UNIVERSITY OF SOUTHAMPTON
FACULTY OF LAW, ARTS & SOCIAL SCIENCES
School of Social Sciences
Economics Division

Three Essays in Imperfect Competition, Political Economy and International Trade

by

Jie Ma

Thesis for the degree of Doctor of Philosophy

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To my parents

"Genius is nothing but a great aptitude for patience." *George Louis De Buffon*

UNIVERSITY OF SOUTHAMPTON

ABSTRACT

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THREE ESSAYS IN IMPERFECT COMPETITION, POLITICAL ECONOMY AND INTERNATIONAL TRADE

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This Thesis has two themes: (1) political economy of international trade and factor mobility policy; (2) the robustness of strategic trade and industrial policy.

Chapter 1 is a non-technical introduction of my research.

In Chapter 2, Double-edged incentive competition for FDI, we study the impact of special interest lobbying on competition between two countries for a multinational in a common agency framework. We address the following questions. On the positive side, is special interest lobbying a determinant of competition for FDI? If so, how does it work? How does it affect the equilibrium price for attracting FDI? On the normative side, what are the welfare effects of FDI competition when special interest lobbying is present? Is allocative efficiency always achieved? We argue that special interest lobbying provides an extra political incentive for a government to attract FDI. We show that compared to the benchmark case when governments maximize national welfare, now (1) an economically disadvantageous country has a chance to win the competition; (2) the equilibrium price for attracting FDI is higher than in the benchmark case; (3) allocative efficiency cannot be always achieved.

In Chapter 3, Advertising in a differentiated duopoly and its policy implications for an open economy, we develop a model of advertising in a differentiated duopoly in which firms first decide how much to invest in cooperative or predatory advertising and then engage in product market competition (Cournot or Bertrand). We then use this model, with the type of advertising endogenously determined, to explore the policy implications in the context of a Brander-Spencer third-country model of strategic trade. Among results derived from this model, most interestingly we show that for a range of parameter values we get robust trade policy in which governments always use a trade subsidy irrespective of the type of advertising or form of market competition.

In Chapter 4, Is export subsidy a robust trade policy recommendation towards a unionized duopoly, we argue that although previous researches imply that the robust trade policy recommendation towards a unionized duopoly is an export subsidy, we cannot get such a result even in the linear case if the opportunity cost of public funds is sufficiently high. However, if we consider the case where the domestic firm and the trade union lobby the government to set their favorable trade policies by giving the government political contributions (modeled in a common agency setting), then the result of robustness will be restored if the government cares about the political contributions sufficiently relative to national welfare.

See Chapter 5 for some technical proofs.

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My first paper, "Double-edged incentive competition for FDI", has been presented at Southampton, CORE, EARIE 2005, ETSG 2005, Nottingham, Beijing University, LSE, 2006 Silvaplana Workshop in Political Economy and EEA 2006. Seminar participants, in particular, Toke Aidt, Daniel Bernhofen, Wilfred Ethier, Rod Falvey, Ben Ferrett, Saileshsingh Gunessee, Carsten Hefeker, Arye Hillman, Miriam Krausz, Douglas Nelson, Peter Nunnenkamp, Emanuel Ornelas, Pascalis Raimondos-Møller, Daniel Seidmann, Jacques-François Thisse, Thierry Verdier, Zhihong Yu, Weiying Zhang, Lian Zhou and Shanli Zhu gave me valuable comments. Discussion with Alistair Ulph and Juuso Välimäki was also helpful.

My second paper, "Advertising in a differentiated duopoly and its policy implications for an open economy", joint with Alistair Ulph, has been presented at Southampton and Beijing University. I thank Jian Tong for detailed comments as well as other seminar participants for helpful discussion.

During the last four years, my family was separated: I studied in Southampton, while my wife worked in Beijing. She endured tough times but without any complaint. I would like to take this opportunity to say "Thank you" to her. In the last year of my study, our son was born in Beijing. This brought greatest happiness to my family. However, I should say "Sorry" to him since I went back to Southampton to submit my Thesis when he was six weeks old. I thank my parents-in-law for looking after my wife and my son during the period of my absence.

I dedicate my Thesis to my parents, who brought me into such a wonderful world.

Chapter 1

Introduction

This Thesis has two themes: (1) political economy of international trade and factor mobility policy; (2) the robustness of strategic trade and industrial policy.

The first paper, "Double-edged incentive competition for FDI", explores the first one in the context of competition between countries for foreign direct investment (hereafter FDI).

The world has witnessed fierce FDI competition between countries during recent years. It is quite natural to ask the following questions. On the positive side, why does a particular country win a particular competition? Is the winner selection determined only by economic factors, such as market scale, labor costs, and so on? On the normative side, is this competition efficient? Is allocative efficiency always achieved?

Almost all of the existing literatures say "yes to all" to the above questions. But consider the case where Portugal, Spain and UK competed for Ford and Volkswagen in 1991. Portugal won the competition. But its market scale is smaller than that of UK and its skilled labor costs are higher than those of UK.

So, how do we explain such a case? One possible answer is that politics matters in competition for FDI.

In this paper, we study the impact of special interest lobbying on competition between two countries for a multinational, using a common agency framework developed by Bernheim and Whinston (1986), and Grossman and Helpman (1994).

The basic idea is as follows. FDI has income redistribution effects. So, in each country, the special interest groups who are the gainers of this redistribution effects have an incentive to lobby the government to attract FDI, whilst the special interest groups who are the losers of this redistribution effects have an incentive to lobby the government not to attract FDI. Governments' objectives are shaped by domestic political competitions. Then they engage in competition for a multinational. National welfare of each country is determined as an equilibrium outcome of this game.

In our model, the gainers of income redistribution effects are trade unions since the demand of labor is increased, whilst domestic firms are losers due to competition effect. (We assume that consumers are not organized into special interest groups.) The domestic political competition between the trade union and the domestic firm in each country is modeled as a common agency situation.

We argue that special interest groups provide a government an extra political incentive to attract FDI via the domestic political competition. If in the economically disadvantageous country, the political incentive provided is great enough to dominate both the other country's economic advantage and the other government's political incentive to attract FDI, then the economically disadvantageous country wins competition for FDI. Otherwise, the economically advantageous country wins the competition. The equilibrium price for attracting FDI is higher than in the case when governments maximize national welfare. We also show that allocative efficiency cannot be always achieved. This happens when the economically disadvantageous country wins the competition.

As an application of the model, we provide a possible explanation of the competition between Portugal, Spain and UK in 1991. Our conjecture is that UK had an economic advantage over Portugal in the competition. But Portugal won the competition, at a 'price' of 250,000 US dollars per job. We think that special interest lobbying mattered there. The Portuguese government was far more influenced by special interest groups than Spanish and UK governments. The trade union won the political competition in Portugal and provided a sufficiently great political incentive for the Portuguese government to dominate its rivals in international arena. Since Spanish and UK governments were also politically-motivated, as a result, the Portuguese government paid a high price to attract the two companies.

This is the first paper studying the effects of special interest politics on competition for FDI and is related to several strands of literatures.

Many papers study competition for FDI from a purely economic angle. For example, Haufler and Wooton (1999), Barros and Cabral (2000), and Fumagalli (2003) study competition for a multinational in the framework of imperfect competition. Barba Navaretti and Venables *et al.* (2004) discuss the implications of policy competition for a multinational in a simple model. Haaparanta (1996) considers the case where the exogenously given FDI is perfectly divisible, and countries compete for their own shares. They all assume that governments seek to maximize national welfare, and study the strategic interactions between governments. They show that on the one hand, an economically advantageous country wins FDI competition in an equilibrium; on the other hand, allocative efficiency is always achieved. However, we show that all of these results can be reversed when special interest politics is present.

To the best of our knowledge, Biglaiser and Mezzetti (1997) is the only other paper to study the bidding war for a firm from a political economy perspective. In their paper, elected officials have re-election concerns, which make their willingness to pay for attracting a firm differ from voters' willingness to pay for that. They derive a similar result to ours: the allocation of FDI may be inefficient. However, this research and theirs are complements rather than substitutes. The driving force of our model is special interest politics, whilst the driving force of their model is politicians' re-election concerns. Our and their papers together send a message that political factors have big impact on competition for FDI. In Biglaiser and Mezzetti (1997) the voters are assumed to be symmetric vis-à-vis the investment project; there are no conflicts of interest among them. Notice that the redistribution effects of FDI are considered explicitly in this paper.

The second theme is the robustness of strategic trade and industrial policy. Strategic trade policy has become a core part of international trade policy analysis since the seminal paper by Brander and Spencer (1985) was published. However, despite a voluminous literature since then, the policy implications remain controversial, mainly because the trade policy recommendation is very sensitive to the market conduct, with an export subsidy being recommended with Cournot competition and an export tax with Bertrand.

Recent studies, such as Bagwell and Staiger (1994), Maggi (1996) and Leahy and Neary (2001) show that if firms engage in strategic investment competition (e.g., for R&D or capacity) prior to product market competition, then industrial policy, in the form of an investment subsidy, would be robust to the form of market conduct. Neary and Leahy (2000) develop a general framework to analyze optimal intervention towards dynamic oligopoly, emphasizing the implications of different kinds of government commitment. They point out that when firms make strategic investments prior to product market competition, the first-best policy combination should be designed for both profit-shifting and correcting the socially wasteful strategic behavior of the domestic firm to influence the decisions of its rival and the domestic government. They also argue that a general model may not be useful in providing a general guide to policy making, and that it might be better to conduct case studies of particular policy combinations. Advertising is a fruitful field for such a case study, since its policy implications in the context of strategic trade policy have not been much explored.

In the second paper, "Advertising in a differentiated duopoly and its policy implications for an open economy", joint with Alistair Ulph, we first construct a model of advertising in a differentiated duopoly. Following Church and Ware (2000) we distinguish between cooperative advertising, which increases demand for rival firms' products as well as those of the advertising firm, and predatory advertising, which increases demand for the advertising firm only by attracting customers away from its rivals. We construct a two-stage game, in which in the first stage firms decide how much to invest in cooperative advertising, or predatory advertising or both. In the second stage they engage in product market competition, either Cournot or Bertrand. We show that whatever the form of product market competition, firms will invest in only one type of advertising, which is determined by the relative effectiveness of the two types of advertising and the degree of product differentiation. In an equilibrium, any combination of market conduct and advertising type is possible except predatory advertising with Cournot and cooperative advertising with Bertrand. Moreover, whatever the form of product market competition, cooperative advertising is a strategic complement and makes the rival's profits increase, while predatory advertising is a strategic substitute and makes the rival's profits decrease.

We then analyze policy setting in the context of a Brander-Spencer third-country model of strategic trade, beginning with the case where governments can set trade and industrial policies, and then considering the cases where they can set only industrial policy or only trade policy. When governments use both trade and industrial policies, these policies are substitutes. When governments can use only industrial policy, it is robust, i.e., governments will always use an advertising subsidy irrespective of the type of advertising and the form of market competition. However the significant new result of this paper is that, when governments can use only trade policy, for a wide range of parameters, trade policy, in the form of a trade subsidy, is similarly robust, i.e., governments always use a trade subsidy irrespective of the type of advertising or the form of market competition.

The second paper studies the robustness of strategic and industrial trade policy from a purely economic angle. In the third paper, "Is export subsidy a robust trade policy recommendation towards a unionized duopoly", I introduce special interest lobbying to a strategic trade policy model. So both themes of this Thesis are developed in an interactive way.

Recently Bandyopadhyay *et al.* (2000) point out that demand linearity ensures that an export subsidy is the optimal trade policy towards a unionized Bertrand duopoly. Brander and Spencer (1988) show that the optimal trade policy towards a unionized Cournot duopoly is an export subsidy. These two papers together imply that an export subsidy is a robust trade policy recommendation towards a linear unionized duopoly.

The objective of my third paper is to assess how robust this result is to two additional factors: an opportunity cost of public funds; and special interest lobbying.

I begin with a linear model in which following Brander and Spencer (1988), I introduce a trade union to a Brander-Spencer third-market model for one of the two exporting countries, say, the 'domestic country', and consider the case of unilateral intervention. In this model, I reproduce the result of robustness implied by the above two papers in a clear-cut way. I.e., the optimal trade policy is an export subsidy irrespective of the form of market conduct. This serves as a benchmark case. Then, I introduce an opportunity cost of public funds to the above setting. Now even in the linear case, an export subsidy cannot be a robust trade policy recommendation if this cost is sufficiently high. Then, I allow the domestic firm and the trade union to lobby for their favorable policies by giving the domestic government political contributions prior to the government setting a trade policy. This is modeled as a common agency framework developed by Bernheim and Whinston (1986), and Grossman and Helpman (1994). I show that an export subsidy is a robust policy recommendation irrespective of the form of market conduct if the government cares about political contributions sufficiently relative to national welfare.

So, what is the main lesson that I have learnt from this simple exercise? First of all, in the absence of political factors, an export subsidy can hardly be a robust trade policy recommendation towards a unionized duopoly: the optimal policy is very sensitive to the opportunity cost of public funds. However, an export subsidy can be a robust policy recommendation when political factors (such as special interest lobbying) are present.

As far as I know, my paper is the first to consider the effect of both an opportunity cost of public funds and special interest lobbying on strategic trade policy towards a unionized duopoly. Matsuyama (1990), followed notably by Neary (1994), introduce a social cost of public funds to the strategic trade policy literature. They do not consider special interest lobbying. Fung and Lin (2000) use a common agency approach to studying strategic trade policy from a political economy perspective. They do not include an opportunity cost of public funds.

Chapter 2

Double-edged Incentive Competition for FDI

Abstract

This paper studies the impact of special interest lobbying on competition between two countries for a multinational in a common agency framework. We address the following questions. On the positive side, is special interest lobbying a determinant of competition for FDI? If so, how does it work? How does it affect the equilibrium price for attracting FDI? On the normative side, what are the welfare effects of FDI competition when special interest lobbying is present? Is allocative efficiency always achieved? We argue that special interest lobbying provides an extra political incentive for a government to attract FDI. We show that compared to the benchmark case when governments maximize national welfare, now (1) an economically disadvantageous country has a chance to win the competition; (2) the equilibrium price for attracting FDI is higher than in the benchmark case; (3) allocative efficiency cannot be always achieved.

Key Words: Coalition-Proof Nash Equilibrium (CPNE), Common agency, FDI, Incentive competition, Multinational, Special interest lobbying

JEL Classification: D72, F23, H25, H71, H73, H87

2.1 Introduction

The world has witnessed fierce FDI competition between countries during recent years. For instance, Table 2.1 lists some of the competitions that have occurred in Europe.[1]

City, State	Year	Plant	Other locations considered	State investment (million \$)	Company's investment (million \$)	Financial incentive per job (\$)
Setubal, Portugal	1991	Ford, Volkswagen	UK, Spain	483.5	2603	254,451
North-East England	1994/95	Samsung	France, Germany, Portugal, Spain	89	690.3	29,675
Castle Bromwich, Birmingham, Whitley, UK	1995	Jaguar	Detroit, USA	128.72	767	128,720
Hambach, Lorraine, France	1995	Mercedes-Benz, Swatch	Belgium, Germany	111	370	?
Newcastle upon Tyne, UK	1995	Siemens	Austria, Germany, Ireland, Portugal, Singapore	76.92	1428.6	51,820

Table 2.1: The cost of attracting investment: Examples of incentives given to investors in Europe

Countries have an economic incentive to attract FDI since possible benefits of FDI include job creation, antitrust, technological spillover and import substitution effects. In order to achieve these potential beneficiary effects, countries tend to give favorable offers to companies. However, in some cases, financial incentives provided were unbelievably high. Consider the case where Portugal, Spain and UK competed for Ford and Volkswagen in 1991. Portugal won the competition but the Portuguese government paid over 250,000 US dollars to companies in order to create one new job. Did Portugal really benefit that much from foreign investments? People have good reason to question whether the Portuguese government behaved efficiently since they can hardly understand why a national-welfare-maximizing government made such a generous offer to foreign investors.[2]

This puzzle stimulates our research. In this paper, we study the impact of special interest lobbying on competition between countries for FDI. We want to address the following questions. On the positive side, is special interest lobbying a determinant of competition

[1]This table is based on Table III.7 of UNCTAD (1996). Competition for FDI is extensively documented by UNCTAD (1996) and Oman (2000).

[2]See Barba Navaretti and Venables *et al.* (2004), Chapter 10, section 10.3.1.

for FDI? If so, how does it work? How does it affect the equilibrium price for attracting FDI? On the normative side, what are the welfare effects of FDI competition when special interest lobbying is present? Is allocative efficiency always achieved?

Our basic idea is as follows. FDI has income redistribution effects in each country. Hence, in each country, the special interest groups who are the gainers of this redistribution have an incentive to lobby the government to attract the FDI, whilst the special interest groups who are the losers of this redistribution have an incentive to lobby the government not to attract the FDI. The government's objective is shaped by this political competition. Governments then engage in competition for FDI. The outcome of this competition determines national welfare of each country. Notice that when the special interest groups in each country engage in political competition, they know that such competition occurs in other countries. Therefore, the optimal lobby behavior should be based on the anticipation of how the special interest groups in other countries lobby their governments, and should take into account the equilibrium outcome of competition for FDI, given that lobby behavior is sunk. This idea is illustrated in Figure 2.1.

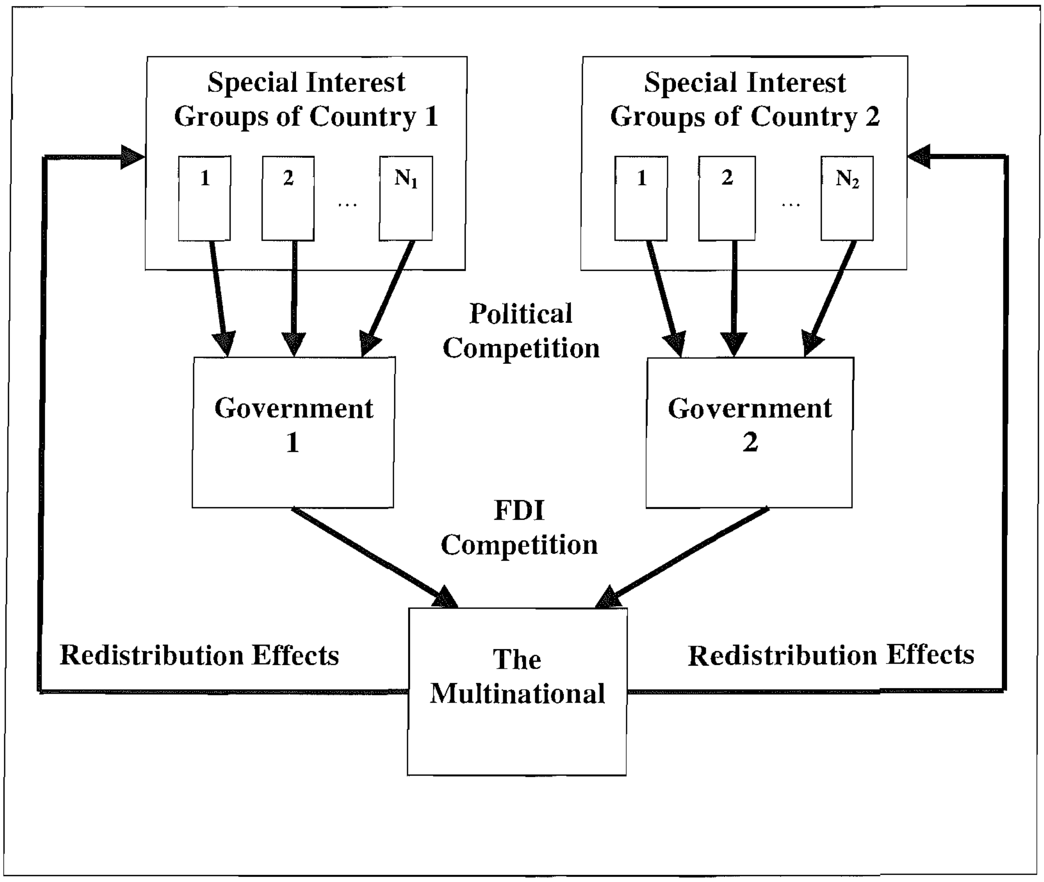


Figure 2.1: Illustration of the basic idea

How do we put this idea to work? We consider the case where two countries compete for a multinational. There is a monopoly market for a homogenous good in each country. The only factor of production is labor, which is unionized, and the wage rate and employment level are determined in a Leontief model. Therefore, in each country, the trade union welcomes the multinational, because it can sell more labor and achieve more economic rents, whilst the domestic firm does not welcome the multinational because its profits will decrease. The shaping of a government's objective by the trade union and the domestic firm via political competition in each country is modelled as a common agency situation based on Bernheim and Whinston (1986), and Grossman and Helpman (1994).[3]

Common agency is initiated by Bernheim and Whinston (1986), and is successfully used to study political economy of trade policy by Grossman and Helpman (1994). Grossman and Helpman (1994) develop a political contributions approach in which at the first place special interest groups acting as principals simultaneously make political contributions, which are functions of trade policies, then after observing political contributions the government acting as the agent chooses trade policies to maximize a weighted sum of political contributions and national welfare with more weight putting on political contributions. Grossman and Helpman (1994) capture the idea that when special interest groups are present, the mechanism of trade policy making would fail to internalize all benefits and costs as the consequence of trade policies. Applying this framework to studying competition for FDI shows the possibility that the cost of subsidizing FDI is not fully internalized and a government's willingness to pay for FDI may be higher than its country's economic incentive to attract FDI.[4]

But a common agency framework *per se* is not sufficient to determine the equilibrium price for attracting FDI since we consider competition between two countries for FDI. As our basic idea shows, we study a situation in which two common agencies compete with each other. This relates to Putnam's idea of a two-level game.[5] Several papers ex-

[3]In our model, we treat the trade union and the domestic firm in each country as special interest groups. Lahiri and Ono (2004) point out that the trade union who wants the government to stipulate that multinationals purchase most their inputs from the local markets, has an incentive to lobby the government, and the purpose is to maximize the income of workers. Kayalica and Lahiri (2003) point out that almost all countries have well-organized local producers, e.g., automobile industry, who lobby the government for higher levels of protection against the goods of foreign-owned plants producing in the country. We suppose that consumers are not organized, and do not form a special interest group in this paper.

[4]Notice that we follow this political contributions framework, but political contributions are not contingent on governments' actions (lump-sum subsidies or taxes) but the outcome of FDI competition in our model.

[5]Putnam (1988) points out that "The politics of many international negotiations can usually be conceived as a two-level game. At the national level, domestic groups pursue their interests by pressuring the government to adopt favorable policies, and politicians seek power by constructing coalitions among those groups. At the international level, national governments seek to maximize their own ability to satisfy domestic pressures, while minimizing the adverse consequences of foreign developments. Neither of the two games can be ignored by central decision-makers, so long as their countries remain interdependent, yet sovereign." See Putnam (1988), pp. 434.

People may argue that in his original work, Putnam (1988) does not suggest whether the idea of a two-level game should be modelled as a sequential game or a simultaneous game. However, in economic analysis, this idea is related to the idea of strategic delegation and is modelled as a sequential game. See plore this idea in different settings. Grossman and Helpman (1995a) study the impact of special interest politics on negotiation of a free-trade agreement between two countries. Grossman and Helpman (1995b) introduce special-interest politics to the analysis of international trade relations, considering both noncooperative tariff setting and negotiated tariffs. Persson and Tabellini (1992) study the effects of election under majority rule on competition for mobile capital between countries in order to shed light on the repercussions of European integration on fiscal policies in different countries.[6] Our paper gives a new application of the idea of a two-level game showing that how it can be used to study competition for FDI when governments are influenced by special interest groups.[7]

Notice that in the benchmark case when governments maximize national welfare, an economically advantageous country wins competition for FDI for sure. The equilibrium price for attracting FDI is equal to the other country's economic incentive to attract FDI minus the multinational's investment premium in the winning country (or plus the multinational's investment premium in the other country). Allocative efficiency is always achieved.

But when special interest lobbying is present, all these results can be changed.

First of all, special interest groups provide a government an extra political incentive to attract FDI via the domestic political competition. If in the economically disadvantageous country, the political incentive provided is great enough to dominate both the other country's economic advantage and the other government's political incentive to attract FDI, then the economically disadvantageous country wins competition for FDI. Otherwise, the economically advantageous country wins the competition.

Two testable hypotheses are derived. First, if the economically disadvantageous country wins FDI competition, then the extent to which its government is influenced by special interest groups must be greater than the extent to which the other government is influenced. Second, if no country has an economic advantage over the other country in FDI competition, then the country whose government is more influenced by special interest groups, wins the competition.

The equilibrium price for attracting FDI is higher than in the case when governments maximize national welfare. The competition for the multinational can be viewed as a Bertrand game. When special interest lobbying is present, each government is provided an extra political inventive to attract FDI besides an economic incentive. So, irrespective

Grossman and Helpman (1995b), and Persson and Tabellini (1995).

[6]Persson and Tabellini (2000) present a slightly different version of this model. See Chapter 12, section 12.4.4.

[7]Notice that in Persson and Tabellini (1992), voters do not vote directly on policy but elect a policy maker who makes policy decision. In Grossman and Helpman (1995a), (1995b) and our paper, special interest groups lobby directly for policies.

of who wins the competition, the payments to the multinational must be higher than before.

We then do welfare analysis. Allocative efficiency cannot be always achieved. This happens when the economically disadvantageous country wins the competition.

As an application of the model, we provide a possible explanation of the competition between Portugal, Spain and UK in 1991. Our conjecture is that UK had an economic advantage over Portugal in the competition. But Portugal won the competition, at a 'price' of 250,000 US dollars per job. We think that special interest lobbying mattered there. The Portuguese government was far more influenced by special interest groups than Spanish and UK governments. The trade union won the political competition in Portugal and provided a sufficiently great political incentive for the Portuguese government to dominate its rivals in international arena. Since Spanish and UK governments were also politically-motivated, as a result, the Portuguese government paid a high price for attracting the two companies.

This is the first paper studying the effects of special interest politics on competition for FDI and is related to several strands of literatures.

Many papers study competition for FDI from a purely economic angle. For example, Haufler and Wooton (1999), Barros and Cabral (2000), and Fumagalli (2003) study competition for a multinational in the framework of imperfect competition. Barba Navaretti and Venables *et al.* (2004) discuss the implications of policy competition for a multinational in a simple model.[8] Haaparanta (1996) considers the case where the exogenously given FDI is perfectly divisible, and countries compete for their own shares. They all assume that governments seek to maximize national welfare, and study the strategic interactions between governments. We have shown that the results obtained under this assumption do not hold when special interest lobbying plays a role in competition for FDI.

To the best of our knowledge, Biglaiser and Mezzetti (1997) is the only other paper to study the bidding war for a firm from a political economy perspective. In their paper, elected officials have re-election concerns, which make their willingness to pay for attracting a firm differ from voters' willingness to pay for that. They derive a similar result to ours: the allocation of FDI may be inefficient. However, this research and theirs are complements rather than substitutes. The driving force of our model is special interest politics, whilst the driving force of their model is politicians' re-election concerns. Our and their papers together send a message that political factors have big impact on competition for FDI. In Biglaiser and Mezzetti (1997) the voters are assumed to be symmetric vis-à-vis the investment project; there are no conflicts of interest among them. Notice that the

[8] See Chapter 10, section 10.3.1.

redistribution effects of FDI are considered explicitly in this paper.

Tax competition for mobile capital, which assuming perfect competition, whilst introducing asymmetries between countries, and studying the interaction between different tax instruments, is one of the most important themes in traditional public finance. However, since profit-maximizing firm is far different from mobile capital, as Fumagalli (2003) notes:[9] this approach is more appropriate when dealing with competition for portfolio investment rather than for FDI.[10] See Wilson (1999), and Wilson and Wildasin (2004) for surveys of tax competition literatures.

Besides the contributions to the existing literatures of competition for FDI, this research has significant policy implications. Recently, José Manuel Barroso, the new president of the European Commission, assailed French and German efforts to end tax competition among European Union countries.

> "Some member countries would like to use tax harmonization to raise taxes in other countries to the high-tax levels in their own countries," Mr. Barroso said in an interview during the World Economic Forum's annual meeting in this Swiss ski resort. "We do not accept that. And member states will not accept it."[11]

His view has been supported by some economists. For example, Milton Friedman said that

> "Competition, not identity, among countries in government taxation and spending is highly desirable. How can competition be good in the provision of private goods and services but bad in the provision of governmental goods and services? A governmental tax and spending cartel is as objectionable as a private cartel."[12]

However, this paper gives a caveat to this optimistic view. We point out that this competition may end up with allocative inefficiency when special interest lobbying is present.

The structure of this paper is as follows. Section 2 sets out the model, which is analyzed in section 3 and section 4. The welfare effects are analyzed in section 5. In section 6, we discuss the robustness of results obtained in this paper, and the final section concludes. See Appendix to Chapter 2 for some technical proofs.

[9] Also see the references she cites.

[10] As noted in the above discussion, Persson and Tabellini (1992), and Persson and Tabellini (2000) explore the political economy implications of competition for mobile capital between countries. But for the same reason, we wonder whether their approach is appropriate for studying competition for FDI from a political economy angle.

[11] *Wall Street Journal Europe*, January 31, 2005. Notice that the tax competition that he mentioned is one form of incentive competition for FDI.

[12] *Wall Street Journal Europe*, July 29, 1998.

2.2 The Model

We set out the model in this section.

Preference:

There are two countries, $i = 1, 2$. The preference of the representative consumer of country i is given by

$$U^i(q_i, m_i) = u^i(q_i) + m_i,$$

where

$$u^i(q_i) = \alpha_i q_i - \frac{1}{2}\beta_i q_i^2.$$

q_i is the consumption of a homogenous good, and m_i is the consumption of a numeraire good. The inverse market demand (market price) is given by

$$p_i = \alpha_i - \beta_i q_i.$$

Production:

Labor, which is immobile between two countries, is the only input for producing q_i, and the technology is a Ricardian one:

$$q_i = \frac{L_i}{\gamma_i},$$

where γ_i is the inverse of the input-output coefficient, and the marginal product of labor is $\frac{1}{\gamma_i}$. We assume that the workers' opportunity wage rate, w_i^c, is equal to the marginal product of labor.[13] Labor is organized and forms a trade union in each country.

Players:

There are three firms: the domestic firm of country 1, the domestic firm of country 2, and a multinational firm; and two trade unions: the trade union of country 1, and the trade union of country 2; and two governments: government 1 and 2.

Timing:

This is a five-stage game.

Stage 1: The trade union and the domestic firm in each country lobby the government simultaneously and noncooperatively by giving the government political contributions contingent on the multinational's location.[14] In particular, trade union i's contribution sched-

[13]We make this assumption in order to simplify analysis. Our key results are not dependent on it. See discussion in section 6.

[14]Notice that in Bernheim and Whinston (1986), (and Grossman and Helpman (1994)), the contract (the contribution schedule) offered to the agent (the government) by a principal (a special interest group) is contingent on the agent's actions (trade policies). Our approach is different from theirs.

ule is given by

$$C_i^T = \begin{cases} C_{ii}^T & if \quad FDI\ in\ country\ i, \\ C_{ij}^T & if \quad FDI\ in\ country\ j; \end{cases} \tag{2.1}$$

where $C_i^T \geq 0$. Domestic firm i's contribution schedule is given by

$$C_i^F = \begin{cases} C_{ii}^F & if \quad FDI\ in\ country\ i, \\ C_{ij}^F & if \quad FDI\ in\ country\ j; \end{cases} \tag{2.2}$$

where $C_i^F \geq 0$. $i = 1, 2,\ j = 1, 2,\ i \neq j$. Notice that the multinational is not allowed to make political contributions.[15]

Stage 2: After observing all contribution schedules, two governments announce simultaneously a lump-sum subsidy b_i to the multinational.[16]

Stage 3: The multinational makes its location choice. We suppose that the multinational wants to establish a subsidiary in country 1 or 2.[17]

Stage 4: The wage rate and the employment level are determined in each country. The trade union moves first and sets the wage rate. After observing the wage rate, the domestic firm decides how much labor to employ when the multinational does not locate in the country; whilst the domestic firm and the multinational make employment decisions simultaneously and noncooperatively when the multinational locates in the country. (We use a Leontief model to characterize the strategic interactions in this stage.)

Stage 5: Product market competition. We assume that if the multinational locates in country i, it will adopt the same technology as firm i's technology. In addition, we suppose that there is no trade between the two countries. In this stage, firm i and the multinational engage in Cournot competition when the multinational locates in country i. Otherwise, firm i sets its monopoly outputs.[18]

Then the game is over.

Payoffs:

A domestic firm receives its profits minus its political contributions. A trade union receives its economic rents minus its political contributions. The economic rents are defined as the product of the difference between the actual wage rate and the opportunity wage rate and the employment level.

[15] See discussion in the Conclusion.

[16] If b_i is negative, it is a lump-sum tax.

[17] We do not consider direct export as one of the multinational's possible options in this paper. See discussion in the Conclusion.

[18] People may argue that a more realistic setting is to consider the case when the multinational is allowed to trade between countries, though domestic firms not. However, we doubt that the basic results derived from the simplest case – the no-trade case – would be changed when considering this more complicated case. See discussion in section 6.

Government i's payoffs are given by

$$G^i = \begin{cases} \lambda^i \left(C_{ii}^T + C_{ii}^F\right) + \left(W_i^i - b_i\right) & if \quad FDI\ in\ country\ i \\ \lambda^i \left(C_{ij}^T + C_{ij}^F\right) + W_j^i & if \quad FDI\ in\ country\ j \end{cases}, \ \lambda^i \geq 0. \tag{2.3}$$

W_i^i is country i's national welfare when it wins the competition for the multinational, whilst W_j^i is its national welfare when it loses the competition. National welfare is defined as the sum of (1) consumers' surplus,[19] (2) domestic firm's profits, and (3) economic rents. When country i wins the competition for the multinational, it pays a lump-sum subsidy b_i to the multinational, which is collected from consumers by lump-sum taxation.[20] λ^i is a parameter that represents the marginal rate of substitution between political contributions and national welfare. The larger is λ^i, the more weight is placed on political contributions relative to national welfare, and the more government i is influenced by trade union i and firm i.[21] When λ^i goes to infinity, government i's payoffs are equivalent to political contributions. When $\lambda^i = 0$, government i's payoffs are national welfare and cannot be influenced by political contributions.[22]

The multinational receives its profits plus the subsidy that it receives (or minus the tax that it is levied).

We solve the model in section 3 and 4 from backward and use a Coalition-Proof Nash Equilibrium (hereafter CPNE) as the solution concept in the first stage of the game.

[19] We assume that workers do not consume the good produced by themselves.

[20] When it collects a lump-sum tax from the multinational, the tax revenue is distributed among consumers by a lump-sum subsidy.

[21] Notice that the coefficient of national welfare is 1, so λ^i is both an absolute weight and a relative weight.

[22] It should be noted that government i's objective takes a linear form. The use of this is initiated by Grossman and Helpman (1994), in which a government's objective is given by

$$G = C + aW,\ a \geq 0,$$

where C is the sum of political contributions that a government receives, W is a country's national welfare, which includes political contributions, and a is the marginal rate of substitution between national welfare and political contributions.

Other authors, for example, Rama and Tabellini (1998), and Kayalica and Lahiri (2003), write a government's objective as follows:

$$G' = (\rho - 1)\, C + W,\ \rho \geq 1.$$

Again, C represents total political contributions that a government receives, and W represents a country's national welfare. $\rho - 1$ is the marginal rate of substitution between political contributions and national welfare. Hence, $\rho - 1$ is the inverse of a.

Define

$$\lambda \equiv \rho - 1,\ \lambda \geq 0.$$

We have the objective function used in our paper.

2.3 Equilibrium Analysis I: The Last Three Stages

Let us consider country i. When the multinational locates in this country, in the last stage of the game, the domestic firm maximizes its profits:

$$\pi_i^i = \left(\alpha_i - \beta_i \left(q_{ii} + q_i^M\right)\right) q_{ii} - \gamma_i w_{ii} q_{ii},$$

whilst the multinational maximizes its profits:

$$\pi_i^M = \left(\alpha_i - \beta_i \left(q_{ii} + q_i^M\right)\right) q_i^M - \gamma_i w_{ii} q_i^M.$$

q_{ii} denotes the domestic firm's sales in country i, q_i^M denotes the multinational's sales in country i, and w_{ii} denotes the wage rate when the multinational locates in country i. The domestic firm's first-order condition for profit maximization and the multinational's first-order condition for profit maximization determine simultaneously the Nash equilibrium:[23]

$$\left(q_{ii}, q_i^M\right) = \left(\frac{\alpha_i - \gamma_i w_{ii}}{3\beta_i}, \frac{\alpha_i - \gamma_i w_{ii}}{3\beta_i}\right).$$

Hence, the equilibrium employment levels are given by

$$\begin{aligned} L_i^i\left(w_{ii}\right) &= \gamma_i \left(\frac{\alpha_i - \gamma_i w_{ii}}{3\beta_i}\right), \\ L_i^M\left(w_{ii}\right) &= \gamma_i \left(\frac{\alpha_i - \gamma_i w_{ii}}{3\beta_i}\right), \end{aligned}$$

where L_i^i denotes firm i's employment levels, and L_i^M denotes the multinational's employment levels.

In the penultimate stage, trade union i maximizes its economic rents:

$$\omega_i^i = \left(w_{ii} - w_i^c\right) \left(L_i^i\left(w_{ii}\right) + L_i^M\left(w_{ii}\right)\right).$$

From the first-order condition for maximization, we can solve for the equilibrium wage rate:

$$w_{ii} = \frac{\alpha_i + 1}{2\gamma_i}. \tag{2.4}$$

[23]Notice that the first-order conditions are also sufficient in this standard Cournot game.

Using expression (2.4), we can show

$$q_{ii} = q_i^M = \frac{\alpha_i - 1}{6\beta_i},$$
$$L_i^i = L_i^M = \gamma_i \left(\frac{\alpha_i - 1}{6\beta_i}\right),$$
$$\pi_i^i = \pi_i^M = \frac{(\alpha_i - 1)^2}{36\beta_i},$$
$$\omega_i^i = \frac{(\alpha_i - 1)^2}{6\beta_i},$$
$$cs_i^i = \frac{(\alpha_i - 1)^2}{18\beta_i},$$
$$W_i^i = cs_i^i + \omega_i^i + \pi_i^i = \frac{(\alpha_i - 1)^2}{4\beta_i}.$$

Notice that cs_i^i denotes the consumers' surplus when the multinational locates in country i.[24]

When the multinational locates in country j, in the last stage of the game, the domestic firm maximizes its profits:

$$\pi_j^i = (\alpha_i - \beta_i q_{ij})\, q_{ij} - \gamma_i w_{ij} q_{ij}.$$

q_{ij} denotes domestic firm's sales when the multinational locates in country j, w_{ij} denotes the wage rate when the multinational locates in country j. From the first-order condition for profit maximization, we can solve

$$q_{ij} = \frac{\alpha_i - \gamma_i w_{ij}}{2\beta_i}.$$

Hence, the equilibrium employment levels are given by

$$L_j^i (w_{ij}) = \gamma_i \left(\frac{\alpha_i - \gamma_i w_{ij}}{2\beta_i}\right),$$

where L_j^i denotes the employment levels when the multinational locates in country j.

In the penultimate stage, trade union i maximizes its economic rents:

$$\omega_j^i = (w_{ij} - w_i^c)\, L_j^i (w_{ij}).$$

[24]It should be noted that q_{ii}, and q_i^M are not functions of γ_i respectively. Why is that? Recall that the production function is $q_i = \frac{L_i}{\gamma_i}$. Therefore, to produce one unit of output requires γ_i units of labor, and the unit production cost is the product of γ_i and the wage rate, which prevails. Here, we consider competitive wage rate, which is equal to $w_i^c = \frac{1}{\gamma_i}$. Hence, the unit production cost is 1. Therefore, γ_i does not appear in the expressions for q_{ii}, and q_i^M respectively. This indicates that in this model, the unit production cost is one of the fundamental parameters. It is 1 in the case that we consider.

From the first-order condition for maximization, we can solve for the equilibrium wage rate:[25]

$$w_{ij} = \frac{\alpha_i + 1}{2\gamma_i}. \tag{2.5}$$

Using expression (2.5), we can show

$$\begin{aligned}
q_{ij} &= \frac{\alpha_i - 1}{4\beta_i}, \\
L_j^i &= \gamma_i \left(\frac{\alpha_i - 1}{4\beta_i} \right), \\
\pi_j^i &= \frac{(\alpha_i - 1)^2}{16\beta_i}, \\
\omega_j^i &= \frac{(\alpha_i - 1)^2}{8\beta_i}, \\
cs_j^i &= \frac{(\alpha_i - 1)^2}{32\beta_i}, \\
W_j^i = cs_j^i + \omega_j^i + \pi_j^i &= \frac{7(\alpha_i - 1)^2}{32\beta_i}.
\end{aligned}$$

Notice that cs_j^i denotes the consumers' surplus when the multinational locates in country j.

We shall use the following Definition.

Definition 2.1

$$\Delta_i \equiv \frac{(\alpha_i - 1)^2}{2\beta_i}.$$

Notice that we are studying an economic environment with a linear inverse market demand and constant returns to scale production, and marketing technologies. Δ_i gives social welfare under perfect competition in this setting. It is straightforward to show that

$$\frac{\partial \Delta_i}{\partial \alpha_i} > 0, \ \frac{\partial \Delta_i}{\partial \beta_i} < 0. \tag{2.6}$$

It is standard that social welfare increases with the market scale, whilst it decreases with the slope of the demand function.

We use Δ_i to normalize consumers' surplus, economic rents, domestic firm's profits and national welfare and the results are summarized in Table 2.2. So, every term in the Table is a relative measure rather than an absolute measure.

Country i's net gain under FDI is $\frac{1}{16}\Delta_i$, which represents government i's economic incentive to attract FDI. Notice that $\pi_i^i = \pi_i^M = \frac{1}{18}\Delta_i$, which represents the multinational's investment incentive in country i.

[25] Notice that $w_{ii} = w_{ij}$, since the equilibrium employment levels when the multinational locates in country i are proportionate to those when the multinational locates in country j.

Term	FDI	NO	WELFARE CHANGE
consumers' surplus	$\frac{1}{9}\Delta_i$	$\frac{1}{16}\Delta_i$	$\frac{7}{144}\Delta_i$
economic rents	$\frac{1}{3}\Delta_i$	$\frac{1}{4}\Delta_i$	$\frac{1}{12}\Delta_i$
domestic firm's profits	$\frac{1}{18}\Delta_i$	$\frac{1}{8}\Delta_i$	$-\frac{5}{72}\Delta_i$
national welfare	$\frac{1}{2}\Delta_i$	$\frac{7}{16}\Delta_i$	$\frac{1}{16}\Delta_i$

Table 2.2: The redistribution effects of FDI in the basic model

Without loss of generality, in the following analysis we make the following Assumption.

Assumption 2.1

$$\Delta_i > \Delta_j.$$

According to Assumption 2.1, $\frac{1}{16}\Delta_i - \frac{1}{16}\Delta_j > 0$. Hence, Assumption 2.1 says that country i benefits more than country j from FDI, and government i has a greater economic incentive to attract FDI. According to Assumption 2.1, $\frac{1}{18}\Delta_i - \frac{1}{18}\Delta_j > 0$. Hence, the multinational's investment incentive in country i is greater than its investment incentive in country j.

