suggestion was that he considered that the theory of areas in Book II of the *Elements* was very hard without using algebra and that students might be ready to study areas after they had studied the properties of circles. However, no other members offered significant opinions as they considered that specifying an order of propositions in geometry was a rather sensitive issue. In fact, Lodge said "the whole subject of rearrangement is too vast to be treated in the course of a paper—it must be settled by a committee" (ibid., p. 131). Accordingly, the focus for subsequent meetings of the MA became tackling the following issues: how and when to introduce practical and experimental tasks, how to address algebra within geometry, and how to rearrange the Euclidean order of theorems. In the next section we examine how the ideas discussed in the annual meeting led to the proposals contained in the MA geometry report published later in 1902.

The geometry report of the MA published in 1902

Soon after the 1902 Annual Meeting, the first Teaching Committee of the MA was established (with the chair being taken by Lodge). There were 26 members of the committee; for their names and affiliations, see Table 1.

This committee published two reports in 1902: the geometry report in May (MA, 1902b, pp. 168–172), and an algebra and arithmetic report in July.

In the geometry report, it was proposed that the teaching of geometry be divided into two stages: first an introductory and experimental course, and, second, a deductive course (MA, 1902b, pp. 168–172). In the first stage (the introductory and experimental course), it was suggested that "a first introduction to Geometry should not be formal but experimental, with use of instruments and numerical measurements and calculations" (ibid., p. 168). In the second stage, the formal course was divided into (i) theorems and (ii) constructions. Overall, the report suggested that related theorems be associated together and that definitions "should not be taught *en bloc* at the beginning of each book, but that each definition should be introduced when required" (ibid., pp. 168–169). The committee recommended use of Euclid's order of theorems as follows: Euclid Book I [geometrical construction, properties of angles, parallel lines, triangles, quadrilaterals, the Pythagorean theorem etc.] → Euclid Book III to proposition 32 [angles in a circle] inclusive → Euclid Book II [areas of rectangles and squares] → Euclid Book III proposition 35 to the end [geometrical construction, tangents of circles, etc.] → Euclid Book IV [constructions of regular polygons]. In addition to making recommendations as to the order of theorems, the use of "riders" (theoretical exercises) during teaching was deemed as important.

Table 1. List of the members of the Teaching Committee of the MA in 1902

Members (by family name)	*Institutional locations*
W.M. Baker (from 2nd meeting)	Cheltenham College (Public school)
S. Barnard	Rugby School (Public school*)
H.D. Drury	Marlborough College (Public school)
J.M. Dyer (from 2nd meeting)	Eton College (Public school*)
T.J. Garstang	Bedales School (Private school)
H.T. Gerrans (from 3rd meeting)	Secretary, Oxford Local Examination Delegacy
C. Godfrey	Winchester College (Public school*)
W.J. Greenstreet	Marling School (Grammar school)
C. Hawkins (from 2nd meeting)	Haileybury College (Public school)
F.W. Hill	City of London School (Private school)
M.J.M. Hill	University College, London (University)
R.W. Hogg	Christ's Hospital (Private school)
H.T. Holmes	Merchant Taylors' School (Public school*)
Prof. Hudson (from 3rd meeting)	King's College, London (University)
E.M. Langley	Bedford Modern School (Public school)
A. Lodge	Royal Indian Engineering College (University)
C.C. Lynam (from 4th meeting)	Oxford Preparatory School (Private school)
Dr. F.S. Macaulay	St. Paul's School (Public school*)
G.M. Minchin	Royal Indian Engineering College (University)
Mr. J. Moulton	(institution not given)
C. Pendlebury	St. Paul's School (Public school*)
H.C. Playne (from 2nd meeting)	Clifton College (Public school)
W.N. Roseveare	Harrow School (Public school*)
C.A. Rumsey	Dulwich College (Public school)
S.A. Saunder	Wellington College (Public school)
H.A. Saunders (2nd meeting)	Haileybury College (Public school)
E.C. Sherwood (2nd meeting)	Westminster School (Public school*)
A.W. Siddons (secretary)	Harrow School (Public school*)
C.O. Tuckey	Charterhouse School (Public school)
E.T. Whittaker (3rd meeting)	Trinity College, Cambridge (University)

* These are the original nine English Public Schools (as defined by the Public Schools Act 1868) which were (and to some extent remain) the most powerful of the English public schools (the one missing from the above list being Shrewsbury School, through representation was invited).

