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Minimum Wage and Tax Evasion: Theory

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Abstract

The paper builds a theoretical model to analyse the interaction between minimum wage legislation and tax evasion by employed labour. The firm and the worker agree on the amount of earnings to report to the fiscal authorities, which possess an imperfect detection technology. The introduction of the minimum wage poses a constraint on the reporting decision and induces an increase in compliance by some agents. As a consequence, a spike at the minimum wage appears in the distribution of declared earnings. Moreover, a nominally neutral fiscal regime becomes regressive, while fiscal revenues may increase.

JEL codes: J38, H26, H32, P2
Keywords: Minimum Wage, Tax Evasion

1 Introduction

What are the fiscal implications of introducing or increasing the minimum wage? How can we explain the very high spike at the minimum wage level appearing in the wage distribution of some countries? This paper contributes to answering these questions by studying the effects of the interaction between tax evasion and minimum wage legislation.

The minimum wage is the subject of a rich literature and policy debate[1], mainly focusing on its effect on employment. The traditional view of adverse labour market effects has been challenged (Card and Krueger, 1995) and, at present, there is no overwhelming consensus on the issue. Potential beneficial effects of the minimum wage for workers through shifts in the composition of jobs toward good (i.e. high-wage) jobs have also been discussed (Acemoglu, 2001.) This paper highlights another aspect of minimum wage policy that has not been considered so far and shows how the minimum wage affects workers and firms through the "fiscal channel".

Large efforts have also been devoted to the theoretical and empirical study of tax evasion and the shadow economy[2]. The study of tax evasion by employed labour is of particular interest as the fiscal imposition on labour in the form of social security contributions (SSC) and personal income tax (PIT) represents the bulk of fiscal revenues in many countries[3]. However, to the best of my knowledge, the effects of the interaction between underreporting of earnings and minimum wage legislation have not previously been addressed

[1] See Brown (1999) for a review.

[2] See Andreoni et al. (1998) or Slemrod and Yitzhaki (2002) for surveys on tax evasion and Schneider and Enste (2000) for a survey on the shadow economy.

[3] Labour taxes are the largest source of tax revenue in the EU-25, representing around half of total tax receipts (Eurostat, 2006).

in a formal model.

Undeclared work is a serious issue in many countries. It is difficult to obtain reliable data on its extension, but raw estimates indicate that the phenomenon is relevant, particularly in transition and developing countries but also in some OECD economies. In a report for the European Commission, the authors stress how the practice of paying "envelope wages" above the officially declared minimum "exists in practically all of the Central and Eastern European countries" (Renooy et al., 2004.) An OECD study of the Baltic countries (OECD, 2003) estimates that in Latvia and Lithuania, 20% of the private-sector employees earn more than what is officially reported[4]. Similar figures have been estimated for Bulgaria (Tomev, 2004.) In Russia, 8% of the employees reported that they received part of their income "under the table" (Petrova, 2005.) The phenomenon is not limited to CEE economies. OECD estimates a 30% shortfall in social security contributions due to undeclared work for Hungary, Mexico and South Korea, and a shortfall above 20% for Italy, Poland, Spain and Turkey[5] (OECD, 2004). According to the World Bank, "in Argentina, roughly 15 percent of workers receive pay partly on the books and partly off the books" (World Bank, 2007). A World Bank study on labour markets in Eastern Europe and the Former Soviet Union (World Bank, 2005) notices how in several countries in the region, "disproportionately high shares of workers cluster on declared wages at or just above the

[4]The Latvian Central Statistical Office publishes data on earnings under the heading "Gross wage of employed *excluding all kinds of irregular payments* by kind of activity" (italics added).

[5]In Turkey, firms belonging to the formal sector are estimated to underreport 28% of their wage bill and for around 50% of the employees enrolled in SSK (Social Security Organization), the wages reported by employers are at the minimum insurable level (World Bank, 2006).

minimum wage (with evidence of additional undeclared incomes above the minimum), creating incentives to sustain a high minimum wage to sustain tax revenue" and calls for further research on this aspect of minimum wage policy. This is indeed the aim of the present paper.

A simple model of the labour market is created where underreporting of earnings is made possible by imperfect detection of tax evasion. The introduction of the minimum wage induces some worker-firm pairs to increase compliance, while pushing others out of the formal labour market into the black economy or into inactivity. The increase in compliance is due to the fact that the minimum wage poses a constraint on reporting behaviour, as agents must choose whether to report nothing or report at least the minimum wage. When faced with such a restriction, agents may prefer to increase their reporting to the minimum wage level rather than decreasing it to zero. The overall effect when enforcement is not too effective is to unambiguously increase fiscal revenues. The distribution of the fiscal burden is also altered, turning a nominally neutral fiscal regime into a regressive one. Moreover, an otherwise smooth distribution of declared earnings is transformed by the introduction of the minimum wage into a distribution presenting a spike at the minimum wage level. The model also predicts a positive correlation between the size of the spike at the minimum wage level and the size of the informal economy. Some supporting evidence on this is presented.

The next section discusses some of the related literature. The model is introduced in the third section. In section 4, the various effects of introducing the minimum wage are explored. Section 5 looks at the model implications for the relationship between the spike at the minimum wage and the underground economy. The following section briefly explores the quantitative implications of the model. In section 7, some extensions of the model are discussed. The

last section concludes.

2 Related literature

The literature on tax evasion has mainly been focused on personal income tax and the compliance decision by an individual filling the tax declaration form. However, due to the tax withholding and information reporting systems present in many countries, this is not an accurate description for the case of employed labour. Indeed, the rate of non-compliance for wages and salaries at the stage of filling the tax declaration form is often negligible. For instance, Klepper and Nagin (1989) report a mere 0.1% of non-compliance for wages and salaries at this stage in the US, i.e. lower than for any other income category. Therefore, to study tax evasion by employed labour it is necessary to take the interaction between the employer and the employee into account.

The literature specifically looking at the labour market effects of tax evasion often considers the formal and informal sections of the labour market as separate, with workers and firms being either completely underground or completely compliant with the regulation. Boeri and Garibaldi (2007) are a recent example of this. Fugazza and Jacques (2003) also take this approach in their study of the effect of labour market institutions when there is an underground sector.

Another strand of the literature, in line with the view taken in this paper, considers that workers' compliance with regulation can also be partial. Sandmo (1981) and Cowell (1985) study models where working time can be allocated between the formal and informal sectors. The former is mainly interested in determining the optimal income tax and enforcement, the lat-

ter in investigating the effects of fiscal and enforcement parameters on the dimension of the informal sector. Kolm and Nielsen (2005) study a search model with wage bargaining, where the worker and the firm agree on the amount of remuneration not to be reported to the fiscal authorities. They find that both higher taxes and weaker enforcement reduce unemployment. Bargaining between the firm and the workers over the true and reported wage is also assumed by Yaniv (1992) who explores the impact of fiscal and detection parameters on tax evasion and contrasts a withholding and a self-declaration system. However, none of the above mentioned studies considers the impact of minimum wage legislation in an economy with underreporting.

The literature on minimum wage deals extensively with its effects on wage distribution and employment. A spike at the minimum wage level has been observed in several instances (see, for instance, DiNardo et al., 1996, Dickens and Manning, 2004). Such a spike has been defined as a "puzzle" for several standard types of labour market models (Brown, 1999) and as an "anomalous finding from the standpoint of the standard model of the low wage labour market" (Card and Krueger, 1995, p. 152). Proposed rationalizations include reductions in non-wage compensation or increases in required effort to offset a binding minimum wage, flatter earnings profiles and adjustments in the amounts of hours worked. The model presented here proposes an alternative rationale for the observed spike in a perfect competition framework. The positive correlation between the size of the spike at the minimum wage and the estimated size of the informal economy in the data presented in the Appendix suggests that the mechanism analysed in this paper indeed contributes to shape the observed distribution of earnings in some countries. Recently, several empirical studies have considered the impact of the minimum wage on other aspects than employment, like fringe

benefits (Simon and Kaestner, 2004), prices (Lemos, 2005), profits (Draca et al., 2006.) The impact of the minimum wage on tax evasion has, to the best of my knowledge, never been investigated.

3 The model without minimum wage

The size of the population is exogenously given and normalized to 1. Every individual has an exogenously given productivity y_i, distributed in the population according to pdf $g(y)$ and cdf $G(y)$ on the support $[\underline{y}, \bar{y}]$, where $\underline{y} \geq 0$. We assume the labour market to be competitive, each firm employs one worker, there is no capital, and production is equal to labour input. Moreover, there is free entry of firms, firms can observe workers' productivity, and workers can move from one firm to another at no cost.

Firms are risk-neutral and maximize expected profits. In an environment without tax evasion, profits for a firm employing a worker with productivity y_i are given by

$$\pi_i = y_i - w_i,$$

where w_i is the gross wage[6]. Firms have an obligation to withhold taxes and social security contributions and transfer them to the authorities. Taxation is at the proportional rate $t \in (0, 1)$. Workers are risk-averse, their (indirect) utility is an increasing function of net income, given by

$$I_i = w_i(1 - t).$$

The wedge between the gross wage paid by the firm and the net wage received by the worker, tw_i, is paid to the fiscal authorities. Free entry of firms implies

[6]No distinction is made between labour cost and gross wage and the two concepts are equivalent in the model.

that in equilibrium, the expected profits are zero which, in turn, in the full compliance case implies that a worker with productivity y_i would receive a gross wage y_i, from which the firm would deduct taxes ty_i, thereby leaving the worker a net wage $(1-t)y_i$.

In this economy, however, it is possible to evade taxes and social security contributions by not reporting part or all of the worker's earnings to the authorities. A firm employing a worker with productivity y_i must therefore decide how much of the worker's production to declare to the tax authorities, x_i, and how much to conceal, $y_i - x_i$. If $x_i = y_i$, the firm is fully compliant with the regulations. If $x_i = 0$, the full product is hidden from the authorities and the firm-worker pair operates completely in the black economy. If $x_i \in (0, y_i)$, there is underreporting. A worker-firm pair can thus operate in the formal economy, by declaring a strictly positive income, or be completely in the black market, by declaring nothing. A worker can also decide to be inactive. In this case, income is normalized to 0.

