Linking Geometry and Algebra with *GeoGebra*

Julie-Ann Edwards and Keith Jones review how geometry and algebra can be linked using a new free software package, with Helen Foster, Neil Gulliver, Peter Ingleby, George Petroudis, Alain Plockyn, James Thomson and Paul Willmott

The renowned UK mathematician, Sir Michael Atiyah, refers to geometry and algebra as "the two formal pillars of mathematics" because they both have been, and remain, fundamental to mathematics. Recently, the mathematics education team at the University of Southampton, together with a large group of students (predominantly secondary mathematics PGCE trainee teachers) were one of three fortunate groups in the UK to benefit from a presentation of *GeoGebra* by its author, Markus Hohenwarter (from the University of Salzburg, Austria). This freeware, downloadable from www.geogebra.at, is so named because it combines geometry and algebra as equal mathematical partners in its representations. What is more, it is written in Java so that it runs on any computer, either direct from the web or by installing it. It is currently available in 15 languages (note that the screen shots below come from the German version).

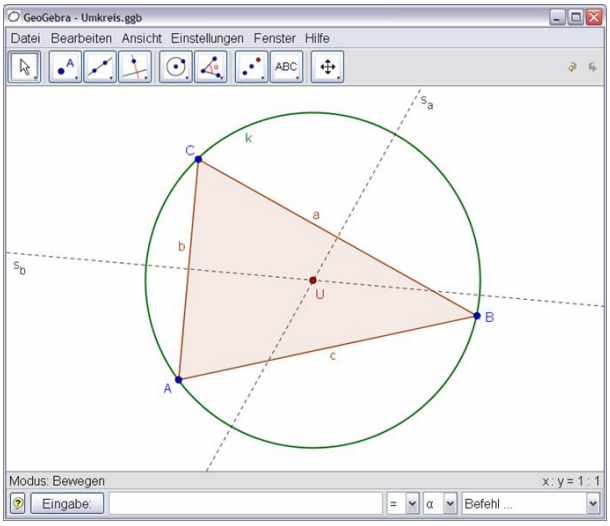

Figure 1

At one level, you can use *GeoGebra* as a dynamic geometry system as in Figure 1, much like others that are available commercially.

And symmetries can be explored just as Peter Ransom did in his article 'Bart's Parts' (*Micromath* 18.3); see Figure 2 (an image from Markus Hohenwarter's file of screenshots on the *GeoGebra* website).

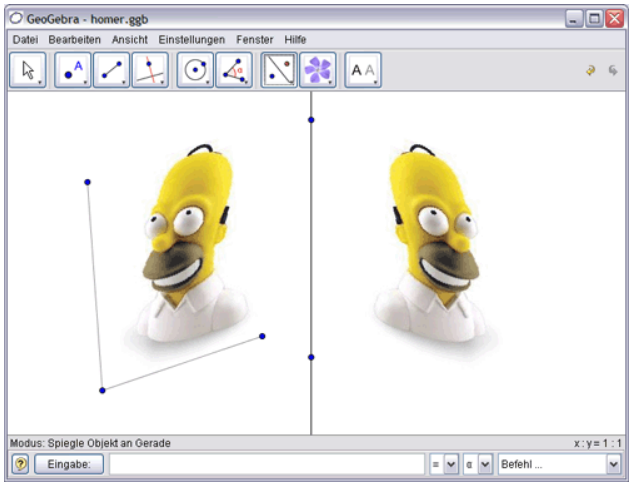

Figure 2

But this is only part of the story. Another window (the algebra part of *Geo**Gebra***) provides an insight into the relationship between the geometric aspects of figures and their algebraic representations. Figure 3, from the *GeoGebra* website, demonstrates this.

Notice the "algebra" window on the left hand side of Figure 3. Here the equations of the ellipse and the straight lines are shown, as are the coordinates of labelled points. Each equation or set of coordinates can be edited in the algebra window and the figure instantly changes. What is more, you can type an equation (or a function) into the space at the foot of the *GeoGebra* interface and the corresponding geometric representation will appear in the geometry window.

