



On the evolution of the snow surface during snowfall

H. Löwe,¹ L. Egli,¹ S. Bartlett,¹ M. Guala,¹ and C. Manes¹

Received 9 August 2007; revised 28 September 2007; accepted 12 October 2007; published 14 November 2007.

[1] The deposition and attachment mechanism of settling snow crystals during snowfall dictates the very initial structure of ice within a natural snowpack. In this letter we apply *ballistic deposition* as a simple model to study the structural evolution of the growing surface of a snowpack during its formation. The roughness of the snow surface is predicted from the behaviour of the time dependent height correlation function. The predictions are verified by simple measurements of the growing snow surface based on digital photography during snowfall. The measurements are in agreement with the theoretical predictions within the limitations of the model which are discussed. The application of ballistic deposition type growth models illuminates structural aspects of snow from the perspective of formation which has been ignored so far. Implications of this type of growth on the aerodynamic roughness length, density, and the density correlation function of new snow are discussed. **Citation:** Löwe, H., L. Egli, S. Bartlett, M. Guala, and C. Manes (2007), On the evolution of the snow surface during snowfall, *Geophys. Res. Lett.*, 34, L21507, doi:10.1029/2007GL031637.

1. Introduction

[2] The physical properties of the natural snowpack are of great importance for many geophysical processes such as heat transfer [Kaempfer *et al.*, 2005; Sturm *et al.*, 2002], interaction with the turbulent boundary layer [Lehning *et al.*, 2002], wind erosion [Clifton *et al.*, 2006], failure and crack propagation [Sigrist and Schweizer, 2007; Heierli and Zaiser, 2006]. However, the ice structure within a natural snowpack is a non-equilibrium system which undergoes metamorphic changes induced by many physical and chemical processes [see, e.g., Arons and Colbeck, 1995; Schweizer *et al.*, 2003, and references therein]. Linking the ice structure to its physical properties represents a major challenge in snow physics.

[3] In contrast to the difficulty of predicting the complete metamorphism dynamics of the ice structure some striking features of its *initial condition* can be observed during snowfall even by eye: due to cohesion a settling snow crystal has the tendency of being attached to the snow surface immediately at first contact rather than being rearranged to a position of minimum potential energy. The immediate attachment of the crystal creates a small overhang of the surface and thus excess pore space directly below the attached crystal. As an obvious consequence of

this attachment mechanism the snow surface develops its irregular, rough appearance. Some degree of order within this irregularity is sometimes revealed when a pattern of bumps with well defined characteristic length scale is clearly visible on the surface during snowfall. As a less obvious consequence of the attachment mechanism, also the ice structure below the snow surface should inherit its structure from the way in which overhangs are buried by successive, random attachment events. While the initially generated ice structure is very early influenced by the aforementioned metamorphism dynamics [Kaempfer *et al.*, 2005], the deposition process provides the initial condition to the dynamics and therefore deserves special attention.

[4] To the best of our knowledge the characteristics of surface roughness and the ice structure of snow have never been addressed from the most obvious perspective of formation itself which is the purpose of this letter. We stress that the purpose is not to give a comprehensive model of snow deposition for “operational use”. We rather demonstrate the wide applicability of a simple but well behaved theoretical framework.

2. Model of Snow Deposition

2.1. Ballistic Deposition

[5] Ballistic deposition (BD) was originally introduced as a model of colloidal aggregation. For a comprehensive overview on the theory presented below we refer to Barabási and Stanley [1995] and Meakin [1998]. For illustration of BD we consider the two-dimensional case on a lattice where the surface height $h(i, n)$ is defined at discrete lattice sites i and discrete times n . At each time step, a particle is released well above a randomly chosen site i . It settles down on a straight vertical trajectory until it encounters an occupied neighboring lattice site. Thus, at each time step the height of the surface is updated according to

$$h(i, n + 1) = \max\{h(i + 1, n), h(i - 1, n), h(i, n) + 1\}. \quad (1)$$

The sticking rule (1) is illustrated in Figure 1a. The restriction to a lattice can be abandoned and the generalization of (1) to the three-dimensional case is straightforward [see Meakin, 1998].

