Ding, L., Fujita, T. and Jones, K. (2005), Developing geometrical reasoning in the classroom: learning from highly experienced teachers from China and Japan. In, Bosch, Marianna (ed.) European Research in Mathematics Education IV. Barcelona, Spain: ERME, pp727-737. ISBN: 8461132823

DEVELOPING GEOMETRICAL REASONING IN THE CLASSROOM: LEARNING FROM HIGHLY EXPERIENCED TEACHERS FROM CHINA AND JAPAN

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Abstract:*Mathematics education has been the subject of considerable international comparative research, mostly focussed on pupil achievement but also examining teaching methods, curricula, and so on. In all this, and perhaps unsurprisingly, the role of teachers has emerged as a key influence on pupil learning. Given that the development of pupils' capability in geometrical reasoning continues to be an issue of considerable international concern, this paper reports an analysis of lower secondary school lesson suggestions prepared by highly experienced "expert teachers" from China and Japan, countries selected because they represent some interesting similarities and contrasts. The paper also gives background to these lesson suggestions in terms of the educational context in which they are presented.*

Introduction

The (recently renamed) *Trends in International Mathematics and Science Study* (TIMSS) is continuing to investigate pupil achievement, the mathematics curricula, teaching methods, and so on, across almost 50 countries around the world (see, for example, Mullis *et al*, 2000). Overall, the results to date of TIMSS suggest that there are significant similarities between the mathematics curricula across countries, especially in terms of topics specified, if not in overall curricular design (Schmidt *et al*, 1997; Valverde *et al*, 2002). Yet these broad correspondences of grade level and content become differences if examined more closely; both in the range of content addressed at a particular grade level and in particular developmental sequences where common content is addressed over several grade levels.

In terms of geometry teaching, while analysis of TIMSS data continues, a detailed comparative study of geometry specifications (Hoyles, Foxman and Küchemann, 2002), though covering fewer countries than TIMSS, found considerable variation in current approaches to the design of the school geometry curriculum. Thus, for example, the study found, a 'realistic' or practical approach apparent in Holland, while a theoretical approach is most evident in France and Japan. The study concludes by noting "there is evidence of a state of flux in the geometry curriculum, with most countries looking to change" (*op cit* p. 121).

As part of TIMSS, or related to it, a number of projects have examined the teaching methods that teachers (typically) use in various countries and, related to this, how teachers structure their lessons (see, for example, Shimizu, 2002; Stigler and Hiebert, 1999). To date there has been little comparative work specifically on how teachers structure mathematics lessons to develop geometrical reasoning. This is despite the issue of geometry teaching being of considerable international concern, especially its role in developing students' powers of reasoning (Mammana and Villani, 1998; Royal Society, 2001).

The analysis presented in this paper compares suggestions from highly experienced "expert teachers" for geometry lessons for lower secondary school classes in China and Japan, countries taken in alphabetic order and selected as they represent an interesting comparison (see methodology section for more on the choice of countries). The paper also analyses the range of influences that impinge on the way lessons are likely to be structured in the selected countries.

Comparative research on geometry teaching

Internationally, on average, it seems that the Grade 8 (UK Year 9) curriculum specifies greatest coverage of topics in fractions and measurement (see Mullis *et al*, 2000, chapter 5). Very few students internationally are given a major emphasis in geometry (three percent, on average), with, it seems, Tunisia the only country where 20 percent or more of the students are in classes that emphasise geometry over other areas of the mathematics curriculum. In terms of what is actually taught, teachers in the TIMSS survey report a range of instructional coverage across topics in geometry. For example, the topic "Simple two dimensional geometry – angles on a straight line, parallel lines, triangles and quadrilaterals" is reportedly taught to 95 percent of students (on average), while "visualization of three-dimensional shapes" is taught to only 57 percent, on average (with a variation across countries from 7 - 99%). Another geometrical topic that shows a large variation across countries is "symmetry and transformations", varying from being taught to 11% to 98% of Grade 8 students. According to their teachers, most students in Grade 8 receive moderate emphasis on geometry. On average internationally, by the end of their eighth grade, it seems that 22 percent of students are yet to be taught 50 percent or more of the geometry topics listed in the TIMSS survey (the list being generated by comparing curricula across countries.

