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ESSAYS ON DYNAMIC MACROECONOMICS

by

REZA BOOSTANI

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ABSTRACT

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This thesis uses the techniques of macroeconomic theory to answer three questions. It is divided in three chapters each focusing on one of these questions. The first chapter investigates the appropriate labor market policy response to two fundamental changes in the economy. I introduce unemployment benefits financed by a proportional payroll tax within a model of directed search on the job. I show that there exists a unique positive level of unemployment benefit which maximizes welfare of individuals. The optimal unemployment benefit level is hump-shaped as a function of the level of idiosyncratic risk. At empirically relevant levels of idiosyncratic risk, a much less generous system than in the economy without uncertainty emerges. Furthermore, the welfare costs of deviating from the optimal level are substantial, and accompanied by high unemployment rates. I also find that while the optimal generosity of the unemployment insurance system declines monotonically with the amount of aggregate risk in the economy, the welfare costs of deviating from the optimal system are rather small.

Chapter two develops a small open economy model with both staggered nominal prices and wages. Then, performances of some alternative simple policy rules are compared by using the welfare loss criterion. It is shown that, firstly, the performance of domestic inflation-targeting or wage inflation-targeting is better than both CPI inflation-targeting and pegged exchange rate. Second, although the performance of simple rules depends on the degree of stickiness in prices and wages, wage inflation-targeting performs better than domestic inflation-targeting for a wide combination of wage and price stickiness.

In chapter three, I develop a model with uninsurable capital-income risk and incomplete markets, and investigate the cyclical properties of the equity premium. Although the model abstracts from some common features of the business cycle model, it can generate a sizable and countercyclical equity premium. Moreover, the model generates relatively more volatile consumption, investment, and equity premium than under complete markets.

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Declaration of Authorship

I, Reza Boostani, declare that the thesis entitled "Essays on Dynamic Macroeconomics" and the work presented in it are my own and has been generated by me as the result of my own original research.

I confirm that:

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7. None of this work has been published before submission.

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Introduction

Over the last decades, a new branch in Macroeconomics has developed rapidly which places household in the core of its analysis. New models are built in order to reproduce stylized facts in the real world, and then if models show success in this test, they could be used for analyzing household welfare. The present thesis places in this school of thought, and it tries to answer three questions by using these models.

This thesis is divided in three chapters each focusing on one question. Chapter I investigates the appropriate labor market policy. I introduce unemployment benefits financed by a proportional payroll tax within a model of directed search on the job. A directed search model is chosen because it shows success in explaining some properties in the labor market. Therefore it is a reliable tool for studying the impact of unemployment benefit on household welfare. It is shown in the chapter that there exists a unique positive level of unemployment benefit which maximizes welfare of individuals.

Moreover, the optimal level of unemployment benefit changes in response to some fundamental changes in economy. For example, the optimal unemployment benefit level is hump-shaped as a function of the level of idiosyncratic risk. At empirically relevant levels of idiosyncratic risk, a much less generous system than in the economy without uncertainty emerges. Furthermore, the welfare costs of deviating from the optimal level are substantial, and accompanied by high unemployment rates. I also find that while the optimal generosity of the unemployment insurance system declines monotonically with the amount of aggregate risk in the economy, the welfare costs of deviating from the optimal system are rather small.

Chapter II examines the appropriate monetary policy in a small open economy. Therefore, it develops a small open economy model with both staggered nominal prices and wages. Monetary policy cannot achieve the efficient allocation that would occur under completely flexible wages and prices. Then, performances of some alternative simple policy rules are compared by using welfare loss criterion. It is shown that the performance of domestic inflation-targeting or wage inflation-targeting is better than both CPI inflation-targeting and pegged exchange rate. Moreover, although the performance of simple rules depends on the degree of stickiness in prices and wages, wage inflation-targeting performs better than domestic inflation-targeting for a wide combination of wage and price stickiness.

