THERMODYNAMIC CONSIDERATIONS IN MICRO HEAT ENGINE DESIGN

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Abstract: The need for compact, high power-density power sources has led to significant research interest in micro heat engines. However, there is a lack of suitable thermodynamic models which can be used to evaluate the power performance of micro heat engines by taking into consideration the effect of leakage and finite heat input. This work is the first to develop such a thermodynamic model to predict the upper limit of performance of micro heat engines. The model allows investigation of the effects of design parameters such as length and material properties on the theoretical output which can be used for design guidance. Results from this model are further illustrated by comparison to the reported P3 micro engine.

Keywords: micro heat engine, power, thermodynamic model

INTRODUCTION

Interest in micro heat engines arises from their potential to meet the need for a compact, high power-density supply for microsystems. Micro heat engines have been touted [1] as a potential replacement for conventional electrochemical batteries as power sources, benefiting from the high energy density available from the combustion of hydrocarbon fuels [2], which is several orders higher than that of conventional batteries. Micro heat engines can also be fabricated using mainstream microfabrication processes and thus can potentially be integrated with other functional microcomponents [3].

Several attempts have been made to develop micro heat engines. Examples include a micro reciprocating engine [1], a micro rotary engine [2], and a micro turbine [3]. However, one of the major difficulties encountered in designing micro heat engines is the relative lack of models for evaluating key performance metrics (e.g. power, power density, efficiency). Thermodynamic models of heat engines can be used to predict the theoretical performance that can be potentially achieved by micro heat engine designs. An appropriate model can not only provide design guidance prior to micro heat engine fabrication but can also indicate the potential room for improvement in existing micro heat engines. This paper presents a new thermodynamic model to realistically predict the upper limit of power performance of micro heat engines.

REVIEW OF HEAT ENGINE MODELS

To the best of our knowledge, only a few works [4,5] reported to date focus on the performance evaluation of micro heat engines. These works are based on a thermodynamic heat engine model with a heat leakage component [6]. This is because heat leakage is a thorny issue in micro heat engine design due to the difficulty in achieving thermal isolation between the hot and cold sections of a micro engine [7], leading to significant losses. Although the efficiency vs. leakage characteristic of a micro heat engine can be derived from this model [4], the effect of heat leakage on power generation can not be inferred.

This is due to an implicit assumption in the model that an infinite amount of input heat can be drawn from the source, leading to generated power being independent from heat leakage [8]. However, in practice, heat flow from the source of a heat engine is necessarily finite [9]. The consideration of this finite heat input is particularly important for micro heat engines due to the down-scaling of the cross-sectional area and thus much reduced heat flow in total. This plays a critical role in determining the power and power density of micro heat engines which was not considered in previous models.
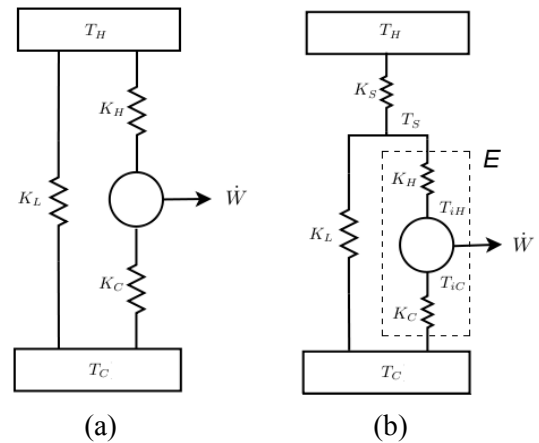
NEW FINITE HEAT MODEL

Figure 1: (a) Previous heat engine model with leakage. (b) New heat engine model with leakage and finite heat input.

Figure 1(a) shows a schematic diagram of the reported micro engine model [6,8] which has a heat leakage conductance (K_L) parallel to the engine. Figure 1(b) shows the new model developed here with an additional source conductance (K_S) between the source and the engine emphasizing the finite heat input in real situations. It also contains a heat leakage component K_L and an engine component E shown inside the dashed box. The leakage component is a thermal conductance K_L running between T_S and T_C, shunting

heat away from the engine component. The engine component E is in turn composed of a reversible Carnot engine running between a hot side temperature of T_{iH} and a cold side temperature of T_{iC}, with finite thermal conductance K_H connecting T_{iH} to the external hot side temperature T_H, and thermal conductance K_C similarly connecting T_{iC} to the external cold side temperature T_C. The component E shown in Figure 1(b) is an endoreversible thermodynamic model that has been used to model both internal and external combustion engines, including gas turbines, Otto engines, Diesel engines, and Stirling engines [5-6,8-9]. Hence, the results from this model could be generally applicable to most micro heat engine types.
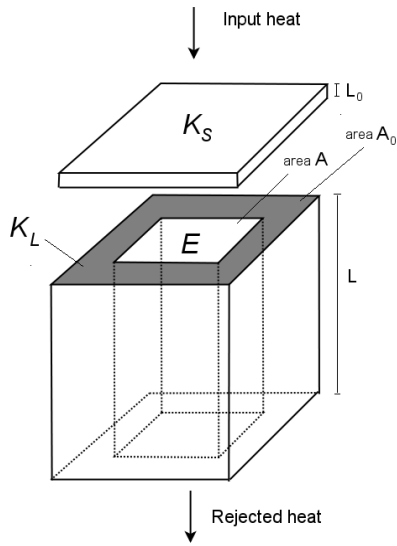