Overall, and perhaps unsurprisingly, the role of the teacher emerges as a key influence on pupil learning. The latest TIMSS research related to the way teachers structure their lessons, the *TIMSS 1999 Video Study* (Hiebert *et al*, 2003), covered seven countries, including a number where students scored highly on the TIMSS achievement tests. This study found that some general features of Grade 8 mathematics lessons (including geometry lessons) were shared across the seven countries studied. For example, lessons were generally organised to include some public whole-class work and some private student work, the latter being mostly individual but with some involving small groups. Most lessons included some review of previous content as well as some attention to new content and, in the majority of cases, made use of a textbook or worksheet of some kind.

Notwithstanding these shared general features, the study reports discernible variation across the countries studies. Distinctions included how new content was introduced, the coherence across mathematical problems and within their presentation (ie the interrelation, both implicit and explicit, of the mathematical components of the lesson), the number and form of topics covered, the procedural complexity of the mathematical problems tackled, and classroom practices regarding individual student work and homework in class (although the report is not detailed enough to say anything specific about geometry lessons).

Overall, as Hiebert *et al* (2003, p149-50) emphasise, the video study found that the countries that show high levels of student achievement in the TIMSS achievement tests do not all employ teaching methods that combine and emphasise features in the same way. As they conclude:

> "The results of this study make it clear that an international comparison of teaching, even among mostly high-achieving countries, cannot, by itself, yield a clear answer to the question of which method of mathematics teaching may be best to implement in a given country".
>
> Hiebert *et al* (2003, p150)

This confirms that further research is needed to shed light on how teachers might best structure their lessons to develop geometrical reasoning.

Aims and theoretical framework

The principal aims of the research project, an initial analysis from which is reported in this paper, are two-fold:

- To determine the influences on the way geometry lessons might be taught in the selected countries;
- To analyse selected suggestions from highly experienced "expert teachers" in these selected countries – suggestions that regular teachers might use as a guide to structuring geometry lessons for lower secondary school students.

At the time of writing the authors are considering a range of theoretical notions with a view to determining which may be appropriate. For the purposes of the analysis presented below, the approach to analysing the lessons is derived in part from the study of textbook 'lessons' by Valverde *et al* (2002) – see next section for more on this.

Research methodology

The countries selected for study are China and Japan, chosen because they represent some interesting similarities and contrasts. Both countries have National Curricula for mathematics that covers geometry, amongst other mathematical topics. Yet, for teachers in the two countries there are different traditions and different ways in which they have responded to international developments over the years.

In terms of the influences on teaching, the sources of primary data selected for analysis in this research include:

- Government guidelines and other official documents
- Guidance documents and /or books for teachers

The specific sources of data providing suggested lessons are as follows:

- China: the data are mainly from the national teaching references (*The Compulsory Education Nine-Year Secondary School Mathematical References, 1995-1996*) and a popular teaching reference, *Master teachers' lessons records (Lower secondary school mathematics), 1992*. Such items are currently in widespread use by secondary school mathematics teachers throughout China.
- Japan: the data are suggested lesson plans by experienced teachers and university researchers (each with more than 10 years experiences, in general). The plans include information on the aims of lessons, problems for students, suggested activities for both teachers and students, time allocations, etc.

The analysis of the lesson suggestions is framed by the following procedure, derived in part from the study of textbook 'lessons' by Valverde *et al* (2002, Appendix A):

- Division of the suggested lesson into 'blocks' in terms of content, focus, and purpose;
- Identification of key features of geometry teaching, especially that focusing on the development of geometrical reasoning.

The analysis of the range of influences on lesson structure is based on a review of the literature.