2.2. Continuum Description

[6] As an alternative to the particle based approach within BD one may apply a related continuum description. It is widely believed but still a matter of debate [cf. Katzav and Schwartz, 2004, and references therein] that the universal properties of BD on length scales large compared to the particle size can be recovered by a continuum growth model, namely the Kardar-Parisi-Zhang (KPZ) equation

¹WSL, Swiss Federal Institute for Snow and Avalanche Research SLF, Davos, Switzerland.

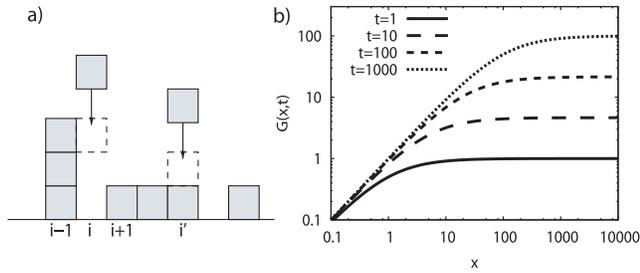


Figure 1. Ballistic deposition: (a) Nearest neighbor sticking for deposition at site i creates an overhang and thus excess pore space which remains inaccessible for other particles. (b) Schematic plot of the correlation function $G(x, t)$ (see equation (3)) for different times $t = 1, 10, 100, 1000$ (arbitrary units).

[Kardar *et al.*, 1986]. For a continuous surface $h(\mathbf{x}, t)$ in three-dimensional space it reads

$$\frac{\partial}{\partial t} h(\mathbf{x}, t) = \nu \nabla^2 h(\mathbf{x}, t) + \lambda |\nabla h(\mathbf{x}, t)|^2 + \xi(\mathbf{x}, t). \quad (2)$$

Time is denoted by t and the gradient by ∇ . The individual terms in (2) can be attributed to particular surface processes: The term $\xi(\mathbf{x}, t) = \eta_0 + \eta(\mathbf{x}, t)$ constitutes the mass supply. It includes a deterministic growth velocity η_0 and a fluctuating term $\eta(\mathbf{x}, t)$ which is zero-mean, Gaussian white noise in space and time. The diffusion term in (2) includes local surface relaxation processes. Most important for the relation of the KPZ equation to BD is the non-linear term of strength λ . It is the leading-order term for growth normal to the surface [Barabási and Stanley, 1995] which is the analog of an overhang in BD.

[7] From the description of a settling snow crystal given in the introduction it is clear that a minimal model of snow deposition should at least include two effects: 1) sticking (which covers both, sintering and interlocking) as the origin of excess pore space and 2) the stochastic nature of the deposition mass flux. Both models, BD and KPZ include these effects. Since we focus on implications and applications of these effects on snow structure we will apply either of them likewise. Obvious limitations of the model can in principle be investigated within extensions of BD/KPZ: the influence of imperfect sticking (e.g., as a consequence of crystal shapes) can be addressed by competitive growth models [Braunstein and Lam, 2005]; non-uniform particle shapes or sizes have been studied by Silveira and Reis [2007]; oblique particle incidence (e.g. as a result of steady, low winds) has been studied by Meakin and Krug [1992]. In contrast, the dynamical evolution of the ice structure below the surface (metamorphism, stress induced re-arrangement) cannot easily be regarded as a generalization of BD/KPZ.