Chapter III develops a model with uninsurable capital-income risk and incomplete markets, and investigates the cyclical properties of the equity premium. Through a calibrated exercise, I show that the model can generate a countercyclical equity premium. In response to a rise in aggregate productivity, the risk-free interest rate and the return on capital increase, but the rise in the former is larger then the latter, so the equity premium decreases. Moreover, higher idiosyncratic risk is associated with relatively more volatile consumption, investment, and equity premium.

Chapter I

Optimal Unemployment Insurance in a Directed Search Model

1 Introduction

Several economies have experienced fundamental changes over the last few decades. For example, Stock and Watson (2005) document a reduction in the volatility of output growth and a moderation of business cycle fluctuation in most G7 economies over the two decades that preceded the last recession. By contrast, Heathcote et al. (2010) document an increase in wage dispersion in the U.S. over the past three decades. Other papers in the same issue of the *Review of Economic Dynamics* document similar facts for other countries. This paper investigates the impact of these fundamental changes on labor market outcomes as well as the appropriate policy response to these changes.

We study these questions within a directed search model of the labor market with on the job search. Specifically, we introduce a simple government-run unemployment insurance system—unemployment benefit financed by a proportional payroll tax—within a model of directed search along the lines of Menzio and Shi (2008, 2010). This framework is particularly well suited for our purpose, for two main reasons. First, this framework has been shown to generate several important features of the labor market.[1] Second, the model lands itself well to the introduction of both idiosyncratic and aggregate uncertainty, which are at the heart of this paper. While it is possible to introduce both types of uncertainty in a random matching model (see for example Moscarini and Postel-Vinay (2010) or Krusell et al. (2010)), directed search models are much more tractable.[2]

This chapter is a joint work with Martin Gervais and Henrey Sui.

[1]For example, these models have been shown to generate the type of residual wage inequality observed in the data, as well as positive returns to tenure and experience, appropriate worker flows between employment, unemployment, and across employers, and so on (see Delacroix and Shi (2006), Menzio and Shi (2008), Shi (2009) and Gonzalez and Shi (2010)).

[2]Technically, this is because equilibria are 'block recursive,' so that individuals' and firms' decisions do not depend directly on the distribution of workers. See Shi (2009) and Menzio and Shi (2008, 2010) for details.

We begin our analysis by studying an environment which abstracts from idiosyncratic and aggregate uncertainty. In this context, we prove existence of an optimal unemployment insurance system which provides positive benefits to unemployed individuals without providing full insurance. The intuition for this result is simple: while an increase in unemployment benefits allows risk-averse individuals to better smooth consumption over time, it also raises the unemployment rate and thus lowers average consumption. The rise in the unemployment rate arises as individuals look for vacancies with higher wages, for which job finding probabilities are lower.

Next we introduce idiosyncratic uncertainty and again find a unique optimal level of unemployment benefit. Interestingly, the optimal unemployment benefit is hump-shaped as a function of the level of risk that individuals face, keeping the mean level of the shock constant at zero. Initially, an increase in idiosyncratic risk calls for a more generous unemployment insurance system in order to provide better insurance for individuals. However, as the level of risk increases further, generous unemployment benefits become unsustainable. This is because matches for workers who experience low productivity shocks are endogenously destroyed, shrinking the pool of employed workers who pay for the benefit. As a result, at high levels of idiosyncratic risk, the optimal level of unemployment benefit falls sharply as risk rises. Indeed, at empirically relevant levels of idiosyncratic risk, a much less generous system than in the economy without such risk emerges. Furthermore, the welfare costs of deviating from the optimal level are substantial, mainly because generous unemployment benefits generate high unemployment rates. These findings offer a similar interpretation of the 'European unemployment dilemma' to that of Ljungqvist and Sargent (1998): when income risk rises, generous welfare states become costly as they increase unemployment duration and the unemployment rate.

