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Measurement: everywhere and nowhere in secondary mathematics

Keith Jones

University of Southampton

School mathematics is commonly structured into number, algebra, geometry and statistics. This raises the issue of where to place ideas within the topic of measurement since some aspects of measurement (such as measuring length or area) have a geometrical component, while other aspects of measurement (such as time or money) are about number. Furthermore, when actual measures are unknown, relationships between measures can be expressed – and this is one of the roots of algebra. Additionally, probability can be thought of as a form of measure (of uncertainty) and the various measures of data variation, such as standard deviation, can also be viewed as a form of measurement. All these considerations mean that the placing of measurement in the mathematics curriculum can be problematic for curriculum designers and policy makers; and equally tricky for teachers to teach in the most effective way. Informed by a review of the research basis for teaching key ideas in secondary school mathematics, this paper argues that measurement is both everywhere and nowhere in secondary mathematics; that is, measurement occurs across the topics that comprise secondary school mathematics, but the ideas of measurement are so scattered that the teaching of measurement in secondary school mathematics may lack some focus that might store up problems for learners as they progress with mathematics.

Keywords: teaching, learning, measurement

Introduction

Amongst the key ideas that comprise secondary school mathematics are those of measurement. With the mathematics curriculum commonly structured within a manageable number of headings such as number, algebra, geometry and statistics, the ideas of measurements are inevitably spread across such headings. This is because some aspects of measurement, such as measuring length or area, clearly relate to the geometric properties of shapes and hence could be listed under the geometry heading, yet at the same time, other aspects of measurement, such as time or money, are about number. Even when actual measures are not known, a relationship between measures can be expressed; this being one of the roots of algebra. What is more, probability can be thought of as a form of measure (of uncertainty) and the various measures of data variation, such as standard deviation, can also be viewed as a type of measurement.

Angle is an example of a mathematical idea that can be a property of a shape, and hence be geometrical, while also being a measure. Decimals, which arise when things are measured, are an example a form of number. Scaling, something which entails both measurement and proportional thinking, is a critical factor for success when learning to use maps.

All this means that positioning measurement in school mathematics curriculum documents can be problematic for curriculum designers and policy makers and equally tricky for teachers to teach in the most effective way. This paper utilises a review of the research basis for teaching key ideas in secondary school mathematics (being funded by the Nuffield Foundation at the time of writing) to examine the extent to which measurement occurs across the secondary school mathematics and the consequences this might have for learners as they progress with mathematics. Using the example of England, the paper begins by examining a specific arrangement of the secondary school mathematics curriculum.

Measurement in the secondary school mathematics curriculum for England

For England, the current version of the National Curriculum (see QCA 2007a) states that the study of mathematics for 11-13 year-olds should include the range of mathematics set out in Figure 1 (for the specification of the curriculum for 14-16 year-olds, see QCA 2007b).

Number and algebra

a rational numbers, their properties and their different representations
b rules of arithmetic applied to calculations and manipulations with rational numbers
c applications of ratio and proportion
d accuracy and rounding
e algebra as generalised arithmetic
f linear equations, formulae, expressions and identities
g analytical, graphical and numerical methods for solving equations
h polynomial graphs, sequences and functions

Geometry and measures

a properties of 2D and 3D shapes
b constructions, loci and bearings
c Pythagoras' theorem
d transformations
e similarity, including the use of scale
f points, lines and shapes in 2D coordinate systems
g units, compound measures and conversions
h perimeters, areas, surface areas and volumes

Statistics

a the handling data cycle
b presentation and analysis of grouped and ungrouped data, including time series and lines of best fit
c measures of central tendency and spread
d experimental and theoretical probabilities, including those based on equally likely outcomes.