Figure 2: Illustration of thermal conductances in a micro heat engine

Practical heat losses in real micro heat engines can come from a variety of sources [5-6], however for this model we focus on K_S and K_L as illustrated on a simplified engine chamber layout, shown in Figure 2. Note that we can match the elements of this layout to the corresponding elements of the model shown in Figure 1(b), which underlines the physical basis of the thermodynamic model. Heat input comes externally from the top through the plate K_S and is conducted to both the engine E and the sidewalls K_L, representing the parallel heat loss through the walls of the engine. The magnitude of the heat conductances mainly depend on the thermal conductivity of the engine construction material, the engine cross-sectional area and the length.

Figure 3 is an analytical result that demonstrates the necessity of the new model for micro heat engines by considering the relationship between K_L and K_S. When $K_S \gg K_L$ or when K_S is effectively infinite, the generated power is independent of K_L as in the case of the previous model [4]. However, significant down-scaling in micro heat engines leads to the decrease of K_S (proportional to s^2) and increase of K_L (proportional to $1/s$), where s is the length scale. The increasing

K_L/K_S ratio with the down-scaling of micro heat engines suggests that heat leakage and finite heat input effect becomes relatively more prominent at the micro scale in comparison with that of macro engines. Figure 3 shows that for a fixed value of K_L, the value of K_S determines how much power is lost to leakage. For example, values of $K_L = 0.015$ W/K and $K_S = 0.263$ W/K can be obtained based on the reported P3 micro heat engine [10]. These values are plotted as dashed lines in Figure 3 and the intersection of the two lines corresponds to the estimated power output. If K_S is further reduced, the power output decreases correspondingly. In comparison, the predicted power of the previous model is indicated by the $K_S = \infty$ line, which clearly shows a significant deviation from the predicted power of the new model. Hence, the consideration of appropriate K_S is crucial for power prediction in micro heat engines, which is, for the first time, undertaken in this work.
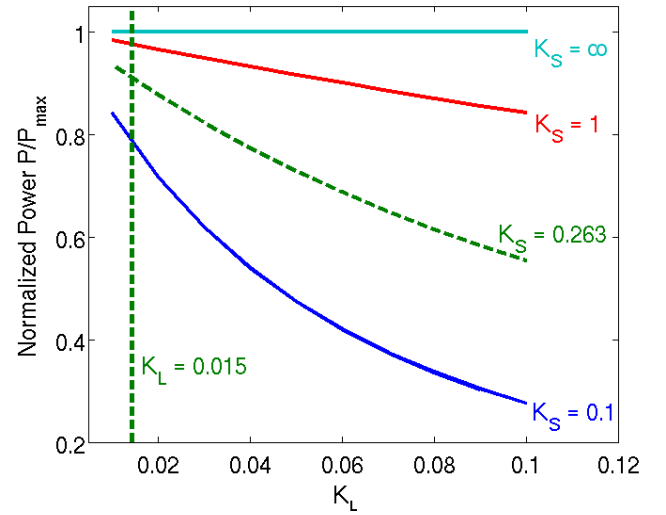

Figure 3: Effect of varying leakage and source conductances K_L and K_S on generated power at $T_H = 400$ K, $T_C = 300$K. P_{max} is calculated for each K_S with $K_L = 0$.

Using thermodynamic relations, the theoretical power can be evaluated, including both the heat leakage and finite heat input elements. Parameter $\tau = T_{iC}/T_{iH}$ is used to express the ratio of internal temperatures. The generated power can be expressed in terms of τ as:

$$\dot{W} = K_H K_C \frac{K_S T_H + 2K_L T_C + K_S T_C - \tau(K_S T_H + K_L T_C) - \frac{1}{\tau}(K_S + K_L)T_C}{K_S K_H + K_S K_C + K_H K_L + K_C K_L + K_H K_C}$$

This power expression is at a maximum when the derivative with respect to τ is equated to zero. Thus, the theoretical maximum power is:

$$\dot{W}_{max} = K_H K_C \frac{(\sqrt{K_S T_H + K_L T_C} - \sqrt{K_S T_C + K_L T_C})^2}{K_S K_H + K_S K_C + K_H K_L + K_C K_L + K_H K_C} \quad (1)$$