Lastly we add aggregate uncertainty to the model. To better understand the impact of aggregate uncertainty, we abstract from idiosyncratic uncertainty. In this case, the optimal generosity of the unemployment insurance system declines monotonically with the amount of aggregate risk, again keeping the average level of productivity constant. Intuitively, with a fixed benefit, unemployment becomes relatively more attractive in bad times than it becomes less attractive in good times. As a result, the unemployment rate increases, and so the pool of employed workers, who pay for the benefit, shrinks. A lower level of unemployment insurance restores the balance between consumption smoothing and the incentive to search for vacancies with relatively high job finding probabilities. For similar reasons, a pro-cyclical benefit is more desirable than a counter-cyclical (or acyclincal) one. It should be pointed out, however, that in the relevant range of aggregate risk, the welfare costs of deviating from the optimal system are rather small.

Literature review: The economic mechanism at the center of our analysis is that higher unemployment benefits induce individuals to look for jobs with higher wages, which, because these jobs are harder to find, increases unemployment duration. As such, our work is related to Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997), who study optimal unemployment insurance in the context of a model where individuals' search effort is private information.[3] Instead of searching for better jobs with a constant search effort, individuals in their environment search for the same job with less intensity when unemployment benefits increase. The idea is nevertheless that optimal unemployment insurance strikes the right balance between risk sharing and unemployment duration.[4]

Unemployment insurance has also been studied in search and matching models of the labor market. In a directed search environment, Acemoglu and Shimer (1999) also find a unique utility maximizing level of unemployment benefit financed by a lump-sum tax. In addition, they prove that a similar result holds for an output-maximizing level of unemployment. While the mechanism for their first result is similar to ours, their latter result is due to the way in which they model the cost of vacancies. They interpret this cost as capital, which firms choose and put in place before the vacancy is filled. Since higher unemployment benefits induce individuals to search for jobs associated with higher wages, which take longer to find, firms have an easier time filling their vacancies. The capital put in place prior to filling the vacancy thus stays idle for a shorter period of time, inducing firms to put more capital in place.

More recently, Krusell et al. (2010) study unemployment insurance in a Bewley-Huggett-Aiyagari type model in which the labor market functions as in Diamond-Mortensen-Pissarides (DMP). One of their main findings is that unemployment insurance provides little value over and above self-insurance. The optimal replacement ratio in their benchmark economy is lower then the 40 percent they use as a proxi for the U.S. economy. Needless to say, the generosity of the optimal unemployment insurance system would clearly decline if individuals could self-insure in our model as well. Krusell et al. (2010) also speculate that the gains from running a cyclical unemployment benefit scheme are rather small, which is confirmed by our model. This is in contrast to Landais et al. (2010), who find large gains from countercyclical benefits when recessions are characterized by 'job rationing' that stems from real wage rigidities. In this context, increasing benefits in recessions improves consumption smoothing without adversely increasing unemployment duration.

As in Acemoglu (2001), changes in UB affect the distribution of workers across wages. A high UB shifts the distribution of workers toward high-wages jobs. Acemoglu (2001) also argues that the UB may improve welfare by changing the composition of jobs toward high-wage ones.

Costain and Reiter (2008) document that while unemployment displays large cyclical fluctuations relative to labor productivity, the response of unemployment to labor market policies (unemployment benefit) is rather small. They argue that the stan-

[3] In the context of our model, one can think of the market (or wage) to which individuals direct their search as private information, creating a moral hazard problem which prevents a private insurer from conditioning benefits on the search strategy that individuals choose.

[4] To better relate our work to this literature, we are currently investigating an environment in which unemployment benefits comprise two parts: a fixed amount, which can be thought of as a welfare payment, and an unemployment benefit which expires with some probability at any time during an individual's unemployment spell.

dard DMP model cannot reproduce these two facts.[5] While we do not focus on business cycle properties *per se*, the model can generate movements in the unemployment rate relative to movements in labor productivity similar to the data.