Analysis

China: As a country with an extensive teaching tradition, teaching practices in China continue to be influenced by the ideas of Confucius (551-479 BCE) and by texts written in subsequent centuries. For example, the distinctive character of Confucianism in respect of learning is to ask questions constantly and to review previous knowledge frequently. In terms of mathematics teaching, the *Arithmetic of Nine Chapters*, a classic Chinese mathematics work of the Tang dynasty (618-907 CE), has greatly affected mathematics teaching and learning in China over centuries. This text lays down rules for solving problems and a sequencing of questions, answers and principles that continue to play an important role in the instructional model of teaching in China (An *et al.,* 2002, p 106). Traditionally, therefore, questioning is a key part in mathematics learning and teachers are likely to use good questions in motivating students to explore new problems. In addition, as Ashmore and Zhen (1997) demonstrate, review and conclusion are indispensable in classroom lessons in China

As is common in education, National Standard Examinations plays a critical role in school mathematics curriculum (Chongqing [China] Conference, 2002). Thus, according to Li (2002), mathematics teachers are likely to carefully select a considerable quality of exercises as one of their main teaching strategies. Consequently, completing exercises is a major feature of mathematics lessons. In addition, national textbooks are an essential teaching and learning resource. Teachers usually plan lessons by referring to such textbooks. The current textbooks in Shanghai, for instance, are arranged as a "spiral" curriculum, with new theorems, rules and formulae appearing in each unit. Consequently, mathematical terms and methods, which have already been taught, have to be frequently repeated through review, conclusion and exercises made by teachers in the lessons. Subsequently, new knowledge often follows introduction or experiment and this usually requires students to review previous knowledge. Given the above, mathematics lessons in China are likely to comprise the following segments:

1. Introduction/review/experiment (about 5 minutes)
2. The teaching of new content (about 25 minutes)
3. Exercises on the content introduced (about 10 minutes)
4. Homework assignment (about 5 minutes)

The case study below is a lesson record of a lesson from what, in China, is referred to as a "master teacher" (the teacher has more than 30 years teaching experience).

Lesson on 'Corresponding Angles, Alternate Angles, Interior Angles at the same side of a line'; grade 7, students aged 13-14, school in SiChuan Province, in south-west of China (Li, 1992, translated by Ding, 2004).

Objectives of teaching and learning of this lesson:

1. To clearly understand the concepts of corresponding angles, alternative angles and interior angles at the same side of a line.
2. To correctly recognise these angles in complex figures;
3. To be fully prepared for further studying about the properties of parallel lines

Introduction (+/- 5 minutes):

Discuss the location relationship of three lines on a plane

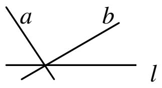


Figure 1

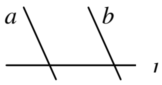


Figure 2

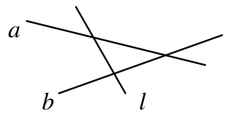
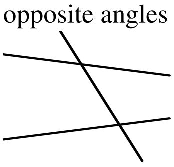


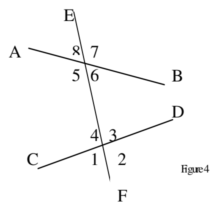
Figure 3

Focus on a figure in which two unparallel lines are crossed by the third line and review the concepts of vertically opposite angles and neighbour complementary angles;

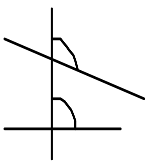
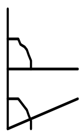
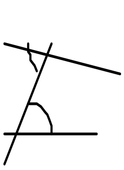
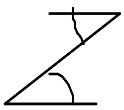
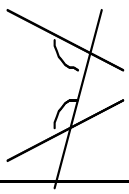
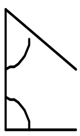


Teaching new knowledge (+/- 20 minutes):

1) Teach the concepts of 'Corresponding Angles, Alternate Angles, Interior Angles at the same side of a line' through observing figures:

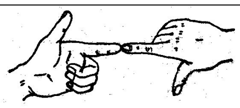


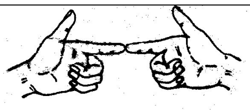
2) Complete the diagram as follows:

The name of angles	Basic figures	The characters of location	One side of the angles on the same cross line	The other side of the angles (which side of the cross line are they?)
Corresponding Angles			The same direction	The same side
Alternate Angles			The opposite direction	The different side
Interior Angles at the same side of a line			The opposite direction	The same side

Conclusion (+/- 5 minutes):

1) Review the concepts of the three types of angles learned in this lesson;
2) Use hands to present the different angles (See pictures below).