3. Surface Roughness

[8] In general, surface roughness can be characterized by studying temporal and spatial correlations of the height $h(\mathbf{x}, t)$ of a growing surface in terms of its dynamic, two-point correlation function [Barabási and Stanley, 1995]

$$G(x, t) := \overline{(h(\mathbf{x}_0 + \mathbf{x}, t) - h(\mathbf{x}_0, t))^2}. \quad (3)$$

Here, \mathbf{x} , \mathbf{x}_0 denote two-dimensional position vectors on the substrate. Assuming statistical homogeneity and isotropy of the growth process $G(x, t)$ is independent of \mathbf{x}_0 and solely a function of the magnitude $x := |\mathbf{x}|$. The overbar in (3) denotes an ensemble average over realizations of the deposition process. For illustration, the correlation function (3) for BD/KPZ type growth is schematically plotted as a function of x in Figure 1b). Such a behaviour on a double logarithmic scale implies that $G(x, t)$ follows the *dynamic scaling* form $G(x, t) \sim x^{2\alpha} g(x/t^{1/z})$ [Barabási and Stanley, 1995; Meakin, 1998]. Here $g(s)$ is a scaling function which is constant for $s \ll 1$ and decreases algebraically $g(s) \sim s^{-2\alpha}$ for $s \gg 1$. This implies that $G(x, t) \sim x^{2\alpha}$ is independent of t if $x \ll t^{1/z}$ and approaches a constant $G(x, t) \sim t^{2\alpha/z}$ for $x \gg t^{1/z}$ when plotted as a function of x (cf. Figure 1b)). The exponents α , z are universal and commonly referred to as roughness- and dynamic exponent, respectively.

[9] The origin of scaling is the combination of random deposition events with a nearest neighbor sticking rule: Initially, different parts of the surface are uncorrelated. After time t regions of correlated surface heights have formed within a spatial extent $x^* \sim t^{1/z}$ from the sticking rule (1) or, likewise, by the non-linear term in (2). Typical numerical estimates for the scaling exponents α, z for KPZ and BD are found to be in a range $0.3 < \alpha < 0.4$ and $1.36 < z < 1.65$ [cf. Katzav and Schwartz, 2004, and references therein].

4. Measurements of the Snow Surface

[10] Quantitative roughness measurements of snow surface outlines at different times were carried out by means of digital photography during several snowfalls in winter 2006/07. Pictures were taken using a 7.0 megapixel digital camera and a scaled target which was carefully inserted within the snow (see Figure 2 for an example). In order to avoid the formation of roughness due to i) wind-induced snow erosion, ii) anomalous deposition processes due to a predominant wind direction, or iii) finite size geometry effects, images of the snow surface outlines were taken during snow falls in the absence of wind, starting from a flat solid surface with dimensions significantly larger than the largest resolved scale which is ≈ 20 cm. The snow roughness measured in these experiments can thus be regarded (in good approximation) solely as the result of the deposition process. From each image, roughness outlines were identi-

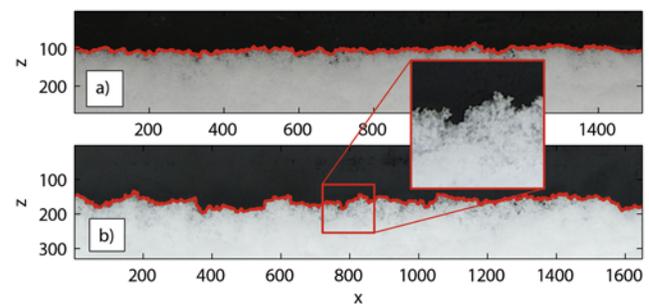


Figure 2. Examples of images taken to estimate snow roughness elevations showing (a) an early and (b) a later stage of the evolution. Estimated outlines are displayed in red. The magnification nicely displays an overhang.

