Jones, K., Fujita, T. and Ding, L. (2005), 'Teaching geometrical reasoning: learning from expert teachers from China and Japan', in D. Hewitt and A. Noyes (Eds), *Proceedings of the sixth British Congress of Mathematics Education* held at the University of Warwick, pp. 89-96. Available from www.bsrlm.org.uk.

TEACHING GEOMETRICAL REASONING: LEARNING FROM EXPERT TEACHERS FROM CHINA AND JAPAN

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International comparative research in mathematics education has found, perhaps unsurprisingly, that teachers are a key influence on pupil learning. Given that the development of pupils' capability in geometrical reasoning continues to be an issue of considerable international concern, this paper reports an analysis of lower secondary school lesson suggestions offered by expert teachers from China and Japan (countries selected because they represent some interesting similarities and contrasts). The analysis indicates some striking similarities between suggested lessons, but some noteworthy differences. Both these may be related to the educational context in which the lesson suggestions are presented.

The (recently renamed) *Trends in International Mathematics and Science Study* (TIMSS) is continuing to investigate pupil achievement, the mathematics curricula, teaching methods, and so on, across almost 50 countries around the world (see, for example, Mullis *et al*, 2000). Overall, the results to date suggest that while there are significant similarities between the mathematics curricula across countries, when these broad correspondences of grade level and content are examined more closely, differences are revealed, both in the range of content addressed at a particular grade level and in particular developmental sequences where common content is addressed over several grade levels (Schmidt *et al*, 1997; Valverde *et al*, 2002). Though not part of TIMSS, a comparative study of geometry specifications (Hoyles, Foxman and Küchemann, 2002) found considerable variation in approaches to the design of the school geometry curriculum. For example, the study found a 'realistic' or practical approach apparent in Holland, while a theoretical approach was most evident in France and Japan. Hoyles *et al* (*op cit* p. 121) conclude by noting "there is evidence of a state of flux in the geometry curriculum, with most countries looking to change".

Whatever the specification of the curriculum, research has found, perhaps unsurprisingly, that teachers are a key influence on pupil learning (see, for example, Stigler and Hiebert, 1999). As part of TIMSS, or related to it, a number of projects have examined the teaching methods that teachers (typically) use in various countries and, related to this, how teachers structure their lessons (see, for example, Shimizu, 2002; Stigler and Hiebert, 1999). To date there has been little in-depth comparative work specifically on how mathematics teachers structure their lessons to develop geometrical reasoning. This is despite the issue of geometry teaching being of considerable international debate, especially how such teaching might best contribute to supporting the development of students' powers of mathematical reasoning (Mammana and Villani, 1998; Royal Society, 2001).

Given the Royal Society report (2001, p11) recommendation that, “we need to develop a completely new pedagogy in geometry”, the research reported in this paper, part of a wider comparative study, is designed to contribute to developing pedagogical ideas by analysing suggestions from expert teachers for geometry lessons for lower secondary school classes in China and Japan (countries taken in alphabetic order and selected as they represent an interesting comparison). In order to appreciate these lesson suggestions, the paper also analyses the range of influences that impinge on the way lessons are likely to be structured in the selected countries.

COMPARATIVE RESEARCH ON GEOMETRY TEACHING

It seems that Grade 8 (UK Year 9) curricula across the world, on average, specify greatest coverage of topics in fractions and measurement, see Mullis *et al* (2000, chapter 5). Very few students internationally are given a major emphasis in geometry at this grade level (three percent, on average), with, it seems, Tunisia the only country where 20 percent or more of the students are in classes that emphasise geometry over other areas of the mathematics curriculum at Grade 8. In terms of what is actually taught, teachers in the survey report a range of coverage across topics in geometry. For example, the topic “angles on a straight line, parallel lines, triangles and quadrilaterals” is reportedly taught internationally, on average, to 95 percent of students, while “visualization of three-dimensional shapes” is taught to some 57 percent, on average, with a variation across countries from 7 - 99%. According to their teachers, most students in Grade 8 receive moderate emphasis on geometry. Even so, by the end of their eighth grade, around 22% of students (on average internationally) are yet to be taught half or more of the geometry topics listed in the TIMSS survey (the list being generated by comparing curricula across countries).