Having made these proposals, the report goes on to make 61 suggestions about axioms and definitions, order of theorems, omission of theorems, notation of theorems, methods of proof, and introduction of new theorems. These suggestions can be summarised as follows:

- Most suggestions were concerned with the methods of proof; for example that Euclid Book I proposition 8 [the congruency condition for two triangles that if the triangles have two corresponding sides equal, and also have the equal bases, then they also have the angles equal which are contained by the equal straight lines] "be proved by placing the triangles in opposition" (in Euclid's *Elements* this proposition is proved by superposing a triangle to another; for the proof suggested in the MA report, see Heath, 1956, p. 263);
- There were suggestions specifying which of Euclid's propositions should be omitted; for example "That [Euclid Book I proposition] 7 [on the same base and on the same side of it, there cannot be two triangles having the two sides which are terminated at one extremity of the base equal to one another …] be omitted";
- In terms of a detailed order of theorems, only one suggestion, "That [Euclid Book I propositions] 13, 14, 15 [vertically opposite angles are equal] be taken first," was proposed. There was no specific order of theorems which the committee thought appropriate to define;
- The report suggested that "illustration from Algebra ought to be given where such is possible" and that the theory of proportion be dealt with only in commensurable magnitudes.

As just noted, a specific detailed order of theorems was not proposed in the MA report, even though this issue was central to the Annual Meeting of the MA earlier in 1902. Furthermore, the geometry report stated that "it is not proposed to interfere with the logical order of Euclid's series of theorems—in other words, it is not proposed to introduce any order of theorems that would render invalid Euclid's proof of any proposition" (MA, 1902b, p. 168). Overall, then, the report was more cautious with regard to the order of theorems than some of the proposals provoked by Perry's address in 1901, but it was more radical than the reforms suggested by the AIGT in the late nineteenth century.

Discussions of the Teaching Committee of the MA in 1902

We now turn to our main focus—*why* the members of the MA Teaching Committee reached such conclusions. In particular, it is of key interest that the geometry report *did not* propose a new order of theorems (remember Lodge's word in the earlier Annual Meeting that the problem was "a fixed ancient model"; see also Price, 1994, p. 56). In what follows, we analyse the discussions held by the Teaching Committee of the MA leading up to the geometry report of 1902. The source of data is the book of minutes of the Teaching Committee of the MA (unpublished), in which is recorded the discussions of the meetings of the committee. This book of minutes, covering five meeting from February 15 to March 22, 1902, can found in the archive of the MA's library in Leicester, UK.

At the first meeting of the MA Teaching Committee (King's College, London; Saturday, February 15, 1902, 3 p.m.), first, the chair and secretary were decided: Minchin and Siddons were chosen, respectively. Then it was decided to write invitation letters to the headmasters of a number of schools—Marlborough, Haileybury, Clifton (these three schools were then leading "private schools" which "developed more modern curricula than the classics-dominated older public schools"; see Price, 1994, p. 15), Cheltenham, Westminster, and Shrewsbury (the last two schools were of the nine old "public schools"). It was also decided that subcommittees would consider drafts of the reports on geometry and algebra. In terms of the geometry subcommittee, the following members were elected: Lodge (Royal Indian Engineering College), Godfrey (Winchester College), Barnard (Rugby School), and Rumsey (Dulwich College).