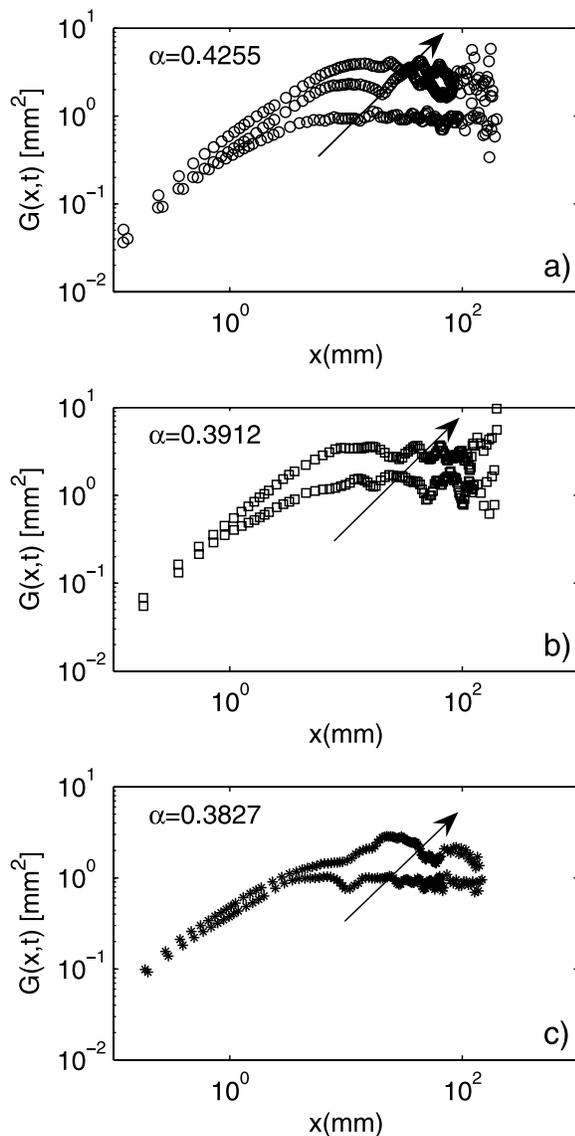


Figure 3. Correlation function for three different snow events. Arrows indicate increasing time, or increasing heights of the mean snow surface, corresponding to (a) 15, 20, 25 mm, (b) 70, 95 mm, and (c) 50, 80 mm.

fied by defining an optimal grey scale threshold value separating the pixels relating to the snow from those to the target. Overall, it was noted that, when inserted into the snowpack, the thin metal target produced very sharp cuts which preserved the shape of the foreground roughness outlines. The accuracy obtained for the roughness elevations depended solely on the camera resolution and the physical size covered by each image. Such an accuracy was estimated to be ≈ 0.1 mm which is fine enough to investigate roughness properties belonging to the sub-crystal scale.

[11] As a typical drawback of natural experiments we acknowledge that some observed snow falls, or parts of them, are not documented here, due to the occurrence of non negligible wind during the snowfall or to significant changes of the crystal sizes in time. The results presented below are thus representative of the time evolution of the

rough snow surface during snowfall where BD/KPZ should be applicable.

5. Results and Discussion

[12] Qualitatively, the measured correlation function $G(x,t)$ (see equation (3)) behaves similarly to what is expected for a BD/KPZ surface. Quantitatively, $G(x,t)$ is plotted for the three sets of experiments as a function of x for different times in Figure 3. The roughness exponent α is obtained by fitting the curves in the ascending range to a power law. The mean values for each set are given in Figure 3, the individual values are $\alpha = 0.36, 0.46, 0.43, \alpha = 0.34, 0.45$ and $\alpha = 0.37, 0.38$ for Figures 3a, 3b and 3c respectively which are in the range of the predictions from BD/KPZ (see section 3).