Tax authorities may inspect firms to find out whether they comply with fiscal regulation. We assume there to be an exogenously given probability of an audit being performed $\gamma \in [0, 1]$. Fines are imposed on firms in case tax evasion is detected and, given the assumption of risk-neutral firms and risk-averse workers, there is no incentive for workers and firms to negotiate a different risk-sharing arrangement. However, the fact that an audit is performed does not imply that the authority with certainty discovers the true tax liability, but it may find evidence to impute an income $\hat{y}_i \in [0, y_i]$, where y_i is the true product. For instance, Feinstein (1991) estimates that IRS examiners on average managed to detect only half of the tax evasion in the forms they audited[7], while Erard (1997) rejects the hypothesis of perfect

[7] An IRS study found that for every dollar of underreported income detected by examin-

detection in his empirical investigation of a model where detection can be either complete or null.

We assume that $\hat{y}_i$ is distributed over the support $[0, y_i]$[8] according to pdf $h(\cdot)$ and cdf $H(\cdot)$, so that $H(0) = 0$ and $H(y_i) = 1$, and $H(\cdot)$ does not depend on x_i. To simplify the discussion, we assume that $h(\cdot) > 0$ within the support, so that $H(\cdot)$ is invertible within $[0, y_i]$.

Given a declaration of x_i and collected evidence of a true tax liability of $\hat{y}_i$, the tax authority imposes on the firm, in case $\hat{y}_i > x_i$, the payment of $\theta t\,(\hat{y}_i - x_i)$, consisting of taxes plus an additional fine proportional to the assessed tax evasion, thus $\theta > 1$. In case $\hat{y}_i \leq x_i$, the tax authority cannot prove any tax evasion, so no fine is imposed[9]. Given a true product y_i and a reported one $x_i \in [0, y_i]$, the expected fine in case of auditing, f_i, is

$$f_i = t\theta \int_{x_i}^{y_i} (\hat{y}_i - x_i) h(\hat{y}_i) d\hat{y}. \tag{1}$$

Below, we determine the equilibrium wage and evasion. For convenience, subscripts are suppressed where not necessary.

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ers without the aid of third-party information documents, another $ 2.28 went undetected (cited in Feldman and Slemrod, 2007).

[8] The assumption is that the tax authority cannot assess and upheld in court a tax liability higher than the true one. To extend the model to situations where this may not be the case, due for instance to ambiguity in the tax code, would be relatively straightforward.

[9] An equivalent narrative is that in an audit, the tax authority may find no evidence at all of tax evasion with probability $H(x_i)$, which is increasing as the tax liability declared to the authorities increases. Conditional on detection taking place, the density for any given level of income $\hat{y}_i \in [x_i, y_i]$ being discovered is given by $h\,(\hat{y}_i)\,/\,[1 - H\,(x_i)]$.

3.1 Equilibrium without minimum wage

For a firm employing a worker with productivity y, declaring x, and paying a gross wage w, the possible realizations of profits are given by[10]

$$\pi = \begin{cases} y - w & \text{with probability } 1 - \gamma \\ y - w - f & \text{with probability } \gamma \end{cases},$$

where f, the expected fine in case an audit is conducted, is given by (1). Therefore, the expected profits for the firm are

$$E(\pi) = y - w - \gamma f. \tag{2}$$

Income I for a worker employed in a firm paying a gross wage w and declaring to the fiscal authorities x is given by

$$I = w - tx. \tag{3}$$

This expression captures the fact that taxes and social security contributions are deducted from the worker's declared gross wage x, not from his true gross wage, w. As income is non-stochastic, income maximization corresponds to utility maximization, given the assumption that (indirect) utility only depends on net income.

The firm and the worker agree to choose x so as to maximize the expected total surplus available to them, equivalent to the product minus total expected payments to fiscal authorities, represented by taxes and social security contributions paid on the declared wage and expected fines. Therefore, the optimal declaration is

$$x^* \quad s.t. \quad \max_{x \in [0,y]} y - \gamma f - tx. \tag{4}$$

[10] Actually, when an audit is performed, possible realizations of profits are a continuum, due to the stochastic nature of the fine. For expositional convenience, the expected value of the fine is considered.

After substituting (1) into (4), the first-order condition is

$$H(x^*) = 1 - \frac{1}{\gamma\theta} \Longleftrightarrow x^* = H^{-1}\left(1 - \frac{1}{\gamma\theta}\right).$$

The second-order condition

$$-t\gamma\theta h(x) < 0$$

is always satisfied. The boundary condition $x \leq y$ is always satisfied. Notice that full compliance (i.e. $x = y$) does not take place unless $\gamma\theta \to +\infty$. The condition $x \geq 0$ implies that full evasion will take place, i.e. $x = 0$, when enforcement is very weak, i.e $\gamma\theta \leq 1$. To simplify the notation, the two enforcement parameters are summarized by $\alpha \equiv 1/(\gamma\theta)$. To summarize, the solution to the reporting problem without minimum wage is given by

$$x^* = \begin{cases} H^{-1}(1-\alpha) & \text{if} \quad \alpha < 1 \\ 0 & \text{if} \quad \alpha \geq 1 \end{cases}. \tag{5}$$

As $\partial\alpha/\partial\gamma < 0$ and $\partial\alpha/\partial\theta < 0$, in an interior solution, the fraction of production that is evaded decreases as enforcement improves.

The equilibrium fine, f^*, is given by substituting (5) into (1). Substituting this into (2) and considering the free entry condition, we get the equilibrium gross wage

$$w^* = y - \gamma f^*,$$

that substituted into (3) gives the equilibrium net income

$$I^* = y - \gamma f^* - tx^*. \tag{6}$$

To simplify the discussion, from now on we will assume $h(\cdot)$ to be uniform in the support $[0, y]$, i.e. $\hat{y}_i \sim U_{[0,y_i]}$. The expression for the expected fine

becomes[11]

$$\gamma f = \gamma t \theta (y - x)^2 / (2y) . \tag{7}$$

The optimal reporting behaviour given by (5) becomes

$$x^* = \begin{cases} (1-\alpha)\, y & \text{if} \quad \alpha < 1 \\ 0 & \text{if} \quad \alpha \geq 1 \end{cases} \tag{8}$$

thus, the model implies that, irrespective of the specific level of productivity, a constant fraction of the true tax liability is revealed to the fiscal authorities. Using (7), the expected fine is given in equilibrium by

$$\gamma f^* = \begin{cases} yt\alpha/2 & \text{if} \quad \alpha < 1 \\ yt/(2\alpha) & \text{if} \quad \alpha \geq 1 \end{cases} \tag{9}$$

and thus, substituting (8) and (9) into (6), we get the worker's equilibrium net income

$$I^* = \begin{cases} y(1-t) + \alpha y t/2 & \text{if} \quad \alpha < 1 \\ y\left[1 - t/(2\alpha)\right] & \text{if} \quad \alpha \geq 1 \end{cases} . \tag{10}$$

Given the detection technology, the expected fraction of unreported tax liability, $y - x^*$, that is discovered in case of auditing is

$$\int_x^y (\hat{y} - x^*) h(\hat{y}) d\hat{y} / (y - x^*) = \alpha/2, \tag{11}$$

i.e. a fraction corresponding to half the ratio of evaded income over true product. The assumption is thus that it is relatively easy to get away with tax-evasion. For example, in an economy where 30% of the income are concealed, only 15% of the evasion are, on average, detected in case of auditing.

[11] The Appendix presents an alternative setting for imperfect detection giving rise to an equivalent expression for the expected fine. It also discusses the case of the probability of an audit being conditioned on declared income.

4 Effects of the minimum wage

In this section, we study what are the effects of introducing a minimum monthly wage ϖ, with universal coverage, in the economy described in the previous section. Workers cannot be legally employed at a wage below the minimum, in the sense that their reported gross wage cannot be below the minimum. The assumption in the model is that the minimum wage is fixed on a monthly basis for full-time work and that no alternative working-time arrangements are available. However, in section 7.1, the model is extended to the case where the minimum wage is fixed on an hourly basis, labour supply can vary across workers and underreporting can involve both hours of work and hourly wage. The results remain qualitatively unchanged. In the following, we focus on the case with partial evasion, i.e. $\alpha \in (0,1)$ [12].

4.1 Effects on the distribution

With the introduction of a minimum wage, (4) becomes

$$x^* \quad s.t. \quad \max_{x \in \{0\} \cup [\varpi, y]} \quad y - \gamma f - tx.$$

The only difference is in the choice set which shrinks from $[0, y]$ to $\{0\} \cup [\varpi, y]$. The introduction of the minimum wage divides worker-firm pairs into three categories:

[12] For this to be the case, we need $\gamma\theta > 1$. By assumption $\theta > 1$, but γ, the probability of being subject to an audit, may be low, so this condition may seem restrictive. Notice, however, that in this model, an audit is extremely ineffective. As already mentioned if, for instance, 30% of the income are evaded, only 15% of the evaded income are, on average, discovered during an audit. Thus, instead of a full-fledged investigation, an audit should in the present set-up rather be interpreted as a routine check by the fiscal authorities, thus occurring much more frequently than a thorough inquiry.

1. High productivity: $y_i > \varpi/(1-\alpha)$

2. Intermediate productivity: $\varpi \leq y_i \leq \varpi/(1-\alpha)$

3. Low productivity: $y_i < \varpi$.

Worker-firm pairs characterized by high productivity would have declared more than the minimum wage anyway, so they are unaffected by it. The minimum wage is instead a binding constraint for worker-firm pairs that would have declared less in its absence. We first analyse the case of low-productivity workers.

Low productivity A worker with productivity below the minimum wage, $y_i < \varpi$, can only work in the black market or be inactive. The possibility of a worker paying back part of his wage to the firm is thus excluded. The main results are qualitatively unaffected by this modelling choice. From (10), we get income in case of work in the black market, i.e. full evasion,

$$I_{bm} \equiv y_i \left[1 - t/(2\alpha)\right]. \tag{12}$$

Income in case of inactivity is assumed to be 0. The labour market status is chosen by comparing income in the two cases, giving the following condition

$$I_{bm} > 0 \Leftrightarrow \alpha > t/2.$$

Then, if $\alpha > t/2$, workers with productivity below the minimum wage work in the black market, otherwise they withdraw from the labour market. Thus, the prediction is that, for a given tax rate, in economies where enforcement is quite effective, i.e. α is low, the minimum wage pushes workers into inactivity and therefore, it has a negative impact on efficiency, as productive labour remains idle. Instead, in economies where enforcement is not very

effective, the minimum wage has no negative impact on efficiency as workers continue to produce in the black market. Naturally, this is true as far as going completely underground does not entail a drop in productivity.