Initial Reactions

In exploring this software with the intention of planning for teaching, some of the secondary mathematics PGCE trainee teachers have recorded their first thoughts about the software.

Helen is excited by its potential. She says, "I am, so far, very impressed with *GeoGebra*. I have found it easy to use (and if I can use it, anyone

can!). I love being able to observe the algebraic representation of an object change as it is moved or deformed dynamically. I have used it to prepare a few resources for use in lessons and think it has great potential to be used in this environment. It supports the idea of pupils discovering certain rules (e.g. circle theorems) for themselves rather than simply being told, a feature I believe will provide a much more powerful learning experience".

George expresses a similar view. "It is extremely easy to navigate around *GeoGebra*, even for people who are not proficient in using ICT. I think it is better than other comparative software in that you can move from one function to another with great ease. This is because the software will modify for the change in function automatically without the need for the user to type in all the new parameters. The program has clear images and it is interesting for pupils to see patterns and shapes and structures which are easy for them to understand".

James' initial reaction was a definite "Wow". "Overall, I was amazed at the efficiency of *GeoGebra* as a learning tool. I was very impressed with its potential and am keen to use it in the classroom myself. I am also keen to know if *GeoGebra* could be expanded to cover other areas of the mathematics curriculum, particularly at primary and lower secondary school levels. For example, what about creating 3D shapes in which you could change the values of the lengths of side, area of faces, volumes, etc?"

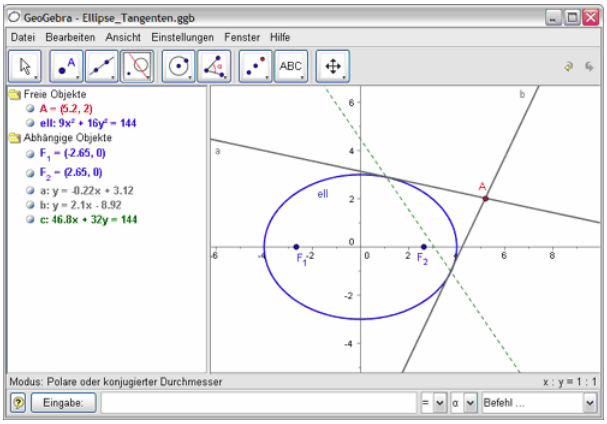

Figure 3

Peter also queries the suitability of the software for the broader range of pupils in lower secondary school. "For me, *GeoGebra* won't replace dynamic geometry software such as Cabri and Sketchpad as I have not found it to be as rich in the features that dynamic geometry software offers. I could see *GeoGebra* being useful for teaching conics and so on, but can't see as much relevance to lower secondary school (which is where most of my current thinking is)".

Neil has demonstrated the software to teachers at his placement school. They had already committed themselves to a commercially available product which provides fairly similar functionality. In situations where other packages are already in use in schools, there is obviously a perceived added time commitment to learn how to use *GeoGebra*. Neil argues that, "an obvious plus in using *GeoGebra* is the fact that it is free to both schools and pupils. This means that pupils can download the software to home PCs and, if the school uses the same software, there is potential for an improved learning environment for the pupils".

In his planning for Year 8 and Year 9 lessons (12-14 year olds) on angles, interior and exterior angles of polygons, and circles, Alain provided some sound practical advice. "It is worth spending time on the default settings such as background colours (anything but white for the interactive whiteboard) and the font size (20 point) for clear visibility from the back of the class. You can then save this as your *GeoGebra* template for all your future *GeoGebra* class presentations". Alain's further advice seems obvious, but indicates the quality of the support given for users of *GeoGebra* by the designer, Markus Hohenwarter. Alain says, "Most of my early problems stem from NOT reading the *GeoGebra* Quickstart which is well worth looking at". This is available online at: www.geogebra.at/help

Paul used the Quickstart advice but found that he had to install the latest version of Java. After that he found *GeoGebra* very easy to use. He says that he's "excited about the possibilities that *GeoGebra* seems to offer and I can't wait to use it with my daughter who is doing A-level Maths!" This reinforces Neil's point about the potential of free accessibility at home to enhance pupils' mathematics learning opportunities.