Overall, and perhaps unsurprisingly, teachers emerge as a key influence on pupil learning. The most recent TIMSS research related to the way teachers actually teach, the *TIMSS 1999 Video Study* (Hiebert *et al*, 2003), focuses on seven countries, including a number where students scored highly on the TIMSS achievement tests. This study found that some general features of Grade 8 mathematics lessons (including geometry lessons) were shared across the seven countries studied. For example, and perhaps unsurprisingly, lessons were generally organised to include some public whole-class work and some private student work, the latter being mostly individual but with some involving small groups. Most lessons included some review of previous content as well as some attention to new content and, in the majority of cases, made use of a textbook or worksheet of some kind.

Notwithstanding these shared general features, the study reports discernible variation across the countries studies. Distinctions included how new content was introduced, the coherence across mathematical problems and within their presentation (ie the interrelation, both implicit and explicit, of the mathematical components of the lesson), the number and form of topics covered, the procedural complexity of the mathematical problems tackled, and classroom practices regarding individual student work and homework in class (although the report is not detailed enough to say anything specific about geometry lessons). This confirms that further research is needed to shed light on how teachers might best structure their lessons to develop geometrical reasoning.

RESEARCH QUESTIONS AND METHODOLOGY

The principle aims of the research reported in this paper are two-fold: to determine the influences on the way geometry lessons might be taught in the selected countries; and to analyse selected suggestions from expert teachers in these countries that regular teachers might use as a guide to structuring geometry lessons. The countries selected for study are China and Japan, chosen because they represent some interesting similarities and contrasts. Both countries have National Curricula for mathematics that covers geometry, amongst other mathematical topics. Yet, for teachers in the two countries there are different traditions and different ways in which they have responded to international developments over the years.

In terms of the influences on teaching, the sources of primary data selected for analysis in this research include Government guidelines and other official documents, and guidance documents and /or books for teachers. The specific sources of data utilised in this paper as providing suggested lessons for lower secondary school students are, for China, a popular teaching reference, *Master teachers' lessons records (Lower secondary school mathematics)*, and, for Japan, *A Collection of Lesson Plans for Lower Secondary Schools.* These resources include information on the aims of lessons, problems for students, suggested activities, time allocations, etc. The analysis of the lesson suggestions is framed by a procedure derived in part from the study of textbook 'lessons' by Valverde *et al* (2002, Appendix A) and includes division of the suggested lesson into 'blocks' in terms of content, focus, and purpose, together with identification of key features of geometry teaching, especially those focusing on the development of geometrical reasoning.

ANALYSIS

The analysis that follows focuses on comparing the context which is likely to influence the structures of lessons in the two selected countries, together with case-study comparisons of one lesson from each country.

China: As a country with an extensive teaching tradition, teaching practices in China continue to be influenced by the ideas of Confucius (551-479 BCE) and by texts written in subsequent centuries (An *et al.,* 2002). For example, the distinctive character of Confucianism in respect of learning is to ask questions constantly and to review previous knowledge frequently. In terms of mathematics teaching, the *Arithmetic of Nine Chapters*, a classic Chinese mathematics work of the Tang dynasty (618-907 CE), has greatly affected mathematics teaching and learning in China over centuries. This text lays down rules for solving problems and a sequencing of questions, answers and principles that continue to play an important role in the instructional model of teaching in China (An *et al.,* 2002, p 106). Traditionally, therefore, questioning is a key part in mathematics learning and teachers are likely to use good questions in motivating students to explore new problems. In addition, as Ashmore and Zhen (1997) demonstrate, review and conclusion are indispensable in classroom lessons in China. As is common in education, examinations play a critical role in school mathematics curriculum and thus, according to Li (2002), mathematics teachers in China are likely to carefully select a considerable quantity of exercises as one of their main teaching strategies. Given the above, mathematics lessons in China are likely to comprise the following segments:

1. Introduction/review/experiment (about 5 minutes)
2. The teaching of new content (about 25 minutes)
3. Exercises on the content introduced (about 10 minutes)
4. Homework assignment (about 5 minutes)