In the third stage, the multinational makes its location choice. Given country i's lump-sum subsidy, b_i, and country j's lump-sum subsidy, b_j, the multinational locates in country i, if and only if

$$\pi_i^M + b_i \geq \pi_j^M + b_j.$$

Otherwise, it locates in country j.[26] Notice that if $b_i = b_j$, it locates in country i.[27]

2.4 Equilibrium Analysis II: The First Two Stages

2.4.1 The second stage

In the second stage, given contribution schedules, government i's objective is given by

$$G^i = \begin{cases} \lambda^i \left(C_{ii}^T + C_{ii}^F\right) + \left(W_i^i - b_i\right) & if \quad FDI\ in\ country\ i, \\ \lambda^i \left(C_{ij}^T + C_{ij}^F\right) + W_j^i & if \quad FDI\ in\ country\ j. \end{cases} \tag{2.7}$$

Setting

$$\lambda^i \left(C_{ii}^T + C_{ii}^F\right) + \left(W_i^i - b_i\right) = \lambda^i \left(C_{ij}^T + C_{ij}^F\right) + W_j^i,$$

[26]We prescribe that the multinational locates in country i if $\pi_i^M + b_i = \pi_j^M + b_j$.

[27]Of course, if $\max\left\{\pi_i^M + b_i,\ \pi_j^M + b_j\right\} \leq 0$, the multinational does not invest in any countries. As we will see, this does not happen in an equilibrium.

we can solve for government i's willingness to pay to attract the multinational, S_i.[28]

$$\begin{aligned} S_i &= \lambda^i \left[\left(C^T_{ii} + C^F_{ii} \right) - \left(C^T_{ij} + C^F_{ij} \right) \right] + \left(W^i_i - W^i_j \right) \\ &= \lambda^i \left[\left(C^T_{ii} + C^F_{ii} \right) - \left(C^T_{ij} + C^F_{ij} \right) \right] + \frac{1}{16}\Delta_i. \end{aligned} \tag{2.8}$$

S_i consists of two terms. The second term is familiar: it represents government i's economic incentive to attract FDI. The first term represents an extra political incentive (or disincentive) for government i to attract FDI, which is provided by special interest groups via the domestic political competition. When the multinational locates in country i, the amount of political contributions that government i receives is equal to $\left(C^T_{ii} + C^F_{ii} \right)$. When the multinational locates in country j, it receives $\left(C^T_{ij} + C^F_{ij} \right)$. So, in case when it attracts FDI, it receives $\left(C^T_{ii} + C^F_{ii} \right)$ at the expense of $\left(C^T_{ij} + C^F_{ij} \right)$. The net political contributions that it receives are equal to $\left(C^T_{ii} + C^F_{ii} \right) - \left(C^T_{ij} + C^F_{ij} \right)$. Since government i's marginal rate of substitution between political contributions and national welfare is λ^i, it is willing to pay an extra amount, $\lambda^i \left[\left(C^T_{ii} + C^F_{ii} \right) - \left(C^T_{ij} + C^F_{ij} \right) \right]$, to the multinational in order to receive $\left(C^T_{ii} + C^F_{ii} \right) - \left(C^T_{ij} + C^F_{ij} \right)$. If $\left(C^T_{ii} + C^F_{ii} \right) - \left(C^T_{ij} + C^F_{ij} \right)$ is positive, so, $\lambda^i \left[\left(C^T_{ii} + C^F_{ii} \right) - \left(C^T_{ij} + C^F_{ij} \right) \right]$ is positive, then government i is provided a political incentive to attract FDI. Otherwise, it is provided a political disincentive to attract FDI. Notice that S_i increases with C^T_{ii} and C^F_{ii}, decreases with C^T_{ij} and C^F_{ij}. So, there is a chance for special interest groups to manipulate government i's willingness to pay to attract the multinational.

Similarly, government j's willingness to pay to attract the multinational is given by

$$\begin{aligned} S_j &= \lambda^j \left[\left(C^T_{jj} + C^F_{jj} \right) - \left(C^T_{ji} + C^F_{ji} \right) \right] + \left(W^j_j - W^j_i \right) \\ &= \lambda^j \left[\left(C^T_{jj} + C^F_{jj} \right) - \left(C^T_{ji} + C^F_{ji} \right) \right] + \frac{1}{16}\Delta_j. \end{aligned} \tag{2.9}$$

And a similar discussion applies.

Therefore, given contribution schedules, and given the governments' anticipation of how the game evolves from the second stage, the equilibrium in this stage is characterized as follows:[29] country i wins the competition, and pays the amount $b_i = S_j - \left(\frac{1}{18}\Delta_i - \frac{1}{18}\Delta_j \right)$,

[28]Notice that the gross value of FDI to government i is $\left[\lambda^i \left(C^T_{ii} + C^F_{ii} \right) + W^i_i \right] - \left[\lambda^i \left(C^T_{ij} + C^F_{ij} \right) + W^i_j \right]$. However, government i pays b_i to the multinational when the multinational locates in country i. Therefore, the net value of FDI to government i is $\left[\lambda^i \left(C^T_{ii} + C^F_{ii} \right) + W^i_i \right] - \left[\lambda^i \left(C^T_{ij} + C^F_{ij} \right) + W^i_j \right] - b_i = \left[\lambda^i \left(C^T_{ii} + C^F_{ii} \right) + \left(W^i_i - b_i \right) \right] - \left[\lambda^i \left(C^T_{ij} + C^F_{ij} \right) + W^i_j \right]$. Let this expression be equal to zero, we can solve for government i's willingness to pay to attract the multinational.

[29]Here we concentrate on the standard Bertrand equilibrium in which players do not play weakly dominated strategies.

to the multinational if and only if

$$S_i + \frac{1}{18}\Delta_i \geq S_j + \frac{1}{18}\Delta_j, \tag{2.10}$$

Otherwise government j wins the competition, and pays the multinational $b_j = S_i + \left(\frac{1}{18}\Delta_i - \frac{1}{18}\Delta_j\right)$.

Notice that the necessary and sufficient condition – condition (2.10) – for country i to win FDI competition in an equilibrium is that government i's political incentive (or disincentive) plus its economic incentive to attract FDI , plus the multinational's investment incentive in country i (weakly) dominates government j's political incentive (or disincentive) plus its economic incentive to attract FDI, plus the multinational's investment incentive in country j. Otherwise, country j wins FDI competition in an equilibrium.

It is useful to note the following Remark.

Remark 2.1 *(Benchmark: No Politics) If government i and j maximize national welfare, i.e.,* $\lambda^i, \lambda^j = 0$, *then a government's political incentive or disincentive to attract FDI disappears. So,* $S_i = \frac{1}{16}\Delta_i$, *and* $S_j = \frac{1}{16}\Delta_j$. *Since* $\frac{1}{16}\Delta_i + \frac{1}{18}\Delta_i > \frac{1}{16}\Delta_j + \frac{1}{18}\Delta_j$, *country i always wins FDI competition. The equilibrium price for attracting the multinational is equal to country j's economic incentive to attract FDI minus the multinational's investment premium in country i,* $b_i = \frac{1}{16}\Delta_j - \left(\frac{1}{18}\Delta_i - \frac{1}{18}\Delta_j\right)$. *This shows a general result that previous literatures had obtained: without political economy, a country wins FDI competition in an equilibrium if and only if its economic incentive to attract FDI plus the multinational's investment incentive in this country is greater than the other country's economic incentive to attract FDI plus the multinational's investment incentive in the other country. In this sense, the difference between these two sums,* $\left(\frac{1}{16}\Delta_i + \frac{1}{18}\Delta_i\right) - \left(\frac{1}{16}\Delta_j + \frac{1}{18}\Delta_j\right) = \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) > 0$, *represents country i's economic advantage over country j in competition for FDI. Now, the result can also be stated as follows: without politics, an economically advantageous country wins the competition in an equilibrium for sure.*

Now, government i's and j's economic incentive to attract FDI, the multinational's investment incentive in country i and j, are summarized by country i's economic advantage in FDI competition. Rearranging condition (2.10), we have the following condition,

$$\lambda^i \left[(C^T_{ii} + C^F_{ii}) - (C^T_{ij} + C^F_{ij}) \right] + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) \geq \lambda^j \left[(C^T_{jj} + C^F_{jj}) - (C^T_{ji} + C^F_{ji}) \right]. \tag{2.11}$$

It implies that given political contributions, whether a country wins FDI competition is determined by the interactions of whether it has an economic advantage in FDI competition, and its government's political incentive (or disincentive) and the other government's political incentive (or disincentive) to attract FDI. With this condition in mind, we turn to analyze how special interest groups play the first stage of the game.

2.4.2 The first stage

First of all, notice that no interest group will make strictly positive political contributions for both locations. Any interest group may gain or lose from FDI, or may be indifferent between the two locations. Obviously, it does not have an incentive to make strictly positive political contributions when its unfavorable outcome occurs, whilst it may do that when its favorable outcome occurs. If this interest group is indifferent between the two outcomes, it surely does not have an incentive to make strictly positive political contributions irrespective of in which country the multinational locates. In addition, it is quite natural to think that the political contributions, which this interest group makes when its favorable outcome occurs, should not be strictly greater than its net gain under that outcome.[30]

See Table 2.2. In country i, trade union i gains, whilst firm i loses from FDI. Trade union i's net gain is $\frac{1}{12}\Delta_i$ if the multinational locates in country i. Hence we have

$$0 \leq C_{ii}^T \leq \frac{1}{12}\Delta_i,\ C_{ij}^T = 0. \tag{2.12}$$

If the multinational locates in country j, firm i's net gain is $\frac{5}{72}\Delta_i$. Hence we have

$$C_{ii}^F = 0,\ 0 \leq C_{ij}^F \leq \frac{5}{72}\Delta_i. \tag{2.13}$$

Country j's case is very much similar to country i's. Replacing subscript i with j, subscript ii with jj, and subscript ij with ji, we have country j's case.

Moreover, condition (2.11) reduces to

$$\lambda^i \left(C_{ii}^T - C_{ij}^F\right) + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) \geq \lambda^j \left(C_{jj}^T - C_{ji}^F\right). \tag{2.14}$$

Whether a government has a political incentive or disincentive to attract FDI is determined by which special interest group wins the domestic political competition in the sense that its political contributions are bigger than its rival's.

The highest incentive that trade union k can provide for government k to attract FDI is given by $\frac{1}{12}\lambda^k\Delta_k$, since $\lambda^k\left(C_{kk}^T - C_{kl}^F\right)$ increases with trade union k's political

[30] By doing this, we assume implicitly that we do not allow players to choose weakly dominated strategies in the first stage of the game. Also see Grossman and Helpman (1995a).

contributions, which is not strictly greater than its net gain under FDI. The highest disincentive that firm k can provide for government k to attract FDI is given by $\frac{5}{72}\lambda^k\Delta_k$, since $\lambda^k\left(C^T_{kk}-C^F_{kl}\right)$ decreases with firm k's political contributions, which is not strictly greater than its net gain when the multinational locating in country l. $k=i,j$, $l=i,j$, $k\neq l$.

This implies that in each country the trade union is always able to win the domestic political competition. Since the trade union gains more than the domestic firm loses from FDI, whatever a disincentive to attract FDI is provided by the domestic firm, it would be beaten by an incentive to attract FDI provided by the trade union if doing so is profitable.

We say that government k's political-competition-proof highest political incentive to attract FDI is given by $\frac{1}{12}\lambda^k\Delta_k-\frac{5}{72}\lambda^k\Delta_k=\frac{1}{72}\lambda^k\Delta_k$, since trade union k cannot increase government k's incentive, at the same time firm k cannot increase government k's disincentive to attract FDI, and trade union k wins the domestic political competition, $k=i,j$.

Before going further, it is useful to note every interest group's payoff function in the first stage of the game.

Trade union i's payoffs are as follows: it gets $\frac{1}{4}\Delta_i+\left(\frac{1}{12}\Delta_i-C^T_{ii}\right)$ if the multinational locates in country i; it gets $\frac{1}{4}\Delta_i$ if the multinational locates in country j.

Firm i's payoffs are as follows: it gets $\frac{1}{18}\Delta_i$ if the multinational locates in country i; it gets $\frac{1}{18}\Delta_i+\left(\frac{5}{72}\Delta_i-C^F_{ij}\right)$ if the multinational locates in country j.

Trade union j's payoffs are as follows: it gets $\frac{1}{4}\Delta_j$ if the multinational locates in country i; it gets $\frac{1}{4}\Delta_j+\left(\frac{1}{12}\Delta_j-C^T_{jj}\right)$ if the multinational locates in country j.

Firm j's payoffs are as follows: it gets $\frac{1}{18}\Delta_j+\left(\frac{5}{72}\Delta_j-C^F_{ji}\right)$ if the multinational locates in country i; it gets $\frac{1}{18}\Delta_j$ if the multinational locates in country j.

Equilibrium characterization

First we derive the best response for each special interest group.

Lemma 2.1 *(Best Response)*

1. *Given the other players' strategies, C^T_i is trade union i's best response, in which*

$$C^T_{ii}=\max\left\{0,z^T_i\right\}\quad if\quad \lambda^i\left(\tfrac{1}{12}\Delta_i-C^F_{ij}\right)+\left(\tfrac{1}{16}+\tfrac{1}{18}\right)(\Delta_i-\Delta_j)\geq\lambda^j\left(C^T_{jj}-C^F_{ji}\right),$$

$$C^T_{ii}\in\left[0,\tfrac{1}{12}\Delta_i\right]\quad if\quad \qquad otherwise$$

where z_i^T is determined by

$$\lambda^i \left(z_i^T - C_{ij}^F\right) + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) = \lambda^j \left(C_{jj}^T - C_{ji}^F\right).$$

2. *Given the other players' strategies, C_i^F is firm i's best response, in which*

$$\begin{array}{lll} C_{ij}^F \in \left[0, \frac{5}{72}\Delta_i\right] & if & \lambda^i \left(C_{ii}^T - \frac{5}{72}\Delta_i\right) + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) \geq \lambda^j \left(C_{jj}^T - C_{ji}^F\right) \\ C_{ij}^F = \max\left\{0, z_i^F\right\} & if & otherwise \end{array},$$

where z_i^F is determined by

$$\lambda^i \left(C_{ii}^T - z_i^F\right) + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) = \lambda^j \left(C_{jj}^T - C_{ji}^F\right).$$

3. *Given the other players' strategies, C_j^T is trade union j's best response, in which*

$$\begin{array}{lll} C_{jj}^T \in \left[0, \frac{1}{12}\Delta_j\right] & if & \lambda^i \left(C_{ii}^T - C_{ij}^F\right) + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) \geq \lambda^j \left(\frac{1}{12}\Delta_j - C_{ji}^F\right) \\ C_{jj}^T = \max\left\{0, z_j^T\right\} & & otherwise \end{array},$$

where z_j^T is determined by

$$\lambda^i \left(C_{ii}^T - C_{ij}^F\right) + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) = \lambda^j \left(z_j^T - C_{ji}^F\right).$$

4. *Given the other players' strategies, C_j^F is firm j's best response, in which*

$$\begin{array}{lll} C_{ji}^F = \max\left\{0, z_j^F\right\} & if & \lambda^i \left(C_{ii}^T - C_{ij}^F\right) + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) \geq \lambda^j \left(C_{jj}^T - \frac{5}{72}\Delta_j\right) \\ C_{ji}^F \in \left[0, \frac{5}{72}\Delta_j\right] & if & otherwise \end{array},$$

where z_j^F is determined by

$$\lambda^i \left(C_{ii}^T - C_{ij}^F\right) + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) = \lambda^j \left(C_{jj}^T - z_j^F\right).$$

Proof. See Appendix to Chapter 2. ■

A combination of special interest groups' contribution schedules is a Nash equilibrium, if and only if given other three special interest groups' contribution schedules, any special interest group's contribution schedule is its best response. But Nash equilibria are too many. So, we characterize the CPNE (CPNEs) in the first stage of the game.[31]

We prove that there are three forms of CPNEs depending on parameter configurations. Firstly, consider the case where

$$-\frac{5}{72}\lambda^i\Delta_i + \left(\frac{1}{16}+\frac{1}{18}\right)(\Delta_i - \Delta_j) \geq \frac{1}{12}\lambda^j\Delta_j. \tag{2.15}$$

Proposition 2.1 *$C_i^T = 0$; $C_j^F = 0$; plus the following contribution schedules*

$$C_i^F = \begin{cases} 0 & if \quad FDI\ in\ country\ i, \\ C_{ij}^F & if \quad FDI\ in\ country\ j, \end{cases}$$

where $C_{ij}^F \in \left[0, \frac{5}{72}\Delta_i\right]$;

$$C_j^T = \begin{cases} 0 & if \quad FDI\ in\ country\ i, \\ C_{jj}^T & if \quad FDI\ in\ country\ j, \end{cases}$$

where $C_{jj}^T \in \left[0, \frac{1}{12}\Delta_j\right]$; constitute a CPNE in the first stage of the game, in which country i wins the competition for the multinational.

Proof. See Appendix to Chapter 2. ∎

Condition (2.15) says that country i's economic advantage in FDI competition minus government i's highest political disincentive to attract FDI (weakly) dominates government j's highest political incentive to attract FDI, when trade union i and firm j do not make political contributions. This happens when both λ^i and λ^j are sufficiently small, in other words, the extent to which each government is influenced by special interest groups is

[31]Bernheim, Peleg and Whinston (1987) develop the concept of a CPNE. See page pp. 6.

"... consider an n-player game $\Gamma = \left[\{g^i\}_{i=1}^n, \{S^i\}_{i=1}^n\right]$, where S^i is player i's strategy set and $g^i\colon \Pi_{j=1}^n S^j \to R$ is player i's payoff function. Let J be the set of proper subsets of $\{1,\ldots,n\}$, and denote an element of J (a 'coalition') as $J \in J$. Let $S^J \equiv \Pi_{i\in J}S^i$; for the case of $\{1,\ldots,n\}$ we will simply write S. Also let $-J$ denote the complement of J in $\{1,\ldots,n\}$. Finally, for each $s_{-J}^0 \in S^{-J}$, let Γ/s_{-J}^0 denote the game induced on subgroup J by the actions s_{-J}^0 for coalition $-J$, i.e.,

$$\Gamma/s_{-J}^0 \equiv \left[\left\{\bar{g}^i\right\}_{i\in J}, \left\{S^i\right\}_{i\in J}\right],$$

where $\bar{g}^i\colon S^J \to R$ is given by $\bar{g}^i(s_J) \equiv g^i\left(s_J, s_{-J}^0\right)$ for all $i \in J$ and $s_J \in S^J$.

...

DEFINITION. (i) In a single player game Γ, $s^* \in S$ is a Coalition-Proof Nash equilibrium if and only if s^* maximizes $g^1(s)$.

(ii) Let $n > 1$ and assume that Coalition-Proof Nash equilibrium has been defined for games with fewer than n players. Then,

(a) For any game Γ with n players, $s^* \in S$ is self-enforcing if, for all $J \in J$, s_J^* is a Coalition-Proof Nash equilibrium in the game Γ/s_{-J}^*.

(b) For any game Γ with n players, $s^* \in S$ is a Coalition-Proof Nash equilibrium if it is self-enforcing and if there does not exist another self-enforcing strategy vector $s \in S$ such that $g^i(s) > g^i(s^*)$ for all $i = 1,\ldots,n$."

sufficiently small; and country i's economic advantage is sufficiently big. As a result, even if pre-play communication is allowed, firm i and trade union j cannot coordinate and help country j win the competition noncooperatively: firm i cannot increase government i's political disincentive, at the same time trade union j cannot increase government j's political incentive enough to offset country i's economic advantage. Clearly trade union i and firm j will not make strictly positive political contributions. Firm i and trade union j can choose arbitrary political contributions.[32]

We have a continuum of equilibria here. Given any equilibrium, country i wins the competition for the multinational, and pays the amount

$$b_{i1} = \lambda^j C_{jj}^T + \frac{1}{16}\Delta_j - \frac{1}{18}(\Delta_i - \Delta_j), \tag{2.16}$$

where $C_{jj}^T \in \left[0, \frac{1}{12}\Delta_j\right]$, to the multinational. b_{i1} takes the minimum value at $C_{jj}^T = 0$, so that the minimum payment to the multinational is given by[33]

$$b_{i1}^{\min} = \frac{1}{16}\Delta_j - \frac{1}{18}(\Delta_i - \Delta_j). \tag{2.17}$$

Secondly, consider the case where

$$\frac{1}{72}\lambda^i\Delta_i + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) \geq \frac{1}{72}\lambda^j\Delta_j, \text{ but } -\frac{5}{72}\lambda^i\Delta_i + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) < \frac{1}{12}\lambda^j\Delta_j. \tag{2.18}$$

Proposition 2.2 *The following contribution schedules*

$$C_i^T = \begin{cases} C_{ii}^T & if \quad FDI \text{ in country } i, \\ 0 & if \quad FDI \text{ in country } j; \end{cases}$$

$$C_i^F = \begin{cases} 0 & if \quad FDI \text{ in country } i, \\ \frac{5}{72}\Delta_i & if \quad FDI \text{ in country } j; \end{cases}$$

$$C_j^T = \begin{cases} 0 & if \quad FDI \text{ in country } i, \\ \frac{1}{12}\Delta_j & if \quad FDI \text{ in country } j; \end{cases}$$

$$C_j^F = \begin{cases} C_{ji}^F & if \quad FDI \text{ in country } i, \\ 0 & if \quad FDI \text{ in country } j; \end{cases}$$

[32] Notice that condition (2.15) is also necessary. Suppose not. Then given trade union i and firm j do not make political contributions, clearly firm i and trade union j can coordinate and help country j win the competition in a noncooperative way if pre-play communication is allowed.

[33] Notice that the multinational receives at least, $\frac{1}{18}\Delta_i + \frac{1}{16}\Delta_j - \frac{1}{18}(\Delta_i - \Delta_j) = \left(\frac{1}{16} + \frac{1}{18}\right)\Delta_j > 0$, in this case.

where C^T_{ii}, and C^F_{ji} satisfy

$$\lambda^i \left(C^T_{ii} - \frac{5}{72}\Delta_i \right) + \left(\frac{1}{16} + \frac{1}{18} \right) (\Delta_i - \Delta_j) = \lambda^j \left(\frac{1}{12}\Delta_j - C^F_{ji} \right), \tag{2.19}$$

constitute a CPNE in the first stage of the game, in which country i wins the competition for the multinational.

Proof. See Appendix to Chapter 2. ∎

The second strict inequality of condition (2.18) implies that the contribution schedules given in Proposition 2.1 cannot form CPNEs now. The first inequality says that government i's political-competition-proof highest political incentive to attract FDI plus country i's economic advantage in FDI competition (weakly) dominates government j's political-competition-proof highest political incentive to attract FDI. In this case, country i still wins the competition since again, even if pre-play communication is allowed, it is impossible for firm i and trade union j to coordinate profitably and help country j win the competition in a noncooperative way. Intuitively, they may form a self-enforcing conspiracy via pre-play communication, but trade union i and domestic firm j can do this also. The above condition guarantees that even if they make their highest political contributions, the self-enforcing conspiracy formed by trade union i and firm j can find a way to defeat them.

Given this form of equilibria, country i wins the competition for the multinational, and pays the amount

$$b_{i2} = \lambda^j \left(\frac{1}{12}\Delta_j - C^F_{ji} \right) + \frac{1}{16}\Delta_j - \frac{1}{18} (\Delta_i - \Delta_j), \tag{2.20}$$

to the multinational. b_{i2} takes the minimum value at $C^F_{ji} = \frac{5}{72}\Delta_j$, so that the minimum payment to the multinational is given by[34]

$$b^{\min}_{i2} = \frac{1}{72}\lambda^j \Delta_j + \frac{1}{16}\Delta_j - \frac{1}{18} (\Delta_i - \Delta_j). \tag{2.21}$$

Finally, consider the case where

$$\frac{1}{72}\lambda^i \Delta_i + \left(\frac{1}{16} + \frac{1}{18} \right) (\Delta_i - \Delta_j) < \frac{1}{72}\lambda^j \Delta_j. \tag{2.22}$$

[34] Notice that the multinational receives at least, $\frac{1}{18}\Delta_i + \frac{1}{72}\lambda^j\Delta_j + \frac{1}{16}\Delta_j - \frac{1}{18}(\Delta_i - \Delta_j) = \frac{1}{72}\lambda^j\Delta_j + \left(\frac{1}{16} + \frac{1}{18}\right)\Delta_j > 0$, in this case.

Proposition 2.3 *The following contribution schedules*

$$C_i^T = \begin{cases} \frac{1}{12}\Delta_i & if \quad FDI \text{ in country } i, \\ 0 & if \quad FDI \text{ in country } j; \end{cases}$$

$$C_i^F = \begin{cases} 0 & if \quad FDI \text{ in country } i, \\ C_{ij}^F & if \quad FDI \text{ in country } j; \end{cases}$$

$$C_j^T = \begin{cases} 0 & if \quad FDI \text{ in country } i, \\ C_{jj}^T & if \quad FDI \text{ in country } j; \end{cases}$$

$$C_j^F = \begin{cases} \frac{5}{72}\Delta_j & if \quad FDI \text{ in country } i, \\ 0 & if \quad FDI \text{ in country } j; \end{cases}$$

where C_{ij}^F, *and* $C_{jj}^T > \frac{5}{72}\Delta_j$ *satisfy*

$$\lambda^i \left(\frac{1}{12}\Delta_i - C_{ij}^F\right) + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) = \lambda^j \left(C_{jj}^T - \frac{5}{72}\Delta_j\right), \tag{2.23}$$

constitute a CPNE in the first stage of the game, in which country j wins the competition for the multinational.

Proof. Using the same type of argument in the Proof of Proposition 2.2, we can establish this result. ∎

Condition (2.22) says that government i's political-competition-proof highest political incentive to attract FDI plus country i's economic advantage in FDI competition is (strictly) dominated by government j's political-competition-proof highest political incentive to attract FDI. Now even if pre-play communication is allowed, there is no chance for trade union i and firm j to coordinate profitably and help country i win the competition noncooperatively. Also, notice that in a CPNE, trade union j always wins the domestic political competition.

Given this form of equilibria, country j wins the competition for the multinational, and pays the amount

$$b_j = \lambda^i \left(\frac{1}{12}\Delta_i - C_{ij}^F\right) + \frac{1}{16}\Delta_i + \frac{1}{18}(\Delta_i - \Delta_j), \tag{2.24}$$

to the multinational. b_j takes the minimum value at $C_{ij}^F = \frac{5}{72}\Delta_i$, so that the minimum payment to the multinational is given by[35]

$$b_j^{\min} = \frac{1}{72}\lambda^i\Delta_i + \frac{1}{16}\Delta_i + \frac{1}{18}(\Delta_i - \Delta_j). \tag{2.25}$$

Remark 2.2 *Before going further, notice that using a CPNE as the solution concept in the first stage of the game helps us eliminate some 'unpleasant' equilibria. For instance, it is easy to show that a combination of contribution schedules, in which every special interest group contributes zero, is a Nash equilibrium, if* $\left(\frac{1}{16}+\frac{1}{18}\right)(\Delta_i-\Delta_j) \geq \max\left\{\frac{5}{72}\lambda^i\Delta_i,\ \frac{1}{12}\lambda^j\Delta_j\right\}$. *However, if* $\frac{1}{72}\lambda^i\Delta_i+\left(\frac{1}{16}+\frac{1}{18}\right)(\Delta_i-\Delta_j) \geq \frac{1}{72}\lambda^j\Delta_j$, *but* $-\frac{5}{72}\lambda^i\Delta_i + \left(\frac{1}{16}+\frac{1}{18}\right)(\Delta_i-\Delta_j) < \frac{1}{12}\lambda^j\Delta_j$, *and pre-play communication is allowed, then this equilibrium is not a CPNE.*

Further discussion

Let us state two technical results first.

Lemma 2.2 *In the first stage of the game, if there exists a CPNE, in which country i wins FDI competition, the following condition*

$$\frac{1}{72}\lambda^i\Delta_i + \left(\frac{1}{16}+\frac{1}{18}\right)(\Delta_i-\Delta_j) \geq \frac{1}{72}\lambda^j\Delta_j,$$

must hold.

Proof. See Appendix to Chapter 2. ∎

Lemma 2.3 *In the first stage of the game, if there exists a CPNE, in which country j wins FDI competition, the following condition*

$$\frac{1}{72}\lambda^i\Delta_i + \left(\frac{1}{16}+\frac{1}{18}\right)(\Delta_i-\Delta_j) < \frac{1}{72}\lambda^j\Delta_j,$$

must hold.

Proof. This Lemma is proved by similar arguments to those in the Proof of Lemma 2.2. ∎

The analysis so far implies immediately the following Theorem, which states the necessary and sufficient condition for a country to win FDI competition in an equilibrium.

[35]Notice that the multinational receives at least, $\frac{1}{18}\Delta_j + \frac{1}{72}\lambda^i\Delta_i + \frac{1}{16}\Delta_i + \frac{1}{18}(\Delta_i-\Delta_j) = \frac{1}{72}\lambda^i\Delta_i + \left(\frac{1}{16}+\frac{1}{18}\right)\Delta_i > 0$, in this case.

Theorem 2.1 *(Winner Selection.) Country i wins the competition for the multinational in a CPNE, if and only if*

$$\frac{1}{72}\lambda^i\Delta_i + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) \geq \frac{1}{72}\lambda^j\Delta_j. \qquad (*)$$

Otherwise, Country j wins the competition for the multinational in a CPNE.

Proof. The necessity part of the Theorem is implied by Lemma 2.2 and 2.3, whilst the sufficiency part of the Theorem is implied by Proposition 2.1, 2.2 and 2.3. ■

Theorem 2.1 says that both countries have a chance to win FDI competition in an equilibrium. If in the economically disadvantageous country, the political incentive provided is great enough to dominate both the other country's economic advantage and the other government's political incentive to attract FDI, then the economically disadvantageous country wins competition for FDI. Otherwise, the economically advantageous country wins the competition.

We can derive two testable implications from Theorem 2.1.

Corollary 2.1 *If country j wins the competition for the multinational in a CPNE, then* $\lambda^i < \lambda^j$.

Proof. Suppose not. According to Theorem 2.1, if country j wins the competition for the multinational in a CPNE, we must have $\frac{1}{72}\lambda^i\Delta_i + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) < \frac{1}{72}\lambda^j\Delta_j$. And this strict inequality holds if and only if $\left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) < \frac{1}{72}\lambda^j\Delta_j - \frac{1}{72}\lambda^i\Delta_i$. Since by Assumption 2.1, $\Delta_i > \Delta_j$, $\left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) > 0$. Now if $\lambda^i \geq \lambda^j$, then $\frac{1}{72}\lambda^j\Delta_j - \frac{1}{72}\lambda^i\Delta_i \leq 0$. A contradiction. ■

If the economically disadvantageous country wins FDI competition, then the extent to which its government is influenced by special interest groups must be greater than the extent to which the other government is influenced.

Corollary 2.2 *When* $\Delta_i = \Delta_j = \Delta$, *country i wins the competition for the multinational in a CPNE, if and only if* $\lambda^i \geq \lambda^j$.

Proof. According to Theorem 2.1, country i wins the competition for the multinational in a CPNE, if and only if

$$\text{condition } (*) \text{ holds}$$

$$\Leftrightarrow$$

$$\frac{1}{72}\lambda^i\Delta \geq \frac{1}{72}\lambda^j\Delta, \text{ since } \Delta_i = \Delta_j = \Delta$$

$$\Leftrightarrow$$

$$\lambda^i \geq \lambda^j.$$

■

If no country has an economic advantage over the other country in FDI competition, then the country whose government is more influenced by special interest groups wins FDI competition.

Next, let us examine boundary cases.

Corollary 2.3 *(Boundary Cases)*

1. *When $\lambda^i = 0$, country j wins the competition for the multinational in a CPNE, if and only if*
$$\lambda^j > \frac{17}{2}\left(\frac{\Delta_i}{\Delta_j} - 1\right);$$
2. *When $\lambda^j = 0$, country i always wins the competition for the multinational in a CPNE.*

Proof. (The first part.) According to Theorem 2.1, country j wins the FDI competition in a CPNE, if and only if

$$\frac{1}{72}\lambda^i\Delta_i + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) < \frac{1}{72}\lambda^j\Delta_j$$

$$\Leftrightarrow$$

$$\left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) < \frac{1}{72}\lambda^j\Delta_j, \text{ since } \lambda^i = 0$$

$$\Leftrightarrow$$

$$\lambda^j > \frac{17}{2}\left(\frac{\Delta_i}{\Delta_j} - 1\right).$$

(The second part.) Condition (*) implies this immediately. ■

The first part of Corollary 2.3 says that when government i maximizes national welfare, country j wins FDI competition if and only if the extent to which government j

is influenced by special interest groups is strictly greater than a threshold value, so that the domestic political competition can provide enough political incentive for government j to attract FDI to dominate country i's economic advantage. The second part says that when government j maximizes national welfare, since country i has an economic advantage in FDI competition, and trade union i is always able to win the domestic political competition, there is no chance for country j to win the competition.

Corollary 2.4 *(An Extreme Case) When $\lambda^i \to \infty$, and $\lambda^j \to \infty$, country i wins the competition for the multinational in a CPNE, if and only if*

$$\frac{\Delta_i}{\Delta_j} \geq \frac{\lambda^j}{\lambda^i}.$$

Otherwise, country j wins the competition for the multinational in a CPNE.

Proof. According to Theorem 2.1, country i wins the competition for the multinational in a CPNE, if and only if

$$\text{condition (*) holds}$$

$$\Leftrightarrow$$

$$\frac{1}{72}\lambda^i\Delta_i + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) \geq \frac{1}{72}\lambda^j\Delta_j$$

$$\Leftrightarrow$$

$$\frac{\frac{1}{72}\lambda^i\Delta_i + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j)}{\frac{1}{72}\lambda^j\Delta_j} \geq 1$$

$$\Leftrightarrow$$

$$\frac{\lambda^i\frac{\Delta_i}{\Delta_j}}{\lambda^j} + \frac{\frac{17}{2}\left(\frac{\Delta_i}{\Delta_j} - 1\right)}{\lambda^j} \geq 1. \tag{2.26}$$

When $\lambda^i \to \infty$, and $\lambda^j \to \infty$, the second term in the LHS of condition (2.26) vanishes. Hence, country i wins the competition in a CPNE, if and only if

$$\frac{\lambda^i\frac{\Delta_i}{\Delta_j}}{\lambda^j} \geq 1 \Leftrightarrow \frac{\Delta_i}{\Delta_j} \geq \frac{\lambda^j}{\lambda^i}.$$

■

Corollary 2.4 says that when both governments maximize political contributions, country i's economic advantage in FDI competition can be neglected. Now, the government with a 'bigger' political incentive to attract FDI, (given by the ratio stated in the Corollary), wins the competition.

We use Figure 2.2 to summarize the above discussion.

Define $\Delta \equiv \frac{\Delta_i}{\Delta_j} > 1$. Now, condition (*) reduces to

$$\lambda^i \Delta + \frac{17}{2}(\Delta - 1) \geq \lambda^j. \tag{*'}$$

Condition (2.15) reduces to

$$-5\lambda^i \Delta + \frac{17}{2}(\Delta - 1) \geq 6\lambda^j. \tag{2.15'}$$

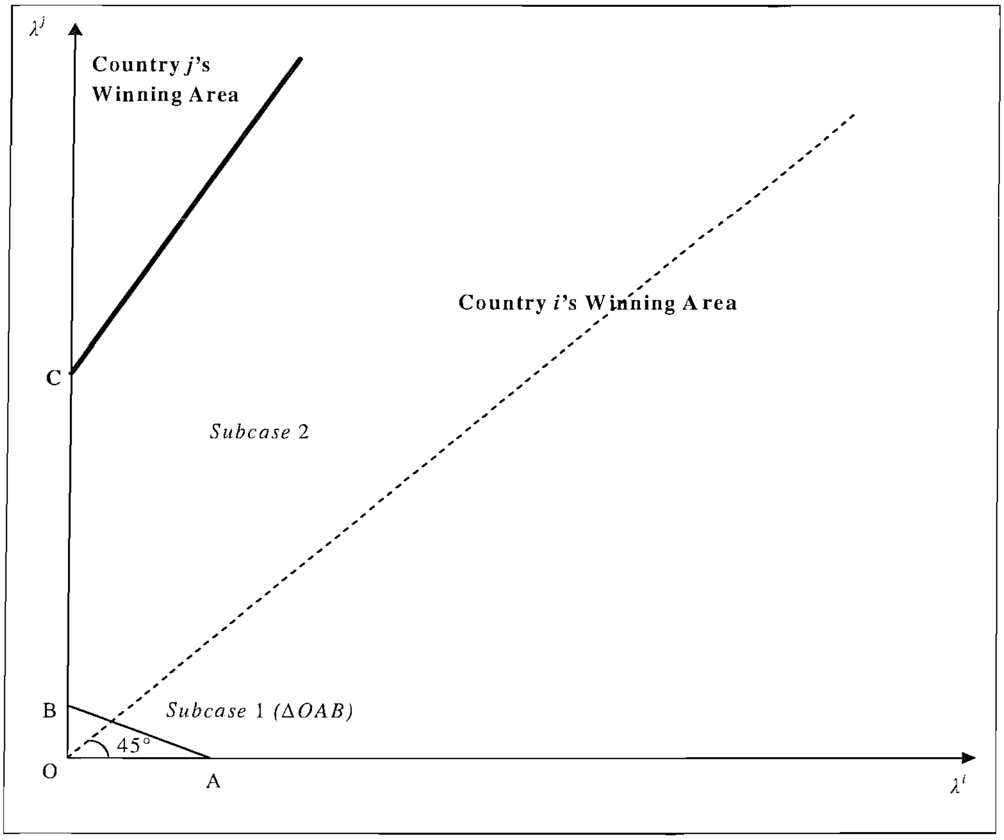


Figure 2.2: Winner selection

See Figure 2.2. The horizontal axis represents λ^i, and the vertical axis represents λ^j. The bold line represents when condition (*') holds with equality. This line divides the nonnegative quadrant into two parts. When parameter configurations fall into the big part, country i wins FDI competition in an equilibrium. There are two subcases. Notice that line segment AB represents when condition (2.15') holds with equality.[36] Now the triangle $\triangle OAB$ represents the case given by Proposition 2.1. Subcase 2 represents the case given by Proposition 2.2. When parameter configurations fall into the small part

[36]The coordinate of point A is given by $(\lambda^i, \lambda^j) = \left(\frac{17}{10\Delta}(\Delta - 1), 0\right)$. The coordinate of point B is given by $(\lambda^i, \lambda^j) = \left(0, \frac{17}{12}(\Delta - 1)\right)$.

above the bold line, country j wins the competition in an equilibrium. This is described in Proposition 2.3.

When country j wins FDI competition in an equilibrium, it must be the case that $\lambda^i < \lambda^j$. This is stated in Corollary 2.1. When one country does not have an economic advantage over the other country, the bold line and the forty-five degree line coincide. Now, the government which is more influenced by special interest groups wins the competition in an equilibrium. This is stated in Corollary 2.2.

As to boundary cases, first of all, notice that the coordinate of point C is given by $(\lambda^i, \lambda^j) = \left(0, \frac{17}{2}(\Delta - 1)\right)$. Now, keeping $\lambda^i = 0$, if λ^j is slightly bigger than $\frac{17}{2}(\Delta - 1)$, parameter configurations fall into country j's winning area. This represents the first part of Corollary 2.3. It is easy to see that the horizontal axis lies in country i's winning area. This represents the second part of Corollary 2.3. It is clear from Figure 2.2 that when λ^i and λ^j go to infinity, which country wins FDI competition in an equilibrium is determined by the relative size of the slope of the bold line, Δ, and the ratio of λ^j to λ^i since country i's economic advantage can be neglected in this case. This is stated in Corollary 2.4.

From Figure 2.2, it is also easy to see that when λ^k goes to infinity, whilst λ^l is bounded, $k = i, j$, $l = i, j$, $i \neq j$, country k always wins the competition in an equilibrium.

Next, we have the following Theorem.

Theorem 2.2 *The equilibrium price for attracting FDI is higher than in the benchmark case.*

Proof. In the benchmark case, which is given by Remark 2.1, country i wins the competition for the multinational, and the equilibrium price for attracting FDI is $b_i = \frac{1}{16}\Delta_j - \left(\frac{1}{18}\Delta_i - \frac{1}{18}\Delta_j\right)$. The Theorem is implied immediately when comparing this price to the prices given by expression (2.17), (2.21) and (2.25). ■

The competition for the multinational can be viewed as a Bertrand game. When special interest lobbying is present, each government is provided an extra political inventive to attract FDI besides an economic incentive. So, irrespective of who wins the competition in an equilibrium, the payments to the multinational must be higher than before.

2.5 Welfare Analysis

We consider welfare effects in this section. Our benchmark is the case discussed in Remark 2.1. In this case country i always wins FDI competition.[37] Country i's national welfare is

[37] Notice that the benchmark case is represented by the origin point in Figure 2.2.

given by $W_i^i = \frac{1}{2}\Delta_i - \left[\frac{1}{16}\Delta_j - \frac{1}{18}(\Delta_i - \Delta_j)\right]$, whilst country j's national welfare is given by $W_i^j = \frac{7}{16}\Delta_j$. Allocative efficiency is always achieved.[38]

Now consider the case where

$$-\frac{5}{72}\lambda^i\Delta_i + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) \geq \frac{1}{12}\lambda^j\Delta_j.$$

Proposition 2.4 *Country i's national welfare is the same as in the benchmark case when it pays $b_{i1}^{\min}$ to the multinational, otherwise its national welfare is strictly smaller than in the benchmark case. Country j's national welfare is the same as in the benchmark case. Allocative efficiency is achieved.*

Proof. According to Proposition 2.1, country i wins the competition in a CPNE in this case. Country i pays the multinational b_{i1}, which is given by expression (2.16). Country i's national welfare, $\frac{1}{2}\Delta_i - b_{i1}$, decreases strictly with b_{i1}. It takes its maximum value at $b_{i1}^{\min}$, which is given by expression (2.17). And $\frac{1}{2}\Delta_i - b_{i1}^{\min} = \frac{1}{2}\Delta_i - \left[\frac{1}{16}\Delta_j - \frac{1}{18}(\Delta_i - \Delta_j)\right]$, which is equal to country i's national welfare in the benchmark case. Otherwise, $\frac{1}{2}\Delta_i - b_{i1} < \frac{1}{2}\Delta_i - \left[\frac{1}{16}\Delta_j - \frac{1}{18}(\Delta_i - \Delta_j)\right]$. Since country j loses the competition for the multinational, it gets $\frac{7}{16}\Delta_j$, which is equal to its national welfare in the benchmark case.

Notice that b_{i1} is a transfer payment. It is straightforward to show that allocative efficiency is achieved. ■

Since country i's payment to the multinational is generally higher than its payment to the multinational in the benchmark case, its national welfare is generally lower than in the benchmark case.

Consider the case where

$$\frac{1}{72}\lambda^i\Delta_i + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) \geq \frac{1}{72}\lambda^j\Delta_j, \text{ but } -\frac{5}{72}\lambda^i\Delta_i + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) < \frac{1}{12}\lambda^j\Delta_j.$$

Proposition 2.5 *Country i's national welfare is strictly smaller than in the benchmark case. Country j's national welfare is the same as in the benchmark case. Allocative efficiency is achieved.*

Proof. According to Proposition 2.2, country i wins the competition in a CPNE in this case. Country i pays the multinational b_{i2}, which is given by expression (2.20). Country i's national welfare, $\frac{1}{2}\Delta_i - b_{i2}$, decreases strictly with b_{i2}. It takes its maximum value at $b_{i2}^{\min}$, which is given by expression (2.21). We have $\frac{1}{2}\Delta_i - b_{i2}^{\min} = \frac{1}{2}\Delta_i - \left[\frac{1}{72}\lambda^j\Delta_j + \frac{1}{16}\Delta_j - \frac{1}{18}(\Delta_i - \Delta_j)\right]$, which is strictly smaller than its national welfare in the

[38] Allocative efficiency requires that the multinational locates in a country such that the country's economic incentive to attract FDI and the multinational's investment incentive in the country are jointly maximized.

benchmark case: $\frac{1}{2}\Delta_i - \left[\frac{1}{16}\Delta_j - \frac{1}{18}(\Delta_i - \Delta_j)\right]$. Since country j loses the competition for the multinational, it gets $\frac{7}{16}\Delta_j$, which is equal to its national welfare in the benchmark case.