This condition occurs when:

$$\tau = \sqrt{\frac{(K_S + K_L)T_C}{K_S T_H + K_L T_C}}$$

In the extreme cases when $K_S = \infty$ or when $K_L = 0$, this condition is reduced to $\tau = \text{sqrt}(T_C/T_H)$, which aligns with the maximum power condition [9] predicted by using the previous model. The power density is just the power output divided by the total volume of the engine.

$$\text{Power density} = \frac{\dot{W}}{V} = \frac{\dot{W}}{AL} \qquad (2)$$

The efficiency at maximum power is the total power output at the maximum power condition divided by the total input heat.

$$\eta = \frac{\dot{W}}{\dot{Q}_S} = \frac{\dot{W}}{K_S(T_H - T_S)} \qquad (3)$$

RESULTS AND DISCUSSION

it is unlikely that real micro heat engines will operate at the theoretical limits predicted by thermodynamics, evaluation of the thermodynamic limit is important for design as well as a measure of the potential room for improvement in fabricated micro heat engines.

Equation (1) demonstrates the effect of length (L) on generated power at different source temperatures, and the results are as shown in Figure 4(a). It is observed that for larger lengths, the power output is close to P_{max} which represents the power generated without any heat leakage. When length is reduced, more of the finite input heat is diverted to the leakage component, causing a rapid fall-off in the generated power. The minimum length for a viable device depends on such factors as thermal conductivity k_L and temperature T_H. If the micro heat engine is designed for an output power level as a percentage of P_{max}, for example, to 90% of P_{max}, then for a given T_H and k_L, the L_{min} to achieve this power output can be determined from equation (1), a typical result of which is shown in Figure 4(b).
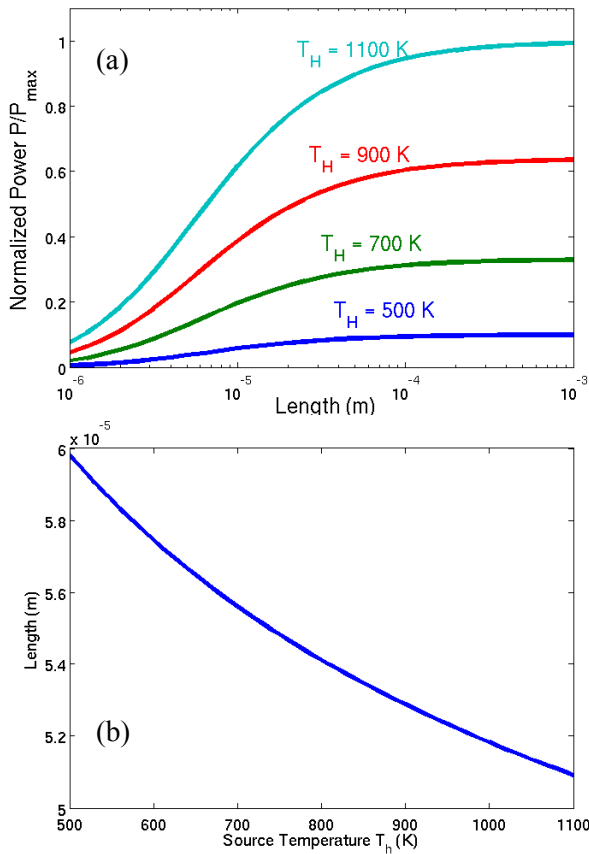

Figure 4: (a) Effect of length on generated power at different temperatures. P_{max} is taken to be P at $K_L = 0$. (b) The minimum length L_{min} at different source temperatures. Values calculated at $T_C = 300K$, $k_L = 0.15$ W/m-K.
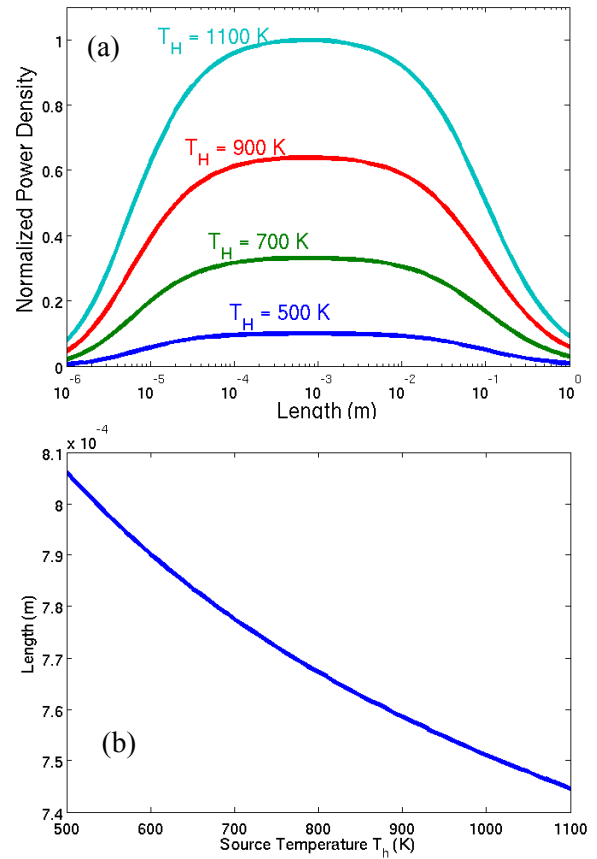