Intermediate productivity The possibility of declaring the minimum wage and thus, participating in the formal labour market, is available for worker-firm pairs whose optimal declaration in case of no minimum wage regulation is less than ϖ, but with a productivity above ϖ, i.e.

$$(1-\alpha)y_i \leq \varpi \leq y_i \Leftrightarrow \varpi \leq y_i \leq \varpi/(1-\alpha). \tag{13}$$

Income in case of declaring ϖ is given by substituting $x = \varpi$ in (7) and (6)

$$I_{mw} \equiv y_i(1-t) + (y_i - \varpi)\,t - t\,(y_i - \varpi)^2/(2\alpha y_i). \tag{14}$$

Declaring a wage higher than the minimum is never optimal for this group. Moreover, as $I_{mw} > 0$ for productivities satisfying (13), these workers will never go into inactivity. The choice is thus between declaring the minimum wage or working in the black market and declaring 0. The comparison between income in case of declaring the minimum wage and income in the black market as given by (12) gives the following condition

$$I_{mw} \geq I_{bm} \Leftrightarrow y_i \geq \varpi/[2(1-\alpha)] \equiv y_{mw}. \tag{15}$$

As the choice between employment at the minimum wage and employment in the black market is only relevant for workers satisfying (13) to determine the behaviour once a minimum wage is introduced, it is necessary to position y_{mw} in the interval $[\varpi, \varpi/(1-\alpha)]$. The threshold y_{mw} is greater than the minimum wage if and only if $\alpha > 1/2$, while it is always the case that $y_{mw} < \varpi/(1-\alpha)$. Thus, if the degree of underreporting is high, i.e. $\alpha > 1/2$,

the threshold y_{mw} is internal to the interval defined by condition (13). This implies that some of the workers affected by the minimum wage and with a productivity higher than the minimum wage prefer to decrease evasion and declare the minimum, while others prefer to go into the black market. If the degree of underreporting is instead low, i.e. $\alpha \leq 1/2$, all workers affected by the minimum wage and with a productivity higher than the minimum wage prefer to increase compliance and declare the minimum.

The results are summarized in the below proposition.

The introduction of the minimum wage in an economy with underreporting of earnings induces some workers to increase compliance by increasing declared earnings to the minimum wage level. Workers with a high productivity are unaffected. Workers with a productivity below the minimum wage work in the black market if enforcement is not too effective, otherwise they withdraw from the labour force.

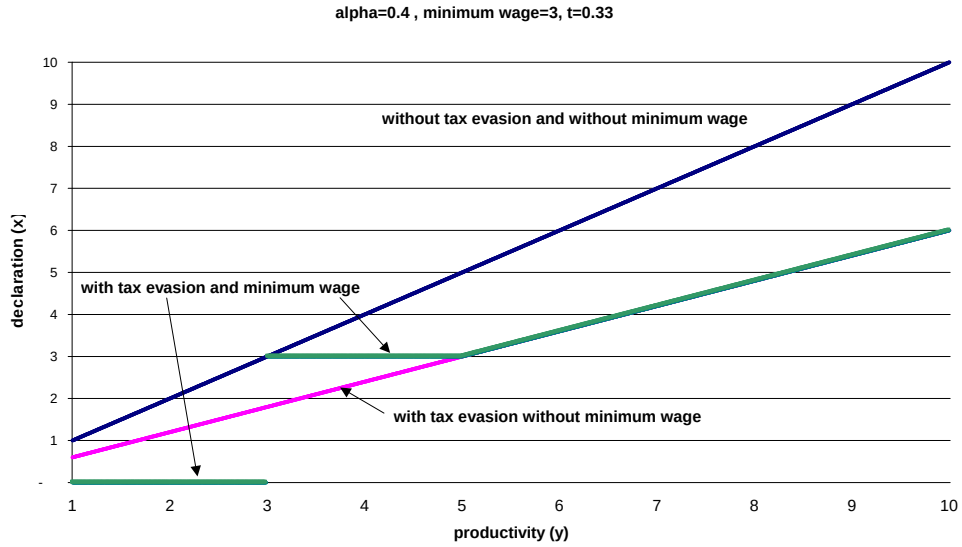
The distribution of declared earnings x before the introduction of the minimum wage is given by

$$g_x(x) = \begin{cases} g\left(\frac{x}{1-\alpha}\right) & \underline{y}(1-\alpha) < x < \bar{y}(1-\alpha) \\ 0 & otherwise \end{cases},$$

where $g(\cdot)$ is the pdf of the productivity distribution. After the introduction of the minimum wage, distribution of declared earnings is given by

$$g_{mw}(x) = \begin{cases} \int_{\underline{y}}^{\varpi \max\left\{\frac{1}{2(1-\alpha)},1\right\}} g(y)dy & if \quad x = 0 \\ \int_{\varpi \max\left\{\frac{1}{2(1-\alpha)},1\right\}}^{\frac{\varpi}{1-\alpha}} g(y)dy & if \quad x = \varpi \\ g\left(\frac{x}{1-\alpha}\right) & if \quad \varpi < x \leq \bar{y}(1-\alpha) \\ 0 & otherwise. \end{cases}.$$

Figure 1: Declared income



Thus, a "smooth" distribution of productivity is associated with a "smooth" distribution of declared earnings without a minimum wage. However, with the introduction of the minimum wage, two spikes appear at the minimum wage level and at zero. Thus, we can state the following:

In a perfectly competitive labour market with underreporting of earnings, a spike at the minimum wage level appears in the distribution of declared earnings.

Figure 1 depicts declared income as a function of productivity with and without the minimum wage. Declared income when there is no tax evasion is also plotted as a reference.

4.2 Fiscal effects

The minimum wage divides worker-firm pairs into three categories: those declaring nothing, those declaring the minimum wage and the unaffected, i.e. those declaring more than the minimum. Here, we first determine payments to fiscal authorities for each category. Then, we use the above analysis of the distribution of declared earnings to find out the effects of the minimum wage on fiscal revenues.

Payments to fiscal authorities Total payments, P, to fiscal authorities include taxes, T, and expected fines, F. For worker-firm pairs not affected by the minimum wage, these quantities are

$$P_1 = (1-\alpha/2)ty \begin{array}{l} \nearrow \\ \searrow \end{array} \begin{array}{l} T_1 = yt(1-\alpha) \\ F_1 = yt\alpha/2 \end{array} .$$

Underreporting gives worker-firm pairs with a relatively high productivity the opportunity to reduce the "effective"[13] tax rate by a factor $\alpha/2$. For worker-firm pairs declaring the minimum wage, fiscal payments are given by

$$P_2 = t\varpi + t(y-\varpi)^2/(2\alpha y) \begin{array}{l} \nearrow \\ \searrow \end{array} \begin{array}{l} T_2 = t\varpi \\ F_2 = t(y-\varpi)^2/(2\alpha y) \end{array} .$$

The remaining category is represented by worker-firm pairs that are either in the black economy (when $\alpha \geq t/2$) or do not participate in the labour market (when $\alpha < t/2$). For workers in the black market, fines are the only type of payment, so that

$$P_3 = F_3 = ty/(2\alpha) .$$

[13] In the sense of total expected payments to fiscal authorities, including fines, over total product, i.e. P/y.

Workers who withdraw from the labour market do not contribute to the public finances, so

$$P_4 = F_4 = 0.$$

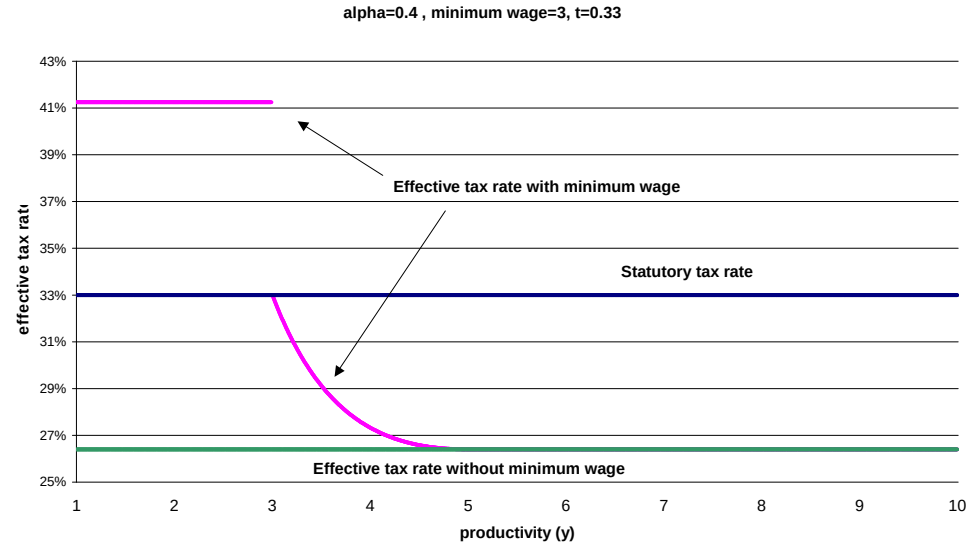
Notice that $P_3/y \geq P_2/y \geq P_1/y$ in the relevant intervals[14]. Expected payments as a portion of income are highest for worker-firm pairs in the black economy and lowest for worker-firm pairs not affected by the minimum wage. Thus, considering expected total payments, it is possible to state the following:

The interaction of minimum wage and underreporting transforms a nominally neutral tax system into a regressive one.

The intuition behind this result is simple: worker-firm pairs try to minimize the share of the product paid to fiscal authorities. The minimum wage is not a binding constraint for high productivity workers who manage to reduce the "effective" tax rate. For instance, if $\alpha = 40\%$, the "effective" tax rate for these workers is 80% of t. For workers with intermediate productivity, the minimum wage is binding. Thus, they are less "successful" in minimizing their "effective" tax rate, even if they still manage to reduce it below t. Low productivity workers are even more constrained, as their only choice is to work in the black market or withdraw from the labour market, and they may end up facing an "effective" tax rate above t. With $\alpha = 40\%$, for instance, the "effective" tax rate for these workers is indeed 125% of t. Figure 2 shows the effective tax rate as a function of productivity.