Notice that b_{i2} is a transfer payment. It is straightforward to show that allocative efficiency is achieved. ■

Since country i's payment to the multinational is strictly higher than its payment to the multinational in the benchmark case, its national welfare is strictly lower than in the benchmark case.

In Propositions 2.4 and 2.5, allocative efficiency is achieved. This is simply because that country i wins FDI competition in an equilibrium.

The remaining case is when country j wins FDI competition. This occurs when

$$\frac{1}{72}\lambda^i\Delta_i + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) < \frac{1}{72}\lambda^j\Delta_j.$$

In this case, Proposition 2.6 holds.

Proposition 2.6 *Country i's national welfare is strictly smaller than in the benchmark case. Country j's national welfare is strictly smaller than in the benchmark case. Allocative efficiency is not achieved.*

Proof. According to Proposition 2.3, country j wins the competition in a CPNE in this case. Country j pays the multinational b_j, which is given by expression (2.24). Country i's national welfare is $\frac{7}{16}\Delta_i$. It is straightforward to show that this is strictly smaller than its national welfare in the benchmark case: $\frac{1}{2}\Delta_i - \left[\frac{1}{16}\Delta_j - \frac{1}{18}(\Delta_i - \Delta_j)\right]$. Country j's national welfare, $\frac{1}{2}\Delta_j - b_j$, decreases strictly with b_j. It takes its maximum value at $b_j^{\min}$, which is given by expression (2.25). And $\frac{1}{2}\Delta_j - b_j^{\min} = \frac{1}{2}\Delta_j - \left[\frac{1}{72}\lambda^i\Delta_i + \frac{1}{16}\Delta_i + \frac{1}{18}(\Delta_i - \Delta_j)\right]$. It is straightforward to show that this is strictly smaller than $\frac{7}{16}\Delta_j$, its national welfare in the benchmark case.

Notice that b_j is a transfer payment. It is straightforward to show that allocative efficiency is not achieved. ■

Given that trade union j wins the domestic political competition in an equilibrium, if government j is far more influenced by special interest groups, then its political incentive to attract FDI may be sufficiently great such that its willingness to pay to attract the multinational can be greater than government i's willingness to pay; country j then wins FDI competition in an equilibrium. Therefore, allocative efficiency is not achieved. Country i's potential gain from FDI is not achieved, at the same time country j makes payment to the multinational. Hence, both country i's and country j's national welfare are strictly smaller than their national welfare in the benchmark case.

2.6 Discussion

This section discusses the robustness of results obtained in the current model.

Firstly, what a trade union gains more than a domestic firm loses from FDI, and therefore the former is always able to win the domestic political competition, is a key point emerging from the current model. But we use a simplest approach to modelling the wage-setting procedure and it has two assumptions: (i) the objective function of a trade union is its economic rents, (ii) a trade union sets the wage rate unilaterally. Keeping the first assumption, consider the case when a trade union bargains over the wage rate with a firm (or firms). Then if the bargaining strength of a trade union is sufficiently great, it still wins the domestic political competition. Keeping the second assumption, consider the case where the objective function of a trade union is a wage bill, which is equal to the actual wage rate times the employment level, or the case where a trade union receives its economic rents plus a share in profits. Then a trade union still wins the domestic political competition.

Secondly, in our model, we treat the marginal product of labor as the opportunity wage rate for workers. The purpose of doing this is to simplify analysis. We can introduce a workers' outside option, which is determined in the rest of the economy, and is not necessarily equal to the marginal product of labor, into the basic model. But our key results are unlikely to change.

Thirdly, our model uses a linear inverse market demand and constant returns to scale production, and marketing technologies. However, we normalize all economic terms in terms of social welfare under perfect competition. Since economic terms appear in relative forms, we doubt whether specific functional forms matter that much in our model. When we use general functional forms, we can do a similar normalization. We may have different coefficients from those obtained in the current model; or coefficients may be functions of fundamental parameters of new models rather than constants. But, notice that provided in general cases, a trade union gains more than a domestic firm loses from FDI, then our key results are unlikely to change.

Fourthly, we consider the no-trade case in this paper. But people may argue that a more realistic setting is to consider the case when the multinational is able to trade between countries, though domestic firms not. But we doubt whether the basic results derived from the no-trade case would be changed when considering this more complicated case. When we allow the multinational to trade between countries, on the one hand, a trade union would gain from FDI more than in the current model, on the other hand a domestic firm would lose from FDI more than in the current model. The status of a trade union, the special interest group lobbying for FDI, in the domestic political competition would be reinforced.

Fifthly, in our model when a country wins the competition for the multinational, its government pays a lump-sum subsidy to the multinational, which is collected from consumers by lump-sum taxation. Now, what will happen when the domestic firm and the trade union share costs for attracting FDI. On the one hand, a trade union's net gains under FDI decrease. On the other hand, a domestic firm's net gains under no FDI increase. But provided a trade union's net gains under FDI are bigger than a domestic firm's net gains under no FDI, then our key results are unlikely to change.

Finally, notice that when both governments maximize political contributions, the equilibrium price for attracting FDI goes to infinity.[39] This unpleasant result is due to the fact that governments' budget constraints are not included in our model. When these constraints are explicitly modeled, an infinite equilibrium price will not appear.

2.7 Conclusion

We have studied the impact of special interest lobbying on competition between two countries for a multinational in a common agency framework. We argue that special interest groups provide a government an extra political incentive to attract FDI via the domestic political competition. If in the economically disadvantageous country, the political incentive provided is great enough to dominate both the other country's economic advantage and the other government's political incentive to attract FDI, then the economically disadvantageous country wins competition for FDI. Otherwise, the economically advantageous country wins the competition. The equilibrium price for attracting FDI is higher than in the case when governments maximize national welfare. We also show that allocative efficiency cannot be always achieved. This happens when the economically disadvantageous country wins the competition.

We may extend the basic model in several ways. First of all, an interesting case is where direct export is one of the multinational's options. Now, a trade union may lobby for a high tariff and a high subsidy; whilst a domestic firm may lobby for a low tariff and a low subsidy. Another possible extension is to consider the case where the multinational is allowed to make political contributions. As a first step, we need to figure out what the multinational's contribution schedule would look like. In addition, notice that people often argue that FDI has a technological spillover effect, which is not considered in our model. What would happen when introducing this effect to the basic model? If the technological

[39]Notice that Corollary 2.4 implies this immediately.

spillover effect is small, then a trade union gains from, whilst a domestic firm loses from FDI. But the more interesting case is when this effect is large enough such that both a trade union and a domestic firm in each country gain from FDI. Now, the political climate changes. As a result, competition for FDI would become more fierce. Finally, in the basic model, the extent to which a government is influenced by domestic special interest groups is exogenously given. An interesting extension is to endogenize this parameter in, say, a probabilistic voting model. At the first place, we need to figure out how to embed this into the basic model.

We plan to analyze these issues in future work.

Chapter 3

Advertising in a Differentiated Duopoly and Its Policy Implications for an Open Economy

Abstract

In this paper, we develop a model of advertising in a differentiated duopoly in which firms first decide how much to invest in cooperative or predatory advertising and then engage in product market competition (Cournot or Bertrand). We then use this model, with the type of advertising endogenously determined, to explore the policy implications in the context of a Brander-Spencer third-country model of strategic trade. We first analyze optimal policies when governments use both trade and industrial policies and show that these policies are substitutes. We then study optimal policy when governments can use only one policy instrument and show that industrial policy is robust, i.e., governments will always use an advertising subsidy irrespective of the type of advertising and form of market competition. More interestingly we show that for a range of parameter values we also get robust trade policy in which governments always use a trade subsidy irrespective of the type of advertising or form of market competition.

Key Words: Cooperative advertising, Predatory advertising; First-best policy combination, Robust industrial policy, Robust trade policy

JEL Classification: F13, L13

This is a joint paper with Alistair Ulph. His contributions include: designing the demand side of the basic model, writing GAUSS program to do simulation. I propose the basic idea of this paper and do the rest of work, including the design of simulation.

3.1 Introduction

Strategic trade policy has become a core part of international trade policy analysis since the seminal paper by Brander and Spencer (1985) was published. However, despite a voluminous literature since then, the policy implications remain controversial, mainly because the *ex ante* trade policy recommendation is very sensitive to the *ex post* market conduct.[1] Recent studies, such as Bagwell and Staiger (1994), Maggi (1996) and Leahy and Neary (2001) show that if firms engage in strategic investment competition (e.g., for R&D or capacity) prior to product market competition, then industrial policy, in the form of an investment subsidy, would be more robust than trade policy. Neary and Leahy (2000) develop a general framework to analyze optimal intervention towards dynamic oligopoly, emphasizing the implications of different kinds of government commitment. They point out that when firms make strategic investments prior to product market competition, the first-best policy combination should be designed for both profit-shifting and correcting the domestic firm's strategic behavior to influence the rival's decision and the domestic government's decision (if possible), which is socially wasteful. They also argue that a general model may not be useful in providing a general guide to policy making, and that it might be better to conduct case studies of particular policy combinations. Advertising is a fruitful field for such a case study, since its policy implications in the context of strategic trade policy have not been much explored.

In this paper, we first construct a model of advertising in a differentiated duopoly. This is modelled as a two-stage game. In the first stage, firms decide how much to invest in cooperative advertising, or predatory advertising or both.[2] In the second stage, they engage in product market competition.

We then analyze policy setting in the context of a Brander-Spencer third-country model of strategic trade, beginning with the case where governments can set trade and industrial policies, and then considering the cases where they can set only industrial policy or only trade policy.

The main results of this paper are as follows.

First, firms will invest only in one type of advertising, which is determined by the relative effectiveness of the two types of advertising and the degree of product differentiation. Second, when governments use both trade and industrial policies, these policies are substitutes. Third, new evidence is found to support trade policy. When governments can

[1] Brander (1995) provides a comprehensive discussion on strategic trade policy models.

[2] According to Church and Ware (2000), pp. 566, "One of the more important distinctions in the study of advertising is between *cooperative advertising*, which increases demand for rival firms' products as well as those of the advertising firm, and *predatory advertising*, which increases demand for the advertising firm only by attracting customers away from its rivals."

use only trade policy, for a range of parameters, which can be wide, trade policy in the form of a trade subsidy is similarly robust, i.e., governments always use that irrespective of the type of advertising or form of market competition.[3] Fourth, further evidence is found to support industrial policy. When governments can use only industrial policy, it is robust, i.e., governments will always use an advertising subsidy irrespective of the type of advertising and form of market competition.

An obvious question is how these results for advertising relate to results for other forms of strategic investment, such as R&D. Cooperative advertising and R&D with spillover effects are similar in that they raise the rival's profit, i.e., they both have positive externality effects. However, predatory advertising decreases the rival's demand, and hence profits and so has a negative externality effect, which has no analogue in R&D.[4]

How do our results relate to other studies of advertising?[5] In the classic paper by Dixit and Norman (1978) advertising shifts utility and demand, which raises problems of evaluating welfare effects.[6] Becker and Murphy (1993) try to solve this problem by treating advertising as a good that consumers purchase. They point out that advertising can increase demand of a product because the advertised good and advertising are complements. This allows a conventional welfare analysis of advertising. In this paper we follow the approach used by Dixit and Norman (1978), and can use this to analyze government policy, because our use of the Brander-Spencer third-country model means that we can ignore the welfare analysis of the effects of advertising on consumers.

Mantovani and Mion (2002) use a similar two-stage game analysis of advertising as in the basic model of this paper. However, our paper differs from theirs in two respects. First this paper uses the basic model to examine the policy implications for an open economy, whereas they use theirs to study the effect of entry deterrence and endogenous exit.[7] Second, this paper considers both quantity and price competition, whereas they consider only price competition.

The paper is organized as follows. The basic model of advertising in a differentiated

[3]Neary and Leahy (2000) argue that an export subsidy may be a practical policy option: "... it may be possible to evade the WTO prohibition on export subsidies (e.g., by providing export credits) but budgetary constraints may preclude direct assistance to investment." Moreover, in practice, WTO does not prohibit the use of an export tax rebate policy, which is equivalent to the effect of an export subsidy.

[4]Intuitively, a firm's production cost could not be increased by its rival's R&D investment. Moreover, studies of R&D and strategic trade have not explored the implications for robustness of strategic trade policy. It is an interesting question of whether the robustness result obtained in the advertising case carries over to the R&D case.

[5]Bagwell (2001) is a good introduction to the literatures on economics of advertising.

[6]The representative criticisms can be found in Schmalensee (1986).

[7]There are some detailed differences. Mantovani and Mion (2002) treat advertising as a discrete variable, while we treat it as a continuous variable. While we distinguish between cooperative and predatory advertising they distinguish between the market enlargement and predatory effects of advertising. But if market enlargement dominates predatory effects in terms of impact on the rival, we call this cooperative, and *vice versa* for predatory.

duopoly is presented in section 2 and analyzed in section 3. In section 4 we examine the policy implications of the basic model for an open economy. Section 5 concludes the paper and points out further extensions. All of the proofs, and the design of simulation, and the discussion on the second-order condition for welfare maximization are presented in Appendix to Chapter 3.

3.2 The Basic Model

The basic model is characterized as a two-stage game. In the first stage, two firms, firm 1 and firm 2, which produce a differentiated product respectively, decide simultaneously how much to invest in cooperative advertising, or predatory advertising, or both. In the second stage they engage in product market competition.

Consumers:

Assume that the representative consumer's preferences are given by the quasilinear utility function

$$U(x_1, x_2, m) = u(x_1, x_2) + m,$$

where x_1 and x_2 are the outputs of the two firms respectively and m is a numeraire good. In addition,

$$u(x_1, x_2) = a_1 x_1 + a_2 x_2 - \frac{1}{2}\left[b(x_1)^2 + 2x_1 x_2 + b(x_2)^2\right],$$

where

$$a_i = a\left[1 + \mu(m_i + m_j) + \nu(n_i - n_j)\right], \tag{3.1}$$
$$a > 0,\ b > 1,\ \mu > 0,\ \nu > 0,\ i = 1,2,\ j = 1,2,\ i \neq j.$$

a_i measures the market scale for firm i. a is the base of market scale: when firms do not make advertising investments, $a_i = a$. b represents the sensitivity of demand to a firm's own product. For simplicity, we assume that the sensitivity of demand to the rival firm's product is equal to 1. m_i and n_i are the respective cooperative and predatory advertising levels of firm i. μ and ν evaluate the effectiveness of the two types of advertising respectively.

Denote by p_i the price for each firm's product. Then the indirect demand system is given by

$$p_i = a_i - bx_i - x_j.$$

The corresponding direct demand system is given by

$$x_i = \alpha_i - \beta p_i + \gamma p_j,$$

where

$$\alpha_i = \alpha \left[1 + \mu (m_i + m_j) + \left(\frac{b+1}{b-1} \right) \nu (n_i - n_j) \right], \tag{3.2}$$

and

$$\alpha = \frac{a}{b+1}, \quad \beta = \frac{b}{b^2-1}, \quad \gamma = \frac{1}{b^2-1}.$$

This formulation implies that one unit of a firm's cooperative advertising investment will increase its own market scale and the rival's market scale by $a\mu$ units when firms play Cournot and by $\alpha\mu = (\frac{a}{b+1})\mu$ units when firms play Bertrand; one unit of a firm's predatory advertising investment will make its own market scale increase and the rival's market scale decrease by $a\nu$ units when firms play Cournot and make its own market scale increase and the rival's market scale decrease by $\alpha(\frac{b+1}{b-1})\nu = (\frac{a}{b-1})\nu$ units when firms play Bertrand.

To simplify notation, without loss of generality, we normalize $\nu = 1$, and henceforth interpret μ as a measure of the *relative* effectiveness of cooperative advertising.

Firms:

Firm i maximizes profit Π_i. We assume that firms have the same CRS production technologies, with cost function: $C_i(x_i) = cx_i$, $i = 1, 2$, where, for the usual reason, $a > c$. The investment cost function of each firm is given by $c_i(m_i, n_i) = \frac{1}{2}k(m_i + n_i)^2$, i.e., we suppose that there exists a joint investment diseconomy for each firm,[8] where

$$\frac{\partial c_i}{\partial m_i} = \frac{\partial c_i}{\partial n_i} = k(m_i + n_i), \quad \frac{\partial^2 c_i}{\partial m_i^2} = \frac{\partial^2 c_i}{\partial n_i^2} = \frac{\partial^2 c_i}{\partial m_i \partial n_i} = k.$$

We make the following assumption on k.

Assumption 3.1 $\underline{k} < k < \overline{k}$, *where*

$$\underline{k} = 2\max\{k_1, k_2, k_3, k_4\},$$

[8]It might be argued that such a formulation cannot capture the potential increasing-return-to-scale effects of advertising. However, the problem might not be so serious as it seems to be. If advertising investment incurs a fixed cost, then there will be an increasing-return-to-scale range for investment. Clearly a firm will invest in advertising only within this range. If we make an appropriate additional assumption on the fixed cost, our analysis will still hold. Of course, fixed costs are one of the important factors determining market structure. However, it is not the focus of this paper and we assume that the market structure is given.

and

$$k_1 = \frac{2a^2b\mu^2}{(2b+1)^2},\ k_2 = \frac{2a^2b}{(2b-1)^2},\ k_3 = \frac{2a^2b(b-1)\mu^2}{(b+1)(2b-1)^2},\ k_4 = \frac{2a^2b(b+1)}{(b-1)(2b+1)^2},$$

and

$$\overline{k} < \infty.$$

It can be easily shown that $k_1 > k_3$, $k_2 < k_4$. Hence $\underline{k} = 2\max\{k_1, k_4\}$.

The first inequality in Assumption 3.1 sets the greatest lower bound on k and ensures that in the investment stage of the game:

1. the profit function of each firm will be a concave function in its own choice,
2. the own effect of any type of advertising will be greater than the corresponding cross effect.

The second inequality sets an upper bound on k to ensure that firms' advertising investments are not so low as to be negligible.

We now solve for the Subgame Perfect Nash Equilibrium (SPNE) of the basic model.

3.3 Analysis of The Basic Model

We first discuss the case where firms in product market play Cournot and then turn to the case where firms play Bertrand.

3.3.1 The Cournot case

In the last stage of the game, firm i maximizes its profit function:

$$\Pi_i^C = (a_i - bx_i - x_j - c)x_i.$$

Note that in this stage the investment costs are sunk and as usual quantities are strategic substitutes. The Nash equilibrium is:

$$x_i^{*C} = \frac{a\left[1 + \mu(m_i + m_j) + \left(\frac{2b+1}{2b-1}\right)(n_i - n_j)\right] - c}{2b+1}. \tag{3.3}$$

Moreover, we have

$$p_i^{*C} - c = bx_i^{*C}. \tag{3.4}$$

The effects of the different types of advertising on the equilibrium quantity x_i^{*C} and equilibrium price p_i^{*C} are as follows.

$$\frac{\partial x_i^{*C}}{\partial m_i} = \frac{\partial x_i^{*C}}{\partial m_j} = \frac{a\mu}{2b+1} > 0, \; \frac{\partial x_i^{*C}}{\partial n_i} = \frac{a}{2b-1}, \; \frac{\partial x_i^{*C}}{\partial n_j} = -\frac{\partial x_i^{*C}}{\partial n_i} < 0. \tag{3.5}$$

$$\frac{\partial p_i^{*C}}{\partial m_i} = \frac{\partial p_i^{*C}}{\partial m_j} = \frac{ab\mu}{2b+1} > 0, \; \frac{\partial p_i^{*C}}{\partial n_i} = \frac{ab}{2b-1}, \; \frac{\partial p_i^{*C}}{\partial n_j} = -\frac{\partial p_i^{*C}}{\partial n_i} < 0. \tag{3.6}$$

The equilibrium profit of firm i is:

$$\Pi_i^{*C} = b\left(x_i^{*C}\right)^2.$$

In the first stage of the game, firm i maximizes its profit function in the reduced extensive form game:

$$\pi_i^C = \Pi_i^{*C} - c_i\left(m_i, n_i\right).$$

By Assumption 3.1, we ensure that the profit function of each firm is concave with respect to its own choice and then there exists a pure strategy Nash equilibrium, which is unique and stable.[9]

Lemma 3.1 *(Cournot Case)*

1. *If $\mu > \frac{2b+1}{2b-1}$, there exists a symmetric equilibrium, in which both firms invest in cooperative advertising. Cooperative advertising is a strategic complement and makes the rival's profit increase.*

2. *If $\mu < \frac{2b+1}{2b-1}$, there exists a symmetric equilibrium, in which both firms invest in predatory advertising. Predatory advertising is a strategic substitute and makes the rival's profits decrease.*[10]

From the specification of the investment cost function, the marginal costs of increasing any type of advertising by one unit are the same. Therefore, in order to make the optimal investment decision, each firm compares the marginal revenues from the two types of advertising and chooses the larger one. Given the equilibrium of the subsequent competition, when $\mu > \frac{2b+1}{2b-1}$, the marginal revenue from cooperative advertising is greater than that from predatory advertising, and *vice versa*.

When firms invest in cooperative advertising, the equilibrium value of cooperative advertising is

$$m_i^{*C} = m^{*C} = \frac{2a\left(a-c\right)b\mu}{\left(2b+1\right)^2 k - 4a^2 b\mu^2}. \tag{3.7}$$

[9]Fudenberg and Tirole (1991) and Nikaido (1968).

[10]Of course, if $\mu = \frac{2b+1}{2b-1}$, there exists a symmetric equilibrium, in which both firms invest in both cooperative advertising and predatory advertising. We omit this knife-edge case.

It is easy to show that:

$$\frac{\partial m^{*C}}{\partial a} > 0, \; \frac{\partial m^{*C}}{\partial b} < 0, \; \frac{\partial m^{*C}}{\partial c} < 0, \; \frac{\partial m^{*C}}{\partial \mu} > 0, \; \frac{\partial m^{*C}}{\partial k} < 0. \tag{3.8}$$

The equilibrium value of cooperative advertising increases with the base of market scale; decreases with the sensitivity of demand to a firm's own product; decreases with the unit production cost; increases with the relative effectiveness of cooperative advertising; and decreases with the coefficient of investment cost.

Substituting expression (3.7) for m^{*C} into expression (3.3) for the equilibrium quantity x_i^{*C}, and expression (3.4) for the equilibrium price p_i^{*C}, we get their equilibrium values respectively,

$$x_i^{*C} = x^{*C}(m) = \frac{(a-c)(2b+1)k}{(2b+1)^2 k - 4a^2 b\mu^2},$$
$$p_i^{*C} = p^{*C}(m) = \frac{ab + (b+1)c}{2b+1} + \frac{4(a-c)a^2b^2\mu^2}{(2b+1)\left[(2b+1)^2 k - 4a^2 b\mu^2\right]}.$$

When firms invest in predatory advertising, the equilibrium value of predatory advertising is

$$n_i^{*C} = n^{*C} = \frac{2a(a-c)b}{(4b^2-1)k}. \tag{3.9}$$

It is easy to show that:

$$\frac{\partial n^{*C}}{\partial a} > 0, \; \frac{\partial n^{*C}}{\partial b} < 0, \; \frac{\partial n^{*C}}{\partial c} < 0, \; \frac{\partial n^{*C}}{\partial k} < 0. \tag{3.10}$$

The equilibrium value of predatory advertising increases with the base of market scale; decreases with the sensitivity of demand to a firm's own product; decreases with the unit production cost; and decreases with the coefficient of investment cost.

Substituting expression (3.9) for n^{*C} into expression (3.3) for the equilibrium quantity x_i^{*C}, and expression (3.4) for the equilibrium price p_i^{*C}, we get their equilibrium values respectively,

$$x_i^{*C} = x^{*C}(n) = \frac{a-c}{2b+1},$$
$$p_i^{*C} = p^{*C}(n) = \frac{ab + (b+1)c}{2b+1}.$$

3.3.2 The Bertrand case

In the last stage of the game, firm i maximizes its profit function:

$$\Pi_i^B = (p_i - c)(\alpha_i - \beta p_i + \gamma p_j).$$

Note that in this stage the investment costs are sunk and as usual prices are strategic complements. The Nash equilibrium is:

$$p_i^{*B} = \frac{\alpha \left[1 + \mu \left(m_i + m_j\right) + \left(\frac{b+1}{b-1}\right)\left(\frac{2\beta-\gamma}{2\beta+\gamma}\right)\left(n_i - n_j\right)\right] + \beta c}{2\beta - \gamma}. \tag{3.11}$$

Moreover, we have

$$x_i^{*B} = \beta \left(p_i^{*B} - c\right). \tag{3.12}$$

The effects of the different types of advertising on the equilibrium price p_i^{*B} and equilibrium quantity x_i^{*B} are as follows.

$$\frac{\partial p_i^{*B}}{\partial m_i} = \frac{\partial p_i^{*B}}{\partial m_j} = \frac{\alpha\mu}{2\beta-\gamma} > 0, \; \frac{\partial p_i^{*B}}{\partial n_i} = \frac{\alpha\left(\frac{b+1}{b-1}\right)}{2\beta+\gamma} > 0, \; \frac{\partial p_i^{*B}}{\partial n_j} = -\frac{\partial p_i^{*B}}{\partial n_i} < 0. \tag{3.13}$$

$$\frac{\partial x_i^{*B}}{\partial m_i} = \frac{\partial x_i^{*B}}{\partial m_j} = \frac{\alpha\beta\mu}{2\beta-\gamma} > 0, \; \frac{\partial x_i^{*B}}{\partial n_i} = \frac{\alpha\beta\left(\frac{b+1}{b-1}\right)}{2\beta+\gamma} > 0, \; \frac{\partial x_i^{*B}}{\partial n_j} = -\frac{\partial x_i^{*B}}{\partial n_i} < 0. \tag{3.14}$$

The equilibrium profit of firm i is:

$$\Pi_i^{*B} = \beta \left(p_i^{*B} - c\right)^2.$$

In the first stage of the game, firm i maximizes its profit function in the reduced extensive form game:

$$\pi_i^B = \Pi_i^{*B} - c_i\left(m_i, n_i\right).$$

By Assumption 3.1, we ensure that the profit function of each firm is concave with respect to its own choice and then there exists a pure strategy Nash equilibrium, which is unique and stable.[11]

Lemma 3.2 *(Bertrand Case)*

1. *If $\mu > \frac{2b^2+b-1}{2b^2-b-1}$, there exists a symmetric equilibrium, in which both firms invest in cooperative advertising. Cooperative advertising is a strategic complement and makes the rival's profit increase.*

2. *If $\mu < \frac{2b^2+b-1}{2b^2-b-1}$, there exists a symmetric equilibrium, in which both firms invest in predatory advertising. Predatory advertising is a strategic substitute and makes the rival's profit decrease.*[12]

[11] Fudenberg and Tirole (1991) and Nikaido (1968).

[12] Of course, if $\mu = \frac{2b^2+b-1}{2b^2-b-1}$, there exists a symmetric equilibrium, in which both firms invest in both cooperative advertising and predatory advertising. We omit this knife-edge case.

The rationale is the same as for Lemma 3.1.

When firms invest in cooperative advertising, the equilibrium value of cooperative advertising is

$$m_i^{*B} = m^{*B} = \frac{2a(a-c)b(b-1)\mu}{(b+1)(2b-1)^2 k - 4a^2 b(b-1)\mu^2}. \tag{3.15}$$

It is easy to show that:

$$\frac{\partial m^{*B}}{\partial a} > 0, \quad \frac{\partial m^{*B}}{\partial c} < 0, \quad \frac{\partial m^{*B}}{\partial \mu} > 0, \quad \frac{\partial m^{*B}}{\partial k} < 0, \tag{3.16a}$$

$$\frac{\partial m^{*B}}{\partial b} > 0, \ if \quad b < 1.6777, \qquad \frac{\partial m^{*B}}{\partial b} < 0, \ if \quad b > 1.6777. \tag{3.16b}$$

The equilibrium value of cooperative advertising increases with the base of market scale; decreases with the unit production cost; increases with the relative effectiveness of cooperative advertising; and decreases with the coefficient of investment cost. In addition, the equilibrium value of cooperative advertising increases with the sensitivity of demand to a firm's own product to a critical value, then decreases with it.

Substituting expression (3.15) for m^{*B} into expression (3.11) for the equilibrium price p_i^{*B}, and expression (3.12) for the equilibrium quantity x_i^{*B}, we get their equilibrium values respectively,

$$p_i^{*B} = p^{*B}(m) = \frac{a(b-1)+bc}{2b-1} + \frac{4a^2(a-c)b(b-1)^2\mu^2}{(2b-1)\left[(b+1)(2b-1)^2 k - 4a^2 b(b-1)\mu^2\right]},$$

$$x_i^{*B} = x^{*B}(m) = \frac{(a-c)b(2b-1)k}{(b+1)(2b-1)^2 k - 4a^2 b(b-1)\mu^2}.$$

When firms invest in predatory advertising, the equilibrium value of predatory advertising is

$$n_i^{*B} = n^{*B} = \frac{2a(a-c)b}{(4b^2-1)k}, \tag{3.17}$$

which is equal to the equilibrium value of predatory advertising in the Cournot case.

Substituting expression (3.17) for n_i^{*B} into expression (3.11) for the equilibrium price p_i^{*B}, and expression (3.12) for the equilibrium quantity x_i^{*B}, we get their equilibrium values respectively,

$$p_i^{*B} = p^{*B}(n) = \frac{a(b-1)+bc}{2b-1},$$

$$x_i^{*B} = x^{*B}(n) = \frac{(a-c)b}{(b+1)(2b-1)}.$$

We summarize the main results of the analysis in the above two subsections in the following Proposition.

Proposition 3.1 *(Results of the Basic Model)*

1. *Whatever the form of product market competition, cooperative advertising will be present in an equilibrium, if* $\mu > \frac{2b^2+b-1}{2b^2-b-1}$. *Cooperative advertising will be present in an equilibrium when firms play Cournot, while predatory advertising will be present in an equilibrium when firms play Bertrand, if* $\frac{2b^2+b-1}{2b^2-b-1} > \mu > \frac{2b+1}{2b-1}$. *Predatory advertising will be present in an equilibrium, if* $\frac{2b+1}{2b-1} > \mu$.

2. *Whatever the form of product market competition, cooperative advertising is a strategic complement and makes the rival's profit increase, while predatory advertising is a strategic substitute and makes the rival's profit decrease.*

The intuition behind the first part of Proposition 3.1 is fairly simple. When b is very small, i.e., the degree of product differentiation is very small,[13] only if the relative effectiveness of cooperative advertising is very large will cooperative advertising be chosen in an equilibrium. Otherwise, firms will invest in predatory advertising. In other words, if the two products are very similar, a firm has a strong incentive to 'steal' its rival's market share. On the other hand, when b is very large, i.e., the degree of product differentiation is very large, even if the relative effectiveness of cooperative advertising is very small, cooperative advertising will be chosen in an equilibrium. The reason is that the incentive to steal the rival's market share diminishes and each firm wants to increase its own market scale.[14]

This completes our discussion of the basic model.

3.4 Policy Implications for an Open Economy

In this section we consider the policy implications of the basic model for an open economy in a Brander-Spencer third-country model. From the viewpoint of an export country, if there is unilateral intervention, what is the optimal policy?

We now consider a three-stage game in which we add an additional stage to the start of the basic model. In this new first stage, the government of country i sets its policy and the potential policy instruments are trade policy, a subsidy s on output, and industrial policy, a subsidy τ on advertising.[15] We assume that the opportunity cost of public funds

[13] According to Singh and Vives (1984), when the market scale of one firm is equal to that of the other, $\frac{1}{b^2}$ is the measure of product differentiation in this case.

[14] See also Mantovani and Mion (2002).

[15] Note that trade policy has two effects on the subsequent game. First, it will directly change the equilibrium outcome of product market competition. Second, it has an indirect effect on the competition as well by changing the rival firm's incentive to invest in advertising. Unlike trade policy, industrial policy cannot directly influence product market competition but has an indirect effect on that by changing the rival firm's investment incentive.

is unity. The representative consumer and the market are now in a third country.[16]

Before going further, note that firms' decisions in the investment stage on whether to invest in cooperative or predatory advertising are not changed by the policy instruments, since, as we have just seen, that decision depends only on the relative effectiveness of cooperative advertising and the degree of product differentiation.

In the following subsections we examine the first-best policy combination, second-best industrial policy and second-best trade policy.

3.4.1 First-best policy analysis

In this case, the government uses different instruments for different targets, in particular, trade policy s towards the domestic firm's quantity or price and industrial policy τ towards the domestic advertising investment. Given the equilibrium outcome in the subsequent game, the government maximizes its welfare:

$$\max_{\{s,\tau\}} W(s,\tau) = \pi_i(s,\tau) - sx_i(s,\tau) - \tau I_i(s,\tau),$$

where π_i is the domestic firm's profit and $I_i \in \{m_i, n_i\}$. We shall assume that the welfare function is strictly concave, so the following two conditions characterize the unique optimal policy combination:

$$\frac{\partial W}{\partial s} = \frac{\partial \pi_i}{\partial s}(\cdot) - x_i - \frac{\partial x_i}{\partial s}(\cdot)\, s - \frac{\partial I_i}{\partial s}\tau = 0, \tag{3.18a}$$

$$\frac{\partial W}{\partial \tau} = \frac{\partial \pi_i}{\partial \tau}(\cdot) - \frac{\partial x_i}{\partial \tau}(\cdot)\, s - I_i - \frac{\partial I_i}{\partial \tau}\tau = 0. \tag{3.18b}$$

Denote

$$D = \frac{\partial x_i}{\partial s}(\cdot)\frac{\partial I_i}{\partial \tau} - \frac{\partial x_i}{\partial \tau}(\cdot)\frac{\partial I_i}{\partial s}, \tag{3.19}$$

$$D_1 = \left[\frac{\partial \pi_i}{\partial s}(\cdot) - x_i\right]\frac{\partial I_i}{\partial \tau} - \left[\frac{\partial \pi_i}{\partial \tau}(\cdot) - I_i\right]\frac{\partial I_i}{\partial s}, \tag{3.20}$$

$$D_2 = \left[\frac{\partial \pi_i}{\partial \tau}(\cdot) - I_i\right]\frac{\partial x_i}{\partial s}(\cdot) - \left[\frac{\partial \pi_i}{\partial s}(\cdot) - x_i\right]\frac{\partial x_i}{\partial \tau}. \tag{3.21}$$

We have[17]

$$s = \frac{D_1}{D},\ \tau = \frac{D_2}{D}. \tag{3.22}$$

[16] According to the terminology of Neary and Leahy (2000), we consider only "Government-Only-Commitment Equilibrium" in this paper.

[17] It should be noted that according to Neary and Leahy (2000) the first-best policy combination should not only do the profit-shifting job but also should correct the domestic firm's strategic behavior to influence the rival's decision and the domestic government's decision (if possible), which is socially wasteful.

We apply these general formulae for the first-best policy combination to the particular cases to obtain the following Proposition.

Proposition 3.2 *(First-best Policy) In a Brander-Spencer third-country world the unique unilateral first-best intervention by the government of country i is as follows.*

1. *If $\mu > \frac{2b+1}{2b-1}$, i.e., in the subsequent game Cournot firms invest in cooperative advertising, both the optimal trade policy and the optimal industrial policy are ambiguous. If $\mu < \frac{2b+1}{2b-1}$, i.e., in the subsequent game Cournot firms invest in predatory advertising, the optimal trade policy is a trade subsidy, whereas the optimal industrial policy is ambiguous.*

2. *If $\mu > \frac{2b^2+b-1}{2b^2-b-1}$, i.e., in the subsequent game Bertrand firms invest in cooperative advertising, the first-best policy is the combination of a trade tax and an advertising subsidy. If $\mu < \frac{2b^2+b-1}{2b^2-b-1}$, i.e., in the subsequent game Bertrand firms invest in predatory advertising, the optimal industrial policy is an advertising subsidy, whereas the optimal trade policy is ambiguous.*

3. *Irrespective of the form of market competition and type of advertising, optimal first-best policy never involves taxes on both exports and advertising.*

We also have the following Corollaries.

Corollary 3.1 *The cross derivative of the welfare function is negative whatever the form of competition and whatever the equilibrium type of advertising investment,*

$$\frac{\partial^2 W}{\partial s \partial \tau} < 0.$$

That is the two policy instruments are substitutes.

Corollary 3.2 *When firms play Cournot,*

$$\left.\frac{\partial W}{\partial s}\right|_{(s,\tau)=(0,0)} > 0.$$

When firms play Bertrand, sign $\frac{\partial W}{\partial s}\big|_{(s,\tau)=(0,0)}$ can be positive or negative depending on parameter configurations.

Corollary 3.3 *Whatever the form of competition and the equilibrium type of advertising investment,*

$$\left.\frac{\partial W}{\partial \tau}\right|_{(s,\tau)=(0,0)} > 0.$$

Corollary 3.4 *(Marginal Rate of Substitution)*

1. *The marginal rate of substitution between trade policy and industrial policy (hereafter MRS), which is defined as*
$$-\frac{d\tau}{ds} = \frac{\frac{\partial W}{\partial s}}{\frac{\partial W}{\partial \tau}},$$
is positive at the non-intervention point, $(s, \tau) = (0, 0)$, *when firms play Cournot, whereas it can be positive or negative at that point when firms play Bertrand (see Figures 3.1 and 3.2).*

2. *The MRS is decreasing at the non-intervention point, i.e.,*
$$-\left.\frac{d^2\tau}{ds^2}\right|_{(s,\tau)=(0,0)} < 0,$$
whatever the form of competition and whatever the equilibrium type of advertising investment.

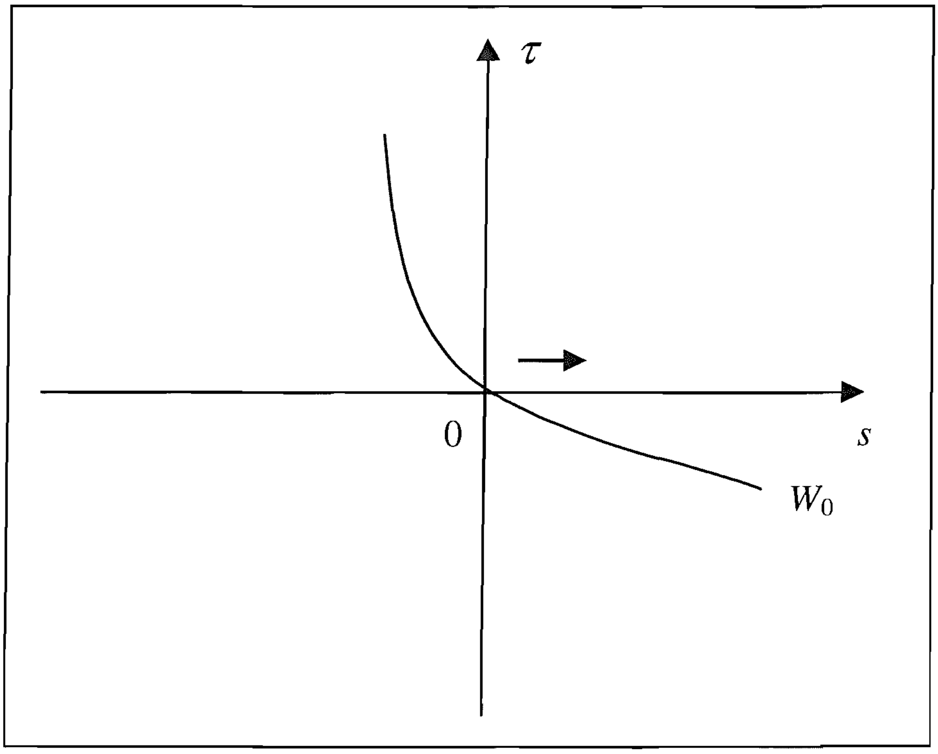


Figure 3.1: Positive MRS at non-intervention point

Obviously, the results on the first-best policy combination are not clear-cut. This is because the two policy instruments are substitutes, as stated in Corollary 3.1. Moreover, the substitutability between them is dependent on the fundamental characteristics of the model, i.e., the degree of product differentiation, the relative effectiveness of cooperative

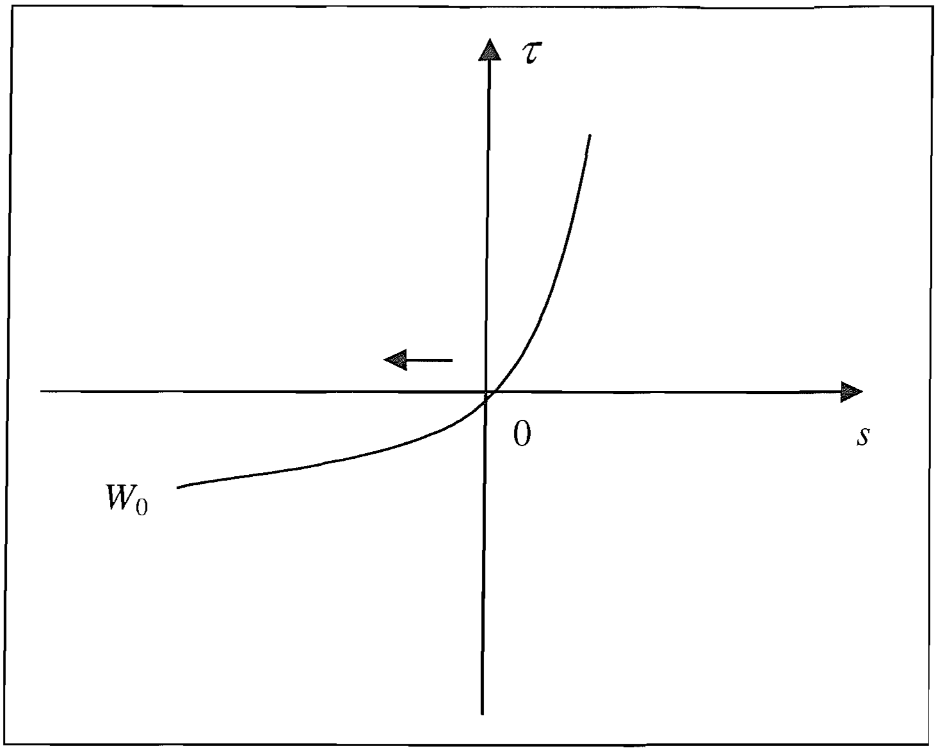


Figure 3.2: Negative MRS at non-intervention point

advertising and the coefficient of investment cost. However, note that whatever the form of competition and whatever the equilibrium type of advertising investment, the two policy instruments cannot both be zero, so the government always wants to play an active role in international competition and its optimal intervention includes at least one positive component, i.e., a subsidy.

Corollary 3.4 is the key to understand the link between the first-best and the second-best results, to which we now turn.

3.4.2 Second-best policy analysis

Trade policy

In this case, the government can use only trade policy to intervene in international competition. Given the equilibrium outcome in the subsequent game, the government maximizes its welfare:

$$\max_{\{s\}} W(s) = \pi_i(s) - s x_i(s).$$

The following condition characterizes the optimal policy:

$$\frac{dW}{ds} = \frac{d\pi_i}{ds}(\cdot) - x_i - \frac{dx_i}{ds}s = 0. \tag{3.23}$$

We have

$$s = \frac{\frac{d\pi_i}{ds}(\cdot) - x_i}{\frac{dx_i}{ds}}. \tag{3.24}$$

Applying this general formula for second-best trade policy to the particular cases, we obtain the following Proposition.