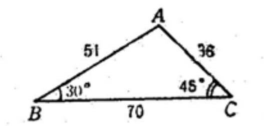
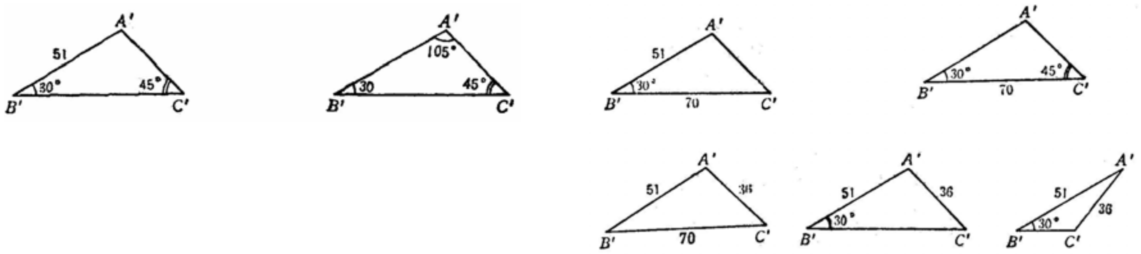
Japan: The way teachers structure their lessons in Japan is influenced (as in China and, no doubt, elsewhere) by the specification of the mathematics curriculum, the demands of examinations, and the design of textbooks. Our analysis also suggests that lesson designs in Japan are also influenced by the occurrence of 'Lesson Studies' and by recent Japanese research into the learning and teaching of mathematics. For example, 'Lesson study', practiced by teachers in Japan for the last several decades, is one of the most common forms of professional development and involves teachers working in small teams collaboratively crafting lesson plans through a cycle of planning, teaching and reviewing (Yoshida 1999). Through this process, Japanese teachers appear to have collaboratively developed a view about 'good lessons of mathematics'. Research that has influenced how teachers structure lessons includes the work on the "Open-ended approach" in which 'the teacher gives the students a problem situation in which the solutions or answers are not necessary determined in only one way' (Sawada, 1997, p. 23). Considering the influences described above, in summary, Japanese teachers tend to structure mathematics lessons as follows (see, also, Stigler and Hiebert, 1999, pp.79-80):

1. Presenting the problem(s) for the day - designed to make students engage in mathematical thinking in challenging (or sometimes open-ended) situations - reviews of previous lessons are sometimes included before the problem(s);
2. Development of the problem(s) - students work on the problem(s) individually or in groups, discussion and presentations of solutions are often included, new (usually related) problems are sometimes introduced
3. Highlighting and summarising the main point(s) - students' ideas are often used, and sometimes students are asked to explain their solutions; solutions are summarised by the teacher with the aim that the students grasp the main goals of the lessons.

Comparison of suggested lessons – China: The case study below is a lesson record taken from Li (1992).

Lesson on 'Verifying congruent triangles' (grade 7, students aged 13-14) (Li, 1992, translated by Ding, 2004).

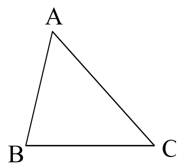
Objective: understanding the axiom and theorems of verifying congruent triangles

Review (+/- 5 minutes): review the concept of congruent figures studied before (distinguishing it from similarity and the same area of figures) by studying the following figures: 
Teaching new knowledge (+/- 35 minutes): question for discussion - how can we verify that two triangles are congruent? Activity: use ruler and compass to draw a triangle that is congruent with the triangle given.  Class discussion: which triangles drawn by students on the blackboard could be congruent with the given one above? 
Conclusion (+/- 3 minutes): the methods to verify congruent triangles
Homework assignment (+/- 2 minutes)

Comparison of suggested lessons – Japan: The case study presented below is a lesson record taken from Haneda (2002):

Perpendicular bisectors of segments; (grade 7, students aged 12-13) (Haneda, 2002, translated by Fujita, 2004).