[13] Instead, a reliable *estimation* of the dynamic exponent z from the standard deviation $\sigma(t) \sim t^{\alpha/z}$ [Barabási and Stanley, 1995] is presently unfeasible. The reason for this is the difficulty to encounter a sufficiently long period of persistent, ideal snowfall conditions (long duration, persistent shapes of the crystals, without wind) which allows to measure the correlation function at many times and estimate the exponent from a single experiment. Likewise, an alternative approach, namely the estimation of z from data collapse of different experiments could only be achieved by explicitly measuring the number flux of settling crystals: the fundamental length scale l_0 of a single experiment is given by $l_0^3 = v/n$ where v is the velocity of the mean surface height in units of ms^{-1} and n is the mean number flux of settling crystals in units of $\text{m}^{-2}\text{s}^{-1}$. By definition l_0^3 is the average volume occupied by a single crystal after its incorporation into the snowpack. Thus, l_0 covers branched crystals as well as those for which sticking can be accompanied by interpenetration. For this reason l_0 is an effective crystal size and not the exact one, which is an ambiguous quantity for non-spherical shapes. The fundamental time scale is then given by $t_0 = l_0/v$. Only by rescaling the variance by l_0 and time by t_0 all data can be combined consistently in a single plot. However, measuring n was clearly beyond the scope of our experimental setup.

[14] For completeness we simply plotted σ of all three experiments as a function of time from the beginning of each experiment in one plot (see Figure 4). As a guide to the eye we added a line of slope $\alpha/z = 0.24$. The increase of the variance is also evident from Figure 3 and follows qualitatively that of BD/KPZ type growth (cf. Figure 1b).

6. Applications

[15] We demonstrate below that, apparently, different aspects involving the structure of snow can be explained from a unifying perspective of BD/KPZ type growth.

6.1. Aerodynamic Roughness Length of New Snow

[16] It has been observed that the aerodynamic roughness length z_0 which determines the logarithmic velocity profile in turbulent boundary layer flows over snow varies even for apparently similar samples of new snow [Clifton *et al.*, 2006]. By dimensional analysis one would expect z_0 to scale on the standard deviation of the surface height. Such a dependence has also been suggested by Lancaster *et al.* [1991] for desert surfaces. Hence, variations of the rough-

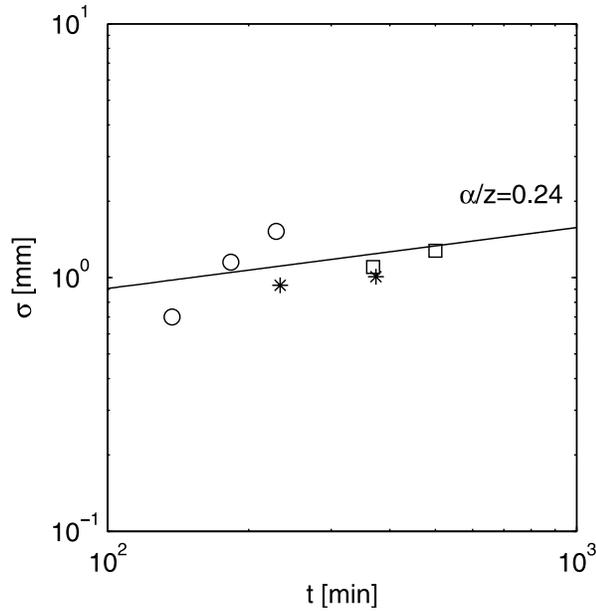


Figure 4. Standard deviation of the surface as a function of time from start of the experiments. Different symbols correspond to the experiments in Figures 3a, 3b, and 3c, respectively.

ness length z_0 of new snow might be attributed to variations of the standard deviation σ as predicted by BD/KPZ.

6.2. Slope Dependent Density of New Snow

[17] It has been observed that the density of new-snow depends on the inclination angle of the slope onto which it is deposited [Endo *et al.*, 1998]. This slope dependence is partly attributed to the mechanism of the deposition process. Such a slope dependence can be predicted by BD/KPZ as follows. For constant vertical mass flux q onto an inclined substrate with slope $m = \tan \theta$ and inclination angle θ one can use the KPZ equation (2) to infer $v(m) = v(0) + \lambda m^2$ [Barabási and Stanley, 1995] for the velocity of the mean surface height in terms of the strength λ of the non-linear term. Thus, the velocity of the mean surface height normal to the surface increases when the inclination angle is increased and the mean density of the deposit given by $\rho(m) = q/v(m)$ decreases. For small slopes m this amounts to $\rho(m) \approx \rho(0)(1 - \lambda m^2)$. Here $\rho(0) = q/v(0)$ is the density of the deposit for a flat substrate. On one hand this can explain the aforementioned claim of Endo *et al.* [1998]. On the other hand it provides an experimental method to estimate λ by fitting the density as a function of slope to a parabola [Barabási and Stanley, 1995].