Proposition 3.3 *(Trade Policy) In a Brander-Spencer third-country world where the government can use only trade policy, the unique unilateral intervention by the government of country i is as follows:*

1. *If $\mu > \frac{2b^2+b-1}{2b^2-b-1}$, i.e., in the subsequent game both Cournot and Bertrand firms invest in cooperative advertising, the optimal trade policy is to implement a trade subsidy whatever the form of product market competition, if and only if*

$$\left[\sqrt{(5b^2-1)} - (b-1)\right] k_3 > k.$$

2. *If $\frac{2b^2+b-1}{2b^2-b-1} > \mu > \frac{2b+1}{2b-1}$, i.e., in the subsequent game Cournot and Bertrand firms will invest in cooperative and predatory advertising respectively, the optimal trade policy is to implement a trade subsidy whatever the form of product market competition, if and only if*

$$\left[\sqrt{(5b^2-1)} + (b+1)\right] k_4 > k.$$

3. *If $\frac{2b+1}{2b-1} > \mu$, i.e., in the subsequent game both Cournot and Bertrand firms invest in predatory advertising, the optimal trade policy is to implement a trade subsidy whatever the form of product market competition, if and only if*

$$\left[\sqrt{(5b^2-1)} + (b+1)\right] k_4 > k.$$

4. *In all three cases if we do not get robust trade policy, then the optimal trade policy is a trade subsidy with Cournot behavior and a trade tax with Bertrand behavior.*

Of course, the magnitude of policy instrument is not necessarily the same in each scenario.

As noted in the above, trade policy has two effects. It directly change the equilibrium outcome of product market competition. In addition, it indirectly affect the competition by changing the rival firm's investment incentive. See the following Table.

	Cooperative advertising	Predatory advertising
Cournot competition	Strategic substitute + Strategic complement	Strategic substitute + Strategic substitute
Bertrand competition	Strategic complement + Strategic complement	Strategic complement + Strategic substitute

It is obvious that the optimal trade policy is a trade subsidy for the case when Cournot firms make predatory advertising investment in an equilibrium. Since both quantity and predatory advertising are strategic substitute, a trade subsidy decreases the rival firm's output and lowers its investment incentive, and therefore benefits the domestic firm.

When Cournot firms make cooperative advertising investment in an equilibrium, again, a trade subsidy decreases the rival firm's output and benefits the domestic firm. But its effect on the rival firm's investment incentive is ambiguous.[18] It proves the direct effect of trade policy to be a dominant one. This justifies a trade subsidy in this case.

In the two Bertrand cases, we cannot get definite results. Since price is a strategic complement, a trade subsidy makes the rival firm's price decrease and hurts the domestic firm. A trade subsidy is observed only if the indirect effect of trade policy dominates its direct effect irrespective of the equilibrium type of advertising investment. For instance, this happens when the investment cost is sufficiently low.

Industrial policy

In this case, the government can use only industrial policy τ to intervene in international competition. Given the equilibrium outcome in the subsequent game, the government maximizes its welfare:

$$\max_{\{\tau\}} W(\tau) = \pi_i(\tau) - \tau I_i(\tau).$$

The following condition characterizes the optimal policy:

$$\frac{dW}{d\tau} = \frac{d\pi_i}{d\tau}(\cdot) - I_i - \frac{dI_i}{d\tau}\tau = 0. \tag{3.25}$$

[18]On the one hand, since quantity is a strategic substitute, a trade subsidy hurts the rival firm, and therefore directly lowers its investment incentive. On the other hand, a trade subsidy makes the domestic firm increase its investment, and therefore raises the rival firm's investment incentive indirectly since cooperative advertising is a strategic complement.

We have

$$\tau = \frac{\frac{d\pi_i}{d\tau}(\cdot) - I_i}{\frac{dI_i}{d\tau}}. \tag{3.26}$$

We apply this general formula for second-best industrial policy to the particular cases to obtain the following Proposition.

Proposition 3.4 *In a Brander-Spencer third-country world where the government can implement only industrial policy the unique unilateral intervention is an advertising subsidy, whatever the form of product market competition and whatever the equilibrium type of advertising investment.*

Of course, the magnitude of policy instrument is not necessarily the same in each scenario.

According to the second part of Proposition 3.1, cooperative advertising is a strategic complement and makes the rival firm's profits increase irrespective of the form of market conduct; while predatory advertising is a strategic substitute and makes the rival firm's profits decrease. So, an advertising subsidy is justified irrespective of the form of market conduct and the equilibrium type of advertising investment.

Discussion

Proposition 3.4 provides further support for the robustness of industrial policy. But the results presented in Proposition 3.3 about the robustness of trade policy seem to be new in the strategic trade policy literature. To see the link between these results we refer again to Figures 3.1 and 3.2. W_0 is the non-intervention welfare level. According to the second part of Corollary 3.4, the upper contour set $\{(s, \tau) \in R^2 : W(s, \tau) \geq W_0\}$ must be 'above' the iso-welfare curve, which passes the non-intervention point in the neighborhood of that point. Therefore, if the government is restrained to maximize its welfare along the vertical axis it is clear that the optimal policy must be an advertising subsidy whatever the form of competition and whatever the equilibrium type of advertising investment.

Things are a bit more complex in the trade policy case. From the above Figures, we see that if the MRS in the neighborhood of the non-intervention point is positive, then there is a robust trade policy, i.e., a trade subsidy always occurs. It turns out that in the Cournot case, the MRSs are always positive, whereas those in the Bertrand case can be positive or negative. When conditions presented in Proposition 3.3 are satisfied, MRSs in the Bertrand case will be positive and the robust trade policy in the form of a trade subsidy will be observed.

In summary, if the MRS is decreasing at the non-intervention point, the robust industrial policy, i.e., an advertising subsidy will be observed. If the MRS is both positive and decreasing at the non-intervention point, both an industrial subsidy and the robust trade policy, i.e., a trade subsidy will be observed. In addition, if MRS is infinity at the non-intervention point, the second-best industrial policy will be non-intervention. If MRS is zero at the non-intervention point, the second-best trade policy will be a free trade one.

Simulation results on robust trade policy

Proposition 3.3 established for each of the three cases a critical value of k, k_c, such that if $k < k_c$ trade policy is robust. Here, we want to ask: how 'large' is the range of k for which trade policy is robust. To answer this question we first make it more precise, and then present some numerical results.

Assumption 3.1 restricted k to lie in the range $\left(\underline{k}, \overline{k}\right)$, where $\underline{k}$ is a function of parameters a, b and μ. However to prove Proposition 3.2 we assumed that the welfare function is strictly concave. In subsection 12 of Appendix to Chapter 3 we show that if the welfare function is strictly concave, we must have $k > \underline{k}'' > \underline{k}$, where $\underline{k}''$ is a function of parameters a, b and μ. So far we have not said anything about what determines $\overline{k}$. In subsection 11 of Appendix to Chapter 3, we derive a value for $\overline{k}$, which depends on parameters a, b, μ and the advertising sales ratio, which we define as ϕ. From Proposition 3.3 we know that k_c depends on parameters a, b and μ. Putting this together, for any set of parameters a, b, μ and ϕ, we can calculate $\underline{k}''$, $\overline{k}$ and k_c, and the question we ask is: for what proportion of the range of feasible values of k, $\left(\underline{k}'', \overline{k}\right)$, is trade policy robust? We denote the proportion as $l \equiv \frac{k_c - \underline{k}''}{\overline{k} - \underline{k}''}$, and so our question is how large is l?

It is shown that l is a function of parameters b, μ and ϕ in the subsection 11 of Appendix to Chapter 3. It is not possible to derive a simple expression for l to show how it relates to these parameter values. So we have used numerical simulations. We take 100 values of b from the interval $(1, 6)$, 100 values of μ from the interval $(1, 2)$ and 100 values of ϕ from the interval $(0, 0.12)$. These intervals are equally-spaced, i.e., we use uniform distributions. For each of the $1,000,000$ combinations of b, μ and ϕ, we used Proposition 3.1 to assign it to one of the three cases. We then calculated $\underline{k}''$, $\overline{k}$, k_c and l. Finally, for the sets of parameter values lying in each of the three cases, we calculated summary statistics of the distribution of l : the average of l, the standard deviation of l, and the maximum and minimum values of l. The results are shown in the following Table.

	Average of l	Standard Deviation of l	Minimum of l	Maximum of l
Case 1	0.462	0.354	0.000	1.000
Case 2	0.450	0.314	0.004	1.000
Case 3	0.671	0.359	0.005	1.000

Table 3.1: Simulation results on robust trade policy

We define the case where both Cournot and Bertrand firms invest in cooperative advertising as Case 1, the case where Cournot and Bertrand firms invest in cooperative and predatory advertising respectively as Case 2 and the case where both Cournot and Bertrand firms invest in predatory advertising as Case 3. The proportion of Case 1 is 0.529, the proportion of Case 2 is 0.033 and the proportion of Case 3 is 0.375.[19]

Two key points emerge from the simulation results. First the average fraction of the feasible range of values for k for which trade policy is robust is not trivial in every case. Second, the robustness results are more likely in Case 3 where firms invest in predatory advertising and there exists a negative externality in investment.

This completes the discussion on the policy implications of the basic model. All of the results presented in this section are summarized in the following Table.

Parameter combination	Equ type of Ad	First-best policy	Second-best industrial policy	Second-best trade policy
$\mu > \frac{2b^2+b-1}{2b^2-b-1}$	C: Coop	$sign\ s = sign\ B_1$ $sign\ \tau = sign\ B_2$	$\tau > 0$	$s > 0$
	B: Coop	$s < 0$ $\tau > 0$	$\tau > 0$	$sign\ s = sign\ B_5$
$\frac{2b^2+b-1}{2b^2-b-1} > \mu > \frac{2b+1}{2b-1}$	C: Coop	$sign\ s = sign\ B_1$ $sign\ \tau = sign\ B_2$	$\tau > 0$	$s > 0$
	B: Pred	$sign\ s = sign\ B_4$ $\tau > 0$	$\tau > 0$	$sign\ s = sign\ B_6$
$\frac{2b+1}{2b-1} > \mu$	C: Pred	$s > 0$ $sign\ \tau = sign\ B_3$	$\tau > 0$	$s > 0$
	B: Pred	$sign\ s = sign\ B_4$ $\tau > 0$	$\tau > 0$	$sign\ s = sign\ B_6$

Table 3.2: Summary of policy implications for an open economy

Equ: Equilibrium

C: Cournot — B: Bertrand

Coop Ad: Cooperative Advertising — Pred Ad: Predatory Advertising

$B_1 = k - (2b+1)\,k_1$ — $B_2 = 2b\,(2b+1)\,k_1 - k$

$B_3 = 2b\,(2b-1)\,k_2 - k$ — $B_4 = (2b+1)\,k_4 - k$

$B_5 = \left[\sqrt{(5b^2-1)} - (b-1)\right] k_3 - k$ — $B_6 = \left[\sqrt{(5b^2-1)} + (b+1)\right] k_4 - k$

[19] If in a combination of parameters we had $\underline{k}'' > \overline{k}$, that combination is invalid. The invalid proportion of observations is 0.063.

3.5 Conclusion and Further Extensions

In this paper, we first construct a model of advertising in a differentiated duopoly. This is modelled as a two-stage game. In the first stage, firms decide how much to invest in cooperative advertising, or predatory advertising or both. In the second stage, they engage in product market competition. We show that firms will invest only in one type of advertising, which is determined by the relative effectiveness of the two types of advertising and the degree of product differentiation. We then use this model to explore the policy implications in the context of a Brander-Spencer third-country model of strategic trade. We first analyze optimal policies when governments use both trade and industrial policies and show that these policies are substitutes. We then study optimal policy when governments can use only one policy instrument and show that industrial policy is robust, i.e., governments will always use an advertising subsidy irrespective of the type of advertising and form of market competition. More interestingly we show that for a range of parameter values we also get robust trade policy in which governments always use a trade subsidy irrespective of the type of advertising or form of market competition.

It might be argued that this paper does not capture the increasing return effect of advertising. However, we can construct a similar model to deal with the increasing return effect of advertising.[20] Consider a three-stage game. In the first stage, firms decide how much to spend on advertising. Then follows the two-stage game described in section 2 with two revisions: the investment cost can be an affine function but cannot exceed the budget set in the first stage. Solving this game is straightforward. Our primary task has been to examine the policy implications for an open economy with a given market structure. With the introduction of a fixed cost, an obvious extension would be to endogenize market structure.[21]

This paper does not discuss bilateral intervention. However, it is a natural extension of the analysis presented in section 4 and we will get symmetric SPNE and the main results will be unchanged. As to other possible extensions, studies of R&D and strategic trade have not explored the implications for robustness of strategic trade policy. So, an obvious question is whether the robustness result we obtained in the advertising case carries over to the R&D case? Finally, Ulph and Ulph (2001) explore the implications for trade and industrial policy of allowing for full government commitment. It would be interesting to reconsider the policy implications of our model using this approach.

We hope to report on the results of these extensions.

[20]See also Footnote 8.

[21]The classic treatment on this topic is Sutton (1991).

Chapter 4

Is an Export Subsidy a Robust Trade Policy Recommendation towards a Unionized Duopoly?

Abstract

Bandyopadhyay *et al.* (2000) and Brander and Spencer (1988) imply that the robust trade policy recommendation towards a unionized duopoly is an export subsidy. In this paper, we show that we cannot get such a result even in the linear case if the opportunity cost of public funds is sufficiently high. However, if we introduce political ingredients to the model, i.e., considering the case where the domestic firm and the trade union lobby the government for setting their favorable trade policies by giving the government political contributions (modeled in a common agency setting), then the result of robustness will be restored if the government cares about political contributions sufficiently relative to national welfare.

Key Words: Opportunity cost of public funds, Special interest politics, Strategic trade policy, Unionized duopoly

JEL Classification: F13, D72

4.1 Introduction

This paper studies whether an export subsidy is a robust trade policy recommendation towards a unionized duopoly.

It is well known that the fundamental problem faced by strategic trade policy models is that the *ex ante* policy recommendation is very sensitive to the *ex post* market conduct. For instance, the optimal trade policy is an export subsidy if firms compete as Cournot competitors in product market (see Brander and Spencer (1985)); whilst it is an export tax if firms engage in Bertrand competition (see Eaton and Grossman (1986)).

Recently Bandyopadhyay *et al.* (2000) point out: demand linearity ensures that an export subsidy is the optimal trade policy towards a unionized Bertrand duopoly.[1] This paper and Brander and Spencer (1988), which show that the optimal trade policy towards a unionized Cournot duopoly is an export subsidy, together imply that an export subsidy is a robust trade policy recommendation towards a linear unionized duopoly.

Is this result a robust one? The aim of this paper is to answer this question.

We begin with a linear model in which following Brander and Spencer (1988), we introduce a trade union to a Brander-Spencer third-market model for one of the two exporting countries, say, the 'domestic country', and consider the case of unilateral intervention. In this model, we reproduce the result of robustness implied by the above two papers in a clear-cut way: the optimal trade policy is an export subsidy irrespective of the form of market conduct. This serves as a benchmark case. Then, we introduce an opportunity cost of public funds to the above setting. Now even in the linear case, an export subsidy is not a robust trade policy recommendation if this cost is sufficiently high. Then, we consider another variation: to allow the domestic firm and the trade union to lobby for their favorable policies by giving the domestic government political contributions prior to the government setting trade policy. This is modeled as a common agency framework due to Bernheim and Whinston (1986), and Grossman and Helpman (1994). We show that an export subsidy is a robust policy recommendation irrespective of the form of market conduct if the government cares about political contributions sufficiently relative to national welfare.[2]

So, what is the main lesson that we have learnt from this simple exercise? First of all, an export subsidy can hardly be a robust trade policy recommendation towards a unionized duopoly, if we consider this problem from a purely economic perspective: it is very sensitive to the opportunity cost of public funds even in the simplest setting. However, an export subsidy as a robust policy recommendation can be supported by political reasons, for

[1] See Bandyopadhyay *et al.* (2000), Proposition 1.

[2] See Helpman (1997) for an excellent introduction to political economy of trade policy.

instance, special interest lobbying.

This research is related to several strands of literatures.

Since Brander and Spencer (1988), many papers have explored the trade policy implications of a unionized duopoly, notably Bandyopadhyay *et al.* (2000), and so on.

Matsuyama (1990) introduces a social cost of public funds to the strategic trade policy literature, and is followed notably by Neary (1994).[3] As far as we know, our paper is the first paper introducing an opportunity cost of public funds to the research of strategic trade policy towards a unionized duopoly.

To the best of our knowledge, Fung and Lin (2000) is the only other paper that uses a common agency approach to studying strategic trade policy from a political economy perspective. But they do not introduce an opportunity cost of public funds to their research, whilst we do.[4]

The paper is organized as follows. Section 2 sets out the basic model. In section 3, we do equilibrium analysis. Next, we introduce an opportunity cost of public funds and special interest politics to the basic model in sequence. In section 6, we discuss the relationship between results obtained in this paper and those in existing literatures. The final section concludes.

4.2 The Basic Model

There are three countries: domestic, foreign and a third country. A domestic firm and a foreign firm produce differentiated goods and sell the goods in the third market. There is no consumption of the goods either in the domestic or in the foreign country.

Technology:

[3] Neary (1994) introduces an opportunity cost of public funds to the Cournot setting in a Brander-Spencer third-market model, whilst introducing an opportunity cost of public funds to the Bertrand setting in the Carmichael-Gruenspecht model. See Carmichael (1987) and Gruenspecht (1988). They consider the case when firms move first, then governments design trade policies, and emphasize the importance of timing in decisions.

We introduce an opportunity cost of public funds to both the Cournot and Bertrand settings in a Brander-Spencer third-market model, in which governments can commit to trade policies.

[4] A second-order difference between our paper and their paper is as follows. In a partial equilibrium version of Grossman and Helpman (1994), they show that "even with political pressure, the ... politically determined export subsidy is identical to the Brander-Spencer rent-shifting export subsidy", and this "highlights the possibility that lobbying can restore the level of trade intervention to a more efficient one in the absence of the benevolent dictator". They do not consider the question of whether an export subsidy is a robust trade policy.

The question of robustness is the main focus of our paper, and in our paper the politically determined export subsidy is always greater than the rent-shifting export subsidy set by a benevolent government. Why do we observe this difference? The answer is that these two papers use different modeling techniques. As indicated above, their model is a slight variation of Grossman and Helpman (1994), whilst we model explicitly the domestic firm and the trade union as special interest groups. In their model, the special interest group that gains from an export subsidy has the same lobbying power as the special interest group that loses from an export subsidy, whilst in our model, both the trade union and the domestic firm gain from an export subsidy. Of course, which modeling technique is appropriate is an empirical question.

In each country, labor is the only input for production. The domestic firm and the foreign firm share the same Ricardian technology: to produce one unit of output needs one unit of labor. However, there are two differences between the two countries. First, the opportunity wage rate for workers in the domestic country is w_0; whilst the opportunity wage rate for workers in the foreign country is w_0^*; they are not necessarily equal. Second, domestic workers are organized and form a trade union, whilst this is not the case in the foreign country.[5] (This implies that the wage rate for the domestic workers is greater than the opportunity wage rate, whilst the wage rate for the foreign workers is the same as the opportunity wage rate.)

Preferences:

In the third country, the representative consumer's preference is given by

$$U(x, x^*; m) = u(x, x^*) + m,$$

where

$$u(x, x^*) = a(x + x^*) - \frac{1}{2}\left(bx^2 + 2xx^* + bx^{*2}\right),$$

and $b > 1$, which is a parameter that measures the degree of product differentiation. The consumption of domestic products is given by x, and the consumption of foreign products, x^*; m represents the consumption of a numeraire good.

The indirect demand system is given by

$$p = a - bx - x^*,\ p^* = a - x - bx^*,$$

where p is the price for domestic products, and p^*, the price for foreign products. The direct demand system is given by

$$x = \alpha - \beta p + \gamma p^*,\ x^* = \alpha + \gamma p - \beta p^*,$$

where

$$\alpha = \frac{a}{b+1},\ \beta = \frac{b}{b^2 - 1},\ \gamma = \frac{1}{b^2 - 1}.$$

Timing:

This is a three-stage game.

In the first stage, the domestic government sets trade policy, s. If $s > 0$, this is an export subsidy; if $s < 0$, this is an export tax; if $s = 0$, this is the non-intervention policy.

In the second stage, the domestic wage rate and the domestic employment levels are

[5] As indicated above, we follow Brander and Spencer (1988) to consider this situation.

determined. The trade union moves first and sets the wage rate. After observing the wage rate, the domestic firm decides how much labor to employ. (We use a Leontief model to characterize the strategic interactions in this stage.)

In the third stage, the domestic firm and the foreign firm engage in product market competition in the third country either as Cournot competitors or as Bertrand competitors.

Then the game is over.

Payoffs:

The domestic firm and the foreign firm receive their profits respectively. The trade union receives its economic rents. The economic rents are defined as the product of the difference between the actual wage rate and the opportunity wage rate and the employment level. The government receives national welfare, which is given by the sum of the domestic firm's profits and the trade union's economic rents subtracting the costs of subsidy.

The solution concept is a Subgame Perfect Nash Equilibrium (SPNE).[6]

Next, we use generalized backward induction to solve this model with the help of the following Assumption.

Assumption 4.1

$$\left(2b^2 - 1\right)(a - w_0) - b\left(a - w_0^*\right) > 0.$$

Assumption 4.1 says that without the government's intervention, the domestic firm can survive in product market competition: either Cournot or Bertrand.[7]

4.3 Equilibrium Analysis

4.3.1 Cournot competition

First of all, let us analyze the case where the domestic firm and the foreign firm compete as Cournot competitors in product market.

In the third stage of the game, the domestic firm maximizes its profits:

$$\pi = (a - bx - x^* - w + s)\, x,$$

whilst the foreign firm maximizes its profits:

$$\pi^* = \left(a - x - bx^* - w_0^*\right) x^*.$$

[6] It should be noted that this model, though simple, is different from Bandyopadhyay *et al.* (2000) in that they consider only the Bertrand competition. On the other hand, this model differs from Brander and Spencer (1988) since they consider only the Cournot competition, and use a Nash bargaining situation to model the process of wage rate determination, and the domestic firm has the full control over the employment levels.

[7] See the first subsection of Appendix to Chapter 4.

The domestic firm's first-order condition for profit maximization and the foreign firm's first-order condition for profit maximization determine simultaneously the Nash equilibrium:[8]

$$
\begin{aligned}
x &= \frac{2b\,(a-w+s)-(a-w_0^*)}{4b^2-1}, && (4.1a)\\
x^* &= \frac{2b(a-w_0^*)-(a-w+s)}{4b^2-1}. && (4.1b)
\end{aligned}
$$

Notice that these are also equilibrium employment levels of the domestic firm and the foreign firm.

In the second stage of the game, the trade union chooses wage rate, w, to maximize its economic rents:

$$
\begin{aligned}
\omega &= (w-w_0)\,x\,(w)\\
&= (w-w_0)\left[\frac{2b\,(a-w+s)-(a-w_0^*)}{4b^2-1}\right]. \qquad (4.2)
\end{aligned}
$$

From the first-order condition for maximization, we can solve for the equilibrium wage rate given the domestic trade policy:[9]

$$
w = \frac{2b\,(a+s+w_0)-(a-w_0^*)}{4b}. \qquad (4.3)
$$

Notice that $\frac{dw}{ds} = \frac{1}{2}$. This means that the trade union skims off one half of the trade policy.

Using expression (4.3), we can show

$$
x = \left[\frac{2b\,(a-w_0+s)-(a-w_0^*)}{2\,(4b^2-1)}\right], \qquad (4.4)
$$

$$
\pi = b\left[\frac{2b\,(a-w_0+s)-(a-w_0^*)}{2\,(4b^2-1)}\right]^2, \qquad (4.5)
$$

$$
\omega = \frac{\left[2b\,(a-w_0+s)-(a-w_0^*)\right]^2}{8b\,(4b^2-1)}. \qquad (4.6)
$$

In the first stage of the game, the government chooses trade policy, s, to maximize national welfare:

$$
G = \pi + \omega - sx,
$$

[8] Notice that the first-order conditions are also sufficient in this standard Cournot game.

[9] It is straightforward to show

$$
\frac{\partial^2 \omega}{\partial w^2} = -\frac{4b}{4b^2-1} < 0.
$$

Therefore, there is a unique interior solution.

where π is given by expression (4.5), ω is given by expression (4.6), and x is given by expression (4.4). From the first-order condition for national welfare maximization, we can solve for the optimal trade policy.[10] It is an export subsidy:

$$s = \frac{b\left[2b\left(a - w_0\right) - \left(a - w_0^*\right)\right]}{2b^2 - 1} > 0. \tag{4.7}$$

How do we explain this result?

Consider the case when domestic workers are not organized. So, the unit production cost is w_0. And we go back to the classic profit-shifting setting of Brander and Spencer (1985). National welfare is defined as

$$G = \pi - sx = \left(a - bx - x^* - w_0 + s\right)x - sx. \tag{4.8}$$

It is easy to show that the optimal trade policy is an export subsidy being equal to

$$s = \frac{\left[2b\left(a - w_0\right) - \left(a - w_0^*\right)\right]}{4b\left(2b^2 - 1\right)}.$$

Now the unit production cost decreases to $w_0 - \frac{\left[2b(a-w_0)-\left(a-w_0^*\right)\right]}{4b(2b^2-1)}$. This gives the domestic firm a Stackelberg quantity leadership position in product market competition.

When domestic workers are organized, the unit production cost is w. As shown above, the optimal trade policy is an export subsidy given by expression (4.7). It can be shown that the unit production cost decreases to $w - s = w_0 - \frac{\left[2b(a-w_0)-\left(a-w_0^*\right)\right]}{4b(2b^2-1)}$, which is equal to the unit production cost after the domestic trade policy intervention in the case of no trade union. National welfare is defined as

$$\begin{aligned} G &= \pi + \omega - sx \\ &= \left(a - bx - x^* - w + s\right)x + \left(w - w_0\right)x - sx \\ &= \left(a - bx - x^* - w_0 + s\right)x - sx, \end{aligned}$$

which is the same as expression (4.8). So, again, the government wants to use trade policy to give the domestic firm a Stackelberg quantity leadership position in product market competition while taking the trade union's best response to the trade policy intervention into account. Since given the trade policy, the trade union skims off a part of it (in the

[10]It is straightforward to show

$$\frac{d^2\left(\pi + \omega - sx\right)}{ds^2} = -\frac{b\left(2b^2 - 1\right)}{\left(4b^2 - 1\right)^2} < 0.$$

Therefore, there is a unique interior solution.

linear case, one half), the government chooses a higher subsidy than in the case of no trade union in order to make the domestic firm commit to a Stackelberg leader's output in product market competition.

See Figure 4.1.

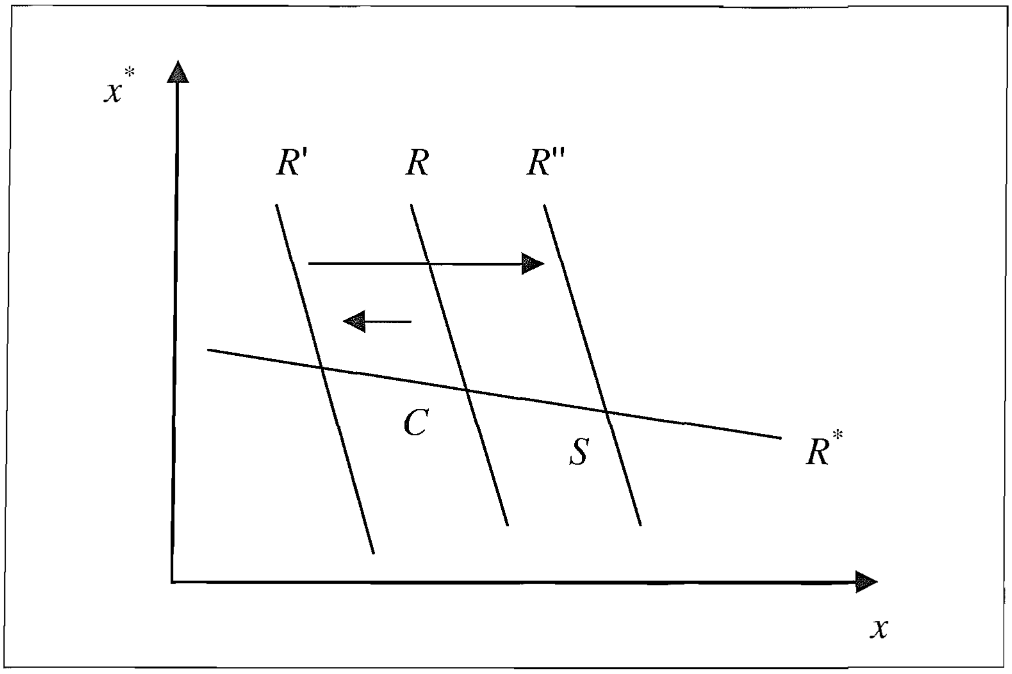


Figure 4.1: Optimal trade policy towards a linear unionized Cournot duopoly

R denotes the domestic firm's reaction function when domestic workers are not organized. R' denotes its reaction function when domestic workers are organized. R'' denotes its export-subsidy-augmented reaction function when domestic workers are organized. R^* denotes the foreign firm's reaction function. The intersection of R and R^* is denoted by C, representing a Cournot equilibrium when domestic workers are not organized. The intersection of R'' and R^* is denoted by S, representing a Stackelberg equilibrium, in which the domestic firm has a leadership position. When domestic workers are organized, without the trade policy intervention, the domestic firm's unit production cost increases and its reaction function moves inward, (see the left arrow in Figure 4.1). When designing an export subsidy, the government knows that the trade union skims off a part of it. So, the government chooses an export subsidy, which is bigger than the export subsidy in the case when domestic workers are not organized, in order to offset this effect and give the domestic firm a Stackelberg quantity leadership position (see the right arrow in Figure 4.1).

4.3.2 Bertrand competition

Next let us analyze the case where the domestic firm and the foreign firm compete as Bertrand competitors in product market.

In the third stage of the game, the domestic firm maximizes its profits:

$$\pi = (p - w + s)(\alpha - \beta p + \gamma p^*),$$

whilst the foreign firm maximizes its profits:

$$\pi^* = (p^* - w_0^*)(\alpha + \gamma p - \beta p^*).$$

The domestic firm's first-order condition for profit maximization and the foreign firm's first-order condition for profit maximization determine simultaneously the Nash equilibrium:[11]

$$p = \frac{(2\beta + \gamma)\alpha + 2\beta^2 w - 2\beta^2 s + \beta\gamma w_0^*}{4\beta^2 - \gamma^2}, \tag{4.9a}$$

$$p^* = \frac{(2\beta + \gamma)\alpha + \beta\gamma w - \beta\gamma s + 2\beta^2 w_0^*}{4\beta^2 - \gamma^2}. \tag{4.9b}$$

Substituting the equilibrium prices into the direct demand for domestic products, we get the domestic firm's equilibrium outputs, and hence equilibrium employment levels:

$$x(w) = \beta\left[\frac{(2\beta + \gamma)\alpha - (2\beta^2 - \gamma^2)w + (2\beta^2 - \gamma^2)s + \beta\gamma w_0^*}{4\beta^2 - \gamma^2}\right]. \tag{4.10}$$

In the second stage of the game, the trade union chooses wage rate, w, to maximize its economic rents:

$$\begin{aligned}\omega &= (w - w_0)\, x(w) \\ &= (w - w_0)\left\{\beta\left[\frac{(2\beta + \gamma)\alpha - (2\beta^2 - \gamma^2)w + (2\beta^2 - \gamma^2)s + \beta\gamma w_0^*}{4\beta^2 - \gamma^2}\right]\right\}. \end{aligned} \tag{4.11}$$

From the first-order condition for maximization, we can solve for the equilibrium wage rate given the domestic trade policy:[12]

$$w = \frac{(2\beta + \gamma)\alpha + (2\beta^2 - \gamma^2)s + (2\beta^2 - \gamma^2)w_0 + \beta\gamma w_0^*}{2(2\beta^2 - \gamma^2)}. \tag{4.12}$$

[11] Notice that the first-order conditions are also sufficient in this standard Bertrand game.

[12] It is straightforward to show

$$\frac{\partial^2 \omega}{\partial w^2} = -\frac{2\beta(2\beta^2 - \gamma^2)}{4\beta^2 - \gamma^2} < 0.$$

Therefore, there is a unique interior solution.

Notice that $\frac{dw}{ds} = \frac{1}{2}$. This means that the trade union skims off one half of the trade policy.

Using expression (4.12), we can show

$$x = \frac{\beta\left[(2\beta+\gamma)\alpha + (2\beta^2-\gamma^2)s - (2\beta^2-\gamma^2)w_0 + \beta\gamma w_0^*\right]}{2(4\beta^2-\gamma^2)}, \tag{4.13}$$

$$\pi = \beta\left[\frac{(2\beta+\gamma)\alpha + (2\beta^2-\gamma^2)s - (2\beta^2-\gamma^2)w_0 + \beta\gamma w_0^*}{2(4\beta^2-\gamma^2)}\right]^2, \tag{4.14}$$

$$\omega = \frac{\beta\left[(2\beta+\gamma)\alpha + (2\beta^2-\gamma^2)s - (2\beta^2-\gamma^2)w_0 + \beta\gamma w_0^*\right]^2}{4(2\beta^2-\gamma^2)(4\beta^2-\gamma^2)}. \tag{4.15}$$

In the first stage of the game, the government chooses trade policy, s, to maximize national welfare:

$$G = \pi + \omega - sx,$$

where π is given by expression (4.14), ω is given by expression (4.15), and x is given by expression (4.13). From the first-order condition for national welfare maximization, we can solve for the optimal trade policy.[13] It is an export subsidy:

$$\begin{aligned} s &= \frac{(2\beta+\gamma)\alpha - (2\beta^2-\gamma^2)w_0 + \beta\gamma w_0^*}{2\beta^2} \\ &= \frac{(2b^2-1)(a-w_0) - b(a-w_0^*)}{2b^2} > 0, \end{aligned} \tag{4.16}$$

since $\alpha = \frac{a}{b+1}$, $\beta = \frac{b}{b^2-1}$, $\gamma = \frac{1}{b^2-1}$.

How do we explain this result?

Consider the case when domestic workers are not organized. So, the unit production cost is w_0. And we go back to the classic profit-shifting setting of Eaton and Grossman (1986). National welfare is defined as

$$G = \pi - sx = (p - w_0 + s)(\alpha - \beta p + \gamma p^*) - s(\alpha - \beta p + \gamma p^*). \tag{4.17}$$

It is easy to show that the optimal trade policy is an export tax being equal to

$$s = -\frac{\gamma^2\left[(2\beta+\gamma)\alpha - (2\beta^2-\gamma^2)w_0 + \beta\gamma w_0^*\right]}{4\beta^2(2\beta^2-\gamma^2)}.$$

[13] It is straightforward to show

$$\frac{d^2(\pi + \omega - sx)}{ds^2} = -\frac{\beta^3(2\beta^2-\gamma^2)}{(4\beta^2-\gamma^2)^2} < 0.$$

Therefore, there is a unique interior solution.

Now the unit production cost increases to $w_0 + \frac{\gamma^2\left[(2\beta+\gamma)\alpha-\left(2\beta^2-\gamma^2\right)w_0+\beta\gamma w_0^*\right]}{4\beta^2\left(2\beta^2-\gamma^2\right)}$. This gives the domestic firm a Stackelberg price leadership position in product market competition.

When domestic workers are organized, the unit production cost is w. As shown above, the optimal trade policy is an export subsidy given by expression (4.16). It can be shown that the unit production cost decreases to $w - s = w_0 + \frac{\gamma^2\left[(2\beta+\gamma)\alpha-\left(2\beta^2-\gamma^2\right)w_0+\beta\gamma w_0^*\right]}{4\beta^2\left(2\beta^2-\gamma^2\right)}$, which is equal to the unit production cost after the domestic trade policy intervention in the case of no trade union. National welfare is defined as

$$\begin{aligned} G &= \pi + \omega - sx \\ &= (p - w + s)(\alpha - \beta p + \gamma p^*) + (w - w_0)(\alpha - \beta p + \gamma p^*) - s(\alpha - \beta p + \gamma p^*) \\ &= (p - w_0 + s)(\alpha - \beta p + \gamma p^*) - s(\alpha - \beta p + \gamma p^*), \end{aligned}$$

which is the same as expression (4.17). So, again, the government wants to use trade policy to give the domestic firm a Stackelberg price leadership position in product market competition while taking the trade union's best response to the trade policy intervention into account. Since given the trade policy, the trade union skims off a part of it (in the linear case, one half), the government chooses an export subsidy in order to make the domestic firm commit to a Stackelberg leader's price in product market competition.

See Figure 4.2.

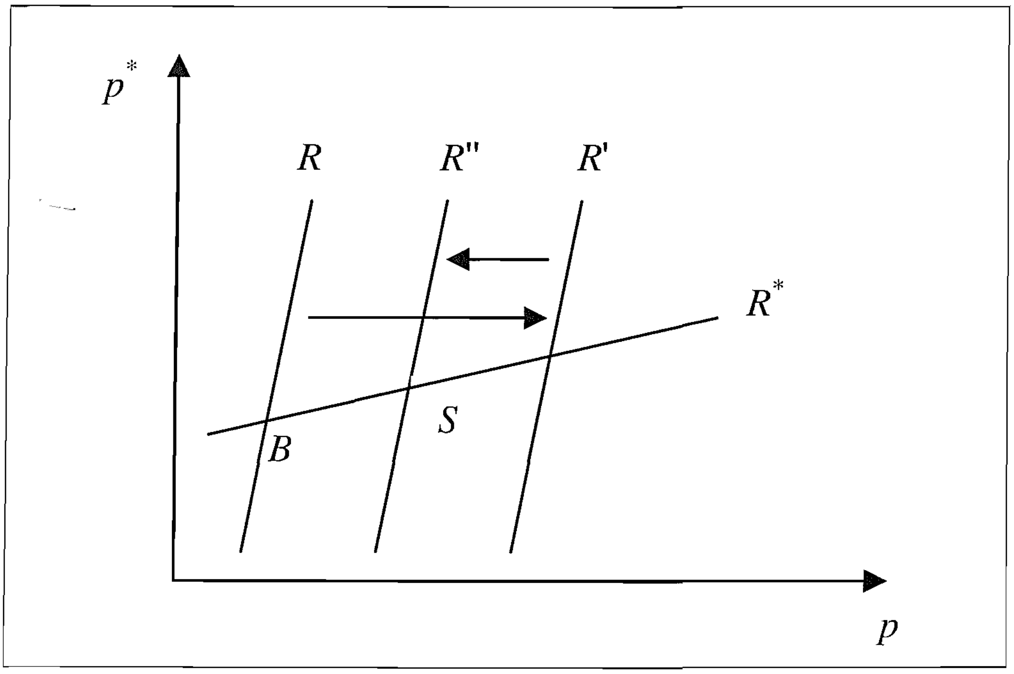


Figure 4.2: Optimal trade policy towards a linear unionized Bertrand duopoly

R denotes the domestic firm's reaction function when domestic workers are not organized. R' denotes its reaction function when domestic workers are organized. R'' denotes its export-subsidy-augmented reaction function when domestic workers are organized. R^* denotes the foreign firm's reaction function. The intersection of R and R^* is denoted by B, representing a Bertrand equilibrium when domestic workers are not organized. The intersection of R'' and R^* is denoted by S, representing a Stackelberg equilibrium, in which the domestic firm has a leadership position. When domestic workers are organized, without the trade policy intervention, the domestic firm's unit production cost increases and its reaction function moves outward, (see the right arrow in Figure 4.2). When designing an export subsidy, the government knows that the trade union skims off a part of it. So, the government chooses an export subsidy rather than an export tax in the case when domestic workers are not organized, to offset this effect and give the domestic firm a Stackelberg price leadership position (see the left arrow in Figure 4.2).

Based on the arguments in the above two subsections, we have the following Proposition.

Proposition 4.1 *An export subsidy is the optimal trade policy towards a linear unionized duopoly irrespective of both the form of market conduct and the degree of product differentiation.*

So far, we have reproduced the result implied by Brander and Spencer (1988), and Bandyopadhyay *et al.* (2000) in a clear-cut way. Is this result a robust one?

4.4 Opportunity Cost of Public Funds

In this section, we introduce an opportunity cost of public funds, $\delta > 1$, to the basic model. Nothing is changed except national welfare function. We have the following Proposition.

Proposition 4.2 *(Opportunity Cost of Public Funds) When the opportunity cost of public funds is strictly greater than 1, the necessary and sufficient condition for the government to set an export subsidy irrespective of the form of market conduct is given by*

$$\delta < \left(\frac{2b^2 - 1}{4b^2 - 1} + 1\right). \tag{4.18}$$

Proof. See Appendix to Chapter 4. ■

Compared to Proposition 4.1, now we see that an export subsidy cannot always be a robust policy recommendation towards a linear unionized duopoly when the opportunity cost of public funds is strictly greater than 1.

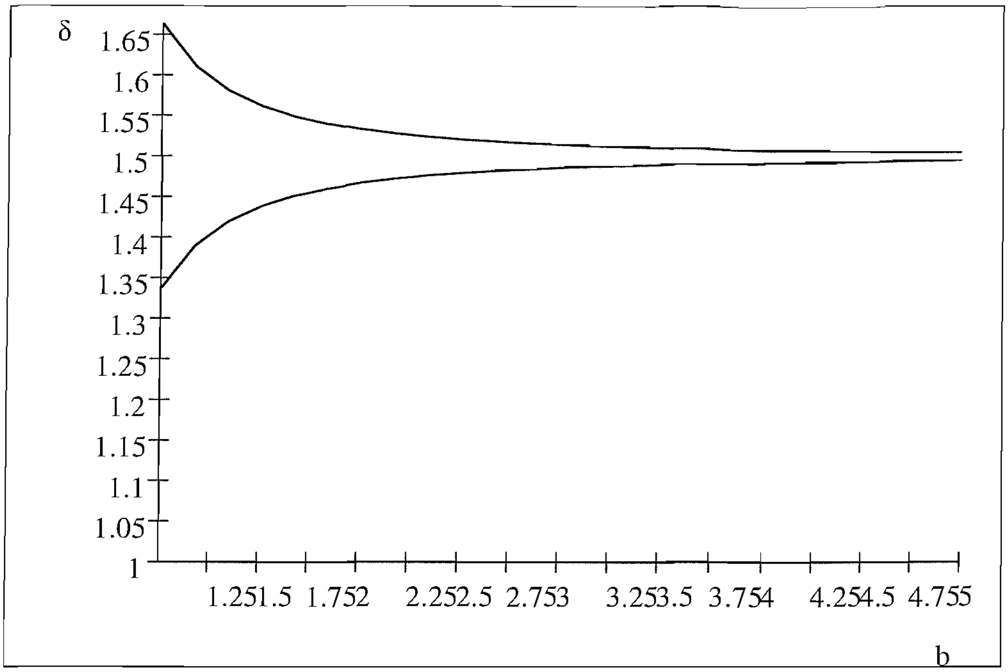


Figure 4.3: Opportunity cost of public funds and optimal trade policy

See Figure 4.3.

Along the downward sloping curve, $\left(\frac{2b^2}{4b^2-1}+1\right)-\delta=0$. Along the upward sloping curve, $\left(\frac{2b^2-1}{4b^2-1}+1\right)-\delta=0$. If parameter configurations are strictly below the upward sloping curve, then the optimal trade policy is an export subsidy irrespective of the form of market conduct. Notice that the horizontal axis, $\delta=1$, represents the case that we discussed in the last section. If parameter configurations are between the two curves, then the optimal trade policy is an export subsidy if firms compete as Cournot competitors, whilst it is an export tax if firms compete as Bertrand competitors. If parameter configurations are strictly above the downward sloping curve, then the optimal trade policy is an export tax irrespective of the form of market conduct. As a result, if the opportunity cost of public funds is sufficiently high, then we would not have an export subsidy as a robust trade policy recommendation. This is mainly because that the gains from the 'strategic use of trade policy' cannot offset the costs of using public funds.

4.5 Special Interest Politics

However, what governments set trade policy to please a particular industry is a frequently observed phenomenon. For instance, see case studies done by Goldstein and McGuire (2004), and Pritchard and MacPherson (2004). This suggests that trade policy making could be influenced by politics, such as special interest lobbing. In particular, the domestic firm and the trade union may lobby the government to shift resources from other sectors to the export sector prior to the stage during which the government sets trade policy.