[14] In particular, $P_2/y \geq P_1/y \quad \forall y$, $P_3/y \geq P_1/y \quad \forall y, P_3/y \geq P_2/y \Leftrightarrow y \geq \frac{\varpi}{2(1-\alpha)}$. As only workers with productivity $y_i \geq \varpi \max\left\{1, \frac{1}{2(1-\alpha)}\right\}$ will declare the minimum wage, $P_3/y \geq P_2/y$ for the relevant interval.

Figure 2: Effective tax rate



Effective Tax Rate = Total Payments to Fiscal Authorities / Productivity

Effects of the minimum wage on revenues When workers with productivity below the minimum wage work in the black market, i.e. when $\alpha \geq t/2$, total revenues R are given by

$$R = \int_0^{\varpi \max\left\{\frac{1}{2(1-\alpha)},1\right\}} ty/\left(2\alpha\right) g\left(y\right) dy + \int_{\varpi \max\left\{\frac{1}{2(1-\alpha)},1\right\}}^{\varpi/(1-a)} \left[t\bar{w} + t\left(y-\varpi\right)^2/\left(2\alpha y\right)\right] g\left(y\right) dy +$$
$$+ \int_{\varpi/(1-a)}^{\bar{y}} \left(1-\alpha/2\right) tyg(y) dy. \tag{16}$$

The marginal worker is indifferent between being employed in the black market or declaring the minimum wage if $\alpha > 1/2$, while he prefers not to be completely underground if $t/2 \leq \alpha \leq 1/2$. In the first case, the only effect of a marginal increase in the minimum wage is to extract higher payments

from workers declaring it while in the second case, there is the additional effect of pushing worker-firm pairs previously in the official economy into the black market. In both cases, total revenues increase with an increase in the minimum wage, i.e.

$$\frac{\partial R}{\partial \varpi} > 0.$$

When workers with a productivity below the minimum wage withdraw from the labour market, i.e. when $\alpha < t/2$, there is no black market from which to extract fines, and total revenues are given by the last two terms in expression (16). Then,

$$\frac{\partial R}{\partial \varpi} = -t\bar{w}g(\varpi) + \int_{\varpi}^{\varpi/(1-a)} [1 - (y - \varpi) / (\alpha y)] t g(y) dy.$$

The first term represents the fiscal loss due to the withdrawal of workers from the labour market, the second term the higher payments by workers declaring the minimum wage. The net effect depends on the shape of the distribution. We can then state the following proposition:

When underreporting is high, revenues increase with the minimum wage. When underreporting is low, the effect of increasing the minimum wage on revenues depends on the productivity distribution.

The intuition is straightforward: maximization of workers' net income is equivalent to minimization of transfers to the government. Choice is limited to the possible declaration space $\{0\} \cup [\varpi, +\infty)$. Increasing the minimum wage shrinks the possible declaration space, so that the newly chosen compliance after the increase in the minimum wage cannot make workers better off. When the increase in the minimum wage does not have a negative impact on production, i.e. it does not "shrink the pie", this implies that the government cannot be made worse off, i.e. revenues cannot decrease. This can be

counterbalanced by a decrease in revenues due to reduced total production when an increase in the minimum wage pushes low productivity workers out of the labour market.

This implies that countries where underreporting is serious because of limited enforcement capacity can use the minimum wage to boost fiscal revenues, without having to worry too much about the impact on efficiency. As enforcement improves, the minimum wage becomes a less effective fiscal instrument and efficiency issues become more prominent. However, equity issues are also at stake, as the minimum wage increases revenues by extracting more payments from low productivity workers.

The revenue boosting effect of the introduction of a minimum wage can be substantial. In Bulgaria, for instance, social security contribution payments increased by almost 20% in 2003 "[a]s a result from the registration of the labour contracts and the introduction of the minimum insurance income upon principal economic activities and qualification groups of professions, as well as from the improved economic situation" (NSSI).

5 Underground economy and minimum wage spike

Both the size of the spike at the minimum wage and the size of the underground economy relative to the economy as a whole are determined by the interplay of the productivity distribution, the fiscal enforcement parameters as summarized by α, and the minimum wage, ϖ. In this section, we study the link between the size of the underground economy and the size of the spike.

The spike at the minimum wage The size of the spike at the minimum wage is given by

$$S = \int_{\varpi \max\left\{\frac{1}{2(1-\alpha)},1\right\}}^{\varpi/(1-a)} g(y)dy.$$

A decrease in enforcement parameters, i.e. an increase in α, induces the minimum wage to be declared by some workers previously declaring more, thereby increasing the size of the spike. If enforcement is sufficiently weak, i.e. if $1/2 < \alpha < 1$, an additional effect plays a role, as some workers previously declaring the minimum wage prefer to go into the black economy, thus reducing the size of the spike. In this case

$$\frac{\partial S}{\partial \alpha} > 0 \Leftrightarrow g\left(\frac{\varpi}{1-a}\right) > \frac{1}{2} g\left(\frac{\varpi}{2\left(1-a\right)}\right).$$

Assuming that the distribution of productivity is single peaked, the above condition is satisfied if the minimum wage is binding for workers with productivity lower than the mode. If this is the case, the spike is always increasing as α increases.

The effect on the size of the spike of a marginal increase in the minimum wage depends on the interplay between two effects: as ϖ increases, some workers previously declaring the minimum wage are pushed out of the formal labour market, thus decreasing the size of the spike, while some, previously declaring more, declare the minimum wage, thus increasing the size of the spike. Given α, the condition for the size of the spike to increase as the minimum wage increases is

$$\frac{\partial S}{\partial \varpi} > 0 \Leftrightarrow g\left(\frac{\varpi}{1-a}\right) > g\left(\varpi\right) \max\left\{1-a, 1/2\right\}.$$

Also in this case are a single peaked productivity distribution and a minimum wage binding for workers with productivity lower than the mode sufficient

conditions for the spike to increase with the minimum wage.[15]

The informal economy When workers with a productivity below the minimum wage work in the black market, i.e. when $\alpha \geq t/2$, the size of the underground economy[16] is given by:

$$U = \underbrace{\int_{\underline{y}}^{\varpi \max\left\{\frac{1}{2(1-\alpha)},1\right\}} yg(y)dy}_{\text{black economy}} + \underbrace{\int_{\varpi \max\left\{\frac{1}{2(1-\alpha)},1\right\}}^{\varpi/(1-a)} (y-\varpi)g(y)dy + \alpha \int_{\varpi/(1-a)}^{\bar{y}} yg(y)dy}_{\text{underreporting}}. \tag{17}$$

A decrease in enforcement, i.e. an increase in α, increases the size of the informal economy as workers unaffected by the minimum wage evade more. Moreover, when enforcement is already low, i.e. $1/2 < \alpha < 1$, some workers previously declaring the minimum wage go into the black economy, thereby further increasing informality.

An increase in the minimum wage pushes some workers previously declaring the minimum wage into the black economy, thus increasing informality, but also forces workers continuing to declare the minimum to declare more of

[15] The analysis can also be conducted in terms of the size of the spike, relative to the size of the officially employed workforce, where the latter is given by:

$$L = \int_{\varpi \max\left\{\frac{1}{2(1-\alpha)},1\right\}}^{\bar{y}} g(y)dy.$$

The conditions for the spike relative to the officially employed workforce, S/L, to increase with α and ϖ are looser than those for S, as the size of the officially employed workforce is not increasing with α and ϖ.

[16] The analysis is made on the size of the informal economy in absolute terms, U. The size of the informal economy relative to the economy as a whole, U/Y, or relative to the size of the formal economy, $U/(Y-U)$, is also of interest. When $\alpha \geq t/2$, the size of the economy is given by $Y = \int_{\underline{y}}^{\bar{y}} yg(y)dy$ and does not depend on α or ϖ. Thus, the derivatives of U, U/Y, $U/(Y-U)$ w.r.t. α and ϖ all have the same sign.

their true income, thus reducing informality. Which effect prevails depends on the shape of the productivity distribution.

When workers with productivity below the minimum wage withdraw from the labour market; i.e. when $\alpha < t/2$, there is no black market, thus the size of the underground economy is given by the last two terms in expression (17). Also in this case does a decrease in enforcement, i.e. an increase in α, increase the size of the informal economy as workers unaffected by the minimum wage evade more[17]. The absolute size of the informal economy decreases with an increase in the minimum wage, as workers declaring the minimum increase their compliance. However, in this case, an increase in the minimum wage reduces the size of the economy that is given by $Y = \int_{\varpi}^{\bar{y}} yg(y)dy$. The effect of an increase in the minimum wage on the size of the informal economy relative to the economy as a whole, U/Y, or relative to the formal economy, $U/(Y-U)$, is ambiguous, as it depends on the shape of the productivity distribution. To summarize:

When enforcement decreases, the size of the informal economy increases, both in absolute terms or relative to the formal economy. Sufficient conditions for the size of the spike at the minimum wage to increase when enforcement decreases are a single peaked productivity distribution combined with a minimum wage binding for workers with productivity lower than the mode or a not too weak enforcement. The effect of an increase in the minimum

[17] There is a discontinuity in the size of the informal economy at $\alpha = t/2$. When enforcement parameters decrease (i.e. α increases), the size of the informal economy jumps up discretely as workers previously withdrawn from the labour market enter into the black market. This jump goes in the same direction as the derivative, so we can state that the size of the informal economy always increases as enforcement decreases. The same is true if we consider the size of the informal economy relative to the whole economy, U/Y, or relative to the formal economy, $U/(Y-U)$.

wage on the size of the informal economy relative to the formal economy is ambiguous. A sufficient condition for the size of the spike at the minimum wage to increase when the minimum wage increases is a single peaked productivity distribution combined with a minimum wage binding for workers with productivity lower than the mode.