Figure 5: (a) Effect of length on power density at different temperatures. P_{max} is taken to be P at $K_L = 0$. (b) The optimum power density length L_{opt} at different source temperatures. Values calculated at $T_C = 300K$, $k_L = 0.15$ W/m-K.

The model equations indicate that design parameters such as thermal conductivity of the engine construction material (k_L), length (L), and source temperature (T_H) all affect the generated power. While

Figure 4 suggests that in order to increase the theoretical power output at a given T_H, the parameters L_{min} and k_L should be taken into consideration during the engine design stage. Taking the P3 as an example

again, by increasing the P3's engine length from 75 μm to 200 μm, the theoretical power output predicted by this model would increase from 9.7 mW to 10.4 mW, a 7% improvement. The relatively small improvement of this design change suggests that heat leakage is not a major concern in the current designs of the P3 engine. Alternatively, a change in the engine material to one of lower thermal conductivity k_L would produce the same effect as an increase in length.

For power applications where mobility, weight, and space are major concerns, e.g. as a portable power supply, the power density is a key performance objective, and can be calculated from equation (2). Since weight is generally proportional to volume, the power density calculated here is taken to be the equivalent of power per unit weight. Typical results obtained from equation (2) are shown in Figure 5(a). It shows that as the length is reduced, power density increases until it reaches an optimum value and then decreases due to the domination of the heat leakage. This optimum length is shown in Figure 5(b). To aim for high power density for a given engine material (k_L) and source temperature (T_H), it is important to design a micro heat engine at its optimum length.

Using the new model, it is also possible to study the relationship between the power performance and efficiency of micro heat engines by using equations (1) and (3), as shown in Figure 6.
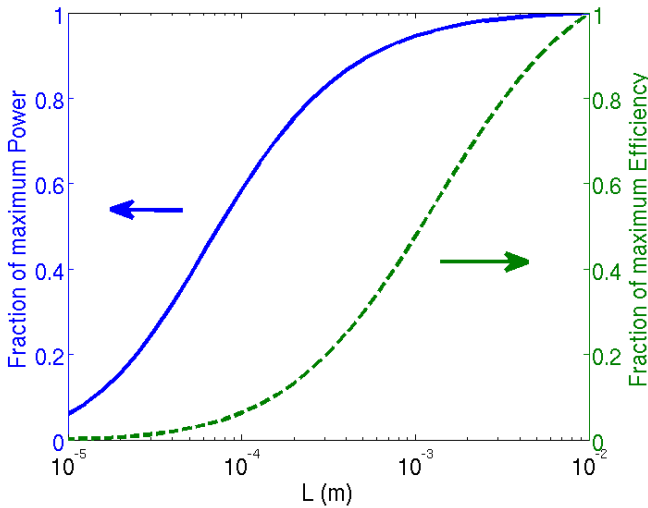

Figure 6: Comparison of power and efficiency vs. length characteristic at $T_H = 400$ K, $T_C = 300K$, $k_L = 1.5$ W/m-K.

This plot shows that for *L* in the range of 10^{-2} to 10^{-3} m, appreciable amounts of power can still be generated, even when the efficiency value is greatly reduced. This comparison study is not possible by using the previous model as that model predicts a constant power output with decreasing length [8]. Thus, for micro heat engines that mainly aim at power generation [2], the minimum length for a viable engine should be determined by the power curve in Figure 6

instead of the efficiency curve as stated in [4]. Since the power curve dies off at a lower length than the efficiency curve, this suggests further room for miniaturization than was previously believed.

CONCLUSION

A new thermodynamic model is developed to evaluate the theoretical output of power, power density, and efficiency of a micro heat engines by considering both heat leakage and finite heat input elements. The theoretical power output of a micro heat engine generally decreases as its length is reduced, and this reduction in power can be quantified by using the new model. For a given engine material and source temperature, an optimal length can be found that corresponds to maximum power density. The length effect on power and efficiency is also investigated, which suggests that for power generation applications, the appropriate length can be designed based on the projected power using the developed new model instead of efficiency.

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