In this section, we use a common agency approach developed by Bernheim and Whinston (1986), and Grossman and Helpman (1994) to studying the impact of special interest lobbying on trade policy towards a unionized duopoly. We want to know whether an export subsidy is a robust policy recommendation when special interest lobbying is present.

We introduce an initial stage to the basic model during which the domestic firm and the trade union simultaneously make political contributions, which are contingent on trade policies, to the government. The domestic firm's political contributions are denoted by $C^F(s)$. The trade union's political contributions are denoted by $C^T(s)$. Then follows the three-stage game described in the basic model. Notice that now the domestic firm receives its profits minus its political contributions, $\pi - C^F(s)$. The trade union receives its economic rents minus its political contributions, $\omega - C^T(s)$. The government receives

$$G = \sum_{i \in \{F,T\}} C^i(s) + \lambda(\pi + \omega - \delta s x), \ \lambda \geq 0.$$

λ is a parameter that represents the marginal rate of substitution between national welfare and political contributions. The larger is λ, the more weight is placed on national welfare relative to political contributions.[14] Hence, the larger is λ, the less the government will be influenced by the domestic firm and the trade union. When $\lambda \to \infty$, the government receives national welfare, and cannot be influenced by the domestic firm and the trade union.

Notice that the domestic firm and the trade union have an incentive to engage in lobbying. When the domestic firm and the foreign firm engage in Cournot competition in the third market, its profits are given by expression (4.5), and

$$\left.\frac{d\pi}{ds}\right|_s = \frac{b^2}{4b^2-1}\left[2b(a-w_0)-(a-w_0^*)\right]\left[\frac{\delta}{2\delta - \left(1+\frac{2b^2}{4b^2-1}\right)}\right] > 0;$$

the trade union's economic rents are given by expression (4.6), and

$$\left.\frac{d\omega}{ds}\right|_s = \frac{1}{2(4b^2-1)}\left[2b(a-w_0)-(a-w_0^*)\right]\left[\frac{\delta}{2\delta - \left(1+\frac{2b^2}{4b^2-1}\right)}\right] > 0,$$

where s is given by expression (5.27).[15] So, in the Cournot case, both the domestic firm and the trade union have an incentive to engage in lobbying.

When the domestic firm and the foreign firm engage in Bertrand competition in the

[14] See Grossman and Helpman (1994).

[15] See Proof of Proposition 4.2 in Appendix to Chapter 4.

third market, its profits are given by expression (4.14), and

$$\left.\frac{d\pi}{ds}\right|_{s} = \frac{\beta\left(2\beta^2-\gamma^2\right)}{2\left(4\beta^2-\gamma^2\right)^2}\left[(2\beta+\gamma)\alpha-\left(2\beta^2-\gamma^2\right)w_0+\beta\gamma w_0^*\right]\left[\frac{\delta}{2\delta-\left(1+\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}\right)}\right]>0;$$

the trade union's economic rents are given by expression (4.15), and

$$\left.\frac{d\omega}{ds}\right|_{s} = \frac{\beta}{2\left(4\beta^2-\gamma^2\right)}\left[(2\beta+\gamma)\alpha-\left(2\beta^2-\gamma^2\right)w_0+\beta\gamma w_0^*\right]\left[\frac{\delta}{2\delta-\left(1+\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}\right)}\right]>0,$$

where s is given by expression (5.29).[16] So, in the Bertrand case, both the domestic firm and the trade union have an incentive to engage in lobbying.

Now, let us turn to how the government design its trade policy. Bernheim and Whinston (1986) uses a Truthful Equilibrium as the solution concept to a common agency game. According to their Theorem 2, in a truthful equilibrium the government chooses trade policy, s, to maximize the sum of the domestic firm's payoff, the trade union's payoff and its own payoff:[17]

$$\left(\pi - C^F(s)\right)+\left(\omega - C^T(s)\right)+G$$
$$\Leftrightarrow$$
$$\pi+\omega+\lambda\left(\pi+\omega-\delta s x\right).$$

Notice that when the domestic firm and the foreign firm engage in Cournot competition in product market, there exists a unique interior solution to the government's maximization

[16]See Proof of Proposition 4.2 in Appendix to Chapter 4.

[17]The government chooses trade policy, s, to maximize G. The first-order condition for maximization is given by

$$\frac{dC^F}{ds}+\frac{dC^T}{ds}+\lambda\frac{d\left(\pi+\omega-\delta s x\right)}{ds}=0.$$

In a truthful equilibrium, the domestic firm's contribution schedule satisfies

$$\frac{dC^F}{ds}=\frac{d\pi}{ds}.$$

The trade union's contribution schedule satisfies

$$\frac{dC^T}{ds}=\frac{d\omega}{ds}.$$

Given these, the first-order condition for maximizing G becomes

$$\frac{d\pi}{ds}+\frac{d\omega}{ds}+\lambda\frac{d\left(\pi+\omega-\delta s x\right)}{ds}=0.$$

This is the first-order condition for maximizing $\pi+\omega+\lambda\left(\pi+\omega-\delta s x\right)$.

problem if and only if[18]

$$\left(\frac{2b^2}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right) < 2\delta. \tag{4.19}$$

When the domestic firm and the foreign firm engage in Bertrand competition in product market, there exists a unique interior solution if and only if[19]

$$\left(\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}+1\right)\left(\frac{\lambda+1}{\lambda}\right) = \left(\frac{2b^2-1}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right) < 2\delta. \tag{4.20}$$

Combining these two facts, there exists a unique interior solution irrespective of the form of market conduct, if and only if[20]

$$\left(\frac{2b^2}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right) < 2\delta. \tag{4.21}$$

We have the following Proposition.

Proposition 4.3 *(Special Interest Politics) In an interior solution, the necessary and sufficient condition for the domestic government to set an export subsidy is as follows:*

1. $\left(\frac{2b^2}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right) < 2\delta$;
2. $\delta < \left(\frac{2b^2-1}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right)$.

Proof. See Appendix to Chapter 4. ■

Look the above Proposition. The first condition just repeats condition (4.21), it guarantees that an interior solution exists. Comparing the second condition to condition (4.18), it is easy to see that the presence of special interest politics weakens the effect of an opportunity cost of public funds on trade policy making.

Remark 4.1 *(i) When* $\delta > \left(\frac{2b^2}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right)$*, the optimal trade policy is an export tax irrespective of the form of market conduct. (ii) When* $\left(\frac{2b^2-1}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right) < \delta <$

[18]Notice that in the Cournot case, we have

$$\frac{d^2\left[\pi+\omega+\lambda\left(\pi+\omega-\delta sx\right)\right]}{ds^2} = \frac{b\lambda}{4b^2-1}\left[\left(\frac{2b^2}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right)-2\delta\right].$$

So, $\left(\frac{2b^2}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right) < 2\delta$ is both necessary and sufficient for the second-order condition to be satisfied.

[19]In the Bertrand case, we have

$$\frac{d^2\left[\pi+\omega+\lambda\left(\pi+\omega-\delta sx\right)\right]}{ds^2} = \frac{\beta\left(2\beta^2-\gamma^2\right)\lambda}{2\left(4\beta^2-\gamma^2\right)}\left[\left(\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}+1\right)\left(\frac{\lambda+1}{\lambda}\right)-2\delta\right].$$

So, $\left(\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}+1\right)\left(\frac{\lambda+1}{\lambda}\right) = \left(\frac{2b^2-1}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right) < 2\delta$ is both necessary and sufficient for the second-order condition to be satisfied.

[20]When λ goes to infinity, the second-order condition is satisfied automatically irrespective of the form of market conduct. This is the case that we discussed in the above section.

$\left(\frac{2b^2}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right) < 2\delta$, *the optimal trade policy is an export subsidy for the Cournot case; whilst it is an export tax for the Bertrand case.*[21]

So far, we have finished discussion of the case when there exists a unique interior solution irrespective of the form of market conduct. However, notice that when special interest lobbying is present, there may not exist an interior solution.

Remark 4.2 *(i) When* $\left(\frac{2b^2-1}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right) < 2\delta < \left(\frac{2b^2}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right)$, *there exists a unique interior solution in the Bertrand case there does not exist an interior solution in the Cournot case. The optimal trade policy is an infinite export subsidy for the Cournot case, whilst it is a finite export subsidy for the Bertrand case. (ii) When* $\left(\frac{2b^2-1}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right) > 2\delta$, *there does not exist an interior solution both in Bertrand case and in Cournot case. So, the optimal trade policy is an infinite export subsidy irrespective of the form of market conduct.*[22]

The results obtained in the above discussion are summarized in the following Corollary.

Corollary 4.1 *The necessary and sufficient condition for the domestic government to choose an export subsidy irrespective of the form of market conduct is given by the second condition in Proposition 4.3.*

Proof. Proposition 4.3 and Remark 4.2 together implies the Corollary immediately. ■

Corollary 4.1 says that whether there is an interior solution, for a pair of b and δ,[23] if the extent to which the government is influenced by the domestic firm and the trade union is sufficiently great, then the government will use an export subsidy irrespective of the form of market conduct. This is a sharp contrast to the result obtained in the last section. To see an extreme example, consider the case when $\lambda = 0$.

4.6 Discussion

So far, we have finished three exercises and derived a number of results. What is the relationship between these results and those obtained in existing literatures? See the following Figure.

[21]See Proof of Proposition 4.3 in Appendix to Chapter 4.

[22]This Remark presents unpleasant results derived from the current model, in which the government's budget constraint is not included. When this constraint is explicitly modeled, an infinite export subsidy will not be an optimal solution: the government chooses an export subsidy such that the total costs of subsidy hit its budget constraint. However, notice that the results obtained in the current model cannot be changed qualitatively.

(Also notice that in the current model, the domestic firm and the trade union do not pay for an export subsidy. Now, what will happen when they need to pay for it? It can be shown that again we can find parameter configurations for which the second-order condition for maximization is not satisfied.)

[23]We treat δ as a finite number. For example, according to Ballard *et al.* (1985), $\delta \in [1.17, 1.56]$.

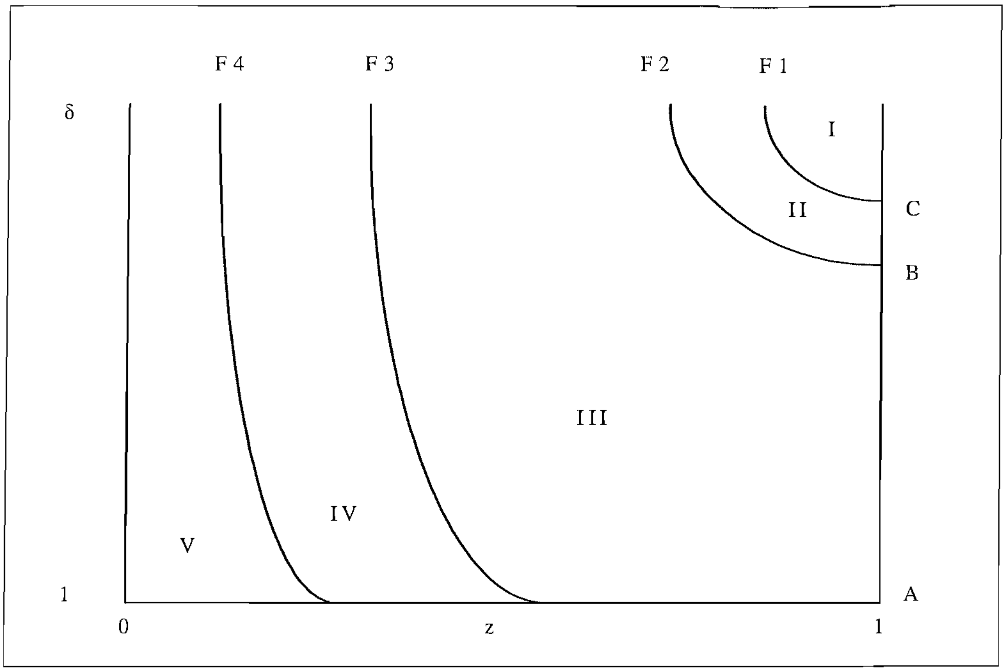


Figure 4.4: Summary of results

Define $z = \frac{\lambda}{\lambda+1}$, since λ is nonnegative, $0 \leq z \leq 1$. The horizontal axis represents z, whilst the vertical axis represents δ, $\delta \geq 1$. Fixing a b, curve F1 represents the case when condition (5.32) holds with equality:[24] $\left(\frac{2b^2}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right) - \delta = 0 \Leftrightarrow \left(\frac{2b^2}{4b^2-1}+1\right)\frac{1}{z} - \delta = 0$. Curve F2 represents the case when condition (5.34) holds with equality:[25] $\left(\frac{2b^2-1}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right) - \delta = 0 \Leftrightarrow \left(\frac{2b^2-1}{4b^2-1}+1\right)\frac{1}{z} - \delta = 0$. Curve F3 represents the case when in condition (4.19), an equality holds: $\left(\frac{2b^2}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right) - 2\delta = 0 \Leftrightarrow \left(\frac{2b^2}{4b^2-1}+1\right)\frac{1}{z} - 2\delta = 0$. Curve F4 represents the case when in condition (4.20), an equality holds: $\left(\frac{2b^2-1}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right) - 2\delta = 0 \Leftrightarrow \left(\frac{2b^2-1}{4b^2-1}+1\right)\frac{1}{z} - 2\delta = 0$. These four curves divide the (z, δ) plane into five regions.

In region I, the optimal trade policy is a finite export tax irrespective of the form of market conduct. See part (i) of Remark 4.1.

In region II, the optimal trade policy is a finite export subsidy for the Cournot case, whilst it is a finite export tax for the Bertrand case. See part (ii) of Remark 4.1.

In region III, the optimal trade policy is a finite export subsidy irrespective of the form of market conduct. See Proposition 4.3.

In region IV, the optimal trade policy is an infinite export subsidy for the Cournot case, whilst it is a finite export subsidy for the Bertrand case. See part (i) of Remark 4.2.

In region V, the optimal trade policy is an infinite export subsidy irrespective of the form of market conduct. See part (ii) of Remark 4.2.

[24]See Proof of Proposition 4.3 in Appendix to Chapter 4.
[25]See Proof of Proposition 4.3 in Appendix to Chapter 4.

In summary, In region III, IV and V, an export subsidy is a robust policy recommendation towards a linear unionized duopoly.

So far, we have explored all of the possibilities in the (z, δ) plane.

The results of Brander and Spencer (1988), and Bandyopadhyay *et al.* (2000) can be represented by point A with coordinate $(1,1)$ in the above Figure: they study optimal trade policy towards a unionized duopoly without an opportunity cost of public funds and political economy. We reproduce their results in a clear-cut way in Proposition 4.1 as a benchmark case.

Neary (1994) can be represented by point C: he studies optimal strategic trade policy for the Cournot case in a Brander-Spencer third-market model with an opportunity cost of public funds but without political economy. We reproduce his result for the Cournot case and go further to consider the Bertrand case, which is represented by point B. See Proposition 4.2.

Fung and Lin (2000) can be represented by the horizontal axis: they study optimal strategic trade policy from a political economy perspective but without introducing an opportunity cost of public funds to their model. See Proposition 4.3. Setting $\delta = 1$ in the two conditions, we get their results.[26]

In summary, all of the results of previous literatures can be viewed as a special case of our research. And our paper explores fully optimal trade policy towards a linear unionized duopoly with an opportunity cost of public funds and special interest lobbying.

4.7 Conclusion

We have studied whether an export subsidy is a robust trade policy recommendation towards a linear unionized duopoly, and two main messages have been derived. First of all, an export subsidy can hardly be a robust trade policy recommendation towards a unionized duopoly, if we consider this problem from a purely economic perspective: it is very sensitive to the opportunity cost of public funds even in the simplest setting. Second, an export subsidy as a robust policy recommendation can be supported by political reasons, for instance, special interest lobbying.

These are fairly robust results. As to the first result, if the result of robustness cannot be obtained in the simplest case, it can hardly be obtained in more sophisticated cases. As to the second result, on the one hand, in reality we observe that governments set trade policies to please a particular industry. (See Goldstein and McGuire (2004).) This

[26]Comparing expression (5.31) to expression (5.27), expression (5.33) to expression (5.29), (see Appendix to Chapter 4), it is easy to see that in our paper, in an interior solution, the politically determined export subsidy is always greater than the rent-shifting export subsidy set by a benevolent government. This result is different from Fung and Lin (2000). We also discuss this point in the Introduction.

means that governments can be very sensitive to special interest lobbying. On the other hand, special interest groups, such as domestic firms and trade unions have an incentive to engage in lobbying. To understand this, notice that when the export subsidy is set sufficiently big, then the foreign firm would be driven out of the third-country market, and domestic firms and trade unions would gain from a monopoly market structure. So, our result derived from the simplest model could be reproduced in a more sophisticated model.

Chapter 5

Appendices

5.1 Appendix to Chapter 2

5.1.1 Proof of Lemma 2.1

First, let us establish trade union i's best response. Given trade union j's political contributions, and firm j's political contributions, government j's political incentive (or disincentive) to attract the multinational is determined. Given that and given firm i's political contributions, can trade union i make country i win the competition? If

$$\lambda^i \left(\frac{1}{12} \Delta_i - C_{ij}^F \right) + \left(\frac{1}{16} + \frac{1}{18} \right) (\Delta_i - \Delta_j) \geq \lambda^j \left(C_{jj}^T - C_{ji}^F \right),$$

this is true. Clearly trade union i will choose the lowest possible political contributions. Hence, trade union i will choose a number, which makes the above inequality hold with equality. Define z_i^T such that

$$\lambda^i \left(z_i^T - C_{ij}^F \right) + \left(\frac{1}{16} + \frac{1}{18} \right) (\Delta_i - \Delta_j) = \lambda^j \left(C_{jj}^T - C_{ji}^F \right).$$

If $z_i^T \geq 0$, trade union i chooses $C_{ii}^T = z_i^T$. However, if $z_i^T < 0$, it chooses $C_{ii}^T = 0$, since it is not allowed to make negative political contributions.

On the other hand, if

$$\lambda^i \left(\frac{1}{12} \Delta_i - C_{ij}^F \right) + \left(\frac{1}{16} + \frac{1}{18} \right) (\Delta_i - \Delta_j) < \lambda^j \left(C_{jj}^T - C_{ji}^F \right),$$

then trade union i cannot make country i win the competition. It can choose arbitrarily its political contributions.

Using the same type of arguments, we can establish the best responses for firm i, trade union j, and firm j respectively. ■

5.1.2 Proof of Lemma 2.2

Suppose that there is such a CPNE $\left(C_i^T, C_i^F, C_j^T, C_j^F\right)$, but $\frac{1}{72}\lambda^i\Delta_i + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) < \frac{1}{72}\lambda^j\Delta_j$. We want to show that $\left(C_i^T, C_i^F, C_j^T, C_j^F\right)$ is not self-enforcing, and hence is not a CPNE since given C_i^T and C_j^F, $\left(C_i^F, C_j^T\right)$ is not a CPNE of the game played by firm i and trade union j.

There are two nonempty proper subcoalitions: one formed by firm i and another formed by trade union j. It is easy to show that $\left(C_i^F, C_j^T\right)$ is self-enforcing. Since by supposition that $\left(C_i^T, C_i^F, C_j^T, C_j^F\right)$ is a CPNE, given C_i^T, C_j^F, and given C_j^T, C_i^F is an optimal strategy for firm i; given C_i^T, C_j^F, and given C_i^F, C_j^T is an optimal strategy for trade union j. Firm i receives $\frac{1}{18}\Delta_i$, and trade union j receives $\frac{1}{4}\Delta_j$.

But there are other self-enforcing strategy profiles, in which C_{ij}^F and C_{jj}^T satisfy

$$\lambda^i\left(C_{ii}^T - C_{ij}^F\right) + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) = \lambda^j\left(C_{jj}^T - C_{ji}^F\right),$$

where $0 < C_{ij}^F < \frac{5}{72}\Delta_i$, and $0 < C_{jj}^T < \frac{1}{12}\Delta_j$. I.e., given C_i^T and C_j^F, firm i and trade union j can coordinate and help country j win FDI competition noncooperatively. Firm i receives $\frac{1}{8}\Delta_i - C_{ij}^F > \frac{1}{18}\Delta_i$, and trade union j receives $\frac{1}{3}\Delta_j - C_{jj}^T > \frac{1}{4}\Delta_j$.

So, $\left(C_i^F, C_j^T\right)$ is strongly Pareto dominated by other self-enforcing strategy profiles described in the above, and hence is not a CPNE of the game played by firm i and trade union j, given C_i^T and C_j^F. Therefore, $\left(C_i^T, C_i^F, C_j^T, C_j^F\right)$ is not self-enforcing, and hence is not a CPNE. A contradiction. ■

5.1.3 Proof of Proposition 2.1

Step 1. We show that any strategy profile is self-enforcing. There are 14 nonempty proper subcoalitions. Four subcoalitions are formed by one player. Six subcoalitions are formed by two players. Four subcoalitions are formed by three players.

1. Let us consider the subcoalitions formed by one player. Given condition (2.15) holds, according to Lemma 2.1, the proposed strategy profiles are Nash equilibria. So, given any other three players' strategies, the strategy prescribed for the left player is a CPNE of the one-player game played by itself.

2. Let us consider the subcoalitions formed by two players.

   a. The subcoalitions formed by trade union i and firm i. Consider the game played by these two players when $C_j^T = \left(0, C_{jj}^T\right)$, where C_{jj}^T is arbitrarily chosen, and $C_j^F = 0$. There are two nonempty proper subcoalitions: one formed by trade union i and another formed by firm i. Given $C_i^F = \left(0, C_{ij}^F\right)$, where C_{ij}^F is arbitrarily chosen, since condition (2.15) holds, we always have $\lambda^i \left(0 - C_{ij}^F\right) + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) \geq \lambda^j \left(C_{jj}^T - 0\right)$, so, $C_i^T = 0$ is a CPNE of the one-player game played by trade union i. Given $C_i^T = 0$, since condition (2.15) holds, we always have $\lambda^i \left(0 - \frac{5}{72}\Delta_i\right) + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) \geq \lambda^j \left(C_{jj}^T - 0\right)$, so, $C_i^F = \left(0, C_{ij}^F\right)$, where C_{ij}^F is arbitrarily chosen, is a CPNE of the one-player game played by firm i. So, the strategy profile consisting of $C_i^T = 0$ and $C_i^F = \left(0, C_{ij}^F\right)$ is self-enforcing. Notice that any strategy profile consisting of $C_i^T = 0$ and $C_i^{F\prime} = \left(0, C_{ij}^{F\prime}\right)$, where $C_{ij}^F \neq C_{ij}^{F\prime}$, is also self-enforcing. But trade union i receives $\frac{1}{3}\Delta_i$, and firm i receives $\frac{1}{18}\Delta_i$, irrespective of self-enforcing strategy profiles. So, $C_i^T = 0$ and $C_i^F = \left(0, C_{ij}^F\right)$ are a CPNE of the game played by trade union i and firm i.

b. The subcoalitions formed by trade union i and trade union j. Using the similar arguments to those in 1.2.a, it proves that $C_i^T = 0$ and $C_j^T = \left(0, C_{jj}^T\right)$, where C_{jj}^T is arbitrarily chosen, are a CPNE of the game played by trade union i and trade union j.

c. The subcoalitions formed by trade union i and firm j. Consider the game played by these two players when $C_i^F = \left(0, C_{ij}^F\right)$, where C_{ij}^F is arbitrarily chosen, and $C_j^T = \left(0, C_{jj}^T\right)$, where C_{jj}^T is arbitrarily chosen. There are two nonempty proper subcoalitions: one formed by trade union i and another formed by firm j. Given $C_j^F = 0$, since condition (2.15) holds, we always have $\lambda^i \left(0 - C_{ij}^F\right) + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) \geq \lambda^j \left(C_{jj}^T - 0\right)$, so, $C_i^T = 0$ is a CPNE of the one-player game played by trade union i. By the same token, given $C_i^T = 0$, $C_j^F = 0$ is a CPNE of the one-player game played by firm j. So, the strategy profile consisting of $C_i^T = 0$ and $C_j^F = 0$ is self-enforcing. This is the only self-enforcing strategy profile since no player has an incentive to make strictly positive political contributions. So, it is a CPNE of the game played by trade union i and firm j.

d. The subcoalitions formed by firm i and trade union j. Consider the game played by these two players when $C_i^T = 0$ and $C_j^F = 0$. There are two nonempty proper subcoalitions: one formed by firm i and another formed by trade union j. Given $C_i^F = \left(0, C_{ij}^F\right)$, where C_{ij}^F is arbitrarily chosen, since condition (2.15) holds, we always have $\lambda^i \left(0 - C_{ij}^F\right) + \left(\frac{1}{16} + \frac{1}{18}\right)(\Delta_i - \Delta_j) \geq \lambda^j \left(\frac{1}{12}\Delta_j - 0\right)$, so, $C_j^T = \left(0, C_{jj}^T\right)$, where C_{jj}^T is arbitrarily chosen, is a CPNE of the one-player game played by trade union j. Given $C_j^T = \left(0, C_{jj}^T\right)$, where C_{jj}^T is arbitrarily chosen, since condition (2.15) holds, we always have $\lambda^i\left(0-\frac{5}{72}\Delta_i\right)+\left(\frac{1}{16}+\frac{1}{18}\right)(\Delta_i-\Delta_j)\geq\lambda^j\left(C_{jj}^T-0\right)$, so, $C_i^F=\left(0,C_{ij}^F\right)$, where C_{ij}^F is arbitrarily chosen, is a CPNE of the one-player game played by firm i. So, the strategy profile consisting of $C_i^F=\left(0,C_{ij}^F\right)$ and $C_j^T=\left(0,C_{jj}^T\right)$ is self-enforcing. Notice that any strategy profile consisting of $C_i^{F\prime}=\left(0,C_{ij}^{F\prime}\right)$, where $C_{ij}^F\neq C_{ij}^{F\prime}$, or $C_j^{T\prime}=\left(0,C_{jj}^{T\prime}\right)$, where $C_{jj}^T\neq C_{jj}^{T\prime}$, or both is also self-enforcing. But firm i receives $\frac{1}{18}\Delta_i$, and trade union j receives $\frac{1}{4}\Delta_j$, irrespective of self-enforcing strategy profiles. So, $C_i^F=\left(0,C_{ij}^F\right)$ and $C_j^T=\left(0,C_{jj}^T\right)$ are a CPNE of the game played by firm i and trade union j.

e. The subcoalitions formed by firm i and firm j. Using the similar arguments to those in 1.2.a, it proves that $C_i^F=\left(0,C_{ij}^F\right)$, where C_{ij}^F is arbitrarily chosen, and $C_j^F=0$, are a CPNE of the game played by firm i and firm j.

f. The subcoalitions formed by trade union j and firm j. Using the similar arguments to those in 1.2.a, it proves that $C_j^T=\left(0,C_{jj}^T\right)$, where C_{jj}^T is arbitrarily chosen, and $C_j^F=0$, are a CPNE of the game played by trade union j and firm j.

3. Let us consider the subcoalitions formed by three players.

a. The subcoalitions formed by trade union i, firm i and trade union j. Consider the game played by these three players when $C_j^F=0$. There are six nonempty proper subcoalitions: three formed by one player and three formed by two players.

i. Let us consider the three subcoalitions formed by one player. According to step 1.1, it is easy to show that fixing $C_j^F=0$, given any other two players' strategies, the strategy prescribed for the left player is a CPNE of the one-player game played by itself.

ii. Let us consider the three subcoalitions formed by two players. According to step 1.2.a, 1.2.b and 1.2.d, it is easy to show that fixing $C_j^F=0$, given any player's strategy, the strategies prescribed for the left two players are a CPNE of the two-player game played by themselves.

iii. So, fixing $C_j^F=0$, the strategies prescribed for the left three players are self-enforcing. Notice that any strategy profile consisting of $C_i^{F\prime}=\left(0,C_{ij}^{F\prime}\right)$, where $C_{ij}^F\neq C_{ij}^{F\prime}$, or $C_j^{T\prime}=\left(0,C_{jj}^{T\prime}\right)$, where $C_{jj}^T\neq C_{jj}^{T\prime}$, or both is also self-enforcing. But trade union i receives $\frac{1}{3}\Delta_i$, firm i receives $\frac{1}{18}\Delta_i$, and trade union j receives $\frac{1}{4}\Delta_j$, irrespective of self-enforcing strategy profiles.

So, $C_i^T = 0$, $C_i^F = \left(0, C_{ij}^F\right)$ and $C_j^T = \left(0, C_{jj}^T\right)$ are a CPNE in this case.

b. The subcoalitions formed by trade union i, firm i and firm j. Consider the game played by these three players when $C_j^T = \left(0, C_{jj}^T\right)$, where C_{jj}^T is arbitrarily chosen. There are six nonempty proper subcoalitions: three formed by one player and three formed by two players.

   i. Let us consider the three subcoalitions formed by one player. According to step 1.1, it is easy to show that fixing $C_j^T = \left(0, C_{jj}^T\right)$, where C_{jj}^T is arbitrarily chosen, given any other two players' strategies, the strategy prescribed for the left player is a CPNE of the one-player game played by itself.

   ii. Let us consider the three subcoalitions formed by two players. According to step 1.2.a, 1.2.c and 1.2.e, it is easy to show that fixing $C_j^T = \left(0, C_{jj}^T\right)$, where C_{jj}^T is arbitrarily chosen, given any player's strategy, the strategies prescribed for the left two players are a CPNE of the two-player game played by themselves.

   iii. So, fixing $C_j^T = \left(0, C_{jj}^T\right)$, where C_{jj}^T is arbitrarily chosen, the strategies prescribed for the left three players are self-enforcing. Notice that any strategy profile consisting of $C_i^{F\prime} = \left(0, C_{ij}^{F\prime}\right)$, where $C_{ij}^F \neq C_{ij}^{F\prime}$, is also self-enforcing. But trade union i receives $\frac{1}{3}\Delta_i$, firm i receives $\frac{1}{18}\Delta_i$, and firm j receives $\frac{1}{8}\Delta_j$, irrespective of self-enforcing strategy profiles. So, the proposed strategy profile is a CPNE in this case.

c. The subcoalitions formed by trade union i, trade union j and firm j. Using the similar arguments to those in step 1.3.b, it proves that the proposed strategies are a CPNE of the game played by themselves.

d. The subcoalitions formed by firm i, trade union j and firm j. Using the similar arguments to those in step 1.3.a, it proves that the proposed strategies are a CPNE of the game played by themselves.

So far, we have established that any strategy profiles prescribed in Proposition 2.1 are self-enforcing.

Step 2. Are there any other self-enforcing strategy profiles? Since given condition (2.15) holds, both trade union i and firm j do not have an incentive to make strictly positive political contributions, there is no other self-enforcing strategy profile.

Step 3. Finally, it is easy to show that given any proposed strategy profile, trade union i receives $\frac{1}{3}\Delta_i$, firm i receives $\frac{1}{18}\Delta_i$, trade union j receives $\frac{1}{4}\Delta_j$, and firm j receives $\frac{1}{8}\Delta_j$.

We conclude that any proposed strategy profile is a CPNE in the first stage of the game. ■

5.1.4 Proof of Proposition 2.2

Step 1. We show that any strategy profile is self-enforcing. There are 14 nonempty proper subcoalitions. Four subcoalitions are formed by one player. Six subcoalitions are formed by two players. Four subcoalitions are formed by three players.

1. Let us consider the subcoalitions formed by one player. Given condition (2.18) holds, according to Lemma 2.1, the proposed strategy profiles are Nash equilibria. So, given any other three players' strategies, the strategy prescribed for the left player is a CPNE of the one-player game played by itself.

2. Let us consider the subcoalitions formed by two players.

   a. The subcoalitions formed by trade union i and firm i. Consider the game played by these two players when $C_j^T = \left(0, \frac{1}{12}\Delta_j\right)$, and $C_j^F = \left(C_{ji}^F, 0\right)$. There are two nonempty proper subcoalitions: one formed by trade union i and another formed by firm i. Given $C_i^F = \left(0, \frac{5}{72}\Delta_i\right)$, it is optimal for trade union i to choose $C_i^T = \left(C_{ii}^T, 0\right)$, such that condition (2.19) holds. Given $C_i^T = \left(C_{ii}^T, 0\right)$, $C_i^F = \left(0, \frac{5}{72}\Delta_i\right)$ is a CPNE of the one-player game played by firm i. So, the strategy profile consisting of $C_i^T = \left(C_{ii}^T, 0\right)$ and $C_i^F = \left(0, \frac{5}{72}\Delta_i\right)$ is self-enforcing. Notice that any other strategy profiles are not self-enforcing since $C_i^T = \left(C_{ii}^T, 0\right)$ and $C_i^F = \left(0, \frac{5}{72}\Delta_i\right)$ are a unique Nash equilibrium of this two-player game. (The nature of this game is a standard Bertrand game with cost asymmetries.) So, it is a CPNE of the game played by trade union i and firm i.

   b. The subcoalitions formed by trade union i and trade union j. Using the similar arguments to those in 1.2.a, it proves that $C_i^T = \left(C_{ii}^T, 0\right)$ and $C_j^T = \left(0, \frac{1}{12}\Delta_j\right)$ are a CPNE of the game played by trade union i and trade union j.

   c. The subcoalitions formed by trade union i and firm j. Consider the game played by these two players when $C_i^F = \left(0, \frac{5}{72}\Delta_i\right)$ and $C_j^T = \left(0, \frac{1}{12}\Delta_j\right)$. There are two nonempty proper subcoalitions: one formed by trade union i and another formed by firm j. Given $C_j^F = \left(C_{ji}^F, 0\right)$, it is optimal for trade union i to choose $C_i^T = \left(C_{ii}^T, 0\right)$, such that condition (2.19) holds. Given $C_i^T = \left(C_{ii}^T, 0\right)$, it is optimal for firm j to choose $C_j^F = \left(C_{ji}^F, 0\right)$, such that condition (2.19) holds.

So, the strategy profile consisting of $C_i^T = \left(C_{ii}^T, 0\right)$ and $C_j^F = \left(C_{ji}^F, 0\right)$ is self-enforcing. Notice that there are other self-enforcing strategy profiles. First of all, any strategy profile consisting of $C_i^{T\prime} = \left(C_{ii}^{T\prime}, 0\right)$ and $C_j^{F\prime} = \left(C_{ji}^{F\prime}, 0\right)$, such that $C_{ii}^{T\prime}$ and $C_{ji}^{F\prime}$ satisfy condition (2.19), is self-enforcing. But $C_{ii}^{T\prime}$ and $C_{ji}^{F\prime}$ cannot be both strictly smaller than C_{ii}^T and C_{ji}^F. Otherwise, condition (2.19) does not hold. So, the proposed strategy profile cannot be strictly Pareto dominated by these self-enforcing strategy profiles. Also we may have a Nash equilibrium, in which trade union i and firm j free-ride on each other. But the payoffs received in this case are strictly smaller than the payoffs received in the case when $C_{ii}^{T\prime}$ and $C_{ji}^{F\prime}$ satisfy condition (2.19), where $0 < C_{ii}^{T\prime} < \frac{1}{12}\Delta_i$, and $0 < C_{ji}^{F\prime} < \frac{5}{72}\Delta_j$. In summary, the proposed strategy profile is a CPNE of the game played by trade union i and firm j.

d. The subcoalitions formed by firm i and trade union j. Consider the game played by these two players when $C_i^T = \left(C_{ii}^T, 0\right)$ and $C_j^F = \left(C_{ji}^F, 0\right)$, such that condition (2.19) holds. It is easy to show that any strategy profiles are Nash equilibria, and hence self-enforcing, since firm i receives $\frac{1}{18}\Delta_i$, and trade union j receives $\frac{1}{4}\Delta_j$, irrespective of strategy profiles. So, $C_i^F = \left(0, \frac{5}{72}\Delta_i\right)$ and $C_j^T = \left(0, \frac{1}{12}\Delta_j\right)$ are self-enforcing and are not strongly Pareto dominated by any other self-enforcing strategy profiles. They are a CPNE of the game played by firm i and trade union j.

e. The subcoalitions formed by firm i and firm j. Using the similar arguments to those in 1.2.a, it proves that $C_i^F = \left(0, \frac{5}{72}\Delta_i\right)$ and $C_j^F = \left(C_{ji}^F, 0\right)$ are a CPNE of the game played by firm i and firm j.

f. The subcoalitions formed by trade union j and firm j. Using the similar arguments to those in 1.2.a, it proves that $C_j^T = \left(0, \frac{1}{12}\Delta_j\right)$ and $C_j^F = \left(C_{ji}^F, 0\right)$ are a CPNE of the game played by trade union j and firm j.

3. Let us consider the subcoalitions formed by three players.

   a. The subcoalitions formed by trade union i, firm i and trade union j. Consider the game played by these three players when $C_j^F = \left(C_{ji}^F, 0\right)$. There are six nonempty proper subcoalitions: three formed by one player and three formed by two players.

      i. Let us consider the three subcoalitions formed by one player. According to step 1.1, it is easy to show that fixing $C_j^F = \left(C_{ji}^F, 0\right)$, given any other two players' strategies, the strategy prescribed for the left player is a CPNE of the one-player game played by itself.

ii. Let us consider the three subcoalitions formed by two players. According to step 1.2.a, 1.2.b and 1.2.d, it is easy to show that fixing $C_j^F = \left(C_{ji}^F, 0\right)$, given any player's strategy, the strategies prescribed for the left two players are a CPNE of the two-player game played by themselves.

iii. So, fixing $C_j^F = \left(C_{ji}^F, 0\right)$, the strategies prescribed for the left three players are self-enforcing. Are there any other self-enforcing strategy profiles? Notice that if a strategy profile is self-enforcing, it must be the case that $C_i^F = \left(0, \frac{5}{72}\Delta_i\right)$ and $C_j^T = \left(0, \frac{1}{12}\Delta_j\right)$. Otherwise, this strategy profile will not induce a CPNE either in the game played by trade union i and firm i, (see step 1.2.a), or the game played by trade union i and trade union j, (see step 1.2.b), or both. Since $C_j^F = \left(C_{ji}^F, 0\right)$ is fixed, given $C_i^F = \left(0, \frac{5}{72}\Delta_i\right)$ and $C_j^T = \left(0, \frac{1}{12}\Delta_j\right)$, it must be the case that trade union i chooses $C_i^T = \left(C_{ii}^T, 0\right)$, such that C_{ii}^T satisfies condition (2.19). So, the self-enforcing strategy profile in this case is unique, and hence a CPNE.

b. The subcoalitions formed by trade union i, firm i and firm j. Consider the game played by these three players when $C_j^T = \left(0, \frac{1}{12}\Delta_j\right)$. There are six nonempty proper subcoalitions: three formed by one player and three formed by two players.

i. Let us consider the three subcoalitions formed by one player. According to step 1.1, it is easy to show that fixing $C_j^T = \left(0, \frac{1}{12}\Delta_j\right)$, given any other two players' strategies, the strategy prescribed for the left player is a CPNE of the one-player game played by itself.

ii. Let us consider the three subcoalitions formed by two players. According to step 1.2.a, 1.2.c and 1.2.e, it is easy to show that fixing $C_j^T = \left(0, \frac{1}{12}\Delta_j\right)$, given any player's strategy, the strategies prescribed for the left two players are a CPNE of the two-player game played by themselves.

iii. So, fixing $C_j^T = \left(0, \frac{1}{12}\Delta_j\right)$, the strategies prescribed for the left three players are self-enforcing. Are there any other self-enforcing strategy profiles? Notice that if a strategy profile is self-enforcing, it must be the case that $C_i^F = \left(\frac{5}{72}\Delta_i, 0\right)$. Otherwise, this strategy profile will not induce a CPNE either in the game played by trade union i and firm i, (see step 1.2.a), or the game played by firm i and firm j, (see step 1.2.e), or both. Since $C_j^T = \left(0, \frac{1}{12}\Delta_j\right)$ is fixed, it must be the case that any strategy profile consisting of $C_i^{T\prime} = \left(C_{ii}^{T\prime}, 0\right)$ and $C_j^{F\prime} = \left(C_{ji}^{F\prime}, 0\right)$, such that $C_{ii}^{T\prime}$ and $C_{ji}^{F\prime}$

satisfy condition (2.19), is self-enforcing. But the proposed strategy profile is not strongly Pareto dominated by any other self-enforcing strategy profiles. Hence, the proposed strategy profile is a CPNE in this case.

c. The subcoalitions formed by trade union i, trade union j and firm j. Using the similar arguments to those in step 1.3.b, it proves that the proposed strategies are a CPNE of the game played by themselves.

d. The subcoalitions formed by firm i, trade union j and firm j. Using the similar arguments to those in step 1.3.a, it proves that the proposed strategies are a CPNE of the game played by themselves.

So far, we have established that any strategy profiles prescribed in Proposition 2.2 are self-enforcing.

Step 2. Are there any other self-enforcing strategy profiles? Since given a self-enforcing strategy profile, it must be the case that $C_{ij}^F = \frac{5}{72}\Delta_i$, $C_{jj}^T = \frac{1}{12}\Delta_j$, and C_{ii}^T and C_{ji}^F satisfy condition (2.19), there is no other self-enforcing strategy profile.

Step 3. Finally, it is easy to show that given any strategy profile, trade union i receives $\frac{1}{3}\Delta_i - C_{ii}^T$, firm i receives $\frac{1}{18}\Delta_i$, trade union j receives $\frac{1}{4}\Delta_j$, and firm j receives $\frac{1}{8}\Delta_j - C_{ji}^F$. Notice that C_{ii}^T and C_{ji}^F cannot be lowered simultaneously. Otherwise, condition (2.19) does not hold. This means that any self-enforcing strategy profile is not strongly Pareto dominated by any other self-enforcing strategy profiles.

We conclude that any proposed strategy profile is a CPNE in the first stage of the game. ■

5.2 Appendix to Chapter 3

5.2.1 Proof of Lemma 3.1

Denote $\sigma_i = (m_i, n_i)$, $i = 1, 2$. We want to find a pair $\left(\sigma_i^*, \sigma_j^*\right)$ satisfying

$$\pi_i^C\left(\sigma_i^*, \sigma_j^*\right) \geq \pi_i^C\left(\sigma_i', \sigma_j^*\right), \ i = 1, 2, \ j = 1, 2, \ i \neq j, \ \forall \sigma_i' \in R_+^2.$$

Given the rival's choice $\sigma_j = (m_j, n_j)$, the optimal response of firm i solves the problem

$$\max_{\{m_i, n_i\}} \pi_i^C, \ s.t. \ m_i \geq 0, \ n_i \geq 0.$$

The Lagrangian is

$$L_i\left(m_i, n_i, \omega_{m_i}, \omega_{n_i}; m_j, n_j\right) = \pi_i^C + \omega_{m_i} m_i + \omega_{n_i} n_i.$$

According to Kuhn-Tucker's Method, the solution is characterized by the following conditions:

$$2bx_i^{*C}\left(\frac{\partial x_i^{*C}}{\partial m_i}\right) = k\left(m_i + n_i\right) - \omega_{m_i},$$
$$2bx_i^{*C}\left(\frac{\partial x_i^{*C}}{\partial n_i}\right) = k\left(m_i + n_i\right) - \omega_{n_i},$$
$$m_i \geq 0,\ n_i \geq 0,$$
$$\omega_{m_i} \geq 0,\ \omega_{n_i} \geq 0,$$
$$\omega_{m_i} m_i = 0,\ \omega_{n_i} n_i = 0.$$

Note first that $m_i = 0$, $n_i = 0$ cannot be a solution because at that point the marginal investment cost is zero while the marginal investment revenue is strictly positive. Second, if $\mu \neq \frac{2b+1}{2b-1}$, we cannot have both $m_i > 0$ and $n_i > 0$ because marginal revenues from the two kinds of advertising would differ while marginal investment costs are the same.