Picture 1


Picture 2

Exercises (+/- 10 minutes):

a) To recognise corresponding angles, alternate angles and interior angles at the same side of a line in figure 7;

b) To discuss whether a pair of alternate angles is equal and the sum of degree of a pair of interior angles at the same side of a line is 180°, when a pair of corresponding angles is equal? Why?

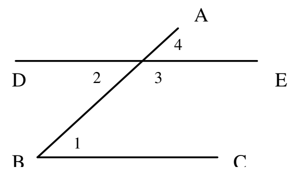


Figure 7

Japan: The way teachers structure their lessons in Japan is influenced by the specification of the mathematics curriculum, the design of textbooks, the occurrence of 'Lesson Studies', and research into the learning and teaching of mathematics. 'Lesson study', practiced in Japan for the last several decades, is one of the most common forms of professional development for Japanese teachers and involves teachers working in small teams collaboratively crafting lesson plans through a cycle of planning, teaching and reviewing (Yoshida, 1999). Through this process, Japanese teachers have collaboratively developed a view about 'good lessons of mathematics'.

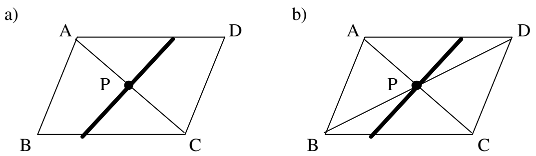
Research in the learning and teaching of mathematics that has influenced how teachers structure lessons includes the "Open-ended approach" in which 'the teacher gives the students a problem situation in which the solutions or answers are not necessary determined in only one way' (Sawada, 1997, p. 23). Considering the influences described above, in summary, Japanese teachers tend to structure mathematics lessons as follows (as also described in other research, including, for example, Stigler and Hiebert, 1999, pp.79-80):

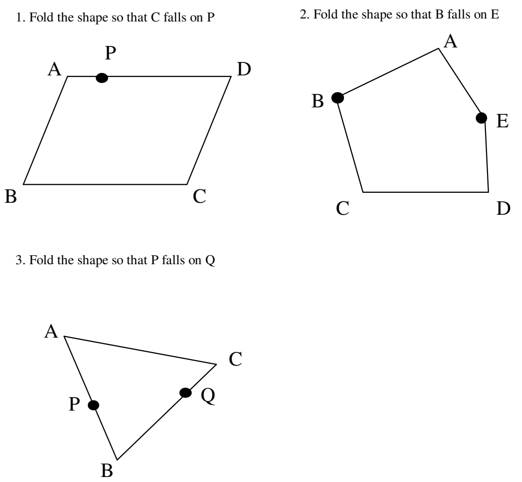
1. Presenting the problem(s) for the day:
    a) The problem(s) selected is/are designed to make students engage in mathematical activity in a challenging (or sometimes open-ended) situation
    b) Reviews of the previous lessons are sometimes included before the problem(s)
2. Development of the problem(s):
    a) Students work the problem(s) individually or in groups
    b) Discussion and presentations of solutions are often included
    c) Teachers clarify and/or extend the mathematical thinking of the students
    d) New problems, usually related to the problems for the day, are sometimes introduced

3. Highlighting and summarising the main point(s):
    a) Students' ideas are often used, and sometimes students are asked to explain their solutions
    b) The solutions of the problem(s) are summarised by the teacher
    c) By the end of the lesson, students would grasp mathematical concepts and deepen their mathematical thinking (often main goals of the lesson)

The case study presented below is a lesson record taken from Haneda (2002):

Perpendicular bisectors of segments; students aged 12-13 (Haneda, 2002, p. 38, translated by Fujita, 2004).