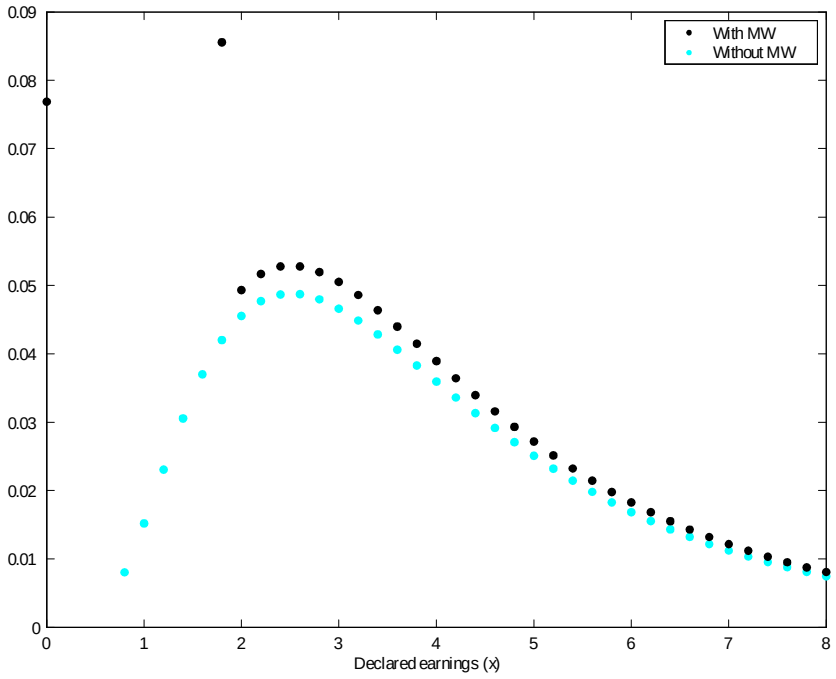
Thus, under mild conditions, the common dependence on α should induce a positive correlation between the spike at the minimum wage and the size of the informal economy. Some evidence on this correlation is presented in the Appendix.

6 A numerical example

In this section, the quantitative properties of the model are briefly explored. Workers' productivity is assumed to be distributed across 37 categories in the range 1-10, with the distance between adjacent productivity categories being 0.25. In the baseline scenario, the distribution of the workforce across the different categories is generated by normalizing the corresponding values of a lognormal with parameters (1.5; 0.6). Tax and social security contributions are assumed to be equivalent to 30% and enforcement parameters are such that without a minimum wage, all agents evade 20% of their income, i.e. $\alpha = 0.2$. The minimum wage is assumed to be equal to the income declared by the 6th productivity category, i.e. 1.8.

Figure 3 shows the distribution of declared earnings among the official workforce before and after the introduction of the minimum wage. Without the minimum wage, declared earnings are in the range 0.8-8, as 20% of the product is evaded. The distribution of declared earnings changes with the

Figure 3: Distribution of declared earnings



introduction of the minimum wage. The minimum wage creates two spikes, at the minimum wage level and at zero. As the minimum wage reduces the size of the official workforce by truncating it from below, the distribution of declared earnings is shifted upward above the minimum wage. Notice that in the figure, the spike at the minimum wage is the percentage of the *official workforce* declaring the minimum wage. Instead, the spike at zero is the percentage of the *population* not participating in the official labour market.

Table 1: Numerical results

Spike ϖ[1]	Spike 0[2]	Kaitz Index[3]	ΔRevenues[4]	Informal Economy[5]
Scenario 1 - Baseline: $\mu = 1.5$ $\sigma = 0.6$ $\alpha = 0.2$ $\varpi = 1.8$				
8.6	7.7	46.5	4.52	28
Scenario 2 - High Evasion: $\mu = 1.5$ $\sigma = 0.6$ $\alpha = 0.3$ $\varpi = 1.575$				
11.5	4.6	47.3	1.41	44
Scenario 3 - High MW: $\mu = 1.5$ $\sigma = 0.6$ $\alpha = 0.2$ $\varpi = 2.2$				
15.3	11.4	55.2	7.58	29.4
Scenario 4 - High Evasion, High MW: $\mu = 1.5$ $\sigma = 0.6$ $\alpha = 0.3$ $\varpi = 1.926$				
18.7	7.7	56.2	2.76	44.6
Scenario 5 - Spread-out Distribution: $\mu = 1.5$ $\sigma = 0.8$ $\alpha = 0.2$ $\varpi = 1.8$				
10.7	14.3	46.2	8.33	30.7

1: as % of workforce in formal employment.

2: as % of total population.

3: minimum wage over average declared wage.

4: % change in total fiscal revenues due to the introduction of the minimum wage.

5: size of the informal economy as % of official economy.

Table 1 reports the size of the two spikes. Other indicators are also calculated. In the model developed in this paper, the minimum wage is assumed to apply to the workforce as a whole; thus the Kaitz index is simply the minimum wage divided by the average declared wage. The percentage increase in total fiscal revenues (taxes and fines) due to the introduction of a minimum wage is also calculated. Finally, the size of the informal economy as a percentage of the formal economy is presented. In the baseline scenario, the informal economy would be 25% of the formal economy without a minimum wage, as 20% of income would be evaded. With the minimum wage, the informal economy is equivalent to 28% of the formal economy. Four other scenarios are explored. In the "high evasion" scenario, enforcement is assumed to be weaker, so that 30% of income would be evaded without a minimum wage constraint, i.e. $\alpha = 0.3$. The minimum wage remains equivalent to the income declared by the sixth productivity category[18]. The share of the population affected by the minimum wage is the same as in the baseline scenario, as only a reshuffle between workers declaring zero and workers declaring the minimum wage takes place[19]. As established in section 5, the size of the spike at the minimum wage level increases, together with the size of the informal economy.

In the "high minimum wage" scenario, the minimum wage is assumed to be equivalent to the income declared by the eighth productivity category, i.e.

[18] Due to the increase in evasion, though, the actual level of the minimum wage is lower than in the baseline scenario.

[19] Notice that the size of the spike at the minimum wage level and the size of the spike at zero do not add up to the same number in scenarios 1 and 2 and in scenarios 3 and 4 only because the reported spike at the minimum wage level is expressed as a percentage of the official workforce, while the spike at zero is expressed as a percentage of the total population.

2. In this case, the minimum wage bites deeper into the wage distribution. As established in section 5, the size of the spike at the minimum wage level increases. The significance from a fiscal point of view is also increased as compared to the baseline scenario, as established by Proposition 4.2.

The "high evasion, high minimum wage" scenario combines the two previous variations. In this case, both the spike at the minimum wage level and the size of the informal economy reach very high levels.

In the last scenario, the distribution generating the frequencies is changed, in particular the standard deviation parameter is increased to 0.8. The resulting sizeable change in some of the indicators points to the fact that the quantitative implications of the model are sensitive to the assumption about the underlying distribution of productivity. However, these simple calculations show that the model is able to match the very high spike at the minimum wage observed in some countries and that the fiscal implications of imposing a minimum wage can be sizeable, even if only people with the lowest productivity are affected.

7 Extensions

In this section, the robustness of the model along several dimensions is discussed and some extensions are proposed. First, we look at the issue of working time. The model is extended to account for the fact that hours can also be underreported. Then, we check the robustness of the model for possible discontinuities arising when a firm-worker pair goes completely underground. In particular, discontinuities in productivity and expected fines are considered. Finally, we look at the implications for the model of accounting for entitlements from social security.

7.1 Working time

A minimum wage fixed on an hourly basis in an environment where firms could declare the amount of hours worked with full flexibility and no risk of detection in case of underreporting would pose an extremely loose constraint on reporting behavior. However, the minimum wage can still play its role against underreporting of earnings if there are legislative constraints on the amount of hours that can be reported or incentives not to declare a minimal amount of hours[20,21] or if misreporting hours of work can also be detected and punished. In this section, we consider the latter case.

Suppose that a worker with hourly productivity y_i inelastically supplies h_i hours of work per period. However, the worker-firm pair can choose to report product per hour $x_i \in [0, y_i]$ and hours of work $\tau_i \in [0, h_i]$. The audit and detection technologies are the same in the two dimensions. In case of audit, the tax authorities manage to impute $\hat{x}_i \in [0, y_i]$ and $\hat{\tau}_i \in [0, h_i]$. For analytical convenience, the probabilities of detection are assumed to be independent and uniformly distributed over the relevant intervals, so that $g_{\hat{x}_i}(\hat{x}_i) = 1/y_i$ and $g_{\hat{\tau}_i}(\hat{\tau}_i) = 1/h_i$. The corresponding c.d.f. are indicated as $G_{\hat{x}_i}$ and $G_{\hat{\tau}_i}$. The imposed fine, f_i, depends on the detected and declared hours of work and product per hour. In particular, it is possible to distinguish four cases:

[20] According to Eurostat data from LFS, the share of part-timers in Central and Eastern European countries is generally low, at around 7% of the employees.

[21] According to OECD "To counter this [under-declaring earnings per employee], the tax authorities may appeal to employment regulations such as the minimum wage and *restrictions on part-time and temporary work*. This issue helps explain why countries with a large informal economy maintain de facto strict employment regulations, even though these regulations are seen by many analysts as a prime cause of informality." (OECD, 2004, page 227, italics added).

1. $\hat{x}_i < x_i$ and $\hat{\tau}_i < \tau_i \Rightarrow f_i = 0$
2. $\hat{x}_i < x_i$ and $\hat{\tau}_i > \tau_i \Rightarrow f_i = t\theta\left(\hat{\tau}_i - \tau_i\right) x_i$
3. $\hat{x}_i > x_i$ and $\hat{\tau}_i < \tau_i \Rightarrow f_i = t\theta\left(\hat{x}_i - x_i\right) \tau_i$
4. $\hat{x}_i > x_i$ and $\hat{\tau}_i > \tau_i \Rightarrow f_i = t\theta\left(\hat{x}_i\hat{\tau}_i - x_i\tau_i\right).$

In cases 2 and 3, underreporting is discovered in one dimension only and the fine is imposed on assessed underreporting in that dimension multiplied by the declared value on the other dimension. Thus, given a declaration (x_i, τ_i)**,** the expected fine is given by (subscripts are suppressed where not necessary):

$$f = t\theta \left[\begin{array}{c} \int_x^y \int_\tau^h \left(\hat{x}\hat{\tau} - x\tau\right) g(\hat{x}, \hat{\tau}) d\hat{\tau} d\hat{x} + \tau G_{\hat{\tau}}(\tau) \int_x^y \left(\hat{x} - x\right) g_{\hat{x}}(\hat{x}) d\hat{x} + \\ + x G_{\hat{x}}(x) \int_\tau^h \left(\hat{\tau} - \tau\right) g_{\hat{\tau}}(\hat{\tau}) d\hat{\tau} \end{array} \right],$$

where $g(\hat{x}, \hat{\tau}) = g_{\hat{x}}(\hat{x}) g_{\hat{\tau}}(\hat{\tau})$. Given the hypothesis on the distributions, the expected fine is equal to:

$$f = t\theta \left[\left(h^2 + \tau^2\right)\left(y^2 + x^2\right) - 4\tau y x h\right] / \left(4yh\right). \tag{18}$$

In what follows, the equilibria with and without the minimum wage are characterized.