During the second meeting of the Teaching Committee (King's College, London, on Saturday, March 1, 1902, at 3 p.m.), various proposals were discussed, including "That the first introduction to Geometry should not be formal but experimental, with use of instruments, and numerical measurements and calculations" and "That in formal Geometry, constructions should not form part of the logical course on theorems, but in proving theorems, hypothetical constructions be permitted." A key part of the discussion focused on the recommendations listed in the first draft report drawn up by the geometry subcommittee for the teaching of the theorems in Euclid Book I. A few amendments were made by the members of the committee with regard to Euclid Book I; for example the recommendation that theorems "29 [alternative angles in parallel lines are equal] and 30 [Straight lines that are parallel to the same straight line are parallel to one another] be proved from Playfair's axiom[4]" was amended to "Playfair's axiom is preferable to Euclid's 12th axiom" (carried by votes 12 to 5). Several proposals were also made by individuals. For example, Playne (Clifton College) proposed, "after §5 the rest of the geometry report be rejected," but this was not seconded. Finally, the following motion was carried unanimously at the end of the meeting: "this committee does not propose to interfere with the logical order of Euclid's theorems; so long as this is retained, the actual order and number is immaterial."

At the third meeting (King's College, London; Saturday, March 15, 1902, 3 p.m.), first, the following people were unanimously elected members of the committee: H.T. Gerrans (Oxford), Hudson (King's College), and E.T. Wittaker (St. Paul's School). Then Gerrans read out the future regulation for the Oxford Local Examination from 1903. These stated that "Any solution which shows an accurate method of geometrical reasoning will be accepted" (it should be noted that the Oxford Local Examination aimed to "confer a great benefit on that large class of persons who cannot afford, or do not require a University education for their children, by undertaking to examine boys, about the time of their leaving school"—quote from Oxford University Archive). Next, the latest draft of the geometry report was considered. The main focus of discussion in this meeting turned on suggestions as to the theorems in Euclid's *Elements* Books I, II, and III. For example: while the proposal to introduce algebraic methods of proof in Book

II was rejected by 6 votes to 9, it was decided that this issue might be considered at the next meeting; next it was decided to retain Euclid Book III proposition 9 [if a point is taken within a circle, and more than two equal straight lines fall from the point on the circle, then the point taken is the centre of the circle], although its omission had been considered in the first draft; then the proposal that “Euclid Book III [proposition] 9 be taken as a corollary to Euclid Book III 1 [how to find the centre of a circle]” received 8 votes in favour and 8 against, with the chair, Minchin, giving his casting vote in favour; the proposal that “Euclid Book III 7–8 be omitted” was rejected by 4 votes to 7; and finally it was decided that “the last parts of 7 and 8 be omitted.”

At the fourth meeting (King’s College, London; Saturday, March 22, 1902, 3 p.m.), the comments from Cambridge Local Examinations and the Civil Service Commission were considered. The former stated that they “would be glad to consider the suggestions made by the committee” and the latter stated that “in Geometry, the demonstrations of sequence of propositions need not be those of Euclid.” Then the draft of the geometry report was considered. First, the algebraic method—something which remained unsolved from the previous meeting—was discussed. Lodge (Royal Indian Engineering School) proposed that “an Algebraical treatment be allowed in [Euclid] Book II except in Prop. 1, Euclid proof being there retained so as to establish rigidly a geometrical analogue of the distributive law,” and Rumsey (Dulwich College) seconded it. Gerstang (Bedales School) proposed an amendment, recommending “after proof [of proposition] 7 [of Euclid Book II], algebraic methods of proof be allowed with a special view to proofs [of propositions] 12 & 13 [Euclid Book II].” Roseveare (Harrow School) seconded the amendment, but it was rejected by 3 votes to 15. The original motion was also rejected by 3 votes to 15. Hill (City of London School) proposed that Euclid Book IV [propositions] 10 and 11 [how to construct a regular pentagon] be omitted, and this was seconded by Saunder (Wellington College). These proposals were carried by 9 votes to 6, and 8 votes to 5 respectively.