Figure 1: the secondary school mathematics curriculum for England for 11-13 year-olds
(an underscored term denotes that there is an explanatory note within the curriculum document)

As can be seen from the specification of the curriculum for 11-13 year-olds given in Figure 1, the word "measures" occurs explicitly under the headings of "geometry and measures" and "statistics". Further insight into the curriculum specification for England can be found in the "Attainment Targets" (see QCA 2007a) that accompany the curriculum (the "Attainment Targets" are in the form of output statements designed to capture the range of knowledge, skills and understanding which pupils are expected to master as they progress through school; with "level 4" being the expected attainment of the modal 11 year old and "level 7" being a high-attaining 13 year old). Aspects of measurement that occur within the "Attainment Targets" are shown in Figure 2.

National Curriculum "level"	Attainment target	Outcome statement
4	Number and algebra	They calculate fractional or percentage parts of quantities and measurements, using a calculator where appropriate.
4	Geometry and measures	They choose and use appropriate units and tools, interpreting, with appropriate accuracy, numbers on a range of measuring instruments.
5	Geometry and measures	When constructing models and drawing or using shapes, pupils measure and draw angles to the nearest degree and use language associated with angles.
5	Geometry and measures	They make sensible estimates of a range of measures in relation to everyday situations
7	Geometry and measures	They appreciate the imprecision of measurement and recognise that a measurement given to the nearest whole number may be inaccurate by up to one half in either direction. They understand and use compound measures, such as speed.
7	Handling data	They use measures of average and range, with associated frequency polygons, as appropriate, to compare distributions and make inferences.
Exceptional performance	Geometry and measures	They appreciate the continuous nature of scales that are used to make measurements.

Figure 2: measurement in the "Attainment Targets" for 11-13 year-olds

As Figure 2 demonstrates, measurement occurs across the three strands of the mathematics National Curriculum for England (that is, across the strands of "Number and algebra", "Geometry and measures", and "Statistics/ Handling data").

Greater detail on the coverage of measurement is provided by the "Progression Maps" devised by the "National Strategies" for England; these "Progression Maps" start with mathematics teaching objectives and provide guidance on the levels of difficulty described by the objectives. The extract in Figure 3 is from the Progression Map for "Measures" (National Strategies, n.d)

Measures

Work on measures involves understanding and using both metric and imperial systems of measurement (length, area, volume, capacity, mass (weight) and time). It also involves using measurements to calculate areas and volumes of common shapes (triangle, square, rectangle, parallelogram, trapezium and circle) and solids (cube, cuboid, pyramid and sphere). A crucial aspect of measurement is the ability to estimate.

Children should be familiar with the following units of measurement:

- length: mm (millimetre), cm (centimetre), m (metre), km (kilometre), inch, foot, mile
- area: cm^2 (square centimetre), m^2 (square metre)
- volume: cm^3 (cubic centimetre), m^3 (cubic metre)
- capacity: l (litre), ml (millilitre), pint, gallon
- mass (weight): g (gram), kg (kilogram), tonne (= 1000 kg), pound
- time: second, minute, hour, week, year
- speed: metres per second
- density: grams per cubic centimetre.

They should be able to convert from one metric unit to another. They should also be able to convert between inches and centimetres, miles and kilometres, litres and pints, and pounds and kilograms.

Figure 3: extract from the Progression Map for "Measures"

The extract in Figure 3 confirms the range of measures that are expected to be taught to 11-13 year-olds in England (even though measurement of angle, for instance, is omitted, presumably in error). This range of measures includes geometrical measures (such as length, area and volume), non-geometrical measures (such as time), and compound measures (such as speed and density).

The example of England illustrates the extent to which measurement occurs across the secondary school mathematics. The next section considers the consequences that this might have for learners as they progress with mathematics.

The progress of learners with ideas of measurement

As the TIMSS study (Schmidt et al, 1997: 64) reveals, in countries across the world the ideas within measurement begin to be introduced in the early years of schooling, become a major focus when pupils are 8 or 9, but are not completed until students are 16 or 17. It is only by the age of 14 that the median pupil attainment across the world extends to understanding measurement in several settings (Kelly, Mullis and Martin, 2000: 13). Not only that, but even at the age of 14 it is only the top 10% of pupils internationally who can solve time-distance-rate problems involving conversion within a system of measures.