In particular, when $\mu > \frac{2b+1}{2b-1}$, the marginal investment revenue from cooperative advertising is the larger one, so firm i invests only in cooperative advertising. In fact, by choosing an appropriate ω_{n_i}, all of the above conditions can be satisfied. When $\mu < \frac{2b+1}{2b-1}$, the marginal investment revenue from predatory advertising is the larger one, so firm i invests only in predatory advertising. In fact, by choosing an appropriate ω_{m_i}, all of the above conditions can be satisfied.

The above arguments are valid for both firms. Therefore, the profit function of firm i in the reduced extensive form game is given by

$$\pi_i^C\left(m_i, n_i, m_j, n_j\right) = \begin{cases} \pi_i^C\left(m_i, m_j\right) & if \quad \mu > \frac{2b+1}{2b-1}, \\ \pi_i^C\left(n_i, n_j\right) & if \quad \mu < \frac{2b+1}{2b-1}. \end{cases}$$

When both firms invest in cooperative advertising, since

$$\frac{\partial^2 \pi_i^C}{\partial m_i \partial m_j} = 2b\left(\frac{\partial x_i^{*C}}{\partial m_i}\right)^2 > 0,$$

cooperative advertising is a strategic complement. We also have

$$\frac{\partial \pi_i^C}{\partial m_j} = 2b\left(\frac{\partial x_i^{*C}}{\partial m_i}\right) > 0.$$

When both firms invest in predatory advertising, since

$$\frac{\partial^2 \pi_i^C}{\partial n_i \partial n_j} = -2b\left(\frac{\partial x_i^{*C}}{\partial n_i}\right)^2 < 0,$$

predatory advertising is a strategic substitute. We also have

$$\frac{\partial \pi_i^C}{\partial n_j} = -2b\left(\frac{\partial x_i^{*C}}{\partial n_i}\right) < 0.$$

Finally, when $\mu > \frac{2b+1}{2b-1}$, the two firms' first-order conditions with respect to cooperative advertising simultaneously determine the equilibrium values of m_i and m_j, and $\left(\sigma_i^*, \sigma_j^*\right) = \left[\left(m_i^{*C}, 0\right), \left(m_j^{*C}, 0\right)\right]$. It is easy to show $m_i^{*C} = m_j^{*C}$. When $\mu < \frac{2b+1}{2b-1}$, the two firms' first-order conditions with respect to predatory advertising simultaneously determine the equilibrium values of n_i and n_j, and $\left(\sigma_i^*, \sigma_j^*\right) = \left[\left(0, n_i^{*C}\right), \left(0, n_j^{*C}\right)\right]$. It is easy to show $n_i^{*C} = n_j^{*C}$. ■

5.2.2 Proof of Lemma 3.2

Denote $\sigma_i = (m_i, n_i)$, $i = 1, 2$. We want to find a pair $\left(\sigma_i^*, \sigma_j^*\right)$ satisfying

$$\pi_i^B\left(\sigma_i^*, \sigma_j^*\right) \geq \pi_i^B\left(\sigma_i', \sigma_j^*\right), \ i = 1, 2, \ j = 1, 2, \ i \neq j, \ \forall \sigma_i' \in R_+^2.$$

Given the rival's choice $\sigma_j = (m_j, n_j)$, the optimal response of firm i solves the problem

$$\max_{\{m_i, n_i\}} \pi_i^B, \ s.t. \ m_i \geq 0, \ n_i \geq 0.$$

The Lagrangian is

$$L_i\left(m_i, n_i, \omega_{m_i}, \omega_{n_i}; m_j, n_j\right) = \pi_i^B + \omega_{m_i} m_i + \omega_{n_i} n_i.$$

According to Kuhn-Tucker's Method, the solution is characterized by the following conditions:

$$\begin{aligned}
2\beta(p_i^{*B} - c)\left(\frac{\partial p_i^{*B}}{\partial m_i}\right) &= k(m_i + n_i) - \omega_{m_i}, \\
2\beta(p_i^{*B} - c)\left(\frac{\partial p_i^{*B}}{\partial n_i}\right) &= k(m_i + n_i) - \omega_{n_i}, \\
m_i \geq 0, \ n_i &\geq 0, \\
\omega_{m_i} \geq 0, \ \omega_{n_i} &\geq 0, \\
\omega_{m_i} m_i = 0, \ \omega_{n_i} n_i &= 0.
\end{aligned}$$

Note first that $m_i = 0$, $n_i = 0$ cannot be a solution because at that point the marginal investment cost is zero while the marginal investment revenue is strictly positive. Second, if $\mu \neq \frac{2b^2+b-1}{2b^2-b-1}$, we cannot have both $m_i > 0$ and $n_i > 0$ because marginal revenues from the two kinds of advertising would differ while marginal investment costs are the same.

In particular, when $\mu > \frac{2b^2+b-1}{2b^2-b-1}$, the marginal investment revenue from cooperative advertising is the larger one, so firm i invests only in cooperative advertising. In fact, by choosing an appropriate ω_{n_i}, all of the above conditions can be satisfied. When $\mu < \frac{2b^2+b-1}{2b^2-b-1}$, the marginal investment revenue from predatory advertising is the larger one, so firm i invests only in predatory advertising. In fact, by choosing an appropriate ω_{m_i}, all of the above conditions can be satisfied.

The above arguments are valid for both firms. Therefore, the profit function of firm i in the reduced extensive form game is given by

$$\pi_i^B(m_i, n_i, m_j, n_j) = \begin{cases} \pi_i^B(m_i, m_j) & if \quad \mu > \frac{2b^2+b-1}{2b^2-b-1}, \\ \pi_i^B(n_i, n_j) & if \quad \mu < \frac{2b^2+b-1}{2b^2-b-1}. \end{cases}$$

When both firms invest in cooperative advertising, since

$$\frac{\partial^2 \pi_i^B}{\partial m_i \partial m_j} = 2\beta \left(\frac{\partial p_i^{*B}}{\partial m_i}\right)^2 > 0,$$

cooperative advertising is a strategic complement. We also have

$$\frac{\partial \pi_i^B}{\partial m_j} = 2\beta \left(\frac{\partial p_i^{*B}}{\partial m_i}\right) > 0.$$

When both firms invest in predatory advertising, since

$$\frac{\partial^2 \pi_i^B}{\partial n_i \partial n_j} = -2\beta \left(\frac{\partial p_i^{*B}}{\partial n_i}\right)^2 < 0,$$

predatory advertising is a strategic substitute. We also have

$$\frac{\partial \pi_i^B}{\partial n_j} = -2\beta \left(\frac{\partial p_i^{*B}}{\partial n_i}\right) < 0.$$

Finally, when $\mu > \frac{2b^2+b-1}{2b^2-b-1}$, the two firms' first-order conditions with respect to cooperative advertising simultaneously determine the equilibrium values of m_i and m_j, and $\left(\sigma_i^*, \sigma_j^*\right) = \left[\left(m_i^{*B}, 0\right), \left(m_j^{*B}, 0\right)\right]$. It is easy to show $m_i^{*B} = m_j^{*B}$. When $\mu < \frac{2b^2+b-1}{2b^2-b-1}$, the two firms' first-order conditions with respect to predatory advertising simultaneously determine the equilibrium values of n_i and n_j, and $\left(\sigma_i^*, \sigma_j^*\right) = \left[\left(0, n_i^{*B}\right), \left(0, n_j^{*B}\right)\right]$. It is easy to show $n_i^{*B} = n_j^{*B}$. ■

5.2.3 Proof of Proposition 3.1

Note that

$$\frac{2b+1}{2b-1}-\frac{2b^2+b-1}{2b^2-b-1}=-\frac{2}{(b-1)(4b^2-1)}<0.$$

Then, given Lemma 3.1 and Lemma 3.2, it is straightforward. ∎

5.2.4 Proof of Proposition 3.2

Part 1

Given trade policy s and industrial policy τ, and firms' advertising investments (m_i, n_i) and (m_j, n_j), in product market firm i maximizes

$$\Pi_i^C = (a_i - bx_i - x_j - c + s)\, x_i,$$

firm j maximizes

$$\Pi_j^C = (a_j - x_i - bx_j - c)\, x_j.$$

The Nash equilibrium is given by

$$x_i = \frac{a\left[1+\mu(m_i+m_j)+\left(\frac{2b+1}{2b-1}\right)(n_i-n_j)\right]-c}{2b+1}+\frac{2bs}{4b^2-1},$$

$$x_j = \frac{a\left[1+\mu(m_i+m_j)+\left(\frac{2b+1}{2b-1}\right)(n_j-n_i)\right]-c}{2b+1}-\frac{s}{4b^2-1}.$$

Obviously, the comparative statics results on advertising investments are unchanged. However, trade policy has a direct impact on the equilibrium outcome:

$$\frac{\partial x_i}{\partial s}=\frac{2b}{4b^2-1}>0,\ \frac{\partial x_j}{\partial s}=-\frac{1}{4b^2-1}<0.$$

Firm i's and j's equilibrium profits are

$$\Pi_i^C = b\,(x_i)^2,\ \Pi_j^C = b\,(x_j)^2,$$

respectively.

We use those to replace the product market competition stage and get the reduced extensive form game. Note that firms' decisions in the investment stage on whether to invest in cooperative or predatory advertising are not changed by the policy instruments, since, as we have just seen, that decision depends only on the relative effectiveness of cooperative advertising and the degree of product differentiation.

Case 1 $\mu > \frac{2b+1}{2b-1}$

If $\mu > \frac{2b+1}{2b-1}$ and both firms invest in cooperative advertising in an equilibrium, the equilibrium values are characterized by the two conditions:

$$2bx_i \left(\frac{\partial x_i}{\partial m_i}\right) - km_i + \tau = 0,$$
$$2bx_j \left(\frac{\partial x_j}{\partial m_j}\right) - km_j = 0.$$

Taking derivatives with respect to s and rearranging the equations, we have

$$\begin{bmatrix} \frac{\partial^2 \pi_i^C}{\partial m_i^2} & \frac{\partial^2 \pi_i^C}{\partial m_i \partial m_j} \\ \frac{\partial^2 \pi_j^C}{\partial m_j \partial m_i} & \frac{\partial^2 \pi_j^C}{\partial m_j^2} \end{bmatrix} \begin{bmatrix} \frac{\partial m_i}{\partial s} \\ \frac{\partial m_j}{\partial s} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 \pi_i^C}{\partial m_i \partial s} \\ -\frac{\partial^2 \pi_j^C}{\partial m_j \partial s} \end{bmatrix}.$$

Using Cramer's Rule,

$$\frac{\partial m_i}{\partial s} = \left[\frac{k_1}{a\mu(2b-1)}\right]\left[\frac{2b(k-k_1)-k_1}{\Delta}\right],$$
$$\frac{\partial m_j}{\partial s} = \left[\frac{k_1}{a\mu(2b-1)}\right]\left[\frac{(2b+1)k_1-k}{\Delta}\right],$$

where $\Delta = k^2 - 2kk_1$.

Taking derivatives with respect to τ and rearranging the equations, we have

$$\begin{bmatrix} \frac{\partial^2 \pi_i^C}{\partial m_i^2} & \frac{\partial^2 \pi_i^C}{\partial m_i \partial m_j} \\ \frac{\partial^2 \pi_j^C}{\partial m_j \partial m_i} & \frac{\partial^2 \pi_j^C}{\partial m_j^2} \end{bmatrix} \begin{bmatrix} \frac{\partial m_i}{\partial \tau} \\ \frac{\partial m_j}{\partial \tau} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 \pi_i^C}{\partial m_i \partial \tau} \\ 0 \end{bmatrix}.$$

Using Cramer's Rule,

$$\frac{\partial m_i}{\partial \tau} = \frac{k-k_1}{\Delta}, \quad \frac{\partial m_j}{\partial \tau} = \frac{k_1}{\Delta},$$

where $\Delta = k^2 - 2kk_1$.

Given the equilibrium of product market competition and cooperative advertising is the equilibrium type of investment, in the first stage of the game the government of country i chooses trade policy s and industrial policy τ to maximize national welfare

$$W(s,\tau) = \pi_i^C - sx_i - \tau m_i.$$

The first-order conditions are as follows.

$$\frac{\partial W}{\partial s}=\left\{\left[\left(\frac{\partial a_i}{\partial m_j}-\frac{\partial x_j}{\partial m_j}\right)\frac{\partial m_j}{\partial s}\right]-\frac{\partial x_j}{\partial s}\right\}x_i-\frac{\partial x_i}{\partial s}(\cdot)\,s-\frac{\partial m_i}{\partial s}\tau=0,$$

$$\frac{\partial W}{\partial \tau}=\left(\frac{\partial a_i}{\partial m_j}-\frac{\partial x_j}{\partial m_j}\right)\frac{\partial m_j}{\partial \tau}x_i-\frac{\partial x_i}{\partial \tau}(\cdot)\,s-\frac{\partial m_i}{\partial \tau}\tau=0.$$

Using matrix notation, we have

$$\begin{bmatrix}\frac{\partial x_i}{\partial s}(\cdot) & \frac{\partial m_i}{\partial s}\\ \frac{\partial x_i}{\partial \tau}(\cdot) & \frac{\partial m_i}{\partial \tau}\end{bmatrix}\begin{bmatrix}s\\ \tau\end{bmatrix}=\begin{bmatrix}\left\{\left[\left(\frac{\partial a_i}{\partial m_j}-\frac{\partial x_j}{\partial m_j}\right)\frac{\partial m_j}{\partial s}\right]-\frac{\partial x_j}{\partial s}\right\}x_i\\ \left(\frac{\partial a_i}{\partial m_j}-\frac{\partial x_j}{\partial m_j}\right)\frac{\partial m_j}{\partial \tau}x_i\end{bmatrix},$$

where $\frac{\partial x_i}{\partial s}(\cdot)=\frac{\partial x_i}{\partial m_i}\frac{\partial m_i}{\partial s}+\frac{\partial x_i}{\partial m_j}\frac{\partial m_j}{\partial s}+\frac{\partial x_i}{\partial s}$, $\frac{\partial x_i}{\partial \tau}(\cdot)=\frac{\partial x_i}{\partial m_i}\frac{\partial m_i}{\partial \tau}+\frac{\partial x_i}{\partial m_j}\frac{\partial m_j}{\partial \tau}$.

We use Cramer's Rule to solve this linear equation system.

Denote the determinant of the coefficient matrix of the above linear equation system by D.

$$\begin{aligned}D &= \frac{\partial x_i}{\partial s}(\cdot)\frac{\partial m_i}{\partial \tau}-\frac{\partial x_i}{\partial \tau}(\cdot)\frac{\partial m_i}{\partial s}\\ &= \frac{\partial x_i}{\partial m_j}\left(\frac{\partial m_j}{\partial s}\frac{\partial m_i}{\partial \tau}-\frac{\partial m_i}{\partial s}\frac{\partial m_j}{\partial \tau}\right)+\frac{\partial x_i}{\partial s}\frac{\partial m_i}{\partial \tau}.\end{aligned}$$

Note that

$$\begin{aligned}&\frac{\partial m_j}{\partial s}\frac{\partial m_i}{\partial \tau}-\frac{\partial m_i}{\partial s}\frac{\partial m_j}{\partial \tau}\\ =&\left[\frac{k_1}{a\mu(2b-1)}\right]\left\{\left[\frac{(2b+1)k_1-k}{\Delta}\right]\left(\frac{k-k_1}{\Delta}\right)-\left[\frac{2b(k-k_1)-k_1}{\Delta}\right]\left(\frac{k_1}{\Delta}\right)\right\}\\ =&-\left[\frac{k_1}{a\mu(2b-1)\Delta}\right]\\ <&\ 0.\end{aligned}$$

Then

$$\begin{aligned}D &= -\left(\frac{a\mu}{2b+1}\right)\left[\frac{k_1}{a\mu(2b-1)\Delta}\right]+\left(\frac{2b}{4b^2-1}\right)\left(\frac{k-k_1}{\Delta}\right)\\ &= \left(\frac{1}{4b^2-1}\right)\left[\frac{2b(k-k_1)-k_1}{\Delta}\right]\\ &> 0.\end{aligned}$$

Next, we have

$$
\begin{aligned}
D_1 &= \left\{\left[\left(\frac{\partial a_i}{\partial m_j}-\frac{\partial x_j}{\partial m_j}\right)\frac{\partial m_j}{\partial s}-\frac{\partial x_j}{\partial s}\right]\frac{\partial m_i}{\partial \tau}-\left(\frac{\partial a_i}{\partial m_j}-\frac{\partial x_j}{\partial m_j}\right)\frac{\partial m_j}{\partial \tau}\frac{\partial m_i}{\partial s}\right\}x_i \\
&= \left[\left(\frac{\partial a_i}{\partial m_j}-\frac{\partial x_j}{\partial m_j}\right)\left(\frac{\partial m_j}{\partial s}\frac{\partial m_i}{\partial \tau}-\frac{\partial m_i}{\partial s}\frac{\partial m_j}{\partial \tau}\right)-\frac{\partial x_j}{\partial s}\frac{\partial m_i}{\partial \tau}\right]x_i \\
&= \left\{-\left(\frac{2ab\mu}{2b+1}\right)\left[\frac{k_1}{a\mu\,(2b-1)\,\Delta}\right]+\left(\frac{1}{4b^2-1}\right)\left(\frac{k-k_1}{\Delta}\right)\right\}x_i \\
&= \left\{\left(\frac{1}{4b^2-1}\right)\left[\frac{k-(2b+1)\,k_1}{\Delta}\right]\right\}x_i,
\end{aligned}
$$

$$
\begin{aligned}
D_2 &= \left\{\frac{\partial x_i}{\partial s}(\cdot)\left(\frac{\partial a_i}{\partial m_j}-\frac{\partial x_j}{\partial m_j}\right)\frac{\partial m_j}{\partial \tau}-\frac{\partial x_i}{\partial \tau}(\cdot)\left[\left(\frac{\partial a_i}{\partial m_j}-\frac{\partial x_j}{\partial m_j}\right)\frac{\partial m_j}{\partial s}-\frac{\partial x_j}{\partial s}\right]\right\}x_i \\
&= \left\{\begin{array}{c}\frac{\partial x_i}{\partial m_i}\left(\frac{\partial a_i}{\partial m_j}-\frac{\partial x_j}{\partial m_j}\right)\left(\frac{\partial m_i}{\partial s}\frac{\partial m_j}{\partial \tau}-\frac{\partial m_j}{\partial s}\frac{\partial m_i}{\partial \tau}\right)+\left(\frac{\partial a_i}{\partial m_j}-\frac{\partial x_j}{\partial m_j}\right)\frac{\partial x_i}{\partial s}\frac{\partial m_j}{\partial \tau} \\ +\left(\frac{\partial x_i}{\partial m_i}\frac{\partial m_i}{\partial \tau}+\frac{\partial x_i}{\partial m_j}\frac{\partial m_j}{\partial \tau}\right)\frac{\partial x_j}{\partial s}\end{array}\right\}x_i \\
&= \left\{\begin{array}{c}\left(\frac{a\mu}{2b+1}\right)\left(\frac{2ab\mu}{2b+1}\right)\left[\frac{k_1}{a\mu(2b-1)\Delta}\right]+\left(\frac{2b}{4b^2-1}\right)\left(\frac{2ab\mu}{2b+1}\right)\left(\frac{k_1}{\Delta}\right) \\ -\left(\frac{a\mu}{2b+1}\right)\left(\frac{1}{4b^2-1}\right)\left(\frac{k}{\Delta}\right)\end{array}\right\}x_i \\
&= \left\{\left[\frac{a\mu}{(2b+1)\,(4b^2-1)}\right]\left[\frac{2b\,(2b+1)\,k_1-k}{\Delta}\right]\right\}x_i.
\end{aligned}
$$

Because

$$s=\frac{D_1}{D},\ \tau=\frac{D_2}{D},$$

we have

$$sign\ s = sign\ \left[k-(2b+1)\,k_1\right],\ sign\ \tau = sign\ \left[2b\,(2b+1)\,k_1-k\right]. \tag{5.1}$$

Case 2 $\mu < \frac{2b+1}{2b-1}$

If $\mu < \frac{2b+1}{2b-1}$ and both firms invest in predatory advertising in an equilibrium, the equilibrium values are characterized by the two conditions:

$$
\begin{aligned}
2bx_i\left(\frac{\partial x_i}{\partial n_i}\right)-kn_i+\tau=0, \\
2bx_j\left(\frac{\partial x_j}{\partial n_j}\right)-kn_j=0.
\end{aligned}
$$

Taking derivatives with respect to s and rearranging the equations, we have

$$
\begin{bmatrix}\frac{\partial^2\pi_i^C}{\partial n_i^2} & \frac{\partial^2\pi_i^C}{\partial n_i\partial n_j} \\ \frac{\partial^2\pi_j^C}{\partial n_j\partial n_i} & \frac{\partial^2\pi_j^C}{\partial n_j^2}\end{bmatrix}\begin{bmatrix}\frac{\partial n_i}{\partial s} \\ \frac{\partial n_j}{\partial s}\end{bmatrix}=\begin{bmatrix}-\frac{\partial^2\pi_i^C}{\partial n_i\partial s} \\ -\frac{\partial^2\pi_j^C}{\partial n_j\partial s}\end{bmatrix}.
$$

Using Cramer's Rule,

$$\frac{\partial n_i}{\partial s} = \left[\frac{k_2}{a(2b+1)}\right]\left[\frac{2b(k-k_2)+k_2}{\Delta}\right],$$
$$\frac{\partial n_j}{\partial s} = \left[\frac{k_2}{a(2b-1)}\right]\left[\frac{-(2b-1)k_2-k}{\Delta}\right],$$

where $\Delta = k^2 - 2kk_2$.

Taking derivatives with respect to τ and rearranging the equations, we have

$$\begin{bmatrix} \frac{\partial^2 \pi_i^C}{\partial n_i^2} & \frac{\partial^2 \pi_i^C}{\partial n_i \partial n_j} \\ \frac{\partial^2 \pi_j^C}{\partial n_j \partial n_i} & \frac{\partial^2 \pi_j^C}{\partial n_j^2} \end{bmatrix} \begin{bmatrix} \frac{\partial n_i}{\partial \tau} \\ \frac{\partial n_j}{\partial \tau} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 \pi_i^C}{\partial n_i \partial \tau} \\ 0 \end{bmatrix}.$$

Using Cramer's Rule,

$$\frac{\partial n_i}{\partial \tau} = \frac{k-k_2}{\Delta},\ \frac{\partial n_j}{\partial \tau} = -\frac{k_2}{\Delta},$$

where $\Delta = k^2 - 2kk_2$.

Given the equilibrium of product market competition and predatory advertising is the equilibrium type of investment, in the first stage of the game the government of country i chooses trade policy s and industrial policy τ to maximize national welfare

$$W(s,\tau) = \pi_i^C - sx_i - \tau n_i.$$

The first-order conditions are as follows.

$$\frac{\partial W}{\partial s} = \left\{\left[\left(\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j}\right)\frac{\partial n_j}{\partial s}\right] - \frac{\partial x_j}{\partial s}\right\} x_i - \frac{\partial x_i}{\partial s}(\cdot)\, s - \frac{\partial n_i}{\partial s}\tau = 0,$$
$$\frac{\partial W}{\partial \tau} = \left(\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j}\right)\frac{\partial n_j}{\partial \tau} x_i - \frac{\partial x_i}{\partial \tau}(\cdot)\, s - \frac{\partial n_i}{\partial \tau}\tau = 0.$$

Using matrix notation, we have

$$\begin{bmatrix} \frac{\partial x_i}{\partial s}(\cdot) & \frac{\partial n_i}{\partial s} \\ \frac{\partial x_i}{\partial \tau}(\cdot) & \frac{\partial n_i}{\partial \tau} \end{bmatrix} \begin{bmatrix} s \\ \tau \end{bmatrix} = \begin{bmatrix} \left\{\left[\left(\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j}\right)\frac{\partial n_j}{\partial s}\right] - \frac{\partial x_j}{\partial s}\right\} x_i \\ \left(\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j}\right)\frac{\partial n_j}{\partial \tau} x_i \end{bmatrix},$$

where $\frac{\partial x_i}{\partial s}(\cdot) = \frac{\partial x_i}{\partial n_i}\frac{\partial n_i}{\partial s} + \frac{\partial x_i}{\partial n_j}\frac{\partial n_j}{\partial s} + \frac{\partial x_i}{\partial s}$, $\frac{\partial x_i}{\partial \tau}(\cdot) = \frac{\partial x_i}{\partial n_i}\frac{\partial n_i}{\partial \tau} + \frac{\partial x_i}{\partial n_j}\frac{\partial n_j}{\partial \tau}$.

We use Cramer's Rule to solve this linear equation system.

Denote the determinant of the coefficient matrix of the above linear equation system by D.

$$\begin{aligned} D &= \frac{\partial x_i}{\partial s}(\cdot)\frac{\partial n_i}{\partial \tau} - \frac{\partial x_i}{\partial \tau}(\cdot)\frac{\partial n_i}{\partial s} \\ &= \frac{\partial x_i}{\partial n_j}\left(\frac{\partial n_j}{\partial s}\frac{\partial n_i}{\partial \tau} - \frac{\partial n_i}{\partial s}\frac{\partial n_j}{\partial \tau}\right) + \frac{\partial x_i}{\partial s}\frac{\partial n_i}{\partial \tau}. \end{aligned}$$

Note that

$$\begin{aligned} & \frac{\partial n_j}{\partial s}\frac{\partial n_i}{\partial \tau} - \frac{\partial n_i}{\partial s}\frac{\partial n_j}{\partial \tau} \\ = & \left[\frac{k_2}{a(2b+1)}\right]\left\{\left[\frac{-(2b-1)k_2 - k}{\Delta}\right]\left(\frac{k-k_2}{\Delta}\right) - \left[\frac{2b(k-k_2)+k_2}{\Delta}\right]\left(-\frac{k_2}{\Delta}\right)\right\} \\ = & -\left[\frac{k_2}{a(2b+1)\Delta}\right] \\ < & \ 0. \end{aligned}$$

Then

$$\begin{aligned} D &= \left(\frac{a}{2b-1}\right)\left[\frac{k_2}{a(2b+1)\Delta}\right] + \left(\frac{2b}{4b^2-1}\right)\left(\frac{k-k_2}{\Delta}\right) \\ &= \left(\frac{1}{4b^2-1}\right)\left[\frac{2b(k-k_2)+k_2}{\Delta}\right] \\ &> 0. \end{aligned}$$

Next, we have

$$\begin{aligned} D_1 &= \left\{\left[\left(\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j}\right)\frac{\partial n_j}{\partial s} - \frac{\partial x_j}{\partial s}\right]\frac{\partial n_i}{\partial \tau} - \left(\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j}\right)\frac{\partial n_j}{\partial \tau}\frac{\partial n_i}{\partial s}\right\} x_i \\ &= \left[\left(\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j}\right)\left(\frac{\partial n_j}{\partial s}\frac{\partial n_i}{\partial \tau} - \frac{\partial n_i}{\partial s}\frac{\partial n_j}{\partial \tau}\right) - \frac{\partial x_j}{\partial s}\frac{\partial n_i}{\partial \tau}\right] x_i \\ &> 0. \end{aligned}$$

This is because

$$\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j} < 0,\ \frac{\partial n_j}{\partial s}\frac{\partial n_i}{\partial \tau} - \frac{\partial n_i}{\partial s}\frac{\partial n_j}{\partial \tau} < 0,\ \frac{\partial x_j}{\partial s} < 0,\ \frac{\partial n_i}{\partial \tau} > 0.$$

$$
\begin{aligned}
D_2 &= \left\{ \frac{\partial x_i}{\partial s} (\cdot) \left(\frac{\partial a_i}{\partial m_j} - \frac{\partial x_j}{\partial m_j} \right) \frac{\partial m_j}{\partial \tau} - \frac{\partial x_i}{\partial \tau} (\cdot) \left[\left(\frac{\partial a_i}{\partial m_j} - \frac{\partial x_j}{\partial m_j} \right) \frac{\partial m_j}{\partial s} - \frac{\partial x_j}{\partial s} \right] \right\} x_i \\
&= \left\{ \begin{array}{c} \frac{\partial x_i}{\partial n_i} \left(\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j} \right) \left(\frac{\partial n_i}{\partial s} \frac{\partial n_j}{\partial \tau} - \frac{\partial n_j}{\partial s} \frac{\partial n_i}{\partial \tau} \right) + \left(\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j} \right) \frac{\partial x_i}{\partial s} \frac{\partial n_j}{\partial \tau} \\ + \left(\frac{\partial x_i}{\partial n_i} \frac{\partial n_i}{\partial \tau} + \frac{\partial x_i}{\partial n_j} \frac{\partial n_j}{\partial \tau} \right) \frac{\partial x_j}{\partial s} \end{array} \right\} x_i \\
&= \left\{ \begin{array}{c} \left(\frac{a}{2b-1} \right) \left(-\frac{2ab}{2b-1} \right) \left[\frac{k_2}{a(2b+1)\Delta} \right] + \left(\frac{2b}{4b^2-1} \right) \left(-\frac{2ab}{2b-1} \right) \left(-\frac{k_2}{\Delta} \right) \\ - \left(\frac{a}{2b-1} \right) \left(\frac{1}{4b^2-1} \right) \left(\frac{k}{\Delta} \right) \end{array} \right\} x_i \\
&= \left\{ \left[\frac{a}{(2b-1)(4b^2-1)} \right] \left[\frac{2b(2b-1)k_2 - k}{\Delta} \right] \right\} x_i.
\end{aligned}
$$

Because

$$
s = \frac{D_1}{D}, \ \tau = \frac{D_2}{D},
$$

we have

$$
s > 0, \ sign \ \tau = sign \ [2b(2b-1)k_2 - k]. \tag{5.2}
$$

Part 2

Given trade policy s and industrial policy τ, and firms' advertising investments (m_i, n_i) and (m_j, n_j), in product market firm i maximizes

$$
\Pi_i^B = (p_i - c + s)(\alpha_i - \beta p_i + \gamma p_j),
$$

firm j maximizes

$$
\Pi_j^B = (p_j - c)(\alpha_j - \beta p_j + \gamma p_i).
$$

The Nash equilibrium is given by

$$
\begin{aligned}
p_i &= \frac{\alpha \left[1 + \mu(m_i + m_j) + \left(\frac{b+1}{b-1} \right) \left(\frac{2\beta-\gamma}{2\beta+\gamma} \right) (n_i - n_j) \right] + \beta c}{2\beta - \gamma} - \frac{2\beta^2 s}{4\beta^2 - \gamma^2}, \\
p_j &= \frac{\alpha \left[1 + \mu(m_i + m_j) + \left(\frac{b+1}{b-1} \right) \left(\frac{2\beta-\gamma}{2\beta+\gamma} \right) (n_j - n_i) \right] + \beta c}{2\beta - \gamma} - \frac{\beta\gamma s}{4\beta^2 - \gamma^2}.
\end{aligned}
$$

Obviously, the comparative statics results on advertising investments are unchanged. However, trade policy has a direct impact on the equilibrium outcome:

$$
\begin{aligned}
\frac{\partial p_i}{\partial s} &= -\frac{2\beta^2}{4\beta^2 - \gamma^2} = -\frac{2b^2}{4b^2 - 1} < 0, \\
\frac{\partial p_j}{\partial s} &= -\frac{\beta\gamma}{4\beta^2 - \gamma^2} = -\frac{b}{4b^2 - 1} < 0.
\end{aligned}
$$

Firm i's and j's equilibrium profits are

$$\Pi_i^B = \beta\left(p_i - c + s\right)^2, \; \Pi_j^B = \beta\left(p_j - c\right)^2,$$

respectively.

We use those to replace the product market competition stage and get the reduced extensive form game. Note that firms' decisions in the investment stage on whether to invest in cooperative or predatory advertising are not changed by the policy instruments, since, as we have just seen, that decision depends only on the relative effectiveness of cooperative advertising and the degree of product differentiation.

Case 1 $\mu > \frac{2b^2+b-1}{2b^2-b-1}$

If $\mu > \frac{2b^2+b-1}{2b^2-b-1}$ and both firms invest in cooperative advertising in an equilibrium, the equilibrium values are characterized by the two conditions:

$$\begin{aligned} 2\beta\left(p_i - c + s\right)\left(\frac{\partial p_i}{\partial m_i}\right) - km_i + \tau &= 0, \\ 2\beta\left(p_j - c\right)\left(\frac{\partial p_j}{\partial m_j}\right) - km_j &= 0. \end{aligned}$$

Taking derivatives with respect to s and rearranging the equations, we have

$$\begin{bmatrix} \frac{\partial^2 \pi_i^B}{\partial m_i^2} & \frac{\partial^2 \pi_i^B}{\partial m_i \partial m_j} \\ \frac{\partial^2 \pi_j^B}{\partial m_j \partial m_i} & \frac{\partial^2 \pi_j^B}{\partial m_j^2} \end{bmatrix} \begin{bmatrix} \frac{\partial m_i}{\partial s} \\ \frac{\partial m_j}{\partial s} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 \pi_i^B}{\partial m_i \partial s} \\ -\frac{\partial^2 \pi_j^B}{\partial m_j \partial s} \end{bmatrix}.$$

Using Cramer's Rule,

$$\begin{aligned} \frac{\partial m_i}{\partial s} &= \left[\frac{k_3}{a\mu\left(b-1\right)\left(2b+1\right)}\right]\left[\frac{\left(2b^2-1\right)\left(k-k_3\right) - bk_3}{\Delta}\right], \\ \frac{\partial m_j}{\partial s} &= \left[\frac{k_3}{a\mu\left(b-1\right)\left(2b+1\right)}\right]\left[\frac{\left(b+1\right)\left(2b-1\right)k_3 - bk}{\Delta}\right], \end{aligned}$$

where $\Delta = k^2 - 2kk_3$.

Taking derivatives with respect to τ and rearranging the equations, we have

$$\begin{bmatrix} \frac{\partial^2 \pi_i^B}{\partial m_i^2} & \frac{\partial^2 \pi_i^B}{\partial m_i \partial m_j} \\ \frac{\partial^2 \pi_j^B}{\partial m_j \partial m_i} & \frac{\partial^2 \pi_j^B}{\partial m_j^2} \end{bmatrix} \begin{bmatrix} \frac{\partial m_i}{\partial \tau} \\ \frac{\partial m_j}{\partial \tau} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 \pi_i^B}{\partial m_i \partial \tau} \\ 0 \end{bmatrix}.$$

Using Cramer's Rule,

$$\frac{\partial m_i}{\partial \tau} = \frac{k - k_3}{\Delta}, \; \frac{\partial m_j}{\partial \tau} = \frac{k_3}{\Delta},$$

where $\Delta = k^2 - 2kk_3$.

Given the equilibrium of product market competition and cooperative advertising is the equilibrium type of investment, in the first stage of the game the government of country i chooses trade policy s and industrial policy τ to maximize national welfare

$$W(s,\tau) = \pi_i^B - sx_i - \tau m_i.$$

The first-order conditions are as follows.

$$\begin{aligned}\frac{\partial W}{\partial s} &= \left\{\left[\left(\frac{\partial \alpha_i}{\partial m_j} + \gamma \frac{\partial p_j}{\partial m_j}\right)\frac{\partial m_j}{\partial s}\right] + \gamma \frac{\partial p_j}{\partial s}\right\}(p_i - c + s) - \frac{\partial x_i}{\partial s}(\cdot)\, s - \frac{\partial m_i}{\partial s}\tau = 0,\\ \frac{\partial W}{\partial \tau} &= \left(\frac{\partial \alpha_i}{\partial m_j} + \gamma \frac{\partial p_j}{\partial m_j}\right)\frac{\partial m_j}{\partial \tau}(p_i - c + s) - \frac{\partial x_i}{\partial \tau}(\cdot)\, s - \frac{\partial m_i}{\partial \tau}\tau = 0.\end{aligned}$$

Using matrix notation, we have

$$\begin{bmatrix} \frac{\partial x_i}{\partial s}(\cdot) & \frac{\partial m_i}{\partial s} \\ \frac{\partial x_i}{\partial \tau}(\cdot) & \frac{\partial m_i}{\partial \tau} \end{bmatrix}\begin{bmatrix} s \\ \tau \end{bmatrix} = \begin{bmatrix} \left\{\left[\left(\frac{\partial \alpha_i}{\partial m_j} + \gamma \frac{\partial p_j}{\partial m_j}\right)\frac{\partial m_j}{\partial s}\right] + \gamma \frac{\partial p_j}{\partial s}\right\}(p_i - c + s) \\ \left(\frac{\partial \alpha_i}{\partial m_j} + \gamma \frac{\partial p_j}{\partial m_j}\right)\frac{\partial m_j}{\partial \tau}(p_i - c + s) \end{bmatrix},$$

where

$$\begin{aligned}\frac{\partial x_i}{\partial s}(\cdot) &= \beta \frac{\partial (p_i - c + s)}{\partial s} = \beta\left(\frac{\partial p_i}{\partial m_i}\frac{\partial m_i}{\partial s} + \frac{\partial p_i}{\partial m_j}\frac{\partial m_j}{\partial s} + \frac{\partial p_i}{\partial s} + 1\right),\\ \frac{\partial x_i}{\partial \tau}(\cdot) &= \beta \frac{\partial (p_i - c + s)}{\partial \tau} = \beta\left(\frac{\partial p_i}{\partial m_i}\frac{\partial m_i}{\partial \tau} + \frac{\partial p_i}{\partial m_j}\frac{\partial m_j}{\partial \tau}\right).\end{aligned}$$

We use Cramer's Rule to solve this linear equation system.

Denote the determinant of the coefficient matrix of the above linear equation system by D.

$$\begin{aligned} D &= \frac{\partial x_i}{\partial s}(\cdot)\frac{\partial m_i}{\partial \tau} - \frac{\partial x_i}{\partial \tau}(\cdot)\frac{\partial m_i}{\partial s} \\ &= \beta\left[\frac{\partial p_i}{\partial m_j}\left(\frac{\partial m_j}{\partial s}\frac{\partial m_i}{\partial \tau} - \frac{\partial m_i}{\partial s}\frac{\partial m_j}{\partial \tau}\right) + \left(\frac{\partial p_i}{\partial s} + 1\right)\frac{\partial m_i}{\partial \tau}\right].\end{aligned}$$

Note that

$$\begin{aligned} & \frac{\partial m_j}{\partial s}\frac{\partial m_i}{\partial \tau} - \frac{\partial m_i}{\partial s}\frac{\partial m_j}{\partial \tau} \\ =\ & \left[\frac{k_3}{a\mu(b-1)(2b+1)}\right]\left[\frac{(b+1)(2b-1)k_3 - bk}{\Delta}\right]\left(\frac{k-k_3}{\Delta}\right) \\ & - \left[\frac{k_3}{a\mu(b-1)(2b+1)}\right]\left[\frac{(2b^2-1)(k-k_3) - bk_3}{\Delta}\right]\left(\frac{k_3}{\Delta}\right) \\ =\ & -\left[\frac{bk_3}{a\mu(b-1)(2b+1)\Delta}\right] \\ <\ & 0.\end{aligned}$$

Then

$$
\begin{aligned}
D &= \left(\frac{b}{b^2-1}\right)\left\{\left[\frac{a\mu(b-1)}{2b-1}\right]\left[-\frac{bk_3}{a\mu(b-1)(2b+1)\Delta}\right]+\left(\frac{2b^2-1}{4b^2-1}\right)\left(\frac{k-k_3}{\Delta}\right)\right\} \\
&= \left(\frac{b}{b^2-1}\right)\left(\frac{1}{4b^2-1}\right)\left[\frac{\left(2b^2-1\right)(k-k_3)-bk_3}{\Delta}\right] \\
&> 0.
\end{aligned}
$$

Next, we have

$$
\begin{aligned}
D_1 &= \left\{\left[\left(\frac{\partial \alpha_i}{\partial m_j}+\gamma\frac{\partial p_j}{\partial m_j}\right)\frac{\partial m_j}{\partial s}+\gamma\frac{\partial p_j}{\partial s}\right]\frac{\partial m_i}{\partial \tau}-\left(\frac{\partial \alpha_i}{\partial m_j}+\gamma\frac{\partial p_j}{\partial m_j}\right)\frac{\partial m_j}{\partial \tau}\frac{\partial m_i}{\partial s}\right\}(p_i-c+s) \\
&= \left[\left(\frac{\partial \alpha_i}{\partial m_j}+\gamma\frac{\partial p_j}{\partial m_j}\right)\left(\frac{\partial m_j}{\partial s}\frac{\partial m_i}{\partial \tau}-\frac{\partial m_i}{\partial s}\frac{\partial m_j}{\partial \tau}\right)+\gamma\frac{\partial p_j}{\partial s}\frac{\partial m_i}{\partial \tau}\right](p_i-c+s) \\
&< 0.
\end{aligned}
$$

This is because

$$
\frac{\partial \alpha_i}{\partial m_j}+\gamma\frac{\partial p_j}{\partial m_j}>0,\ \frac{\partial m_j}{\partial s}\frac{\partial m_i}{\partial \tau}-\frac{\partial m_i}{\partial s}\frac{\partial m_j}{\partial \tau}<0,\ \frac{\partial p_j}{\partial s}<0,\ \frac{\partial m_i}{\partial \tau}>0.
$$

$$
\begin{aligned}
D_2 &= \left\{\begin{array}{c}\frac{\partial x_i}{\partial s}(\cdot)\left(\frac{\partial \alpha_i}{\partial m_j}+\gamma\frac{\partial p_j}{\partial m_j}\right)\frac{\partial m_j}{\partial \tau}- \\ \frac{\partial x_i}{\partial \tau}(\cdot)\left[\left(\frac{\partial \alpha_i}{\partial m_j}+\gamma\frac{\partial p_j}{\partial m_j}\right)\frac{\partial m_j}{\partial s}+\gamma\frac{\partial p_j}{\partial s}\right]\end{array}\right\}(p_i-c+s) \\
&= \beta\left\{\begin{array}{c}\frac{\partial p_i}{\partial m_i}\left(\frac{\partial \alpha_i}{\partial m_j}+\gamma\frac{\partial p_j}{\partial m_j}\right)\left(\frac{\partial m_i}{\partial s}\frac{\partial m_j}{\partial \tau}-\frac{\partial m_j}{\partial s}\frac{\partial m_i}{\partial \tau}\right)+ \\ \left(\frac{\partial \alpha_i}{\partial m_j}+\gamma\frac{\partial p_j}{\partial m_j}\right)\left(\frac{\partial p_i}{\partial s}+1\right)\frac{\partial m_j}{\partial \tau}-\left(\frac{\partial p_i}{\partial m_i}\frac{\partial m_i}{\partial \tau}+\frac{\partial p_i}{\partial m_j}\frac{\partial m_j}{\partial \tau}\right)\gamma\frac{\partial p_j}{\partial s}\end{array}\right\}(p_i-c+s) \\
&> 0.
\end{aligned}
$$

This is because

$$
\frac{\partial p_i}{\partial m_i}>0,\ \frac{\partial p_i}{\partial m_j}>0,\ \frac{\partial \alpha_i}{\partial m_j}+\gamma\frac{\partial p_j}{\partial m_j}>0,\ \frac{\partial m_j}{\partial s}\frac{\partial m_i}{\partial \tau}-\frac{\partial m_i}{\partial s}\frac{\partial m_j}{\partial \tau}<0,
$$
$$
\frac{\partial p_i}{\partial s}+1>0,\ \frac{\partial p_j}{\partial s}<0,\ \frac{\partial m_i}{\partial \tau}>0,\ \frac{\partial m_j}{\partial \tau}>0.
$$

Because

$$
s=\frac{D_1}{D},\ \tau=\frac{D_2}{D},
$$

we have

$$
s<0,\ \tau>0. \tag{5.3}
$$

Case 2 $\mu<\frac{2b^2+b-1}{2b^2-b-1}$

If $\mu<\frac{2b^2+b-1}{2b^2-b-1}$ and both firms invest in predatory advertising in an equilibrium, the equilibrium values are characterized by the two conditions:

$$2\beta\left(p_i - c + s\right)\left(\frac{\partial p_i}{\partial n_i}\right) - kn_i + \tau = 0,$$
$$2\beta\left(p_j - c\right)\left(\frac{\partial p_j}{\partial n_j}\right) - kn_j = 0.$$

Taking derivatives with respect to s and rearranging the equations, we have

$$\begin{bmatrix} \frac{\partial^2 \pi_i^B}{\partial n_i^2} & \frac{\partial^2 \pi_i^B}{\partial n_i \partial n_j} \\ \frac{\partial^2 \pi_j^B}{\partial n_j \partial n_i} & \frac{\partial^2 \pi_j^B}{\partial n_j^2} \end{bmatrix} \begin{bmatrix} \frac{\partial n_i}{\partial s} \\ \frac{\partial n_j}{\partial s} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 \pi_i^B}{\partial n_i \partial s} \\ -\frac{\partial^2 \pi_j^B}{\partial n_j \partial s} \end{bmatrix}.$$

Using Cramer's Rule,

$$\frac{\partial n_i}{\partial s} = \left[\frac{k_4}{a\left(b+1\right)\left(2b-1\right)}\right]\left[\frac{\left(2b^2-1\right)\left(k-k_4\right)+bk_4}{\Delta}\right],$$
$$\frac{\partial n_j}{\partial s} = \left[\frac{k_4}{a\left(b+1\right)\left(2b-1\right)}\right]\left[\frac{-\left(b-1\right)\left(2b+1\right)k_4 - bk}{\Delta}\right],$$

where $\Delta = k^2 - 2kk_4$.