At the final meeting (King’s College, London; Saturday, May 10, 1902, 3 p.m.), the discussion focused on recommendations as to Euclid Book VI, in particular the applications of the theory of proportion (involving the similarity of figures). It was proposed by Macaulay “That in the opinion of the Committee, the course in commensurables might, with advantage, be followed, in the case of advanced students, by a general theory etc.” Garstang (Bedales School) seconded this proposal but the motion was rejected by six votes to nine. Siddons (of Harrow School) then read a letter from Professor Hill (University College), who pointed out that, “in §55, the following assumption was made that was not justified; viz rect BC.AG/rect EF.DH=BC/EF.AG/DH.” With this, and with several changes and additions having been made to matters regarding similar figures, finally Lodge proposed that the whole report be passed; Godfrey seconded this proposal. The motion was carried unanimously.

Analysis of the process of redesigning the geometry curriculum: The case of the MA

Analysis framework

Having described the discussions leading up to the geometry report of the MA, we now analyse the process of drawing up the report in more detail, and our approach is to examine social factors around the MA members and report. Existing research on educational reforms show that such reforms usually involve not just a few individuals, but, rather that various people and organisations from both inside and outside the subject are involved (for example, see Griffiths and Howson, 1974, p. 135). Cooper (1985, p. 31) argues that the process of changing school mathematics is "characteristically a compromise between different demands of various powerful groups," and reveals why and how, in the 1950–1960s, a traditional approach in mathematics in schools was replaced by content that was more based on contemporary mathematics. During these decades, Cooper found that the nature of the "mathematics" to be taught in schools was discussed by people from inside and outside various "mathematical communities," and that several curriculum projects were founded to replace the "traditional" mathematics curriculum. Of these projects, the School Mathematics Project (SMP; of the University of Southampton, UK), was particularly successful. Cooper concludes that SMP's "success," relative to such projects as the MME (Midlands Mathematical Experiment), "must be understood, at least partially, in terms of the differential availability of such resources as status, academic legitimacy and finance ..." (op. cit, p. 265).

Cooper's study reveals the factors to be examined in order to understand the complex process of reform. Of these factors, and in terms of the 1902 MA report, first, it should be appreciated that various opinions were expressed in the process of drawing up the report among the members of the MA. Therefore, it is reasonable to expect to see that the MA report would reflect the various opinions of the different members, even though the focus is solely the teaching of geometry. A second factor is the "power" of each opinion, and, as in Cooper's model, that the availability of "resources" includes not only money, but also academic authority and prestige, time, and so on. Therefore, the institutional and academic locations of reformers also have to be examined, because locations can be seen as a factor contributing to the possibility of access to "resources." For example, Godfrey, one of the MA committee members, had gained "wrangler" status at Cambridge (that is, he completed Part II of the Mathematical Tripos with first-class honours; see Howson, 1982, p. 143), a position occupied by highly respected mathematicians (such as De Morgan, Whitehead, and Hardy). Thus it is likely that Godfrey's reputation would be well known. Finally, interactions among members, particularly conflicts, are also important, because, in Cooper's model, such interactions relate to changes in "what counts as school mathematics" (Cooper, 1985, p. 31).

In what follows, our analysis focuses on what took place during the discussions of the Teaching Committee of the MA from the following points of view: that the MA report is an amalgam of various ideas of the different members; that various "resources," and the strategies used to obtain them, were employed by the members to justify their proposal; that there were conflicts among the MA members; and that there was a complex relationship between the teaching committee and outside interested parties. By paying attention to these points, our aim is to give a comprehensive account of *why* the MA 1902 report can be judged a modest reform.