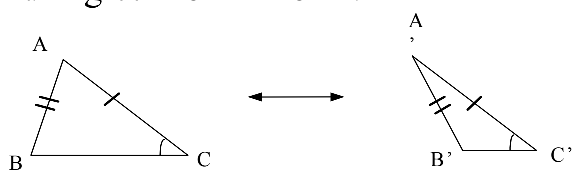
Objective: understand the conditions for congruent triangles

Introduction 5 min.	Review of the properties in congruent shapes such as sizes of angles, and lengths of lines Introduction of the problem for the day: draw triangles which are congruent to the triangle ABC	
Development 32 min.	Undertaking the problem by students individually. Discussion by students in small groups Presentation from each group	

Summary 13 min.	Summary by teachers (focusing on the conditions for congruent triangles) Students write summary Teacher refers to topic for next lesson	

DISCUSSION

In each country, the lesson structure, not surprisingly, follows the pattern expected for that country. Thus in the lesson from China, there are number of short tasks and questions in each segment of the lesson. In the lesson from Japan, the three-part structure is followed with a problem introduced in the first part and developed in the second before the main teacher explanation is given in the third. Yet despite what may appear as different structures, there are key similarities to the lesson. For example, instead of teaching the conditions of congruent triangles directly, the suggested structure of both lessons is that students should investigate the conditions by themselves as much as possible. Both lessons start from an exploration of how to draw (ie verify) a triangle which is congruent to a given triangle. Students' methods are shared and confirmed by classroom discussions and presentations. In the discussions, it is expected that students would start considering why they cannot always draw congruent triangles if they focus on, for example, AB=A′B′, AC=A′C′, and angles ACB=A′C′B′.



The case of AB=A'B', AC=A'C', and angles ACB=A'C'B'

Finally the conditions are summarised by teachers, probably by using more precise mathematical languages.

As was found in the *TIMSS video studies* (Stigler and Hiebert, 1999; Hiebert *et al*, 2003), notwithstanding these shared general features, there is a modicum of variation across the two lessons. For example, in the lesson from China, the dimensions of the triangle are given whereas in the lesson from Japan, no dimensions are given. There is some variation in how the new content is introduced – in the Chinese lesson through the teacher asking many questions, in the Japanese lesson through the teacher posing fewer questions but emphasising student-student discussion and student presentation.

The tendency in Japan for lessons to be structured around fewer, more substantial questions is reflected in other analyses of lessons from China and Japan (eg Jones *et al*, 2004), and from other data from the TMSS video study. The TIMSS data, for example, suggest that mathematics lessons in Japan place a greater emphasis on introducing new content, while lessons in Hong Kong SAR placed a greater emphasis on practicing new content (note that the bulk of the People's Republic of China did not take part in TIMSS and that, perhaps due in part to its western colonial history, it may be a special case and be unlike the bulk of mainland China). In terms of procedural complexity (defined in terms of the number of steps it takes to solve a problem using a common solution method), the TIMSS study found that 39% of the mathematics problems studied per lesson in Japanese classrooms were of high procedural complexity, a greater percentage than in any of the other six countries studied.

Whilst data from mainland China remains relatively scarce, in a lesson analysis described by Mok (2003), most of the lesson (taught in the city of Shanghai) was made up of segments of either teacher-talk or students answering questions raised by the teacher (76.4% of the lesson time). In between these whole-class segments, the teacher might ask the students to work on a mathematical problem individually (13.2% of lesson time) or in small groups (10.3% of lesson time). In the whole class interaction segments, the teacher frequently asked questions and expected the students to answer them. According to Mok, there were hardly any instances of students raising their own questions. The segments of pupil work, either individual or as groups, were all very short (about 1 to 3 minutes long each) but quite frequent.

CONCLUDING COMMENT

What the research presented in this paper has not, as yet, been able to ascertain are what the implications might be for student achievement in geometry in the countries under consideration. Overall, as Hiebert *et al* (2003, p149-50) emphasise, research has found that the countries that show high levels of student achievement (in, say, the TIMSS achievement tests) do not all employ teaching methods that combine and emphasise features in the same way. As they conclude:

> "The results of this study [the TIMSS video study] make it clear that an international comparison of teaching, even among mostly high-achieving countries, cannot, by itself, yield a clear answer to the question of which method of mathematics teaching may be best to implement in a given country". Hiebert *et al* (2003, p150)

This confirms that further work is needed in order to refine what might be a suitable pedagogy for geometry and that, in any case, any pedagogic theory is likely to need to take account of the local historical and cultural setting in which the teaching is taking place.

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