Equilibrium without minimum wage If the worker-firm pair chooses to declare τ hours and a product per hour x, the total surplus remaining within the firm-work pair, equivalent to the worker's net income because of the free entry assumption, is given by

$$I = yh - x\tau t - \gamma f, \tag{19}$$

where f is given by (18.) Therefore, the optimal declaration, equivalent to (4), is given by:

$$(x^*, \tau^*) \qquad s.t. \max_{x\in[0,y],\tau\in[0,h]} yh - x\tau t - \gamma f. \tag{20}$$

The first-order conditions are simultaneously satisfied iff

$$\tau^* = h\sqrt[2]{1-2\alpha} \qquad x^* = y\sqrt[2]{1-2\alpha},$$

where $\alpha = 1/(\gamma\theta)$. To have an interior solution, it is necessary that $\alpha < 1/2$, otherwise full evasion in both dimensions takes place. In what follows, it is assumed that $\alpha < 1/2$, i.e. enforcement is sufficiently strong to avoid full evasion. The maximand is locally concave at (x^*, τ^*); however, it is not globally concave. To establish whether (x^*, τ^*) is indeed the global maximum point, it is necessary to check the value of the function along the boundaries. As a reference, the income corresponding to reporting (x^*, τ^*) is

$$I^* = yh(1-t) + \alpha yht. \tag{21}$$

First, we analyse the boundaries within the axes, i.e. with full evasion in at least one dimension.

1 . Substituting $x = 0$ in (19), we get $I|_{x=0} = yh - t\left(h^2 + \tau^2\right) y/\left(4\alpha h\right)$, that is maximized for $\tau = 0$;

2. Substituting $\tau = 0$ in (19), we get $I|_{\tau=0} = yh - t(y^2 + x^2)h/\left(4\alpha y\right)$, that is maximized for $x = 0$;

Thus, when there is total evasion in one dimension, then it is also optimal to have total evasion in the other dimension. A positive declaration would only represent a lower bound on the fine to be paid. Therefore, we need to compare I^* given by (21) with the income corresponding to total evasion given by substituting $x = 0, \tau = 0$ in (19):

$$I^*_{bm} = yh - \gamma t\theta hy/4. \tag{22}$$

For $\alpha < 1/2$, we always have that $I^* > I^*_{bm}$.

The case with full compliance in at least one dimension is parallel to the case analyzed in the main model, where indeed there is assumed to be full reporting of the amount of hours worked.

3. In case $x = y$, then I is maximized for $\tau = (1-\alpha)h$, resulting in an income $I^*|_{x=y} = yh(1-t) + \alpha tyh/2$;

4. In case $\tau = h$, then I is maximized for $x = (1-\alpha)y$, resulting in the same income as in the previous case.

Thus, the income when there is total compliance in one dimension is $I^*_{fc} = I^*|_{x=y} = I^*|_{\tau=h}$. It is straightforward to show that $I^* > I^*_{fc}$.

So, the analysis at the boundaries shows that (x^*, τ^*) is indeed the global maximum point.

Equilibrium with a minimum hourly wage Given an hourly minimum wage ϖ, problem (20) becomes:

$$(x^*, \tau^*) \qquad s.t. \max_{x \in \{0\} \cup [\varpi, y], \tau \in [0,h]} yh - x\tau t - \gamma f.$$

Parallel to the main model, workers split into three categories:

1. High productivity: $y_i > \varpi / \sqrt[2]{1-2\alpha}$;

2. Intermediate productivity: $\varpi \leq y_i \leq \varpi / \sqrt[2]{1-2\alpha}$;

3. Low productivity: $y_i < \varpi$.

High productivity workers are unaffected by the introduction of the minimum wage as they would have declared higher hourly earnings anyway. Low productivity workers are expelled from the formal labour market and can choose black market activity or inactivity. The choice is made by comparing income in the two cases, given by (22) and 0, respectively. This gives rise to the following condition:

$$I^*_{bm} > 0 \Leftrightarrow \alpha > t/4. \tag{23}$$

As in the main model, if enforcement is very effective (low α), then the minimum wage has an efficiency cost as workers with positive productivity withdraw into idleness. If enforcement is instead not too effective, workers with an hourly productivity below the minimum wage work completely underground.

To analyse the behaviour of workers with intermediate productivity, we need to compare the income when declaring the minimum wage to the income when being completely underground and when fully reporting.

When declaring the minimum wage, i.e. $x = \varpi$, the amount of declared hours maximizing income is given by $\tau_{mw} = 2yh\varpi(1-\alpha)/\left(y^2+\varpi^2\right)$, giving an income:

$$I^*_{mw} = yh - th\left[(y^2+\varpi^2)^2 - (2y\varpi)^2(1-\alpha)^2\right] / \left[4\alpha y\left(y^2+\varpi^2\right)\right].$$

A worker firm pair can always choose to be completely in the informal economy, i.e. $x = \tau = 0$. We have seen that this is the best that can be done when there is full evasion in at least one dimension. Income in case of full evasion is given by (22).

The choice between full evasion and declaring the minimum wage is made by comparing income in the two cases. It turns out that:

$$I^*_{mw} > I^*_{bm} \Leftrightarrow y_i > \varpi/\sqrt[2]{4(1-\alpha)^2 - 1} \equiv y_{mw}.$$

As the minimum wage constraint is binding only if $y_i < \varpi/\sqrt[2]{1-2\alpha}$ and $y_{mw} < \varpi/\sqrt[2]{1-2\alpha} \quad \forall\alpha < 1/2$, there is always a productivity interval where workers prefer increasing their compliance to the minimum wage rather than decreasing it by declaring zero.

To complete the analysis, we need to analyse the remaining boundaries, i.e. the case with full reporting in at least one dimension.

In case $x = y$, the maximum income that can be achieved is I^*_{fc} , where $I^*_{mw} > I^*_{fc}$ and $I^*_{bm} > I^*_{fc}$ for workers whose productivity is such that they are affected by the minimum wage. In case $\tau = h$, the maximum income that can be achieved is certainly less than I^*_{fc} and thus less than I^*_{mw} and I^*_{bm}. Thus, the choice faced by this type of worker is indeed between increasing compliance to the minimum wage level or decreasing it to zero.

In this section, the model has been extended by allowing hours of work to be underreported, subject to the same detection technology as earnings. Also in this case does the introduction of the minimum wage induce some workers to increase compliance, thereby producing a spike at the minimum wage level. Proposition 4.1 is thus robust to this extension. As the minimum wage acts as an effective constraint for the low-productivity part of the workforce, Propositions 4.2 and 4.2 extend to this more general setting.

7.2 The black economy

The model presents no discontinuity when a firm-worker pair leaves the formal economy and goes completely underground. It may, however, be argued that being completely in the black economy is substantially different than being part of the official economy. In particular, we analyse the implication of possible discontinuities in two key variables: productivity and expected fines. In the analysis, we assume that enforcement parameters are such that there is underreporting.

Productivity discontinuity While it seems unlikely that the product generated by a firm-worker pair is dependent on the reporting behavior in case of simple underreporting, it is more plausible that completely entering into the black economy may have an effect. More difficult access to the

legal protection system to enforce contracts and property rights, inability to tap formal credit, restricted possibility to advertise, no access to support programmes (like training schemes, subsidies to R&D) for enterprises are some of the factors that may cause a decrease in the surplus once a firm goes underground. On the other side, the avoidance of official regulation and red tape may boost the product of firms fully in the underground economy (see Loayza, 1996, for a review). The relative relevance of the pros and cons depends on the specific situation of a country. For instance, an ineffective court system and a credit market that is not accessible for some types of enterprises (like SME) even if registered may decrease the disadvantage of being underground.

Extending the model to take this potential discontinuity into account is straightforward. Assume that productivity is

$$\begin{cases} y_i & if \quad x_i > 0 \\ y_i + d & if \quad x_i = 0 \end{cases} \text{ or } \begin{cases} y_i & if \quad x_i > 0 \\ \eta y_i & if \quad x_i = 0 \end{cases}.$$

In case $d < 0$ or $\eta < 1$, the cons of being in the black market outweight the pros. When there is no minimum wage nothing changes. When there is a minimum wage ϖ, then the worker-firm pair has a greater incentive to increase compliance to the minimum wage level, instead of going into the black market, thus reinforcing the tendency to show a spike at the minimum wage level and the positive impact of minimum wage on fiscal revenues.

In case $d > 0$ or $\eta > 1$ (and $\alpha > t/2$), being in the black market provides an advantage as compared to being in the official economy. In case of an addictive productivity difference, when there is no minimum wage, worker-firm pairs characterized by low productivity, i.e. with $y_i < d(2\alpha - t)/\left[t\left(1-\alpha\right)^2\right]$, will go into the black market, while nothing changes for higher productivity pairs. When there is a minimum wage, a positive productivity advantage

of being in the black market reduces the incentive for firms to declare the minimum wage level instead of going into the black economy, but as long as the minimum wage is sufficiently high as compared to the productivity differential, in particular for $d < t\varpi\left[2y\left(1-\alpha\right)-\varpi\right]/\left[\left(2\alpha-t\right)y\right]$, there is still a spike at the minimum wage level. In case the productivity difference is multiplicative, for the no minimum wage case, a sufficiently low productivity advantage, i.e. $\eta < 1 + t(1-\alpha)^2/\left(2\alpha - t\right)$, is necessary for avoiding that all agents go into the black market. In such circumstances, the incentives to declare the minimum wage are reduced, but do not disappear. In particular, a spike at the minimum wage level will be present anyway.