The geometry report of the MA as a collection of diverse ideas

As we have seen in the previous section, various ideas, such as the omission of theorems and the methods of proofs, were discussed in the meetings of the MA Teaching Committee. Furthermore, several matters not included in Euclid—such as practical work, and the introduction of algebra—were proposed. The conclusions of the committee were not reached by only one person—all proposals needed to be seconded and carried by a vote. Therefore, in brief, it can be said that the report reflects a compromise of the members' opinions.

Yet it should be noted that the weight of the committee's different conclusions was not equal. The conclusion that the teaching of geometry should be based on Euclid was still held by members of the committee. In the end, their report confirmed that the Teaching Committee members would not violate the logical order of theorems in Euclid. Nevertheless, the introduction of the algebra, though discussed during several meetings, remained an ambiguous, and therefore weak, conclusion.

Members' institutional/academic locations and power against traditional Euclidean-style teaching of geometry

Using Cooper's (1985) framework for analysis, it is vital to pay attention to the location and affiliations of the members. In general, people's educational beliefs and attitudes are often influenced by "where they belong to," and this is an important aspect in the nineteenth- and twentieth-century reform, given the major force of change was "educational" at that time. From Table 1, we can see an interesting, yet very limited "mixture" of the committee members, i.e., leading "public schools," leading "private schools," and leading universities. It is noticeable that people from outside "pure mathematics" played important roles in the MA, in general, and on the Teaching Committee in particular. For example, Minchin, who chaired the Annual Meeting of the committee, was a professor of Applied Mechanics at Royal Indian Engineering College. His comment on the problems in teaching geometry was that "the cause was the adoption of Euclid's language and method. The schoolboy is not taught geometry; he is taught to remember the words of Euclid" (MA, 1902a, p. 132). Lodge, a colleague of Minchin, also considered that a more practical approach would be appropriate to the teaching of geometry, stating that "The pupil should learn at an early stage to measure angles in degrees, and to learn by experiment such things as that the angles of a triangle add up to two right angles. The angles of various triangles could be estimated by eye and then measured" (ibid., p. 130).

A conservative character of the committee can be inferred from Table 1. Although there were people from "private schools" which used more "modern" curricula (such as Marlborough College, Haileybury College, or Clifton College), the majority of Teaching Committee members were teachers from prominent private schools (such as Dulwich College, Eton College, Harrow School, Merchant Taylors' School, Wellington College, Winchester College, and so on) or were university mathematicians. There were no committee members from either the grammar schools or any major girls' school (such as Cheltenham Ladies' College). Given that Siddons later wrote that when he started teaching at Harrow in 1899, he had no special directions about what Algebra and Arithmetic he should teach but that he was told that "at half-term they would have a paper on Euclid Book III, the paper consisting entirely only of propositions," it might be surmised that many of the committee were expected to teach the traditional Euclidean approach to geometry for the university entrance examinations (particularly for entrance to Oxford and Cambridge) and probably respected such an approach as well (also remember the AIGT member's view in 1873 cited in the previous section).

As such, it would be difficult for the MA committee members to do away completely with traditional Euclidean-style geometry teaching. That the committee unanimously agreed "this committee does not propose to interfere with the logical order of Euclid's theorems" is evidence of this.

Availability of resources

From inside and outside the MA Teaching Committee, the main "resources" which might be used against traditional Euclidean-style geometry teaching are as follows: Perry's address in Glasgow, the "letter of the 22 schoolmasters," and the stance of the examination boards of some universities. For example, while the MA Teaching Committee members considered Perry's ideas as probably not achievable in secondary schools, his address encouraged them to express their opinions regarding at least the introduction of practical and experimental tasks. In addition to these forces from the educational arena, some influential scholars also started attacking the status of Euclid. For example, Bertrand Russell, one of the most prominent scholars in the world at the time, wrote on "The teaching of Euclid" in the *Mathematical Gazette* in 1902, stating that:

> It has been customary when Euclid, considered as a text-book, is attacked for his verbosity or his obscurity or his pedantry, to defend him on the ground that his logical excellence is transcendent, and affords an invaluable training to the youthful powers of reasoning. This claim, however, vanishes on a close inspection. His definitions do not always define, his axioms are not always indemonstrable, his demonstrations require many axioms of which he is quite unconscious,. … (Russell, 1902, p. 165)

Of these "resources," the most important one would be the stance of the universities because the universities had strong control over the syllabi and textbooks in the teaching of geometry at that time. The MA Teaching Committee members definitely knew how important the university examinations were, and hence they approached Gerrans (Secretary of the Oxford Local Examination Delegacy) to join them. In fact, as we have seen, the committee members learned that concessions had been made by some university examiners, thanks to pressure from reformers such as Perry and the 22 schools' masters. As a consequence, any proof would now be accepted in geometry examinations for the Oxford Local Examination Delegacy. Thus, the MA Teaching Committee members could recommend different methods of proofs of Euclid's propositions for at least the schools which used the Oxford Local Examination.

However, the committee members did not have enough "resources" to give more weight to radical ideas in geometry teaching. It is also possible that the members did not make full use of all the resources that they did have at their disposal. For example, although they had a contact with the Oxford Local Examination, they did not yet have information of the Oxford and Cambridge Examination Board, which had responsibilities for the examinations that most of the leading schools used at that time. In the end, the committee did not advocate a detailed revised order of theorems, nor was there strong advocacy for the use of algebra in geometry, even though both were discussed and a detailed order of theorems was included in earlier drafts of the report (for example, in terms of the propositions in Euclid Book I, the following order was proposed: theorems 13, 14, 15; 4, 5, 6, 8, 16, 17, [18, 19], 20, 21, [24, 25], 26; [27, 28], 29, 30, and 32—with the theorems in brackets being optional).[5]

A tactic for education reformers, even today, is to refer to education in other countries. During the MA Annual Meeting of 1902, mention was made of the teaching of geometry in France, Cremona's geometry from Italy (see, for example, Menghini, 1996), and a "Belgian book for the secondary teaching of young girls." According to Mahoney (1980), French educators had, by the sixteenth century, already attempted to integrate algebraic methods into geometry, with Ramus (1515–1572) seemingly being "the first to suggest that algebra deserved greater importance," maintaining that "algebra underlay certain parts of the second and sixth books of Euclid's *Elements*, as well as the famous geometrical analysis of the Greek writers" (note that we are aware of debates about "geometric algebra" in Greek mathematics—see, for example, Unguru, 1975, or van der Waerden, 1976— but we do not have space here to elucidate such matters).

According to Stamper (1909, pp. 110–112), the books by Charles Meray (in 1873) were officially recognised in France in 1904 (see also Howson, 1982, p. 163). Such developments were not discussed by the members (or if such matters were discussed, they were not recorded by the MA Teaching Committee members). Apart from the brief mentions listed prior, the MA minute book reveals no strong evidence that committee members gave lengthy consideration to syllabi in existence in France, or other countries, in which the theorems were arranged differently from Euclid, or where algebraic methods were in use in the teaching of geometry.

Internal and external interactions

From the descriptions of Teaching Committee meetings in the previous section, it is clear that disagreements and conflicts between members took place. For example, Professor Hill (University College, London) was not happy and eventually resigned from the committee (see Siddons, 1952). Playne (Clifton College) considered most of the proposals unacceptable. The reason why Playne disagreed with the draft report is not clear from the historic record, but he was from Clifton College which used more technical and scientific curricula. His proposal (at the second meeting of the committee) was not seconded by anybody, and this might imply that the other members were more cautious about radical changes.