In terms of limitations with the software for planning lessons, Alain observes, "The only thing I cannot do at the moment is to hide the coordinates of the points (the vertices of my geometric shapes) so as not to introduce too much information to pupils at once. The only option is to close the algebra window. However, if I keep the coordinates visible, I can always reveal the x- and y-axes later to link the algebraic and graphical aspects of the image".

One thing Helen would like to be able to do is related to displaying data. "For example, when demonstrating circle theorems, the software can measure a particular angle, and the size of this angle is displayed in the algebra window. This number then changes as the angle is modified dynamically. I would like to be able to create a box within the geometry window to display this single piece of information, as it can get 'lost' amongst the other formulae in the algebra window".

These minor features aside, the overall opinion of those PGCE trainee teachers who engaged with *GeoGebra* for planning lessons is summed up by Amanda Ladbury's comment that "I wish *GeoGebra* had been around when I was at school".

Further ideas

An opportunity arose a couple of weeks after our introduction to the software to share *GeoGebra* with practicing mathematics teachers from the Southampton LEA at a professional development day held at Cantell Maths and Computing College, a newly built specialist school a few minutes walk from the University of Southampton campus. Keith used *GeoGebra* in a session on working with gifted pupils in mathematics (the "gifted", according to the DfES, are the top 5-10% of pupils *per school* - as measured by actual or potential achievement - in curriculum areas such as mathematics). The features of *GeoGebra* that appealed to teachers, when thinking about providing for gifted pupils, included the visibility of the links between geometry and algebra and the fact that:

- it is free for educational use so that pupils can install and use it at home (or elsewhere, such as in a library with internet access);
- it is available in a range of languages so can be useful for pupils in UK schools who do not have English as their first language;
- it permits activities that need high level thinking;
- it entails pupils engaging with the potential that ICT brings, such as users learning from feedback, seeing patterns, making connections, working with dynamic images, etc.

Imagine the potential for developing "What if …" questions with the dynamic image in Figure 4 and its corresponding algebraic representation.

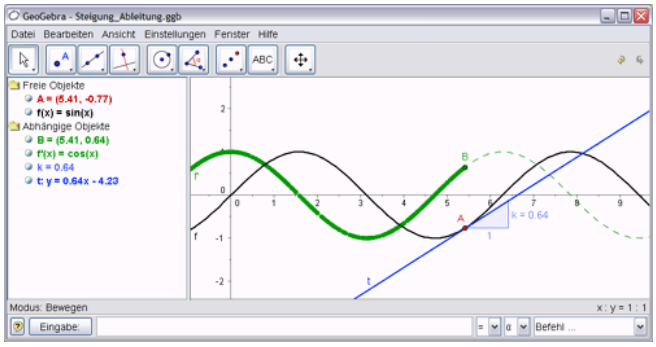

Figure 4

The main theme of the teacher workshop was that *GeoGebra* could be very useful when planning for gifted pupils. This is because given that, in lesson planning, a common technique is to provide three levels of differentiation (namely, core objectives with enrichment/ extension for the more able and support for the less able), using *GeoGebra* may help in stimulating the planning of enrichment/ extension activity that goes beyond the provision of a worksheet of more difficult questions. Perhaps utilising *GeoGebra* could inspire a change from regular forms of enrichment/ extension activity to things that need *high level thinking*, and things that pupils may find themselves *wanting to follow-up* outside school lessons.

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Postscript from Markus Hohenwarter, designer of *GeoGebra*:

GeoGebra is free software because I believe education should be free. This philosophy makes it easy to convince teachers to give this tool a try, even if they haven't used ICT in their classrooms before. Moreover, some of them translate the software into other languages, share their own materials on the web (www.geogebra.at/en/wiki) and answer questions in the user forum (www.geogebra.at/forum) – for free, of course. Future versions of *GeoGebra* will cover all the classic features of dynamic geometry (like macros and animations) and bring new possibilities to the fields of algebra and calculus. Maybe there will even be a *GeoGebra* 3D one day…