Discontinuity in expected fines A discontinuity at zero declaration may also exist with regard to the expected fine. Once more, it is not a priori obvious in which direction such a discontinuity may work. On the one hand, the non-existence of a company in official registers may make it more difficult to localize it and perform an audit. On the other hand, once an audit is performed, proving underreporting is much more difficult than proving non-reporting, as in the latter case the operation of a firm without registration constitutes evidence in itself. Discontinuities may also exist in the fine applied in case of detection, with complete underreporting being likely to be punished more harshly than partial underreporting. Assume the expected fine to be:

$$\begin{cases} \gamma f & if \quad x_i > 0 \\ \rho\gamma f & if \quad x_i = 0 \end{cases}, \text{ where } f \text{ is given by (1).}$$

In case $\rho > 1$**,** being in the black market gives rise to higher expected fines due to a higher probability of auditing or higher fines imposed in case of detection. Without a minimum wage, nothing changes. With a minimum wage, the incentives to declare the minimum are stronger.

In case $\rho \in (0,1)$, being in the black market gives rise to lower expected fines due to a lower probability of auditing. Unless the advantage of being in the black market is not too high, every agent goes underground. In particular for $\rho > (2-\alpha)\,\alpha$, the equilibrium without a minimum wage will not change, while in case of a minimum wage, the incentives to declare the minimum wage instead of going into the black economy are reduced, but do not disappear, with a spike remaining at the minimum wage level.

7.3 Entitlements from social security

Social security contributions usually provide entitlements in the form of pensions, unemployment benefits, health insurance, maternity benefits and so on. If workers value such entitlements, their existence represents an incentive to contribute and should be taken into account when analyzing the evasion decision. Entitlements are usually partly linked to contributions and partly independent of them. Below, the implications for the model are analyzed for each case.

Proportional transfers Suppose that workers receive a transfer proportional to their declared wage, ϑx, from social security institutions. In theory, their value of this could be more than its cost, i.e. $\vartheta > t$. This may be the case when social security funds run a deficit or are subsidized by the general budget (and thus by fiscal imposition on a different tax base) or when workers value these transfers highly (for instance because they provide some insurance that, due to some market failure, cannot be purchased separately.) In this case, however, there is no reason to evade taxes, so we assume, more realistically, that $\vartheta < t$. Equation (3) becomes:

$$I = w - tx + \vartheta x.$$

In case equation (1) is also modified, so that fines are paid only on the amount of evasion net of foregone benefits, the model is simply modified by substituting $(t - \vartheta)$ to t. In case fines continue to be paid on evaded taxes, the solution to (4) becomes

$$x = (1 - \alpha + \alpha\vartheta/t)\, y.$$

Not surprisingly, evasion declines, while a positive correlation between the tax rate and the portion of income that is evaded appears. This is consistent with the results reported by Alm et al. (1990) in their study about Jamaican employees' tax evasion and avoidance. They find that "the tax base rises with higher benefit for payroll tax contributions and falls with higher marginal tax rates", albeit estimated elasticities are small. As for the effects of the minimum wage, the productivity threshold above which workers prefer to declare the minimum wage is lower in case of transfers proportional to contributions, thus possibly increasing the size of the spike.

Lump-sum transfers Here, the case of a lump-sum transfer δ is analyzed. The transfer is assumed to be conditional on formal working status. In the absence of a minimum wage, the only effect of a lump-sum transfer is to displace complete evasion emerging when enforcement is weak with a minimal declaration, so as to qualify for the transfer by formally being part of the workforce. More interestingly, in case of a minimum wage, a transfer conditional on formal working status represents a further incentive to declare the minimum wage instead of going into the black market and thus reduces the productivity threshold above which workers prefer to declare the minimum wage. In particular, the threshold (15) becomes

$$y_{mw} = \varpi / \left[2(1 - \alpha) + 2\alpha\delta / (t\varpi)\right].$$

The lump-sum transfer δ should be intended as the difference between transfers conditional on being employed and transfers conditional on not being employed (unemployment benefits or other forms of social support.) In case $\delta < 0$, then the threshold would be higher as being formally employed would mean giving up some net transfer, but the effects of the minimum wage will not disappear as far as the monetary loss in case of official employment status is sufficiently low compared to the minimum wage, in particular for $|\delta| / \varpi < t(1-\alpha)/(2\alpha)$.

8 Conclusions

The paper develops a tractable model of underreporting of earnings by employed labour and works out the implications of introducing minimum wage regulation in such an environment.

A contribution of the paper to the literature on tax evasion is to show that imperfect detection alone is able to generate an internal solution to the tax evasion decision, even with a fixed probability of an audit and risk neutrality by the agent subjected to this.

The interaction between tax evasion and minimum wage gives rise to a spike at the minimum wage level. This is a mechanism that has never been proposed in the literature, that works in a perfectly competitive labour market and that can account for the double digit spike present in some countries.

In addition, the model contributes to the policy discussion on minimum wage in countries where underreporting of earnings is a relevant phenomenon. In particular, it is shown that introducing or increasing the minimum wage can boost fiscal revenues. The discussion of the fiscal impact of the minimum wage has usually focused on the expenditure side. The role of the state as

an employer or the fact that, in some countries, social benefits are indexed to the minimum wage are two reasons why a higher minimum wage might deteriorate the fiscal balance. This paper claims that this may not be the case, if the effect on revenues is sufficiently large to counterbalance the higher spending. However, the boost in revenues is due to extracting more resources from the lower end of the productivity distribution and introduces some degree of regressivity in the fiscal system.

The model also makes a new prediction about the correlation between the size of the spike at the minimum wage level and the size of the informal economy that finds some support in the data.

The optimal auditing strategy by a tax authority in case it possesses an imperfect detection technology is subject to ongoing research.

References

[1] Acemoglu, D. (2001), "Good jobs versus bad jobs", *Journal of Labor Economics*, vol. 19, pp. 1-21.

[2] Alm, J., Bahl, R., and Murray, M. (1990), "Tax structure and tax compliance", *The Review of Economics and Statistics*, vol. 72(4), pp. 603-613.

[3] Andreoni, J., Erard, B., and Feinstein, J. (1998), "Tax compliance", *Journal of Economic Literature*, Vol. 36, pp. 818-860.

[4] Boeri, T., and Garibaldi, P. (2007), "Shadow Sorting", in: Pissarides, C. and Frenkel, J. (eds.) *NBER International Seminar on Macroeconomics 2005*, MIT Press, Cambridge, MA.

[5] Brown, C. (1999), "Minimum wages, employment, and the distribution of income", *Handbook of Labor Economics*, vol. III, Elsevier, Amsterdam.

[6] Card, D., and Krueger, A. (1995), *Myth and measurement*, Princeton University Press, Princeton, NJ.

[7] Cowell, F. (1985), "Tax evasion with labour income", *Journal of Public Economics*, vol. 26, pp. 19-34.

[8] Dickens, R., and Manning, A. (2004), "Spikes and spill-overs: the impact of the national minimum wage on the wage distribution in a low-wage sector", *Economic Journal*, vol. 114, pp. 95-101.

[9] DiNardo, J., Fortin, N., and Lemieux, T. (1996), "Labor market institutions and the distribution of wages, 1973-1992: a semiprametric approach", *Econometrica*, vol. 64(5), pp. 1001-1044.

[10] Draca, M., Machin, S., and Van Reenen, J. (2006), "Minimum wage and firm profitability", IZA DP 1913.

[11] Erard, B. (1997), "Self-selection with measurement errors. A microeconometric analysis of the decision to seek tax assistance and its implications for tax compliance", *Journal of Econometrics*, vol. 81, pp. 319-356.

[12] Eurostat (2004), "Minimum Wages - EU Member States, Candidate Countries and the US", *Statistics in focus 10/2004*.

[13] Eurostat (2006), "Small decline in overall taxation to 39.3% of GDP in 2004", *News release 62/2006*.

[14] Feinstein, J. (1991), "An econometric analysis of income tax evasion and its detection", *RAND Journal of Economics*, vol. 22, pp. 14-35.

[15] Feldman, N., and Slemrod, J. (2007), "Estimating tax noncompliance with evidence from unaudited tax returns", *The Economic Journal*, vol. 117(518), pp. 327-352.

[16] Fugazza, M., and Jacques, J. (2003), "Labor market institutions, taxation and the underground economy", *Journal of Public Economics*, vol. 88, pp. 395-418.

[17] Klepper, S., and Nagin, D. (1989), "The anatomy of tax evasion", *Journal of Law, Economics, and Organization*, vol. 5(1), pp. 1-24.

[18] Kolm, A., and Nielsen, S.B. (2005), "Under-Reporting of Income and Labor Market Performance", WP 2005:5, Stockholm University.

[19] Lemos, S. (2006), "A Survey of the Effects of the Minimum Wage on Prices", forthcoming *Journal of Economic Surveys*.

[20] Loayza, N. (1996), "The economics of the informal sector: a simple model and some empirical evidence from Latin America", *Carnegie-Rochester Conference Series on Public Policy*, vol. 45, pp 129-162.

[21] NSSI (Undated), "Report from Jordan Hristoskov - governor of the National Social Security Institute, concerning of the State Social Security Budget, 2003", *National Social Security Institute*, Sofia.

[22] OECD (2003), *Labour market and social policies in the Baltic countries*, OECD, Paris.

[23] OECD (2004), *Employment Outlook*, OECD, Paris.

[24] Petrova, A. (2005), "Wages: On the Payroll or Under the Table?", *Public Opinion Foundation*, Moscow.

[25] Renooy, P., Ivarsson, S., van der Wusten-Gritsai, O. and Meijer, R. (2004), "Undeclared work in an enlarged Union", European Commission, Bruxelles.

[26] Sandmo, A. (1981), "Income tax evasion, labour supply, and the equity-efficiency tradeoff", *Journal of Public Economics*, vol. 16, pp. 265-288.

[27] Schneider, F. (2005), "Shadow economies of 145 countries all over the world: estimation results over the period 1999 to 2003", Working Paper.

[28] Schneider, F., and Enste, D. (2000), "Shadow economies: size, causes, and consequences", *Journal of Economic Literature*, vol. 38(1), pp. 77-114.

[29] Simon, K.I., and Kaestner, R. (2004), "Do minimum wages affect non-wage job attributes? Evidence on fringe benefits", *Industrial and Labor Relations Review*, vol. 58, pp. 52-70.