Year 7 (students 12~13)	The lesson on perpendicular bisectors of segments
Aim of the lesson	By the end of the lesson, students will be able to a) grasp the meaning of perpendicular bisectors of segments, and b) grasp the method of the construction, and be able to construct perpendicular bisectors of segments
Segment	**Description**
1 : Introduction	**Introducing problem 1** Problem 1: Let us fold a parallelogram ABCD so that C will fall on A, and consider how to draw the folded line.  a) Solution: drawing the perpendicular bisector of AC b) Solution: taking the intersection P of AC and BD, and drawing a perpendicular line to AC **Undertaking the construction by students**
	Notes for teachers - Give paper parallelograms and worksheet - Encourage students to try various ways of solutions - It is expected that students would notice the solutions a) or b) by looking at the facts that APC, 180 degree, is bisected when they actually fold paper parallelograms - In addition to the solutions a) and b), it is expected that students would use congruent quadrilaterals or angle bisectors which they have learnt to draw the line.

2: Development	**Introducing similar problems** Problem 2: Also consider how to draw folded lines in the following case  Undertaking the constructions by students
	Notes for teachers - Give worksheet for students - Give further tasks to students who finished the three problems - It is expected that students would use the construction of angle bisectors
3: Summary	**Summary** Knowing the lines which students drew are perpendicular bisectors of the segments Clarifying how to draw perpendicular bisectors of segments
	Notes for teachers - Explain clearly and precisely the words such as the mid-point or perpendicular bisectors - Clarify the simplest methods of the construction

Discussion

In each of the countries, the lesson structure followed the pattern expected for that country, something not altogether surprising given the evidence from existing research. Thus, in the lesson from China, new content is introduced and a considerable number of short tasks and questions are included in each segment of the lesson. In the lesson from Japan, the three-part structure is followed with a problem introduced in the first part and developed in the second before the main teacher explanation is given in the third.

As was found in the *TIMSS video studies* (Stigler and Hiebert, 1999; Hiebert *et al*, 2003), notwithstanding these shared general features, there is variation across the countries studied. For example, there is some variation in how new content is introduced – in the Chinese lesson through the teacher asking many questions, in the Japanese lesson through the teacher posing fewer, but perhaps more substantial, problems. Variation occurred, as in the *TIMSS video studies*, in the coherence of the lesson (ie the interrelation, both implicit and explicit, of the mathematical components of the lesson) and the procedural complexity of the mathematical problems tackled. There was also variation in the type of individual student work and the sort of homework set (if any).

Concluding comment

What this study has not been able to ascertain as yet are what the implications might be for student achievement in geometry in the countries under consideration. This is as an area for future research. Further research also needs to focus on what teachers actually do in lessons and whether, if, or how, they may make use of the advice that is available on how they might structure their geometry lessons.

References

An, Sh.H., Kulm, G., Wu, Zh., Ma, F. and Wang L.: 2002, The Impact of Cultural Differences on Middle School Mathematics Teachers' Beliefs in the U.S. and China. In: *Pre-conference Proceedings (Supplement) of the ICMI Comparative Study Conference, 20th-25th October, 2002, Hong Kong*. Hong Kong: The University of Hong Kong, Faculty of Education.

Ashmore, R.A. and Zhen, C.: 1997, *Teacher Education in the People's Republic of China.* Bloomington: Phi Delta Kappa Educational Foundation.

Chongqing [China] Conference: 2002, An Overview of the Mathematics Curricula in the West and East - summary of findings at the Chongqing conference in 2002. In: *Pre-conference Proceedings (Supplement) of the ICMI Comparative Study Conference, 20th-25th October, 2002, Hong Kong.* Hong Kong: Faculty of Education, The University of Hong Kong.