Taking derivatives with respect to τ and rearranging the equations, we have

$$\begin{bmatrix} \frac{\partial^2 \pi_i^B}{\partial n_i^2} & \frac{\partial^2 \pi_i^B}{\partial n_i \partial n_j} \\ \frac{\partial^2 \pi_j^B}{\partial n_j \partial n_i} & \frac{\partial^2 \pi_j^B}{\partial n_j^2} \end{bmatrix} \begin{bmatrix} \frac{\partial n_i}{\partial \tau} \\ \frac{\partial n_j}{\partial \tau} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 \pi_i^B}{\partial n_i \partial \tau} \\ 0 \end{bmatrix}.$$

Using Cramer's Rule,

$$\frac{\partial n_i}{\partial \tau} = \frac{k-k_4}{\Delta},\ \frac{\partial n_j}{\partial \tau} = -\frac{k_4}{\Delta},$$

where $\Delta = k^2 - 2kk_4$.

Given the equilibrium of product market competition and predatory advertising is the equilibrium type of investment, in the first stage of the game the government of country i chooses trade policy s and industrial policy τ to maximize national welfare

$$W\left(s,\tau\right) = \pi_i^B - sx_i - \tau n_i.$$

The first-order conditions are as follows.

$$\frac{\partial W}{\partial s} = \left\{\left[\left(\frac{\partial \alpha_i}{\partial n_j} + \gamma\frac{\partial p_j}{\partial n_j}\right)\frac{\partial n_j}{\partial s}\right] + \gamma\frac{\partial p_j}{\partial s}\right\}\left(p_i - c + s\right) - \frac{\partial x_i}{\partial s}\left(\cdot\right)s - \frac{\partial n_i}{\partial s}\tau = 0,$$
$$\frac{\partial W}{\partial \tau} = \left(\frac{\partial \alpha_i}{\partial n_j} + \gamma\frac{\partial p_j}{\partial n_j}\right)\frac{\partial n_j}{\partial \tau}\left(p_i - c + s\right) - \frac{\partial x_i}{\partial \tau}\left(\cdot\right)s - \frac{\partial n_i}{\partial \tau}\tau = 0.$$

Using matrix notation, we have

$$\begin{bmatrix} \frac{\partial x_i}{\partial s}(\cdot) & \frac{\partial n_i}{\partial s} \\ \frac{\partial x_i}{\partial \tau}(\cdot) & \frac{\partial n_i}{\partial \tau} \end{bmatrix} \begin{bmatrix} s \\ \tau \end{bmatrix} = \begin{bmatrix} \left\{\left[\left(\frac{\partial \alpha_i}{\partial n_j} + \gamma\frac{\partial p_j}{\partial n_j}\right)\frac{\partial n_j}{\partial s}\right] + \gamma\frac{\partial p_j}{\partial s}\right\}(p_i - c + s) \\ \left(\frac{\partial \alpha_i}{\partial n_j} + \gamma\frac{\partial p_j}{\partial n_j}\right)\frac{\partial n_j}{\partial \tau}(p_i - c + s) \end{bmatrix},$$

where $\frac{\partial x_i}{\partial s}(\cdot) = \beta\frac{\partial(p_i-c+s)}{\partial s} = \beta\left(\frac{\partial p_i}{\partial n_i}\frac{\partial n_i}{\partial s} + \frac{\partial p_i}{\partial n_j}\frac{\partial n_j}{\partial s} + \frac{\partial p_i}{\partial s} + 1\right)$, $\frac{\partial x_i}{\partial \tau}(\cdot) = \beta\frac{\partial(p_i-c+s)}{\partial \tau} = \beta\left(\frac{\partial p_i}{\partial n_i}\frac{\partial n_i}{\partial \tau} + \frac{\partial p_i}{\partial n_j}\frac{\partial n_j}{\partial \tau}\right)$.

We use Cramer's Rule to solve this linear equation system.

Denote the determinant of the coefficient matrix of the above linear equation system by D.

$$\begin{aligned} D &= \frac{\partial x_i}{\partial s}(\cdot)\frac{\partial n_i}{\partial \tau} - \frac{\partial x_i}{\partial \tau}(\cdot)\frac{\partial n_i}{\partial s} \\ &= \beta\left[\frac{\partial p_i}{\partial n_j}\left(\frac{\partial n_j}{\partial s}\frac{\partial n_i}{\partial \tau} - \frac{\partial n_i}{\partial s}\frac{\partial n_j}{\partial \tau}\right) + \left(\frac{\partial p_i}{\partial s} + 1\right)\frac{\partial n_i}{\partial \tau}\right]. \end{aligned}$$

Note that

$$\begin{aligned} & \frac{\partial n_j}{\partial s}\frac{\partial n_i}{\partial \tau} - \frac{\partial n_i}{\partial s}\frac{\partial n_j}{\partial \tau} \\ = & \left[\frac{k_4}{a(b+1)(2b-1)}\right]\left[\frac{-(b-1)(2b+1)k_4 - bk}{\Delta}\right]\left(\frac{k-k_4}{\Delta}\right) \\ & - \left[\frac{k_4}{a(b+1)(2b-1)}\right]\left[\frac{\left(2b^2-1\right)(k-k_4) + bk_4}{\Delta}\right]\left(-\frac{k_4}{\Delta}\right) \\ = & -\left[\frac{bk_4}{a(b+1)(2b-1)\Delta}\right] \\ < & \ 0. \end{aligned}$$

Then

$$\begin{aligned} D &= \left(\frac{b}{b^2-1}\right)\left\{\left[-\frac{a(b+1)}{2b+1}\right]\left[-\frac{bk_4}{a(b+1)(2b-1)\Delta}\right] + \left(\frac{2b^2-1}{4b^2-1}\right)\left(\frac{k-k_4}{\Delta}\right)\right\} \\ &= \left(\frac{b}{b^2-1}\right)\left(\frac{1}{4b^2-1}\right)\left[\frac{\left(2b^2-1\right)(k-k_4) + bk_4}{\Delta}\right] \\ &> 0. \end{aligned}$$

Next, we have

$$\begin{aligned}
D_1 &= \left\{\left[\left(\frac{\partial \alpha_i}{\partial n_j}+\gamma\frac{\partial p_j}{\partial n_j}\right)\frac{\partial n_j}{\partial s}+\gamma\frac{\partial p_j}{\partial s}\right]\frac{\partial n_i}{\partial \tau}-\left(\frac{\partial \alpha_i}{\partial n_j}+\gamma\frac{\partial p_j}{\partial n_j}\right)\frac{\partial n_j}{\partial \tau}\frac{\partial n_i}{\partial s}\right\}(p_i-c+s)\\
&= \left[\left(\frac{\partial \alpha_i}{\partial n_j}+\gamma\frac{\partial p_j}{\partial n_j}\right)\left(\frac{\partial n_j}{\partial s}\frac{\partial n_i}{\partial \tau}-\frac{\partial n_i}{\partial s}\frac{\partial n_j}{\partial \tau}\right)+\gamma\frac{\partial p_j}{\partial s}\frac{\partial n_i}{\partial \tau}\right](p_i-c+s)\\
&= \left(-\frac{2ab}{(b-1)(2b+1)}\right)\left(-\frac{bk_4}{a(b+1)(2b-1)\Delta}\right)(p_i-c+s)\\
&\quad+\left(\frac{1}{b^2-1}\right)\left(-\frac{b}{4b^2-1}\right)\left(\frac{k-k_4}{\Delta}\right)(p_i-c+s)\\
&= \left[\frac{b}{(b^2-1)(4b^2-1)}\right]\left[\frac{(2b+1)k_4-k}{\Delta}\right],
\end{aligned}$$

$$\begin{aligned}
D_2 &= \left\{\frac{\partial x_i}{\partial s}(\cdot)\left(\frac{\partial \alpha_i}{\partial n_j}+\gamma\frac{\partial p_j}{\partial n_j}\right)\frac{\partial n_j}{\partial \tau}-\frac{\partial x_i}{\partial \tau}(\cdot)\left[\left(\frac{\partial \alpha_i}{\partial n_j}+\gamma\frac{\partial p_j}{\partial n_j}\right)\frac{\partial n_j}{\partial s}+\gamma\frac{\partial p_j}{\partial s}\right]\right\}(p_i-c+s)\\
&= \beta\left\{\begin{array}{c}\frac{\partial p_i}{\partial n_i}\left(\frac{\partial \alpha_i}{\partial n_j}+\gamma\frac{\partial p_j}{\partial n_j}\right)\left(\frac{\partial n_i}{\partial s}\frac{\partial n_j}{\partial \tau}-\frac{\partial n_j}{\partial s}\frac{\partial n_i}{\partial \tau}\right)+\\ \left(\frac{\partial \alpha_i}{\partial n_j}+\gamma\frac{\partial p_j}{\partial n_j}\right)\left(\frac{\partial p_i}{\partial s}+1\right)\frac{\partial n_j}{\partial \tau}-\left(\frac{\partial p_i}{\partial n_i}\frac{\partial n_i}{\partial \tau}+\frac{\partial p_i}{\partial n_j}\frac{\partial n_j}{\partial \tau}\right)\gamma\frac{\partial p_j}{\partial s}\end{array}\right\}(p_i-c+s)\\
&= \beta\left\{\begin{array}{c}\left[\frac{a(b+1)}{2b+1}\right]\left[-\frac{2ab}{(b-1)(2b+1)}\right]\left[\frac{bk_4}{a(b+1)(2b-1)\Delta}\right]+\\ \left[-\frac{2ab}{(b-1)(2b+1)}\right]\left(\frac{2b^2-1}{4b^2-1}\right)\left(-\frac{k_4}{\Delta}\right)-\left[\frac{a(b+1)}{2b+1}\right]\left(\frac{k}{\Delta}\right)\left(\frac{1}{b^2-1}\right)\left(-\frac{b}{4b^2-1}\right)\end{array}\right\}(p_i-c+s)\\
&= \beta\left\{\left[\frac{ab}{(b-1)(2b+1)(4b^2-1)}\right]\left[\frac{2(b-1)(2b+1)k_4+k}{\Delta}\right]\right\}(p_i-c+s)\\
&> 0.
\end{aligned}$$

Because

$$s=\frac{D_1}{D},\ \tau=\frac{D_2}{D},$$

we have

$$sign\ s = sign\ \left[(2b+1)k_4-k\right],\ \tau>0. \tag{5.4}$$

Part 3

This is implied by the above two Parts. ■

5.2.5 Proof of Corollary 3.1

By the assumption that the welfare function is strictly concave we must have[1]

$$k>\underline{k}'=\max\left\{\left(\frac{3}{2}+\frac{1}{2}\sqrt{5}\right)k_1,\left(\frac{3}{2}+\frac{1}{2}\sqrt{13}\right)k_4\right\}. \tag{5.5}$$

[1]See the last subsection of Appendix to Chapter 3.

Cournot competition

First consider Cournot competition.

Case 1 $\mu > \frac{2b+1}{2b-1}$

Step 1. Note that in this case,

$$\begin{aligned}\frac{\partial^2 W}{\partial s \partial \tau} &= \frac{\partial x_i}{\partial \tau}(\cdot)\left[\left(\frac{\partial a_i}{\partial m_j} - \frac{\partial x_j}{\partial m_j}\right)\frac{\partial m_j}{\partial s} - \frac{\partial x_j}{\partial s}\right] - \frac{\partial m_i}{\partial s} \\ &= \frac{\partial x_i}{\partial s}(\cdot)\left[\left(\frac{\partial a_i}{\partial m_j} - \frac{\partial x_j}{\partial m_j}\right)\frac{\partial m_j}{\partial \tau}\right] - \frac{\partial x_i}{\partial \tau}(\cdot),\end{aligned}$$

and

$$sign\ \frac{\partial^2 W}{\partial s \partial \tau} = sign\ \left[\left(1-4b^2\right)k^2 + \left(12b^2-2\right)k_1 k - 2b\left(2b+1\right)\left(k_1\right)^2\right].$$

Let

$$\left(1-4b^2\right)k^2 + \left(12b^2-2\right)k_1 k - 2b\left(2b+1\right)\left(k_1\right)^2 = 0.$$

The solutions to this quadratic equation in k are as follows.

$$k(0) \in \left\{\begin{array}{l} \frac{1}{2(-1+4b^2)}\left(12b^2 - 2 - 2\sqrt{(20b^4 - 8b^2 + 1 + 2b - 8b^3)}\right)k_1, \\ \frac{1}{2(-1+4b^2)}\left(12b^2 - 2 + 2\sqrt{(20b^4 - 8b^2 + 1 + 2b - 8b^3)}\right)k_1 \end{array}\right\}.$$

Note that the coefficient of k^2 is $1-4b^2<0$, since $b>1$. So, for an arbitrarily chosen b, when k is smaller than the minimum of these two roots, or when it is bigger than the maximum of these two roots, $\frac{\partial^2 W}{\partial s \partial \tau} < 0$. (The arguments in other cases are similar to the arguments in this case.)

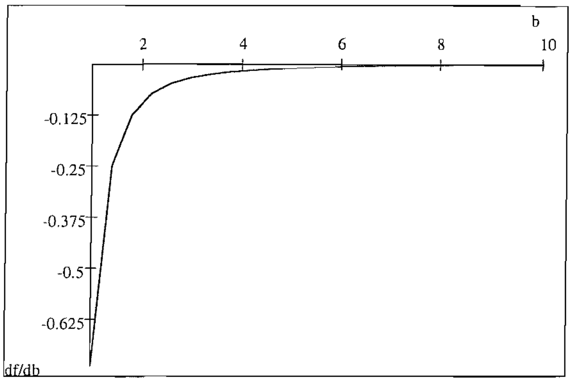
Step 2. Consider the first root. Let

$$f = \frac{1}{2\left(-1+4b^2\right)}\left(12b^2 - 2 - 2\sqrt{(20b^4 - 8b^2 + 1 + 2b - 8b^3)}\right).$$

Then,

$$\begin{aligned}\frac{df}{db} &= \frac{b}{64b^4 - 32b^2 + 4}\left(32\sqrt{2b - 8b^2 - 8b^3 + 20b^4 + 1} - 192b^2 + 32\right) \\ &+ \frac{1}{8b^2 - 2}\left(24b - \frac{80b^3 - 24b^2 - 16b + 2}{\sqrt{2b - 8b^2 - 8b^3 + 20b^4 + 1}}\right).\end{aligned}$$

From the graph of $\frac{df}{db}$,



we know that f is a decreasing function. Take the following limit,

$$\lim_{b\to 1^+} \frac{1}{2(-1+4b^2)}\left(12b^2 - 2 - 2\sqrt{(20b^4 - 8b^2 + 1 + 2b - 8b^3)}\right) = \frac{5}{3} - \frac{1}{3}\sqrt{7}.$$

So, the first root is smaller than $\underline{k}'$.

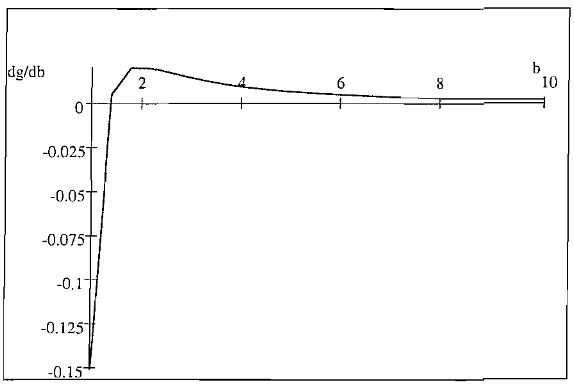
Step 3. Consider the second root. Let

$$g = \frac{1}{2(-1+4b^2)}\left(12b^2 - 2 + 2\sqrt{(20b^4 - 8b^2 + 1 + 2b - 8b^3)}\right).$$

Then,

$$\begin{aligned}\frac{dg}{db} \quad &= \quad \frac{b}{64b^4 - 32b^2 + 4}\left(32 - 32\sqrt{2b - 8b^2 - 8b^3 + 20b^4 + 1} - 192b^2\right) \\ &\quad + \frac{1}{8b^2 - 2}\left(24b + \frac{80b^3 - 24b^2 - 16b + 2}{\sqrt{2b - 8b^2 - 8b^3 + 20b^4 + 1}}\right).\end{aligned}$$

From the graph of $\frac{dg}{db}$,



we know that g has a critical point and it is a minimizer. Take the following limits,

$$\begin{aligned}
& \lim_{b\to 1^+} \frac{1}{2\left(-1+4b^2\right)}\left(12b^2-2+2\sqrt{(20b^4-8b^2+1+2b-8b^3)}\right) \\
= & \frac{5}{3}+\frac{1}{3}\sqrt{7}, \\
& \lim_{b\to +\infty} \frac{1}{2\left(-1+4b^2\right)}\left(12b^2-2+2\sqrt{(20b^4-8b^2+1+2b-8b^3)}\right) \\
= & \frac{3}{2}+\frac{1}{2}\sqrt{5}.
\end{aligned}$$

It is easy to show that $\frac{3}{2}+\frac{1}{2}\sqrt{5} > \frac{5}{3}+\frac{1}{3}\sqrt{7}$.

Step 4. Therefore, we have

$$\frac{\partial^2 W}{\partial s \partial \tau} < 0, \tag{5.6}$$

if $k > \underline{k}'$.

Case 2 $\mu < \frac{2b+1}{2b-1}$

Step 1. Note that in this case,

$$\begin{aligned}
\frac{\partial^2 W}{\partial s \partial \tau} &= \frac{\partial x_i}{\partial \tau}(\cdot)\left[\left(\frac{\partial a_i}{\partial n_j}-\frac{\partial x_j}{\partial n_j}\right)\frac{\partial n_j}{\partial s}-\frac{\partial x_j}{\partial s}\right]-\frac{\partial n_i}{\partial s} \\
&= \frac{\partial x_i}{\partial s}(\cdot)\left[\left(\frac{\partial a_i}{\partial n_j}-\frac{\partial x_j}{\partial n_j}\right)\frac{\partial n_j}{\partial \tau}\right]-\frac{\partial x_i}{\partial \tau}(\cdot),
\end{aligned}$$

and

$$sign\ \frac{\partial^2 W}{\partial s \partial \tau} = sign\ \left[\left(1-4b^2\right)k^2+\left(12b^2-2\right)k_2 k-2b\left(2b-1\right)\left(k_2\right)^2\right].$$

Let

$$\left(1-4b^2\right)k^2+\left(12b^2-2\right)k_2 k-2b\left(2b-1\right)\left(k_2\right)^2=0.$$

The solutions are as follows.

$$k(0) \in \left\{ \begin{array}{l} \frac{1}{2(-1+4b^2)}\left(12b^2-2-2\sqrt{(20b^4-8b^2+1-2b+8b^3)}\right)k_2, \\ \frac{1}{2(-1+4b^2)}\left(12b^2-2+2\sqrt{(20b^4-8b^2+1-2b+8b^3)}\right)k_2 \end{array} \right\}.$$

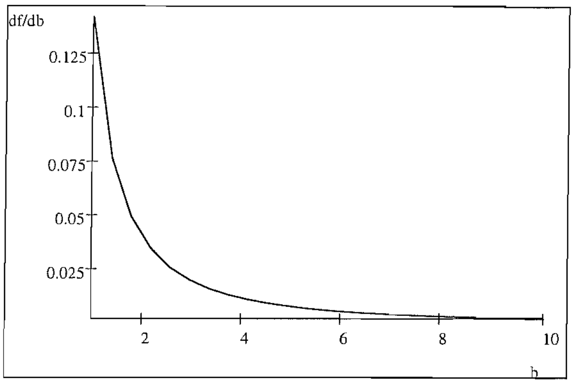
Step 2. Consider the first root. Let

$$f = \frac{1}{2\left(-1+4b^2\right)}\left(12b^2-2-2\sqrt{(20b^4-8b^2+1-2b+8b^3)}\right).$$

Then

$$\begin{aligned}
\frac{df}{db} = & \frac{b}{64b^4-32b^2+4}\left(32\sqrt{8b^3-8b^2-2b+20b^4+1}-192b^2+32\right) \\
& +\frac{1}{8b^2-2}\left(24b-\frac{24b^2-16b+80b^3-2}{\sqrt{8b^3-8b^2-2b+20b^4+1}}\right).
\end{aligned}$$

From the graph of $\frac{df}{db}$,



we know that f is an increasing function. Take the following limit,

$$\lim_{b\to+\infty}\frac{1}{2\left(-1+4b^2\right)}\left(12b^2-2-2\sqrt{(20b^4-8b^2+1-2b+8b^3)}\right)=\frac{3}{2}-\frac{1}{2}\sqrt{5}.$$

So, the first root is smaller than $\underline{k}'$.

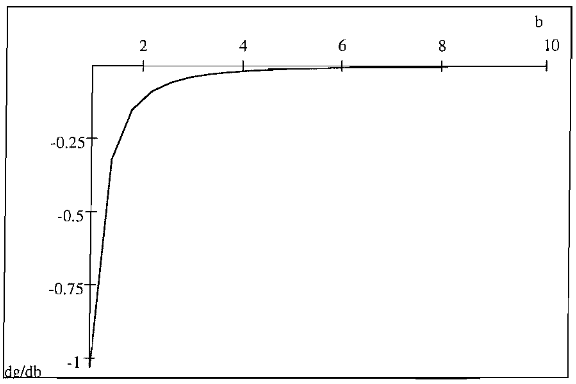
Step 3. Consider the second root. Let

$$g=\frac{1}{2\left(-1+4b^2\right)}\left(12b^2-2+2\sqrt{(20b^4-8b^2+1-2b+8b^3)}\right).$$

Then

$$\begin{aligned}\frac{dg}{db} &= \frac{b}{64b^4-32b^2+4}\left(32-32\sqrt{8b^3-8b^2-2b+20b^4+1}-192b^2\right)\\ &\quad+\frac{1}{8b^2-2}\left(24b+\frac{24b^2-16b+80b^3-2}{\sqrt{8b^3-8b^2-2b+20b^4+1}}\right).\end{aligned}$$

From the graph of $\frac{dg}{db}$,



we know that g is a decreasing function. Take the following limit,

$$\lim_{b\to1^+}\frac{1}{2\left(-1+4b^2\right)}\left(12b^2-2+2\sqrt{(20b^4-8b^2+1-2b+8b^3)}\right)=\frac{5}{3}+\frac{1}{3}\sqrt{19}.$$

Step 4. Since

$$\left(\frac{3}{2}+\frac{1}{2}\sqrt{13}\right)k_4 > \left(\frac{3}{2}+\frac{1}{2}\sqrt{13}\right)k_2 > \left(\frac{5}{3}+\frac{1}{3}\sqrt{19}\right)k_2,$$

we have

$$\frac{\partial^2 W}{\partial s \partial \tau} < 0, \tag{5.7}$$

if $k > \underline{k}'$.

Bertrand competition

Second consider Bertrand competition.

Case 1 $\mu > \frac{2b^2+b-1}{2b^2-b-1}$

Step 1. Note that in this case,

$$\begin{aligned}\frac{\partial^2 W}{\partial s \partial \tau} &= \frac{\partial p_i}{\partial \tau}(\cdot)\left\{\left[\left(\frac{\partial \alpha_i}{\partial m_j}+\gamma\frac{\partial p_j}{\partial m_j}\right)\frac{\partial m_j}{\partial s}\right]+\gamma\frac{\partial p_j}{\partial s}\right\}-\frac{\partial m_i}{\partial s} \\ &\quad \left[\frac{\partial p_i}{\partial s}(\cdot)+1\right]\left(\frac{\partial \alpha_i}{\partial m_j}+\gamma\frac{\partial p_j}{\partial m_j}\right)\frac{\partial m_j}{\partial \tau}-\frac{\partial x_i}{\partial \tau}(\cdot),\end{aligned}$$

and

$$sign\ \frac{\partial^2 W}{\partial s \partial \tau} = sign\ \left[\left(1-4b^2\right)k^2+\left(12b^2-4\right)k_3 k-2\left(b+1\right)\left(2b-1\right)\left(k_3\right)^2\right].$$

Let

$$\left(1-4b^2\right)k^2+\left(12b^2-4\right)k_3 k-2\left(b+1\right)\left(2b-1\right)\left(k_3\right)^2=0.$$

The solutions are as follows.

$$k(0) \in \left\{\begin{array}{l}\frac{1}{2(-1+4b^2)}\left(-4+12b^2-2\sqrt{(2-12b^2+20b^4+2b-8b^3)}\right)k_3, \\ \frac{1}{2(-1+4b^2)}\left(-4+12b^2+2\sqrt{(2-12b^2+20b^4+2b-8b^3)}\right)k_3\end{array}\right\}.$$

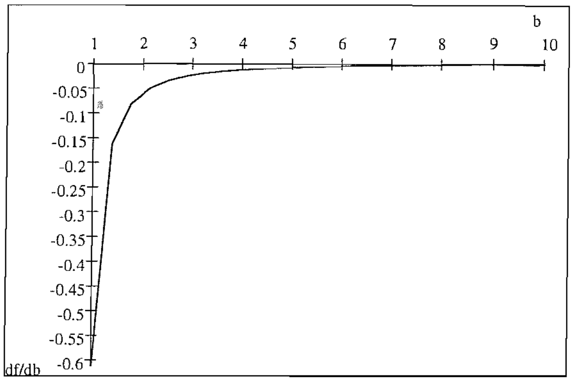
Step 2. Consider the first root. Let

$$f=\frac{1}{2\left(-1+4b^2\right)}\left(-4+12b^2-2\sqrt{(2-12b^2+20b^4+2b-8b^3)}\right).$$

Then

$$\begin{aligned}\frac{df}{db} &= \frac{b}{64b^4-32b^2+4}\left(32\sqrt{2}\sqrt{b-6b^2-4b^3+10b^4+1}-192b^2+64\right) \\ &\quad +\frac{1}{8b^2-2}\left(24b-\sqrt{2}\frac{40b^3-12b^2-12b+1}{\sqrt{b-6b^2-4b^3+10b^4+1}}\right).\end{aligned}$$

From the graph of $\frac{df}{db}$,



we know that f is a decreasing function. Take the following limit,

$$\lim_{b\to 1^+} \frac{1}{2\left(-1+4b^2\right)}\left(-4+12b^2-2\sqrt{(2-12b^2+20b^4+2b-8b^3)}\right)=\frac{2}{3}.$$

So, the first root is smaller than $\underline{k}'$.

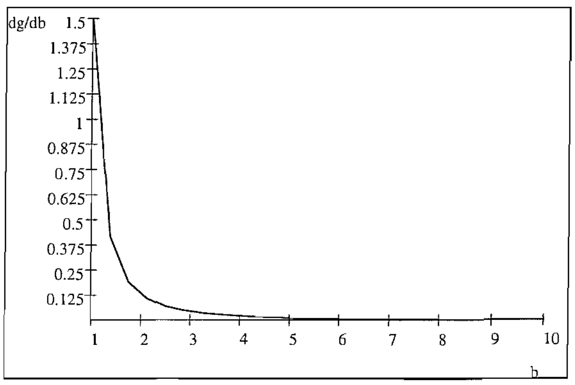
Step 3. Consider the second root. Let

$$g=\frac{1}{2\left(-1+4b^2\right)}\left(-4+12b^2+2\sqrt{(2-12b^2+20b^4+2b-8b^3)}\right).$$

Then

$$\begin{aligned}\frac{dg}{db} &= \frac{b}{64b^4-32b^2+4}\left(64-32\sqrt{2}\sqrt{b-6b^2-4b^3+10b^4+1}-192b^2\right)\\ &+\frac{1}{8b^2-2}\left(24b+\sqrt{2}\frac{40b^3-12b^2-12b+1}{\sqrt{b-6b^2-4b^3+10b^4+1}}\right).\end{aligned}$$

From the graph of $\frac{dg}{db}$,



we know that g is an increasing function. Take the following limit,

$$\lim_{b\to +\infty} \frac{1}{2\left(-1+4b^2\right)}\left(-4+12b^2+2\sqrt{(2-12b^2+20b^4+2b-8b^3)}\right)=\frac{3}{2}+\frac{1}{2}\sqrt{5}.$$

Step 4. Since

$$\left(\frac{3}{2}+\frac{1}{2}\sqrt{5}\right)k_1 > \left(\frac{3}{2}+\frac{1}{2}\sqrt{5}\right)k_3,$$

we have

$$\frac{\partial^2 W}{\partial s \partial \tau} < 0, \tag{5.8}$$

if $k > \underline{k}'$.

Case 2 $\mu < \frac{2b^2+b-1}{2b^2-b-1}$

Step 1. Note that in this case,

$$\begin{aligned}\frac{\partial^2 W}{\partial s \partial \tau} &= \frac{\partial p_i}{\partial \tau}(\cdot)\left\{\left[\left(\frac{\partial \alpha_i}{\partial n_j}+\gamma\frac{\partial p_j}{\partial n_j}\right)\frac{\partial n_j}{\partial s}\right]+\gamma\frac{\partial p_j}{\partial s}\right\}-\frac{\partial n_i}{\partial s} \\ &\left[\frac{\partial p_i}{\partial s}(\cdot)+1\right]\left(\frac{\partial \alpha_i}{\partial n_j}+\gamma\frac{\partial p_j}{\partial n_j}\right)\frac{\partial n_j}{\partial \tau}-\frac{\partial x_i}{\partial \tau}(\cdot),\end{aligned}$$

and

$$sign\ \frac{\partial^2 W}{\partial s \partial \tau} = sign\ \left[\left(1-4b^2\right)k^2+\left(12b^2-4\right)k_4 k-2\left(b-1\right)\left(2b+1\right)\left(k_4\right)^2\right].$$

Let

$$\left(1-4b^2\right)k^2+\left(12b^2-4\right)k_4 k-2\left(b-1\right)\left(2b+1\right)\left(k_4\right)^2=0.$$

The solutions are as follows.

$$k(0) \in \left\{\begin{array}{l} \frac{1}{2(-1+4b^2)}\left(-4+12b^2-2\sqrt{(2-12b^2+20b^4-2b+8b^3)}\right)k_4, \\ \frac{1}{2(-1+4b^2)}\left(-4+12b^2+2\sqrt{(2-12b^2+20b^4-2b+8b^3)}\right)k_4 \end{array}\right\}.$$

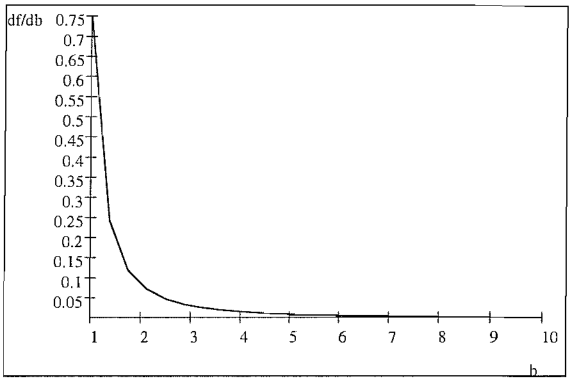
Step 2. Consider the first root. Let

$$f=\frac{1}{2\left(-1+4b^2\right)}\left(-4+12b^2-2\sqrt{(2-12b^2+20b^4-2b+8b^3)}\right).$$

Then

$$\begin{aligned}\frac{df}{db} &= \frac{b}{64b^4-32b^2+4}\left(32\sqrt{2}\sqrt{4b^3-6b^2-b+10b^4+1}-192b^2+64\right) \\ &+\frac{1}{8b^2-2}\left(24b-\sqrt{2}\frac{12b^2-12b+40b^3-1}{\sqrt{4b^3-6b^2-b+10b^4+1}}\right).\end{aligned}$$

From the graph of $\frac{df}{db}$,



we know that f is an increasing function. Take the following limit,

$$\lim_{b\to+\infty} \frac{1}{2(-1+4b^2)}\left(-4+12b^2-2\sqrt{(2-12b^2+20b^4-2b+8b^3)}\right)=\frac{3}{2}-\frac{1}{2}\sqrt{5}.$$

So, the first root is smaller than $\underline{k}'$.

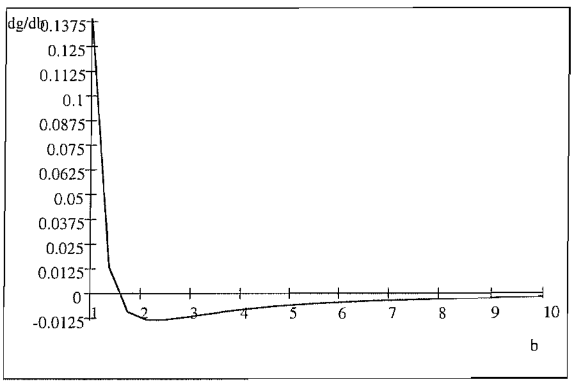
Step 3. Consider the second root. Let

$$g=\frac{1}{2(-1+4b^2)}\left(-4+12b^2+2\sqrt{(2-12b^2+20b^4-2b+8b^3)}\right).$$

Then

$$\begin{aligned}\frac{dg}{db} &= \frac{b}{64b^4-32b^2+4}\left(64-32\sqrt{2}\sqrt{4b^3-6b^2-b+10b^4+1}-192b^2\right)\\ &\quad+\frac{1}{8b^2-2}\left(24b+\sqrt{2}\frac{12b^2-12b+40b^3-1}{\sqrt{4b^3-6b^2-b+10b^4+1}}\right).\end{aligned}$$

From the graph of $\frac{dg}{db}$,



we know that g has a critical point and it is a maximizer. In addition, the critical point is $b = 1.5253$, and

$$g|_{b=1.5253} = 2.6891.$$

Step 4. Since

$$\left(\frac{3}{2} + \frac{1}{2}\sqrt{13}\right) k_4 > (2.6891)\, k_4,$$

we have

$$\frac{\partial^2 W}{\partial s \partial \tau} < 0, \tag{5.9}$$

if $k > \underline{k}'$. ■

5.2.6 Proof of Corollary 3.2

First consider Cournot case.

1. When both firms invest in cooperative advertising in an equilibrium,

$$\begin{aligned} \left.\frac{\partial W}{\partial s}\right|_{(s,\tau)=(0,0)} &= \left\{ \left[\left(\frac{\partial a_i}{\partial m_j} - \frac{\partial x_j}{\partial m_j} \right) \frac{\partial m_j}{\partial s} \right] - \frac{\partial x_j}{\partial s} \right\} x_i \\ &= \left(\frac{1}{4b^2 - 1} \right) \left\{ \frac{2bk_1 \left[(2b+1)\, k_1 - k \right] + \Delta}{\Delta} \right\} x_i > 0, \end{aligned}$$

where $\Delta = k^2 - 2kk_1$, and note that

$$2bk_1 \left[(2b+1)\, k_1 - k \right] + \Delta = \left[k - (b+1)\, k_1 \right]^2 + \left(3b^2 - 1 \right) (k_1)^2 > 0.$$

2. When both firms invest in predatory advertising in an equilibrium,

$$\left.\frac{\partial W}{\partial s}\right|_{(s,\tau)=(0,0)} = \left\{ \left[\left(\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j} \right) \frac{\partial n_j}{\partial s} \right] - \frac{\partial x_j}{\partial s} \right\} x_i > 0,$$

because $\left(\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j} \right) < 0$, $\frac{\partial n_j}{\partial s} < 0$, $\frac{\partial x_j}{\partial s} < 0$.

Next, consider Bertrand case.

1. When both firms invest in cooperative advertising in an equilibrium,

$$\begin{aligned} \left.\frac{\partial W}{\partial s}\right|_{(s,\tau)=(0,0)} &= \left\{ \left[\left(\frac{\partial \alpha_i}{\partial m_j} + \gamma \frac{\partial p_j}{\partial m_j} \right) \frac{\partial m_j}{\partial s} \right] + \gamma \frac{\partial p_j}{\partial s} \right\} (p_i - c + s) \\ &= \left[\frac{b}{(b^2 - 1)(4b^2 - 1)} \right] \left\{ \frac{2k_3 \left[(b+1)(2b-1)\, k_3 - bk \right] - \Delta}{\Delta} \right\} (p_i - c + s), \end{aligned}$$

where $\Delta = k^2 - 2kk_3$. Because

$$2k_3 \left[(b+1)(2b-1)\, k_3 - bk \right] - \Delta = \left(5b^2 - 1 \right) (k_3)^2 - \left[k + (b-1) k_3 \right]^2,$$

we have

$$\begin{cases} sign\ \frac{\partial W}{\partial s}\Big|_{(s,\tau)=(0,0)} > 0 & \Leftrightarrow \quad \left[\sqrt{(5b^2-1)} - (b-1)\right] k_3 > k, \\ sign\ \frac{\partial W}{\partial s}\Big|_{(s,\tau)=(0,0)} < 0 & \Leftrightarrow \quad \left[\sqrt{(5b^2-1)} - (b-1)\right] k_3 < k. \end{cases}$$

2. When both firms invest in predatory advertising in an equilibrium,

$$\begin{aligned} \frac{\partial W}{\partial s}\bigg|_{(s,\tau)=(0,0)} &= \left\{\left[\left(\frac{\partial \alpha_i}{\partial n_j} + \gamma\frac{\partial p_j}{\partial n_j}\right)\frac{\partial n_j}{\partial s}\right] + \gamma\frac{\partial p_j}{\partial s}\right\}(p_i - c + s) \\ &= \left[\frac{b}{(b^2-1)(4b^2-1)}\right]\left\{\frac{2k_4\left[(b-1)(2b+1)k_4 + bk\right] - \Delta}{\Delta}\right\}(p_i - c + s), \end{aligned}$$

where $\Delta = k^2 - 2kk_4$. Because

$$2k_4\left[(b-1)(2b+1)k_4 + bk\right] - \Delta = \left(5b^2-1\right)(k_4)^2 - \left[k - (b+1)k_4\right]^2,$$

we have

$$\begin{cases} sign\ \frac{\partial W}{\partial s}\Big|_{(s,\tau)=(0,0)} > 0 & \Leftrightarrow \quad \left[\sqrt{(5b^2-1)} + (b+1)\right] k_4 > k, \\ sign\ \frac{\partial W}{\partial s}\Big|_{(s,\tau)=(0,0)} < 0 & \Leftrightarrow \quad \left[\sqrt{(5b^2-1)} + (b+1)\right] k_4 < k. \end{cases}$$

■

5.2.7 Proof of Corollary 3.3

First consider Cournot case.

1. When both firms invest in cooperative advertising in an equilibrium,

$$\frac{\partial W}{\partial \tau}\bigg|_{(s,\tau)=(0,0)} = \left(\frac{\partial a_i}{\partial m_j} - \frac{\partial x_j}{\partial m_j}\right)\frac{\partial m_j}{\partial \tau}x_i > 0,$$

because $\left(\frac{\partial a_i}{\partial m_j} - \frac{\partial x_j}{\partial m_j}\right) > 0$, $\frac{\partial m_j}{\partial \tau} > 0$.

2. When both firms invest in predatory advertising in an equilibrium,

$$\frac{\partial W}{\partial \tau}\bigg|_{(s,\tau)=(0,0)} = \left(\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j}\right)\frac{\partial n_j}{\partial \tau}x_i > 0,$$

because $\left(\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j}\right) < 0$, $\frac{\partial n_j}{\partial \tau} < 0$.

Next consider Bertrand case.

1. When both firms invest in cooperative advertising in an equilibrium,

$$\left.\frac{\partial W}{\partial \tau}\right|_{(s,\tau)=(0,0)} = \left(\frac{\partial \alpha_i}{\partial m_j} + \gamma\frac{\partial p_j}{\partial m_j}\right)\frac{\partial m_j}{\partial \tau}(p_i - c + s) > 0,$$

because $\left(\frac{\partial \alpha_i}{\partial m_j} + \gamma\frac{\partial p_j}{\partial m_j}\right) > 0$, $\frac{\partial m_j}{\partial \tau} > 0$.

2. When both firms invest in predatory advertising in an equilibrium,

$$\left.\frac{\partial W}{\partial \tau}\right|_{(s,\tau)=(0,0)} = \left(\frac{\partial \alpha_i}{\partial n_j} + \gamma\frac{\partial p_j}{\partial n_j}\right)\frac{\partial n_j}{\partial \tau}(p_i - c + s) > 0,$$

because $\left(\frac{\partial \alpha_i}{\partial n_j} + \gamma\frac{\partial p_j}{\partial n_j}\right) < 0$, $\frac{\partial n_j}{\partial \tau} < 0$.

■

5.2.8 Proof of Corollary 3.4

1. It is implied by Corollary 3.2 and 3.3.

2. Note that

$$-\frac{d^2\tau}{ds^2} = \frac{\frac{\partial^2 W}{\partial s^2}\left(\frac{\partial W}{\partial \tau}\right)^2 - 2\frac{\partial^2 W}{\partial s \partial \tau}\frac{\partial W}{\partial s}\frac{\partial W}{\partial \tau} + \frac{\partial^2 W}{\partial \tau^2}\left(\frac{\partial W}{\partial s}\right)^2}{\left(\frac{\partial W}{\partial \tau}\right)^3}$$

   By the assumption that the welfare function is strictly concave it is strictly quasiconcave. Hence, $\frac{\partial^2 W}{\partial s^2}\left(\frac{\partial W}{\partial \tau}\right)^2 - 2\frac{\partial^2 W}{\partial s \partial \tau}\frac{\partial W}{\partial s}\frac{\partial W}{\partial \tau} + \frac{\partial^2 W}{\partial \tau^2}\left(\frac{\partial W}{\partial s}\right)^2 < 0$. This fact and Corollary 3.3 together imply that the marginal rate of substitution is decreasing at the non-intervention point.

■

5.2.9 Proof of Proposition 3.3

Note that the second-best trade policy analysis is equivalent to the constraint-augmented first-best policy analysis where the constraint is $\tau = 0$. Therefore, this Proposition is implied by Corollary 3.2. ■

5.2.10 Proof of Proposition 3.4

Note that the second-best industrial policy analysis is equivalent to the constraint-augmented first-best policy analysis where the constraint is $s = 0$. Therefore, this Proposition is implied by Corollary 3.3. ■

5.2.11 The design of simulation

The general form of 'robust' fraction

Denote l the fraction of the feasible range of values for k for which trade policy is robust. The general form of l can be written as follows.

$$l = \frac{k_c - \underline{k}''}{\overline{k} - \underline{k}''} = \begin{cases} 0 & if \quad k_c < \underline{k}'', \\ l \in (0,1) & if \quad \underline{k}'' < k_c < \overline{k}, \\ 1 & if \quad k_c > \overline{k}. \end{cases}$$

We have

$$k_c \in \{k_5, k_6\},$$

where

$$k_5 = \left[\sqrt{(5b^2-1)} - (b-1)\right] k_3, \ k_6 = \left[\sqrt{(5b^2-1)} + (b+1)\right] k_4,$$

and[2]

$$\underline{k}'' = \max\left\{\left(1+\sqrt{2}\right) k_1, \left(\frac{3}{2} + \frac{1}{2}\sqrt{13}\right) k_4\right\}.$$

Note that when $l = 0$, we cannot get robust trade policy in an equilibrium. In particular, when firms play Cournot, the optimal trade policy is a trade subsidy while firms play Bertrand, it is a trade tax. When $l = 1$, we can definitely get robust trade policy in an equilibrium and it is a trade subsidy whatever the form of product market competition.