[30] Slemrod, J., and Yitzhaki, S. (2002), "Tax avoidance, evasion, and administration", in: Auerbach, A., Feldstein, M. (eds.) *Handbook of Public Economics*, vol. III, Elsevier, Amsterdam.

[31] Tomev, L. (2004), "Thematic feature - industrial relations and undeclared work", EIRO, www.eiro.eurofound.eu.int.

[32] Yaniv, G. (1992), "Collaborated employee-employer tax evasion", *Public Finance/ Finances Publiques*, vol. 47(2), pp. 312-321.

[33] World Bank (2005), *Enhancing Job Opportunities: Eastern Europe and the Former Soviet Union*, World Bank, Washington, DC.

[34] _. (2006), *Turkey. Country Economic Memorandum*, World Bank, Washington, DC.

[35] _. (2007), *Informality: Exit and Exclusion*, World Bank, Washington., DC.

Appendix

A1 - An alternative setting for imperfect detection

The tax authority devotes $\gamma \geq 0$ units of "auditing resources" to every firm-worker pair. The more resources, the more income is discovered in expectation. In particular, if γ units of resources are used, then $\hat{y}$, the income for which the tax authority can find evidence, is distributed with uniform probability over the interval $[(1-a^{-\gamma})y, y]$, where $a > 1$ measures the effectiveness of auditing. Thus,

- if $\gamma = 0$, i.e. no resources are used, the interval is $[0, y]$. The fact that even with no resources there is the possibility of discovering some evasion may be interpreted as the emergence of evidence from other investigations or from receiving denunciation or by other costless means;
- if $\gamma \to +\infty$ the (degenerated) interval is $[y, y] = \{y\}$, i.e. full income is discovered with certainty;

The pdf of the distribution over the interval $[(1-a^{-\gamma})y, y]$ is $h(\hat{y}) = a^{\gamma}/y$. Given that the tax authority devotes resources γ to a taxpayer characterized

by true income y and declared income x, then the expected fine is

$$f = \begin{cases} t\theta \int_x^y (\hat{y} - x) h(\hat{y}) d\hat{y} & if \qquad x \geq (1 - a^{-\gamma}) y \\ \left[(1 - a^{-\gamma}) y - x \right] t\theta + t\theta \int_x^y (\hat{y} - x) h(\hat{y}) d\hat{y} & if \qquad x < (1 - a^{-\gamma}) y \end{cases}.$$

As the part of undeclared income below $(1-a^{-\gamma})y$ is discovered with certainty and a fine is imposed on it, it will never be the case that $x < (1 - a^{-\gamma})y$, provided that the taxpayer knows the detection technology and γ. Thus, concentrating on $x \geq (1 - a^{-\gamma})y$ we have

$$f = t\theta \int_x^y (\hat{y} - x) h(\hat{y}) d\hat{y} = a^{\gamma} t\theta (y - x)^2 / (2y),$$

which is equivalent to (7), where the probability of an audit being performed, $\gamma \in [0, 1]$, is substituted by the coefficient $a^{\gamma} \geq 0$.

A2 - Audit conditional on report x

The probability of performing an audit can be conditioned on declared income x, so that $\gamma = \gamma(x)$

As far as $\gamma(x)\theta < +\infty$, it is impossible to induce full compliance.

Proof. Given a tax liability y and a probability of an audit $\gamma(x) \in [0, 1]$ full compliance is preferred to declaring $x \in [0, y)$ iff

$$(1 - t)y > y - \gamma(x) f - tx.$$

Using (7), this becomes

$$(1 - t)y > y - \gamma(x) \frac{t\theta}{2y} (y - x)^2 - tx \Leftrightarrow \theta\gamma(x) > \frac{2y}{y - x} \equiv \gamma^*_{x,y}.$$

As $\lim_{x \to y^-} \gamma^*_{x,y} = +\infty$, then, as far as $\gamma\theta < +\infty$, there is a neighborhood of y in which the above condition cannot hold and thus underreporting is preferred to full compliance. ■

In the alternative setting for imperfect detection proposed in this appendix, the equivalent condition not to have full compliance even in case of devoted "auditing resources" conditional on declared income is $\alpha^{\gamma(x)}\theta < +\infty$.

The above proposition implies that whatever auditing policy is implemented, there will be some evasion at any income level. So, for any auditing policy, there is room for the minimum wage to exert its influence. However, a fixed cost for the taxpayer of being subject to an audit, together with a higher probability of being audited in case of non-compliance than in case of full compliance, would undo the result.

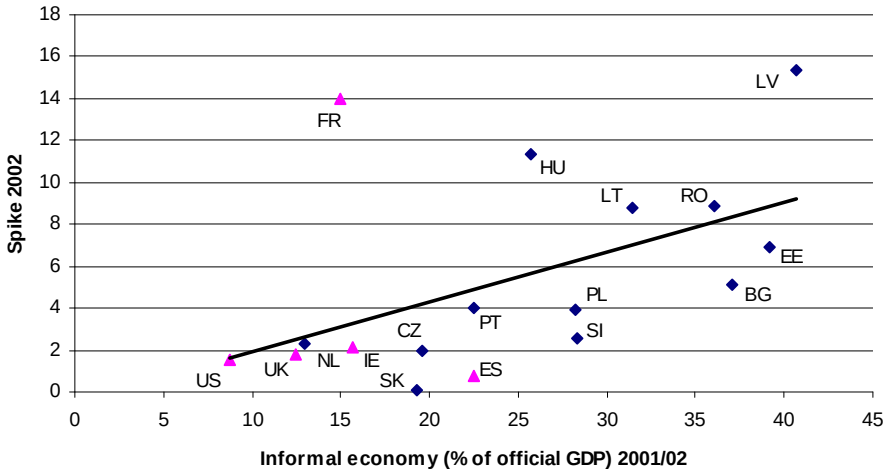
A3 - Evidence on underground economy and minimum wage spike

As stated in Proposition 5, a prediction of the model is that enforcement parameters (as summarized by α) should induce a positive correlation between the spike at the minimum wage and the size of the informal economy relative to the formal economy. In this section, some supporting evidence is presented.

The two figures in this section present the relationship of the spike at the minimum wage[22] with the size of the informal economy relative to the

[22]The proportion of full-time employees with earnings exactly equal to the monthly minimum wage (source: Eurostat). Notice that the data collected by Eurostat are obtained from administrative sources. For data points indicated with a triangle, the definition is different: part-time workers are included (France, Spain), minimum wage is fixed on an hourly basis (France, Ireland, UK, USA), earnings below the minimum wage are also

Figure 4: Informal economy and minimum wage spike



Sources: informal economy: Schneider (2005); spike: Eurostat

formal economy[23] and the ratio of the minimum wage to the average wage[24] (in what follows, this measure is indicated as the Kaitz index.) The countries included are all countries for which Eurostat reports data on the minimum wage spike and Schneider (2005) reports estimates of the informal economy. The sample includes 16 European countries and the US. Ten of the European countries are Central and Eastern European, where statutory minimum wage arrangements are common.

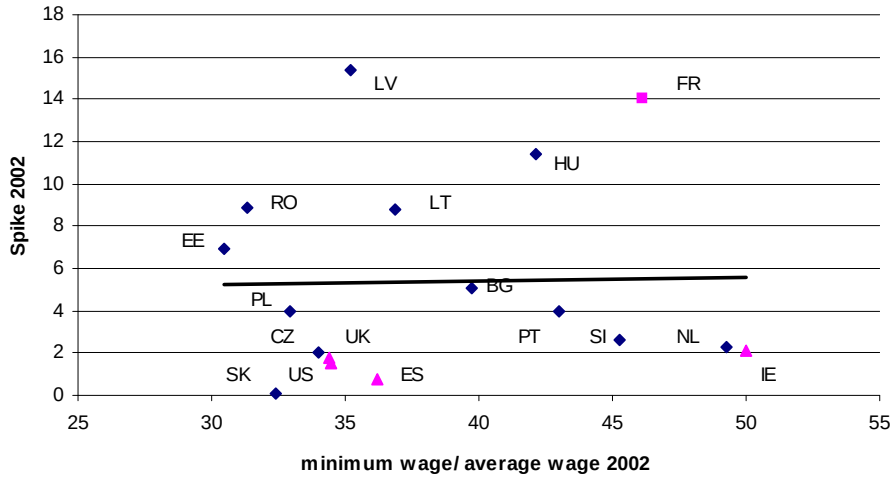
A positive correlation clearly appears between the size of the spike at the

included (UK, USA). See Eurostat (2004) for details.

[23]Informal economy as % of official GDP (source: Schneider 2005).

[24]Minimum monthly wage as a proportion of average monthly earnings in industry and services (source: Eurostat). For France, the figure has been calculated by the author dividing the hourly gross wage by the average gross hourly wage for a full-time employee in industry, trade and services (data source: INSEE.)

Figure 5: Kaitz index and minimum wage spike



Source: Eurostat, except France - Kaitz Index: own calculations

minimum wage level and the estimated size of the informal economy (see figure 4). As mentioned in the introduction, other mechanisms have been proposed to explain the existence of a spike at the minimum wage level and one natural "culprit" for a high spike would be a minimum wage "biting" deeply into the wage distribution. However, no clear relationship appears between a measure of this "bite", the Kaitz index, and the size of the spike (see figure 5).

Regression analysis (see table 2) confirms that the positive relationship between the spike and the informal economy is not driven by a high minimum wage resulting in both a high spike and a sizeable informal economy. Regressing the size of the spike on the size of the informal economy and the Kaitz index, the former is significant, while the latter is not. The model suggests that the positive correlation between the size of the spike and the

size of the informal economy is instead driven by the common dependence on enforcement parameters. The regression implies that a 1% increase in the size of the informal economy is associated with a 0.28% increase in the share of employees earning the minimum wage.

Table 2: Determinants of minimum wage spike

Informal Economy as % of Official GDP (2002)	0.279^{**}
	(0.113)
Minimum Wage / Average Wage (2002)	0.179
	(0.179)
Constant	-8.337
	(8.381)
R^2	0.30
Observations	17

a. Dependent variable is spike at minimum wage level in 2002.

b. OLS estimation. Standard errors in parenthesis.

c. *** [**] (*) denote significance at 1, [5], and (10) percent level.