Haneda, A.: 2002, Chapter 3 Lesson plans in geometry in the 1st grade. *In*: S. Kunimune *et al*. (Eds.), *A Collection of Lesson Plans for Lower Secondary Schools, Geometry 2*, Tokyo Meiji Tosho publisher [in Japanese].

Hiebert, J., Gallimore, R., Garnier, H., Bogard Giwin, K., Hollingsworth, H., Jacobs, J., Miu-Ying Chui, A., Wearne, D., Smith, M., Kersting, N., Manaster, A., Tseng, E., Etterbeek, W., Manaster, C., Gonzales, P., & Stigler, J.: 2003, *Teaching Mathematics in Seven Countries: Results From the TIMSS 1999 Video Study* (NCES 2003-013). United States Department of Education, National Center for Education Statistics. Washington, DC.

Hoyles, C., Foxman, D. and Küchemann, D.: 2002, *A Comparative Study of Geometry Curricula*. London: Qualifications and Curriculum Authority.

Li, R.: 1992, Corresponding Angles, Alternate Angles, Interior Angles at the same side of a line. *In*: Master teachers' lessons records (Secondary school mathematics) editors association (Eds), *Master Teachers' Lessons Records (secondary school mathematics).* Shanghai: Shanghai Education Press. Lower Secondary School Edition [in Chinese].

Li, Sh.Q.: 2002, Practice Makes Perfect: a key belief in China. In: *Pre-conference Proceedings of the ICMI Comparative Study Conference, 20th-25th October, 2002, Hong Kong*. Hong Kong: The University of Hong Kong, Faculty of Education.

Mammana, C. and Villani, V. (Eds.): 1998, *Perspective on the Teaching of Geometry for the 21st Century*. Dordrecht: Kluwer.

Mullis, I. V. S., Martin, M. O., Gonzales, E. J., Gregory, K. D., Garden, R. A., O'Connor, K. M., Chrostowski, S. J., & Smith, T. A., 2000, *TIMSS 1999 International Mathematics Report: Findings from IEA's Repeat of the Third International Mathematics and Science Study at the Eighth Grade*. Chestnut Hill, MA: Boston College.

Royal Society/Joint Mathematical Council: 2001, *Teaching and Learning Geometry 11-19*, London, Royal Society/Joint Mathematical Council.

Sawada, T.: 1997, Developing Lesson Plans. *In*: Becker, J. & Shimada, S. (Eds.), *The Open-Ended Approach: a new proposal for teaching mathematics* (pp.1-9). Reston, VA: National Council of Teachers of Mathematics.

Schmidt, W. H., Jorde, D., Cogan, L. S., Barrier, E., Gonzalo, I., Moser, U., Shimizu, K., Sawada, T., Valverde, G. A., McKnight, C., Prawat, R. S., Wiley, D. E., Raizen, S. A., Brotton, E. D., & Wolfe, R. G.: 1996, *Characterizing Pedagogical Flow: an investigation of mathematics and science teaching in six countries*. Dordrecht: Kluwer Academic Publishers.

Shimizu, Y.: 2002, Capturing the Structure of Japanese Mathematics Lessons: some findings of the international comparative studies. Paper presented at the *ICMI-Second East Asia Regional Conference on Mathematics Education and Ninth Southeast Asian Conference on Mathematics Education*, May 27-31, 2002, National Institute of Education, Nanyang Technological University, Singapore.

Stigler, J. W. & Hiebert, J.: 1999, *The Teaching Gap: best ideas from the world's teachers for improving education in the classroom*. New York: Free Press.

Valverde, G. A. Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T.: 2002, *According to the Book: using TIMSS to investigate the translation of policy into practice through the world of textbooks.* Dordrecht: Kluwer Academic Press.

Yoshida, M.: 1999, Lesson study [Jugyokenkyu] in elementary school mathematics in Japan: a case study. Paper presented at the *American Educational Research Association* (1999 Annual Meeting), Montreal, Canada.