Calibrating $\overline{k}$

We use the advertising to sales ratio to calibrate $\overline{k}$.

First, define a firm's advertising cost to profit ratio κ_i as the proportion of total advertising investment cost in its profit earned in product market.

1. When firms play Cournot and in an equilibrium invest in cooperative advertising, the advertising cost to profit ratio is

$$\kappa_i^C(m) = \kappa^C(m) = \frac{k_1}{k}.$$

2. When firms play Cournot and in an equilibrium invest in predatory advertising, the advertising cost to profit ratio is

$$\kappa_i^C(n) = \kappa^C(n) = \frac{k_2}{k}.$$

[2]See the last subsection of Appendix to Chapter 3.

3. When firms play Bertrand and in an equilibrium invest in cooperative advertising, the advertising cost to profit ratio is

$$\kappa_i^B(m) = \kappa^B(m) = \frac{k_3}{k}.$$

4. When firms play Bertrand and in an equilibrium invest in predatory advertising, the advertising cost to profit ratio is

$$\kappa_i^B(n) = \kappa^B(n) = \frac{k_4}{k}.$$

It can be easily shown that

$$\kappa^C(m) > \kappa^B(m), \ \kappa^C(n) < \kappa^B(n).$$

Next, how do we use the above results to impose an 'appropriate' upper bound on k? First, in empirical work, industrial organization economists often care about the advertising to sales ratio, which is smaller than the advertising cost to profit ratio. Given this fact, it is possible to calibrate four upper bounds on k using the data collected from the real world or the estimation results of empirical researches.

In particular, if in a given industry, the advertising to sales ratio is ϕ, then we have the following results.

1. If firms play Cournot and in an equilibrium invest in cooperative advertising, we must have $\frac{k_1}{k} > \phi$. So, the upper bound calibrated should be $\frac{k_1}{\phi}$,

2. If firms play Cournot and in an equilibrium invest in predatory advertising, we must have $\frac{k_2}{k} > \phi$. So, the upper bound calibrated should be $\frac{k_2}{\phi}$,

3. If firms play Bertrand and in an equilibrium invest in cooperative advertising, we must have $\frac{k_3}{k} > \phi$. So, the upper bound calibrated should be $\frac{k_3}{\phi}$,

4. If firms play Bertrand and in an equilibrium invest in predatory advertising, we must have $\frac{k_4}{k} > \phi$. So, the upper bound calibrated should be $\frac{k_4}{\phi}$.

In the simulation, we treat

$$\bar{k} = \frac{\min\{k_2, k_3\}}{\phi},$$

as the upper bound on k.

In general, we do not know the market conduct and the equilibrium investment behavior of firms and what is available is the data or the estimation result on advertising to sales ratios. Hence, we should follow a prudential strategy that given an observed ϕ, whatever the form of competition and whatever the equilibrium type of advertising, the advertising cost to profit ratio should be greater than ϕ.

Using k_1 to represent k_i, k_c, $\underline{k}''$ and $\overline{k}$

According to Assumption 3.1, it is easy to show that

$$k_2 = \left[\frac{(2b+1)^2}{(2b-1)^2 \mu^2}\right] k_1,$$
$$k_3 = \left[\frac{(b-1)(2b+1)^2}{(b+1)(2b-1)^2}\right] k_1,$$
$$k_4 = \left[\frac{(b+1)}{(b-1)\mu^2}\right] k_1,$$

and

$$k_5 = \left[\sqrt{(5b^2-1)} - (b-1)\right]\left[\frac{(b-1)(2b+1)^2}{(b+1)(2b-1)^2}\right] k_1,$$
$$k_6 = \left[\sqrt{(5b^2-1)} + (b+1)\right]\left[\frac{(b+1)}{(b-1)\mu^2}\right] k_1.$$

Note that we can write k_c as follows.

$$k_c = k_1 \delta,$$

where

$$\delta \in \left\{ \begin{array}{c} \left[\sqrt{(5b^2-1)} - (b-1)\right]\left[\frac{(b-1)(2b+1)^2}{(b+1)(2b-1)^2}\right], \\ \left[\sqrt{(5b^2-1)} + (b+1)\right]\left[\frac{(b+1)}{(b-1)\mu^2}\right] \end{array} \right\}.$$

In addition,

$$\underline{k}'' = k_1 \max\left\{\left(1+\sqrt{2}\right), \left(\frac{3}{2}+\frac{1}{2}\sqrt{13}\right)\left[\frac{(b+1)}{(b-1)\mu^2}\right]\right\},$$
$$\overline{k} = k_1 \left[\frac{\min\left\{\frac{(2b+1)^2}{(2b-1)^2\mu^2}, \frac{(b-1)(2b+1)^2}{(b+1)(2b-1)^2}\right\}}{\phi}\right].$$

Simulation on the robust proportion

Given the above results, we have

$$\begin{aligned} l &= \frac{k_c - \underline{k}''}{\overline{k} - \underline{k}''} \\ &= \frac{\delta - \max\left\{(1+\sqrt{2}),\left(\frac{3}{2}+\frac{1}{2}\sqrt{13}\right)\left[\frac{(b+1)}{(b-1)\mu^2}\right]\right\}}{\frac{\min\left\{\frac{(2b+1)^2}{(2b-1)^2\mu^2},\frac{(b-1)(2b+1)^2}{(b+1)(2b-1)^2}\right\}}{\phi} - \max\left\{(1+\sqrt{2}),\left(\frac{3}{2}+\frac{1}{2}\sqrt{13}\right)\left[\frac{(b+1)}{(b-1)\mu^2}\right]\right\}}. \end{aligned}$$

Furthermore, given the values of parameters b and μ, and the data or estimation results on ϕ, we can calculate the two potential values of δ and the other three numbers presented in the above formula. In addition, according to Proposition 3.1, we can infer from the values of b and μ that in an equilibrium, whether firms make cooperative or predatory advertising investments. Hence, we can decide in that case which value of δ we should use to calculate l.

According to the relationship between b and μ, we can calculate l in three cases, i.e.,

1. $\mu > \frac{2b^2+b-1}{2b^2-b-1}$, and whatever the form of product market competition, cooperative advertising will be present in an equilibrium,

2. $\frac{2b^2+b-1}{2b^2-b-1} > \mu > \frac{2b+1}{2b-1}$, and cooperative advertising will be present in an equilibrium when firms play Cournot, while predatory advertising will be present in an equilibrium when firms play Bertrand.

3. $\frac{2b+1}{2b-1} > \mu$, and whatever the form of product market competition, predatory advertising will be present in an equilibrium.

Note that, given b and μ, if we find a ϕ such that the calibrated $\overline{k}$ is smaller than $\underline{k}''$, then that case should be ignored.

5.2.12 Further discussion on the second-order condition for welfare maximization

In the text we directly assume that the welfare function is strictly concave because it is not straightforward to identify the condition that guarantees strict concavity. However, it is quite helpful to explore the implications of this assumption. In particular, strict concavity implies that

$$\frac{\partial^2 W}{\partial s^2} < 0, \ \frac{\partial^2 W}{\partial \tau^2} < 0.$$

These conditions could enable us to identify a 'reasonable' lower bound on k in each case of policy analysis. As subsection 5.2.5 shows, in the case of first-best policy analysis, such a lower bound helps us prove Corollary 3.1; As subsection 5.2.11 shows, in the case of second-best trade policy analysis, such a lower bound helps us do simulation. Of course, like before, we also have $k < \overline{k}$.

Cournot competition

First consider Cournot competition.

Case 1 $\mu > \frac{2b+1}{2b-1}$

Step 1. Note that in this case,

$$\frac{\partial^2 W}{\partial s^2} = \frac{\partial x_i}{\partial s}(\cdot)\left[\left(\frac{\partial a_i}{\partial m_j} - \frac{\partial x_j}{\partial m_j}\right)\frac{\partial m_j}{\partial s} - \frac{\partial x_j}{\partial s} - 1\right],$$

and

$$sign\ \frac{\partial^2 W}{\partial s^2} = sign\ \left[\left(1-2b^2\right)k^2 + \left(4b^2-b-2\right)k_1 k + \left(2b^2+b\right)(k_1)^2\right].$$

Let

$$\left(1-2b^2\right)k^2 + \left(4b^2-b-2\right)k_1 k + \left(2b^2+b\right)(k_1)^2 = 0.$$

The solutions to this quadratic equation in k are as follows.

$$k^s(0) \in \left\{ \begin{array}{l} \frac{1}{2(-1+2b^2)}\left(-b+4b^2-2-\sqrt{(-23b^2+32b^4+4)}\right)k_1, \\ \frac{1}{2(-1+2b^2)}\left(-b+4b^2-2+\sqrt{(-23b^2+32b^4+4)}\right)k_1 \end{array} \right\}.$$

Note that the coefficient of k^2 is $1-2b^2<0$, since $b>1$. So, for an arbitrarily chosen b, when k is smaller than the minimum of these two roots, or when it is bigger than the maximum of these two roots, $\frac{\partial^2 W}{\partial s^2} < 0$. (The arguments in other cases are similar to the arguments in this case.)

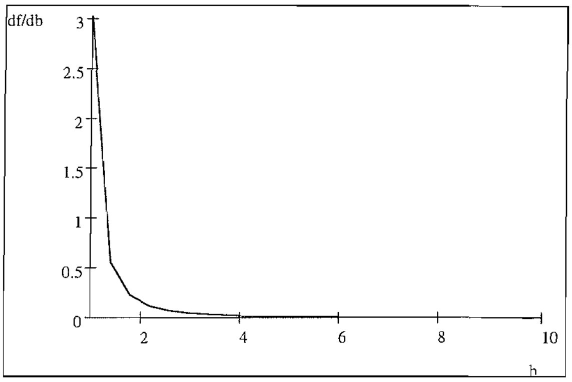
Step 2. Consider the first root. Let

$$f = \frac{1}{2(-1+2b^2)}\left(-b+4b^2-2-\sqrt{(-23b^2+32b^4+4)}\right).$$

Then

$$\begin{aligned}\frac{df}{db} &= \frac{b}{16b^4-16b^2+4}\left(8b-32b^2+8\sqrt{32b^4-23b^2+4}+16\right) \\ &\quad + \frac{1}{4b^2-2}\left(8b-\frac{1}{2}\frac{128b^3-46b}{\sqrt{32b^4-23b^2+4}}-1\right).\end{aligned}$$

From the graph of $\frac{df}{db}$,



we know that it is an increasing function. Take the following limit,

$$\lim_{b\to+\infty} \frac{1}{2\left(-1+2b^2\right)}\left(-b+4b^2-2-\sqrt{\left(-23b^2+32b^4+4\right)}\right) = 1-\sqrt{2} < 0.$$

This implies that we always have

$$k = \frac{1}{2\left(-1+2b^2\right)}\left(-b+4b^2-2-\sqrt{\left(-23b^2+32b^4+4\right)}\right)k_1 < 0.$$

But k cannot be smaller than the first root. Otherwise Assumption 3.1 is violated.

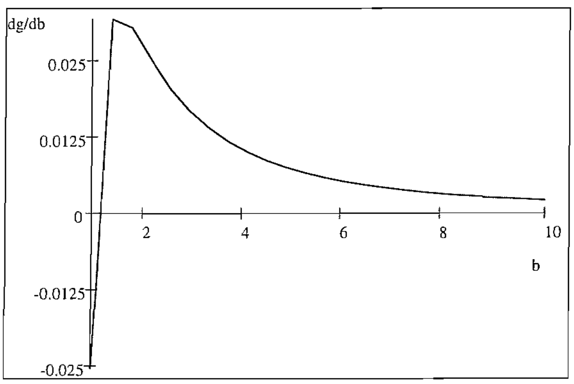
Step 3. Consider the second root. Let

$$g = \frac{1}{2\left(-1+2b^2\right)}\left(-b+4b^2-2+\sqrt{\left(-23b^2+32b^4+4\right)}\right).$$

Then

$$\begin{aligned}\frac{dg}{db} &= \frac{b}{16b^4-16b^2+4}\left(8b-32b^2-8\sqrt{32b^4-23b^2+4}+16\right)\\ &\quad+\frac{1}{4b^2-2}\left(8b+\frac{1}{2}\frac{128b^3-46b}{\sqrt{32b^4-23b^2+4}}-1\right).\end{aligned}$$

From the graph of $\frac{dg}{db}$,



we know that g has a critical point and it is a minimizer. Take the following limits,

$$\lim_{b\to 1^+} \frac{1}{2\left(-1+2b^2\right)}\left(-b+4b^2-2+\sqrt{(-23b^2+32b^4+4)}\right)=\frac{1}{2}+\frac{1}{2}\sqrt{13},$$
$$\lim_{b\to +\infty} \frac{1}{2\left(-1+2b^2\right)}\left(-b+4b^2-2+\sqrt{(-23b^2+32b^4+4)}\right)=1+\sqrt{2}.$$

It could be easily shown that $1+\sqrt{2}>\frac{1}{2}+\frac{1}{2}\sqrt{13}>2$.

Hence, we have

$$\frac{\partial^2 W}{\partial s^2}<0 \Rightarrow k>\left(1+\sqrt{2}\right)k_1. \tag{5.10}$$

Step 4. Note that in this case,

$$\frac{\partial^2 W}{\partial \tau^2}=\frac{\partial x_i}{\partial \tau}(\cdot)\left[\left(\frac{\partial a_i}{\partial m_j}-\frac{\partial x_j}{\partial m_j}\right)\frac{\partial m_j}{\partial \tau}\right]-\frac{\partial m_i}{\partial \tau},$$

and

$$sign\ \frac{\partial^2 W}{\partial \tau^2}=sign\ \left[-k^2+3k_1k-\left(k_1\right)^2\right].$$

Let

$$k^2-3k_1k+\left(k_1\right)^2=0.$$

The solutions to this quadratic equation in k are as follows.

$$k^{\tau}(0)\in\left\{\left(\frac{3}{2}-\frac{1}{2}\sqrt{5}\right)k_1,\ \left(\frac{3}{2}+\frac{1}{2}\sqrt{5}\right)k_1\right\}.$$

Note that the coefficient of k^2 is $-1<0$. So, for an arbitrarily chosen b, when k is smaller than the minimum of these two roots, or when it is bigger than the maximum of these two roots, $\frac{\partial^2 W}{\partial \tau^2}<0$. (The arguments in other cases are similar to the arguments in this case.)

Obviously, k cannot be smaller than the first root. Otherwise, Assumption 3.1 is violated. Hence, we have

$$\frac{\partial^2 W}{\partial \tau^2}<0 \Rightarrow k>\left(\frac{3}{2}+\frac{1}{2}\sqrt{5}\right)k_1. \tag{5.11}$$

Step 5. Note that

$$\left(\frac{3}{2}+\frac{1}{2}\sqrt{5}\right)k_1>\left(1+\sqrt{2}\right)k_1.$$

Therefore,

$$\left\{\begin{array}{l}\frac{\partial^2 W}{\partial s^2}<0\\ \frac{\partial^2 W}{\partial \tau^2}<0\end{array}\right. \Rightarrow k>\left(\frac{3}{2}+\frac{1}{2}\sqrt{5}\right)k_1. \tag{5.12}$$

Case 2 $\mu < \frac{2b+1}{2b-1}$

Step 1. Note that in this case,

$$\frac{\partial^2 W}{\partial s^2} = \frac{\partial x_i}{\partial s}(\cdot)\left[\left(\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j}\right)\frac{\partial n_j}{\partial s} - \frac{\partial x_j}{\partial s} - 1\right],$$

and

$$sign\ \frac{\partial^2 W}{\partial s^2} = sign\ \left[\left(1-2b^2\right)k^2 + \left(4b^2+b-2\right)k_2 k + \left(2b^2-b\right)(k_2)^2\right].$$

Let

$$\left(1-2b^2\right)k^2 + \left(4b^2+b-2\right)k_2 k + \left(2b^2-b\right)(k_2)^2 = 0.$$

The solutions are as follows.

$$k^s(0) \in \left\{ \begin{array}{l} \frac{1}{2(-1+2b^2)}\left(4b^2-2+b-\sqrt{(32b^4-23b^2+4)}\right)k_2, \\ \frac{1}{2(-1+2b^2)}\left(4b^2-2+b+\sqrt{(32b^4-23b^2+4)}\right)k_2 \end{array} \right\}.$$

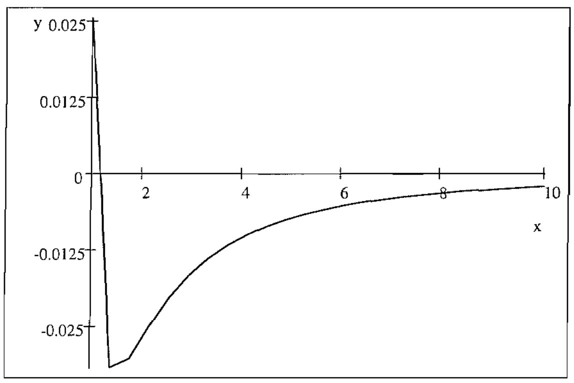
Step 2. Consider the first root. Let

$$f = \frac{1}{2\left(-1+2b^2\right)}\left(4b^2-2+b-\sqrt{(32b^4-23b^2+4)}\right).$$

Then

$$\begin{aligned} \frac{df}{db} &= \frac{b}{16b^4-16b^2+4}\left(8\sqrt{32b^4-23b^2+4}-32b^2-8b+16\right) \\ &\quad + \frac{1}{4b^2-2}\left(8b-\frac{1}{2}\frac{128b^3-46b}{\sqrt{32b^4-23b^2+4}}+1\right). \end{aligned}$$

From the graph of $\frac{df}{db}$,



we know that f has a critical point and it is a maximizer. In addition, the critical point is $b = 1.0679$ and

$$f|_{b=1.0679} = -0.302.$$

But k cannot be smaller than the first root. Otherwise Assumption 3.1 is violated.

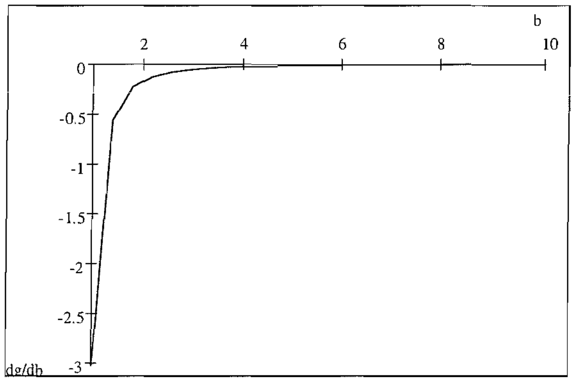
Step 3. Consider the second root. Let

$$g = \frac{1}{2(-1+2b^2)}\left(4b^2 - 2 + b + \sqrt{(32b^4 - 23b^2 + 4)}\right).$$

Then

$$\begin{aligned}\frac{dg}{db} &= \frac{b}{16b^4 - 16b^2 + 4}\left(16 - 32b^2 - 8\sqrt{32b^4 - 23b^2 + 4} - 8b\right) \\ &\quad + \frac{1}{4b^2 - 2}\left(8b + \frac{1}{2}\frac{128b^3 - 46b}{\sqrt{32b^4 - 23b^2 + 4}} + 1\right).\end{aligned}$$

From the graph of $\frac{dg}{db}$,



we know that g is a decreasing function. Take the following limit,

$$\lim_{b\to 1^+} \frac{1}{2(-1+2b^2)}\left(4b^2 - 2 + b + \sqrt{(32b^4 - 23b^2 + 4)}\right) = \frac{3}{2} + \frac{1}{2}\sqrt{13}.$$

Hence, we have

$$\frac{\partial^2 W}{\partial s^2} < 0 \Rightarrow k > \left(\frac{3}{2} + \frac{1}{2}\sqrt{13}\right) k_2. \tag{5.13}$$

Step 4. Note that in this case,

$$\frac{\partial^2 W}{\partial \tau^2} = \frac{\partial x_i}{\partial \tau}(\cdot)\left[\left(\frac{\partial a_i}{\partial n_j} - \frac{\partial x_j}{\partial n_j}\right)\frac{\partial n_j}{\partial \tau}\right] - \frac{\partial n_i}{\partial \tau},$$

and

$$sign\ \frac{\partial^2 W}{\partial \tau^2} = sign\ \left[-k^2 + 3k_2 k - (k_2)^2\right].$$

Let

$$k^2 - 3k_2 k + (k_2)^2 = 0.$$

The solutions are as follows.

$$k^{\tau}(0) \in \left\{ \left(\frac{3}{2} - \frac{1}{2}\sqrt{5} \right) k_2,\ \left(\frac{3}{2} + \frac{1}{2}\sqrt{5} \right) k_2 \right\}.$$

Obviously, k cannot be smaller than the first root. Otherwise, Assumption 3.1 is violated. Hence, we have

$$\frac{\partial^2 W}{\partial \tau^2} < 0 \Rightarrow \left(\frac{3}{2} + \frac{1}{2}\sqrt{5} \right) k_2. \tag{5.14}$$

Step 5. Note that

$$\left(\frac{1}{2}\sqrt{13} + \frac{3}{2} \right) k_2 > \left(\frac{3}{2} + \frac{1}{2}\sqrt{5} \right) k_2.$$

Hence,

$$\left\{ \begin{array}{l} \frac{\partial^2 W}{\partial s^2} < 0 \\ \frac{\partial^2 W}{\partial \tau^2} < 0 \end{array} \right. \Rightarrow k > \left(\frac{1}{2}\sqrt{13} + \frac{3}{2} \right) k_2. \tag{5.15}$$

Bertrand competition

Second consider Bertrand competition.

Case 1 $\mu > \frac{2b^2+b-1}{2b^2-b-1}$

Step 1. Note that in this case,

$$\frac{\partial^2 W}{\partial s^2} = \left[\frac{\partial p_i}{\partial s}(\cdot) + 1 \right] \left\{ \left[\left(\frac{\partial \alpha_i}{\partial m_j} + \gamma \frac{\partial p_j}{\partial m_j} \right) \frac{\partial m_j}{\partial s} \right] + \gamma \frac{\partial p_j}{\partial s} \right\} - \frac{\partial x_i}{\partial s}(\cdot),$$

and

$$sign\ \frac{\partial^2 W}{\partial s^2} = sign \left[-2b^2k^2 + \left(4b^2 - b\right) k_3 k + (b+1)(2b-1)(k_3)^2 \right].$$

Let

$$-2b^2k^2 + \left(4b^2 - b\right) k_3 k + (b+1)(2b-1)(k_3)^2 = 0.$$

The solution is as follows.

$$k^s(0) \in \left\{ \left(b - \frac{1}{4} - \frac{1}{4}\sqrt{(32b^2-7)} \right) \frac{k_3}{b}, \left(b - \frac{1}{4} + \frac{1}{4}\sqrt{(32b^2-7)} \right) \frac{k_3}{b} \right\}.$$

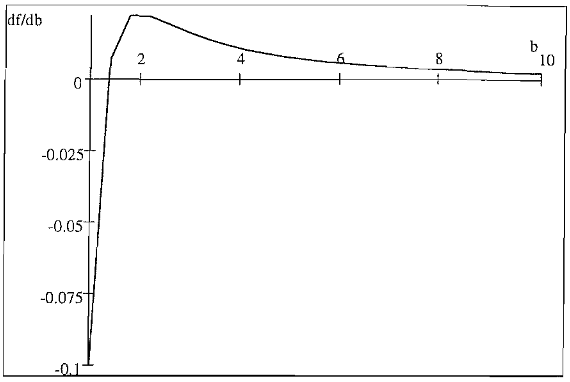
Step 2. Consider the first root. Let

$$f = \left(b - \frac{1}{4} - \frac{1}{4}\sqrt{(32b^2-7)} \right) \frac{1}{b}.$$

Then,

$$\frac{df}{db} = \frac{1}{b}\left(1 - 8\frac{b}{\sqrt{32b^2-7}} \right) + \frac{1}{b^2}\left(\frac{1}{4}\sqrt{32b^2-7} - b + \frac{1}{4} \right).$$

From the graph of $\frac{df}{db}$,



we know that f has a critical point and it is a minimizer. Take the following limits,

$$\lim_{b\to 1^+}\left(b-\frac{1}{4}-\frac{1}{4}\sqrt{(32b^2-7)}\right)\frac{1}{b}=-\frac{1}{2},$$
$$\lim_{b\to +\infty}\left(b-\frac{1}{4}-\frac{1}{4}\sqrt{(32b^2-7)}\right)\frac{1}{b}=1-\sqrt{2}.$$

These results implies that we always have

$$k=\left(b-\frac{1}{4}-\frac{1}{4}\sqrt{(32b^2-7)}\right)\frac{k_3}{b}<0.$$

But k cannot be smaller than the first root. Otherwise Assumption 3.1 is violated.

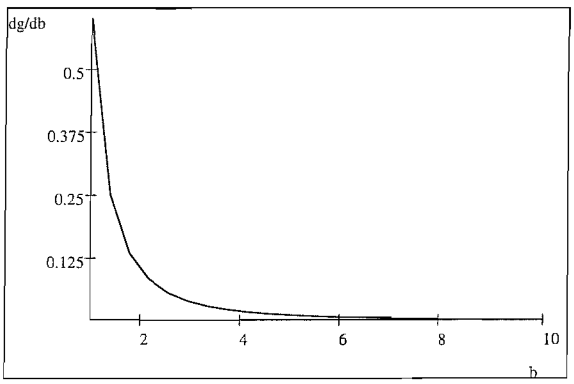
Step 3. Consider the second root. Let

$$g=\left(b-\frac{1}{4}+\frac{1}{4}\sqrt{(32b^2-7)}\right)\frac{1}{b}.$$

Then

$$\frac{dg}{db}=\frac{1}{b}\left(8\frac{b}{\sqrt{32b^2-7}}+1\right)+\frac{1}{b^2}\left(\frac{1}{4}-\frac{1}{4}\sqrt{32b^2-7}-b\right).$$

From the graph of $\frac{dg}{db}$,



we know that g is an increasing function. Take the following limit,

$$\lim_{b \to +\infty} \left(b - \frac{1}{4} + \frac{1}{4}\sqrt{(32b^2 - 7)} \right) \frac{1}{b} = 1 + \sqrt{2}.$$

Hence, we have

$$\frac{\partial^2 W}{\partial s^2} < 0 \Rightarrow k > \left(1 + \sqrt{2}\right) k_3. \tag{5.16}$$

Step 4. Note that in this case,

$$\frac{\partial^2 W}{\partial \tau^2} = \frac{\partial p_i}{\partial \tau} (\cdot) \left(\frac{\partial \alpha_i}{\partial m_j} + \gamma \frac{\partial p_j}{\partial m_j} \right) \frac{\partial m_j}{\partial \tau} - \frac{\partial m_i}{\partial \tau},$$

and

$$sign\ \frac{\partial^2 W}{\partial \tau^2} = sign \left[-k^2 + 3k_3 k - (k_3)^2 \right].$$

Let

$$k^2 - 3k_3 k + (k_3)^2 = 0.$$

The solutions are as follows.

$$k^{\tau}(0) \in \left\{ \left(\frac{3}{2} - \frac{1}{2}\sqrt{5} \right) k_3,\ \left(\frac{3}{2} + \frac{1}{2}\sqrt{5} \right) k_3 \right\}.$$

Obviously, k cannot be smaller than the first root. Otherwise, Assumption 3.1 is violated. Hence, we have

$$\frac{\partial^2 W}{\partial \tau^2} < 0 \Rightarrow \left(\frac{3}{2} + \frac{1}{2}\sqrt{5} \right) k_3. \tag{5.17}$$

Step 5. Note that

$$\left(1 + \sqrt{2}\right) k_3 < \left(\frac{3}{2} + \frac{1}{2}\sqrt{5} \right) k_3.$$

Hence,

$$\left\{ \begin{array}{l} \frac{\partial^2 W}{\partial s^2} < 0 \\ \frac{\partial^2 W}{\partial \tau^2} < 0 \end{array} \right. \Rightarrow k > \left(\frac{3}{2} + \frac{1}{2}\sqrt{5} \right) k_3. \tag{5.18}$$

Case 2 $\mu < \frac{2b^2+b-1}{2b^2-b-1}$

Step 1. Note that in this case,

$$\frac{\partial^2 W}{\partial s^2} = \left[\frac{\partial p_i}{\partial s} (\cdot) + 1 \right] \left\{ \left[\left(\frac{\partial \alpha_i}{\partial n_j} + \gamma \frac{\partial p_j}{\partial n_j} \right) \frac{\partial n_j}{\partial s} \right] + \gamma \frac{\partial p_j}{\partial s} \right\} - \frac{\partial x_i}{\partial s} (\cdot),$$

and

$$sign\ \frac{\partial^2 W}{\partial s^2} = sign \left[-2b^2 k^2 + \left(4b^2 + b\right) k_4 k + (b-1)(2b+1)(k_4)^2 \right].$$

Let

$$-2b^2k^2 + \left(4b^2 + b\right) k_4 k + (b-1)(2b+1)(k_4)^2 = 0.$$

The solutions are as follows.

$$k^s(0) \in \left\{ \begin{array}{c} \left(b + \frac{1}{4} - \frac{1}{4}\sqrt{(32b^2 - 7)}\right) \frac{k_4}{b}, \\ \left(b + \frac{1}{4} + \frac{1}{4}\sqrt{(32b^2 - 7)}\right) \frac{k_4}{b} \end{array} \right\}.$$

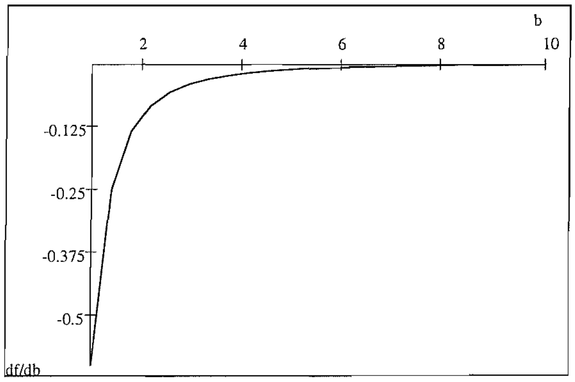
Step 2. Consider the first root. Let

$$f = \left(b + \frac{1}{4} - \frac{1}{4}\sqrt{(32b^2 - 7)}\right) \frac{1}{b}.$$

Then

$$\frac{df}{db} = \frac{1}{b}\left(1 - 8\frac{b}{\sqrt{32b^2 - 7}}\right) + \frac{1}{b^2}\left(\frac{1}{4}\sqrt{32b^2 - 7} - b - \frac{1}{4}\right).$$

From the graph of $\frac{df}{db}$,



we know that f is a decreasing function. Take the following limit,

$$\lim_{b \to 1^+} \left(b + \frac{1}{4} - \frac{1}{4}\sqrt{(32b^2 - 7)}\right) \frac{1}{b} = 0.$$

But k cannot be smaller than the first root. Otherwise, Assumption 3.1 is violated.

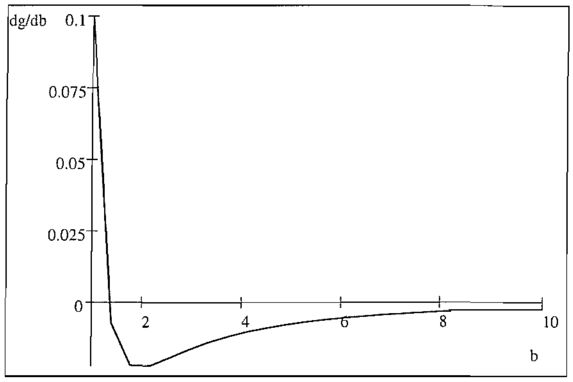
Step 3. Consider the second root. Let

$$g = \left(b + \frac{1}{4} + \frac{1}{4}\sqrt{(32b^2 - 7)}\right) \frac{1}{b}.$$

Then

$$\frac{dg}{db} = \frac{1}{b}\left(8\frac{b}{\sqrt{32b^2 - 7}} + 1\right) + \frac{1}{b^2}\left(-b - \frac{1}{4}\sqrt{32b^2 - 7} - \frac{1}{4}\right).$$

From the graph of $\frac{dg}{db}$,



we know that g has a critical point and it is a maximizer. In addition, the critical point is $b = 1.3229$ and

$$g|_{b=1.3229} = 2.5119.$$

Hence, we have

$$\frac{\partial^2 W}{\partial s^2} < 0 \Rightarrow k > (2.5119)\, k_4. \tag{5.19}$$

Step 4. Note that in this case,

$$\frac{\partial^2 W}{\partial \tau^2} = \frac{\partial p_i}{\partial \tau} (\cdot) \left(\frac{\partial \alpha_i}{\partial n_j} + \gamma \frac{\partial p_j}{\partial n_j} \right) \frac{\partial n_j}{\partial \tau} - \frac{\partial n_i}{\partial \tau},$$

and

$$sign\ \frac{\partial^2 W}{\partial \tau^2} = sign \left[-k^2 + 3k_4 k - (k_4)^2 \right].$$

Let

$$k^2 - 3k_4 k + (k_4)^2 = 0.$$

The solutions are as follows.

$$k^{\tau}(0) \in \left\{ \left(\frac{3}{2} - \frac{1}{2}\sqrt{5} \right) k_4,\ \left(\frac{3}{2} + \frac{1}{2}\sqrt{5} \right) k_4 \right\}.$$

Obviously, k cannot be smaller than the first root. Otherwise, Assumption 3.1 is violated. Hence, we have

$$\frac{\partial^2 W}{\partial \tau^2} < 0 \Rightarrow \left(\frac{3}{2} + \frac{1}{2}\sqrt{5} \right) k_4. \tag{5.20}$$

Step 5. Note that

$$(2.5119)\, k_4 < \left(\frac{3}{2} + \frac{1}{2}\sqrt{5} \right) k_4.$$

Hence,

$$\begin{cases} \frac{\partial^2 W}{\partial s^2} < 0 \\ \frac{\partial^2 W}{\partial \tau^2} < 0 \end{cases} \Rightarrow k > \left(\frac{3}{2} + \frac{1}{2}\sqrt{5}\right) k_4. \tag{5.21}$$

Summary

Based on the above discussion, we have the following results.

1. In the first-best policy analysis, the 'reasonable' lower bound on k is

$$\underline{k}' = \max\left\{\left(\frac{3}{2} + \frac{1}{2}\sqrt{5}\right) k_1, \left(\frac{3}{2} + \frac{1}{2}\sqrt{13}\right) k_4\right\}. \tag{5.22}$$

   So, in fact, we require in this case $\underline{k}' < k < \overline{k}$.

2. In the second-best trade policy analysis, the 'reasonable' lower bound on k is

$$\underline{k}'' = \max\left\{\left(1 + \sqrt{2}\right) k_1, \left(\frac{3}{2} + \frac{1}{2}\sqrt{13}\right) k_4\right\}. \tag{5.23}$$

   So, in fact, we require in this case $\underline{k}'' < k < \overline{k}$.

3. In the second-best industrial policy analysis, the 'reasonable' lower bound on k is

$$\underline{k}''' = \max\left\{\left(\frac{3}{2} + \frac{1}{2}\sqrt{5}\right) k_1, \left(\frac{3}{2} + \frac{1}{2}\sqrt{5}\right) k_4\right\}. \tag{5.24}$$

   So, in fact, we require in this case $\underline{k}''' < k < \overline{k}$.

5.3 Appendix to Chapter 4

5.3.1 Discussion of Assumption 4.1

In this subsection, we discuss the case of no intervention and show the reason why we made Assumption 4.1 in the text.

Cournot competition

It is straightforward to show that the SPNE is given by

$$x = \frac{2b\,(a - w) - (a - w_0^*)}{4b^2 - 1},\; x^* = \frac{2b\,(a - w_0^*) - (a - w)}{4b^2 - 1};$$
$$w = \frac{2b\,(a + w_0) - (a - w_0^*)}{4b}.$$

In an equilibrium,

$$x = \frac{2b\left(a-w_0\right)-\left(a-w_0^*\right)}{2\left(4b^2-1\right)},$$
$$\pi = b\left[\frac{2b\left(a-w_0\right)-\left(a-w_0^*\right)}{2\left(4b^2-1\right)}\right]^2,$$
$$\omega = \frac{\left[2b\left(a-w_0\right)-\left(a-w_0^*\right)\right]^2}{8b\left(4b^2-1\right)}.$$

We assume that in an equilibrium, $x>0$. It is straightforward to show

$$x>0$$
$$\Leftrightarrow$$
$$2b\left(a-w_0\right)-\left(a-w_0^*\right)>0. \tag{5.25}$$

I.e., without the government's intervention, the domestic firm can survive in product market competition.

Bertrand competition

It is straightforward to show that the SPNE is given by

$$p = \frac{\left(2\beta+\gamma\right)\alpha+2\beta^2 w+\beta\gamma w_0^*}{4\beta^2-\gamma^2},\ p^* = \frac{\left(2\beta+\gamma\right)\alpha+\beta\gamma w+2\beta^2 w_0^*}{4\beta^2-\gamma^2};$$
$$w = \frac{\left(2\beta+\gamma\right)\alpha+\left(2\beta^2-\gamma^2\right)w_0+\beta\gamma w_0^*}{2\left(2\beta^2-\gamma^2\right)}.$$

In an equilibrium,

$$x = \frac{\beta\left[\left(2\beta+\gamma\right)\alpha-\left(2\beta^2-\gamma^2\right)w_0+\beta\gamma w_0^*\right]}{2\left(4\beta^2-\gamma^2\right)},$$
$$\pi = \beta\left[\frac{\left(2\beta+\gamma\right)\alpha-\left(2\beta^2-\gamma^2\right)w_0+\beta\gamma w_0^*}{2\left(4\beta^2-\gamma^2\right)}\right]^2,$$
$$\omega = \frac{\beta\left[\left(2\beta+\gamma\right)\alpha-\left(2\beta^2-\gamma^2\right)w_0+\beta\gamma w_0^*\right]^2}{4\left(2\beta^2-\gamma^2\right)\left(4\beta^2-\gamma^2\right)}.$$

We assume that in an equilibrium, $x > 0$. It is straightforward to show

$$x > 0$$
$$\Leftrightarrow$$
$$(2\beta + \gamma)\,\alpha - \left(2\beta^2 - \gamma^2\right) w_0 + \beta\gamma w_0^* > 0$$
$$\Leftrightarrow$$
$$\left(2b^2 - 1\right)(a - w_0) - b\,(a - w_0^*) > 0. \tag{5.26}$$

I.e., without the government's intervention, the domestic firm can survive in product market competition.

Finally, notice that $\left(2b^2 - 1\right)(a - w_0) - b\,(a - w_0^*) > 0$ is sufficient for $2b\,(a - w_0) - (a - w_0^*) > 0$.

5.3.2 Proof of Proposition 4.2

Cournot competition

Now, national welfare is given by

$$G = \pi + \omega - \delta s x.$$

And it is straightforward to show that the optimal trade policy is given by[3]

$$s = \frac{\left[2b\,(a - w_0) - (a - w_0^*)\right]\left[\left(\frac{2b^2}{4b^2-1} + 1\right) - \delta\right]}{2b\left[2\delta - \left(\frac{2b^2}{4b^2-1} + 1\right)\right]}. \tag{5.27}$$

By Assumption 4.1, $2b\,(a - w_0) - (a - w_0^*) > 0$. Since $\delta > 1$, $2\delta - \left(\frac{2b^2}{4b^2-1} + 1\right) > 0$. Therefore,

$$sign\ s = sign\left[\left(\frac{2b^2}{4b^2 - 1} + 1\right) - \delta\right]. \tag{5.28}$$

Bertrand competition

Now, national welfare is given by

$$G = \pi + \omega - \delta s x.$$

[3]Notice that first of all, since $\delta > 1$, we have

$$\frac{d^2\left(\pi + \omega - \delta s x\right)}{ds^2} = -\frac{b\left[2\delta - \left(\frac{2b^2}{4b^2-1} + 1\right)\right]}{4b^2 - 1} < 0,$$

so, there is a unique interior solution. Next, when $\delta = 1$, s is given by expression (4.7).

And it is straightforward to show that the optimal trade policy is given by[4]

$$s = \frac{\left[(2\beta+\gamma)\,\alpha - \left(2\beta^2-\gamma^2\right)w_0 + \beta\gamma w_0^*\right]\left[\left(\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}+1\right)-\delta\right]}{\left(2\beta^2-\gamma^2\right)\left[2\delta-\left(\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}+1\right)\right]}. \tag{5.29}$$

By Assumption 4.1, $(2\beta+\gamma)\,\alpha - \left(2\beta^2-\gamma^2\right)w_0 + \beta\gamma w_0^* = \left(2b^2-1\right)(a-w_0) - b\,(a-w_0^*) > 0$. Since $\delta > 1$, $2\delta - \left(\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}+1\right) > 0$. Therefore,

$$\begin{aligned} sign\ s &= sign\left[\left(\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}+1\right)-\delta\right] \\ &= sign\left[\left(\frac{2b^2-1}{4b^2-1}+1\right)-\delta\right]. \end{aligned} \tag{5.30}$$

Notice that Condition (5.28) and (5.30) imply the Proposition immediately. ■

5.3.3 Proof of Proposition 4.3

Cournot competition

It is straightforward to show that the optimal trade policy is given by

$$s = \frac{\left[2b\,(a-w_0)-(a-w_0^*)\right]\left[\left(\frac{2b^2}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right)-\delta\right]}{2b\left[2\delta-\left(\frac{2b^2}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right)\right]}. \tag{5.31}$$

By Assumption 4.1, $2b\,(a-w_0)-(a-w_0^*) > 0$, and since condition (4.21) holds,

$$sign\ s = sign\left[\left(\frac{2b^2}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right)-\delta\right]. \tag{5.32}$$

Bertrand competition

It is straightforward to show that the optimal trade policy is given by

$$s = \frac{\left[(2\beta+\gamma)\,\alpha - \left(2\beta^2-\gamma^2\right)w_0 + \beta\gamma w_0^*\right]\left[\left(\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}+1\right)\left(\frac{\lambda+1}{\lambda}\right)-\delta\right]}{\left(2\beta^2-\gamma^2\right)\left[2\delta-\left(\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}+1\right)\left(\frac{\lambda+1}{\lambda}\right)\right]}. \tag{5.33}$$

[4]Notice that first of all, since $\delta > 1$, we have

$$\frac{d^2\left(\pi+\omega-\delta sx\right)}{ds^2} = -\frac{\beta\left(2\beta^2-\gamma^2\right)}{2\left(4\beta^2-\gamma^2\right)}\left[2\delta-\left(\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}+1\right)\right] < 0,$$

so, there is a unique interior solution. Next, when $\delta = 1$, s is given by expression (4.16).

By Assumption 4.1, $(2\beta+\gamma)\,\alpha-\left(2\beta^2-\gamma^2\right)w_0+\beta\gamma w_0^* = \left(2b^2-1\right)(a-w_0)-b\,(a-w_0^*) > 0$, and since condition (4.21) holds,

$$\begin{aligned} sign\ s &= sign\left[\left(\frac{2\beta^2-\gamma^2}{4\beta^2-\gamma^2}+1\right)\left(\frac{\lambda+1}{\lambda}\right)-\delta\right] \\ &= sign\left[\left(\frac{2b^2-1}{4b^2-1}+1\right)\left(\frac{\lambda+1}{\lambda}\right)-\delta\right]. \end{aligned} \tag{5.34}$$

Combining the arguments in the above two subsections, we establish the Proposition.

■

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