In terms of the order of theorems, the detailed revised order included in an earlier draft was omitted from the final version of the Teaching Committee's report. This issue of the order of theorems turned out to be controversial. Whereas the order of the first three books was proposed Book I 32 [the sum of the interior angles of triangles is 180 degrees] → Book III 32 → Book I 33 [properties of parallelograms] to end [the Pythagorean theorem] → Book II [area] → Book III 35 to end, it was amended to Book I → Book III 32 → Book II → Book III 35 to end, and still four people opposed this decision at the fourth committee meeting (held on March 22, 1902). Furthermore, even the proposal to omit certain theorems caused controversy at this meeting, and it can be inferred that these conflicts took up precious time (one of the "resources") in discussions. In fact, Siddons was of the view that "the standard order had not been sufficiently discussed" (Siddons, 1902, p. 253).

Another example of disagreement was, as noted earlier, the role of algebra. The final recommendation on algebra is rather ambiguous in the report as it simply states "That illustration from Algebra ought to be given where such is possible" (MA, 1902b, p. 170). As we have seen, the methods of algebra caused discussions and conflicts in the meetings. In summary:

- The proposal "Introduction of algebraic methods of proof in Book II" was rejected by 6 votes to 9 at the 3rd meeting, but it was decided that comments about this might be considered at the 4th meeting.
- The proposal "After proof [proposition] 7 [of Euclid Book II], algebraic methods of proof be allowed with a special view to proofs [proposition] 12 & 13 [Euclid Book II]" was rejected by 3 votes to 15 at the 4th meeting.
- The proposal "An Algebraical treatment be allowed in [Euclid] Book II except in Prop. 1, Euclid proof being there retained so as to establish rigidly a geometrical analogue of distributive law" was rejected by 3 votes to 15 at the 4th meeting.

There are no detailed records in the book of minutes of actual opinions made by the members of the committee during these votes; only the total votes are recorded.

As to external interactions, it can be considered that the climate of the reform at that time was not ready for the complete abolition of traditional Euclidean-style geometry teaching. In particular, the issue of the order of theorems was a cause of controversy. For example, whereas Perry (1902) severely attacked Euclidean-style geometry teaching in his address in Glasgow in 1901, Forsyth, Lamb, and Larmor voiced their dissatisfaction with Perry's view (Howson, 1982, p. 149). Not only that, but the "letter of the 22 schoolmasters" stated "it may be felt convenient to retain Euclid" (Godfrey, Siddons et al., 1902, p. 258). All this shows the modest attitude to reform at the time. In 1902, before the publication of the geometry report of the MA, Lodge proposed a detailed revised order of theorems contained in Book I of Euclid's *Elements*, and suggested that the order be rearranged from angles, parallel lines, and congruent triangles to inequalities of triangles (Lodge, 1902, p. 534). This caused an immediate response from W.C. Fletcher, E.T. Dixon, T. Petch, R.B. Hayward, and G.H. Bryan, published in *Nature* in 1902 (Bryan, 1902; Dixon, 1902; Fletcher, 1902; Hayward, 1902; Petch, 1902). Some of this group of people agreed with Lodge's order, while others proposed different orders or disagreed with Lodge's suggestion. Given that even the revised ordering of the theorems in Euclid Book I was controversial, the committee members never reached a position where they could advocate a detailed revised order for the whole of the six books of Euclid's *Elements*. The committee members must have been aware of the fairly anxious climate towards change, and considered that radical reform would be unlikely to have widespread support.

Conclusion

In this paper, we have focused our analysis on the 1902 geometry report of the Mathematical Association in England, a landmark document in the history of mathematics education. Although the report recognised that a form of Euclid's *Elements* was no longer suitable as a textbook in secondary schools, and the necessity of rearranging the order of theorems was advocated, the recommendations of the report were quite modest. In summary, the causes of the rather conservative character of the report relate to several issues and this is our answer for our question concerning *why* the MA report 1902 can be seen as a modest reform. First, because of the nature of the members' institutional and academic locations and affiliations, the members still respected Euclid as a basis of school geometry; the "resources" available to the Teaching Committee were not sufficient to devise the complete replacement of the traditional Euclidean approach, but they were enough to at least support the recommendation covering different methods of proof. Second, the conflicts among the members prevented a more radical option. Third, the climate outside the Teaching Committee was not ready for radical reform at that time. Hence, the 1902 MA geometry report can be ascribed to an inability to determine a radical consensus with the consequence of resorting to a compromise.