6.3. Anisotropy of Density Correlations in New Snow

[18] It has been observed that the ice structure of snow might exhibit an anisotropy [Flin *et al.*, 2004] as a result of metamorphism dynamics under isothermal conditions. A different anisotropy naturally emerges from BD/KPZ solely as a result of the deposition process and irrespective of possible anisotropic crystal shapes. To explain this we will extend recent two-dimensional results on properties of deposits below the growing surface from Katzav *et al.* [2006].

[19] Fluctuations of the density around the ensemble mean $\bar{\rho}$ in a porous medium are commonly studied by means of the two-point density correlation function

$$C(\mathbf{x}, x_3) := \overline{(\rho(\mathbf{x}, x_3) - \bar{\rho})(\rho(\mathbf{0}, 0) - \bar{\rho})}, \quad (4)$$

where x_3 is the vertical component of the three-dimensional lag vector (\mathbf{x}, x_3) and again $\mathbf{x} = (x_1, x_2)$. We note that the density $\rho(\mathbf{x}, x_3)$ has to be understood as a microscopic quantity. It is given by the indicator function of the porous medium, i.e. $\rho(\mathbf{x}, x_3) = \rho_{\text{ice}}$ if (\mathbf{x}, x_3) lies in the ice phase and $\rho(\mathbf{x}, x_3) = 0$ if (\mathbf{x}, x_3) lies in the pore space.

[20] Katzav *et al.* [2006] calculated the correlation function (4) from the time dependent structure factor of the surface. Their results can easily be generalized to three dimensional space by employing the scaling form equation (A.11) of Barabási and Stanley [1995] for the structure factor, yielding

$$C(\mathbf{x}, x_3) \sim x_3^{-4(1-\alpha)/z} \phi(|\mathbf{x}|/x_3^{1/z}). \quad (5)$$

[21] Here, $\phi(s)$ is a scaling function which vanishes for $s \gg 1$ and approaches a constant for $s \ll 1$. In other words, for $x_3 \gg |\mathbf{x}|$, that is, in a thin vertical strip the correlation function has an algebraic tail $C(\mathbf{x}, x_3) \sim x_3^{-4(1-\alpha)/z}$ which is missing for $x_3 \ll |\mathbf{x}|$, that is, in a thin horizontal strip. Using, e.g., $\alpha = 0.4$ and $z = 1.6$, the power law decay is governed by an exponent -1.5 . The anisotropy, a result of formation, becomes manifest in (5) since it is not solely a function of the magnitude $(x_1^2 + x_2^2 + x_3^2)$ of the lag.

[22] The great importance of density correlation functions stems from the fact that they arise in rigorous expressions for the effective transport and mechanical properties of porous media [Torquato, 2002].

7. Conclusions

[23] With simple experiments and analytical predictions, we suggest that Ballistic Deposition/Kardar-Parisi-Zhang (BD/KPZ) processes i) well describe the growth of the snow surface during snowfall, ii) allow for the interpretation of previously observed behaviour of the density of new snow and iii) can predict density correlations within new snow. Despite the fact that the BD/KPZ model cannot always be applied to natural snowfalls (limitations are discussed), we emphasize its consequences on the aerodynamic roughness length or the anisotropy of snow as a porous medium. It remains an open question, however, how long the signatures of BD/KPZ persist after snowfall.

[24] **Acknowledgments.** We thank Charles Fierz and Michi Lehning for a careful reading of the manuscript.