In terms of measuring lengths, pupils generally do succeed with this prior to tackling the measurement of area and volume. Even so, length measuring poses particular problems for children despite them being likely to understand the underlying idea. One source of difficulty for pupils is in grasping how to imagine correspondence between the iterated units on the ruler and imagined equivalent units on the line being measured. One common mistake is to set the 1 cm ruler mark at one end of the line segment rather than the 0 cm mark. Research evidence (for example, Sowder, Wearne, Martin and Strutchens 2004) indicates that pupils may apply a poorly-understood procedure when they measure length rather than focusing on the correspondence between the units on the ruler and the length being measured.

Area measurement poses further challenges for pupils. In order to measure area, spaces are divided into equal units and this turns out to be quite hard for pupils. A particular source of difficulty is that area is often *calculated* from lengths, rather than "measured". So, although the measurement is in one kind of unit (e.g. centimetres), the final calculation is in another (e.g. square centimetres). While calculations of area are generally multiplicative (to find the area of a rectangle, one multiplies the figure's base by its height), evidence suggests (eg Hart, 1981) that many pupils attempt to calculate area by adding parts of the perimeter, rather than by multiplying. One consequence of the multiplicative nature of area calculations is that doubling a figure's dimensions more than doubles its area - and this can be hard for pupils to understand.

Volume introduces even more complexity, not only in adding a third dimension and thus presenting a significant challenge to students' spatial structuring, but also in the very nature of the objects that are measured using volume. This leads to two ways to measure volume (or capacity) - "packing" a space with cubic units, and "filling" with iterations of units of a fluid that take the shape of the container. In the latter case, the process might seem to pupils to be simple iterative counting that is not processed as geometric 3-D. This can especially happen, for example, in filling a cylindrical jar in which the linear height of the jar corresponds to the volume (see Curry and Outhred, 2005).

The formal concept of angle is another serious stumbling block for pupils - even though they are familiar enough with angles in their everyday spatial environment. One of the problems they have is that they find it hard to grasp that two angles in very different contexts are the same, e.g. themselves turning 90 degrees and the corner of a page in a book. Similarly, pupils can fail to see the connection between angles in dissimilar contexts, like the steepness of a slope and how much a person has to turn at a corner.

Concluding comments

Research demonstrates that measurement is best taught not as a simple skill – rather, it is a complex combination of concepts and skills that develops over years. The available evidence indicates that the principles of measurement are difficult for many pupils, require more attention in school than is usually given, and that the transition from informal to formal measurement needs considerable time and care. The precision (or level of precision) of measures may be encountered at the intuitive level, as might compound measures such as speed, but all such coverage is likely to be limited.

Battista (2007) argues that measurement in secondary school mathematics lacks focus and that this might be "the tip of a huge learning-difficulty iceberg" (p902). Battista's contention, given the pervasiveness of measurement across all the strands of the school mathematics curriculum, is that "poor understanding of measure might be a major cause of learning problems for numerous advanced mathematical concepts" (*op cit*), including graphs of functions, locus problems, vectors, and so on.

This paper argues that measurement is both everywhere and nowhere in the secondary mathematics curriculum. That is, that measurement occurs across the topics that comprise secondary school mathematics, but that the ideas of measurement are so scattered that the teaching of measurement in secondary school mathematics may well lack some focus that might store up problems for learners as they progress with mathematics. Accumulated research evidence, such as that being reviewed by the project examining the research basis for the teaching of the key ideas in secondary school mathematics (being funded by the Nuffield Foundation at the time of writing) could well usefully inform any subsequent revision of the school mathematics curriculum. As Ainley concluded (1991: 76) "There is mathematics in measurement; but it does not happen to be in the bits which currently get priority in mathematics lessons".

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