These results suggest that we must be aware that various people are likely to be involved in the reform of an academic subject, and if we want to make a successful reform, we have to consider various reformers' institutional and academic locations, the availability of "resources" and interactions. In fact, both Godfrey and Siddons used their locations (Winchester College and Harrow College, respectively), and their strong Cambridge connections, to achieve a more radical reform when writing *Elementary Geometry* in 1903 (see Howson, 1982; Fujita, 2001; Fujita and Jones, 2003). This use of power, in all its senses, is likely still to be the case when curriculum policy is decided and is applicable to the interpretation and analysis of current curricula in schools in that "Proposals for curriculum reform come from various sources including: teachers, teacher unions, policy-makers, academics, politicians, media and pressure groups" (Marsh, 1997, p. 211). For example, Graham (1993) documents the intrigues and pressures that surrounded the development of the contemporary national curriculum for England that was introduced in 1988. Similarly, Ellerton and Clements (1994) expose the machinations behind the attempts to develop a national curriculum in Australia over the period 1987–1993. Returning to the period that is the focus of this paper, the beginning of the twentieth century, Howson (2010) relates how, after 1902, a state secondary school system was established in England and this led to the issue of which of the two established curricula was to be encouraged: the DSA curriculum (which was technical and scientific) or the endowed grammar school curriculum (which centred on "classical" subjects such as Latin and Greek). Eventually, Howson goes on to explain, a short-lived national curriculum that set out the number of hours to be allocated to different subjects was designed by those educated in public and endowed schools and so the result was not in doubt: it was not to be the DSA curriculum.

The implication of our conclusion is that our understanding of curricula, and curriculum policy, can be enhanced by examining not only what is changing but also who was involved in the decision-making process behind any particular curriculum, whose were the strongest opinions and what did they advocate, and what resources did those people use to promote their views.

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Notes

[1] Euclid's *Elements* was originally written in about 300 B.C. This text consists of the theory of plane geometry, proportion, numbers, and solid geometry in a systematic way in a total of 13 "books" (Meserve, 1983, p. 229, Heath, 1956 et al). The first six "books" in the *Elements* are concerned with plane geometry and cover the theorems studied in elementary geometry: Book I is about the properties of plane figures and the Pythagorean theorem; Book II is about area and applications of the Pythagorean theorem; Books III and IV are about the properties of circles and constructions of regular polygons; Book V is about the theory of

proportion, dealing with both commensurable and incommensurable; and Book VI is the application of the theory of proportion to plane figures, i.e., the properties of similar figures.

[2] For example, if one proves Euclid I Proposition 5 (in the triangle ABC, AB=AC → ∠ABC=∠ACB) by drawing an angle bisector from a vertex to the base line of the triangle, then this is logical circularity, because Euclid I Proposition 5 is necessary to prove the existence of an angle bisector. This kind of logical circularity was carefully avoided in Euclid's *Elements*.

[3] "If, a, b, and c are the lengths of the side of a triangle and if the sides of lengths a and b form a right angle, then $c^2 = a^2 + b^2$, … if a=b=1, then $c = \sqrt{2}$ and c cannot be written as an integer or as a quotient of the integers. The number $c = \sqrt{2}$ is irrational and the corresponding line segment is said to be incommensurable with respect to the segments of length 1" (Meserve, 1983, p. 225).

[4] Playfair's axiom states, "through a given point only one parallel line can be drawn to a given straight line" (Heath, 1956, p. 220).

[5] This order suggests that students first learn the properties of angles at a point and vertically opposite angles (13–15), congruent triangles (4–8), properties of angles and sides in triangles (17–21), congruent triangles (26), properties of parallel lines (29–30), and then the angles in triangles (32).

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