References

- Arons, E. M., and S. C. Colbeck (1995), Geometry of heat and mass transfer in dry snow: A review of theory and experiment, *Rev. Geophys.*, 33, 463–493.
- Barabási, A. L., and H. E. Stanley (1995), *Fractal Concepts in Surface Growth*, Cambridge Univ. Press, Cambridge, U. K.
- Braunstein, L. A., and C. H. Lam (2005), Exact scaling in competitive growth models, *Phys. Rev. E*, 72, 026128.
- Clifton, A., J.-D. Ruedi, and M. Lehning (2006), Snow saltation threshold measurements in a drifting-snow wind tunnel, *J. Glaciol.*, 52, 585–596.

- Endo, Y., Y. Kominami, and S. Niwano (1998), Dependence of new-snow density on slope angle, *Ann. Glaciol.*, *26*, 14–18.
- Flin, F., J. B. Brzoska, and B. Lesaffre (2004), Three-dimensional geometric measurements of snow microstructural evolution under isothermal conditions, *Ann. Glaciol.*, *38*, 39–44.
- Heierli, J., and M. Zaiser (2006), An analytical model for fracture nucleation in collapsible stratifications, *Geophys. Res. Lett.*, *33*, L06501, doi:10.1029/2005GL025311.
- Kaempfer, T. U., M. Schneebeli, and S. A. Sokratov (2005), A microstructural approach to model heat transfer in snow, *Geophys. Res. Lett.*, *32*, L21503, doi:10.1029/2005GL023873.
- Kardar, M., G. Parisi, and Y. Zhang (1986), Dynamic scaling of growing interfaces, *Phys. Rev. Lett.*, *56*, 889–892.
- Katzav, E., and M. Schwartz (2004), What is the connection between ballistic deposition and the Kardar-Parisi-Zhang equation?, *Phys. Rev. E*, *70*, 061608.
- Katzav, E., S. F. Edwards, and M. Schwartz (2006), Structure below the growing surface, *Europhys. Lett.*, *75*, 29–35.
- Lancaster, N., R. Greeley, and K. R. Rasmussen (1991), Interaction between unvegetated desert surfaces and the atmospheric boundary layer: A preliminary assessment, *Acta Mech.*, *2*, 89–102.
- Lehning, M., P. Bartelt, B. Brown, and C. Fierz (2002), A physical SNOWPACK model for the Swiss avalanche warning part III: Meteorological forcing, thin layer formation and evaluation, *Cold Reg. Sci. Technol.*, *35*, 169–184.
- Meakin, P. (1998), *Fractals, Scaling and Growth Far From Equilibrium*, Cambridge Univ. Press, Cambridge, U. K.
- Meakin, P., and J. Krug (1992), Three-dimensional ballistic deposition at oblique incidence, *Phys. Rev. A*, *46*, 3390–3399.
- Schweizer, J., J. B. Jamieson, and M. Schneebeli (2003), Snow avalanche formation, *Rev. Geophys.*, *41*(4), 1016, doi:10.1029/2002RG000123.
- Sigrist, C., and J. Schweizer (2007), Critical energy release rates of weak snowpack layers determined in field experiments, *Geophys. Res. Lett.*, *34*, L03502, doi:10.1029/2006GL028576.
- Silveira, F. A., and Aarão Reis (2007), Surface and bulk properties of deposits grown with a bidisperse ballistic deposition model, *Phys. Rev. E*, *75*, 061608.
- Sturm, M., D. K. Perovich, and J. Holmgren (2002), Thermal conductivity and heat transfer through the snow on the ice of the Beaufort Sea, *J. Geophys. Res.*, *107*(C10), 8043, doi:10.1029/2000JC000409.
- Torquato, S. (2002), *Random Heterogeneous Materials*, Springer, Heidelberg, Germany.
-
- S. Bartlett, L. Egli, M. Guala, H. Löwe, and C. Manes, WSL, Swiss Federal Institute for Snow and Avalanche Research SLF, Flüelastr. 11, CH-7260 Davos Dorf, Switzerland. (loewe@slf.ch)