The rest of the paper is organized as follows. The next section presents the Model: we introduce a simple unemployment insurance system in a directed search model of on the job search. In section 3, we characterize the optimal unemployment system in an economy which abstracts from (aggregate and idiosyncratic) uncertainty. In section 4 we present numerical results, successively studying the impact of introducing idiosyncratic then aggregate uncertainty. A short conclusion is offered in section 5.

2 Model

The economy is populated by a continuum of risk-averse workers with measure one and a continuum of firms with positive measure.[6] Workers maximize the expected sum of per-period utilities discounted at factor $\beta \in (0, 1)$, with a periodical utility function $\nu(c)$ which is strictly increasing, strictly concave, continuously differentiable, and satisfies Inada conditions. Employed workers' consumption is equal to after-tax labor income, and unemployed workers' consumption equals unemployment benefit paid by the government, b.

Each firm produces output through a constant return to scale production function, $(y + z)n$, with $n \in \{0, 1\}$. When a firm is matched with a worker, its output depends on aggregate and idiosyncratic productivities. Aggregate productivity y is common across all firms, and its values lie in the set $y \in Y = \{y_1, y_2, ..., y_{N_Y}\}$, where $\underline{y} \equiv y_1 \leq y_2 \leq ... \leq y_{N_Y} \equiv \bar{y}$ and $N_Y \geq 1$ is an integer. The idiosyncratic productivity is $z \in Z = \{z_1, z_2, ..., z_{N_Z}\}$, where $\underline{z} \equiv z_1 \leq z_2 \leq ... \leq z_{N_Z} \equiv \bar{z}$ and $N_Z \geq 1$ is an integer. The firm maximizes the expected sum of profits discounted at factor β.

The labor market is organized as a continuum of submarkets $x \in X = [\underline{x}\,, \overline{x}]$, where $\underline{x} = \nu(b)/(1-\beta)$ and $\overline{x} = \nu(\overline{y}+\overline{z})/(1-\beta)$. If a firm meets a worker in submarket x, the firm offers the worker expected lifetime utility x, and we refer to it as the value of that submarket. The probability of finding a job and the probability of filling a vacancy in a submarket depend on the tightness ratio in that submarket. The tightness ratio in submarket x is defined as the ratio of the number of vacancies created by firms to the number of individuals who search for jobs in that particular submarket, denoted $\theta(x, \psi) \geq 0$, where $\psi \in \Psi$ is the aggregate state of the economy at the beginning of each period.

The aggregate state of the economy ψ consists of aggregate productivity y and the distribution of workers among employment positions (u, e): $u \in [0, 1]$ is the measure of unemployed individuals, and the distribution of employed workers is $e : X \times Z \to [0, 1]$

[5]We suspect that their critique also applies to Krusell et al. (2010), who need to rely on the Hagedorn and Manovskii (2008) calibration in order to generate plausible unemployment volatility.

[6]Free entry determines the measure of firms in the economy endogenously.

with pdf $e(x,z)$, and $u + \sum_z \int_x e(x,z)dx = 1$. Menzio and Shi (2010) show that the agents' value and policy functions as well as tightness ratios depend on the aggregate state of the economy ψ only through aggregate productivity y, and not through the distribution of workers across different employment state, (u,e). Therefore, we write, hereafter, tightness ratios and values as function of aggregate productivity, not the aggregate state of economy.[7]

Time is discrete and continues forever. Each period consists of four stages: separation, search, matching, and production. In the separation stage, workers get separated from their match with probability $d \in \{\delta, 1\}$, where δ denotes the exogenous probability of separation. Matches can also be endogenously destroyed by setting $d = 1$. When a worker loses his job, he must remain unemployed until the start of the searching stage next period.

During the search stage, an individual who has the opportunity to search first decides in which submarket to direct his search. While all individuals who have been unemployed for at least one period have the opportunity to search, employed workers only have the opportunity to search with probability $\lambda_e \in [0,1]$. As mentioned above, workers who's unemployment spell started this period cannot search this period. Firms also choose submarkets in which to open vacancies during the search stage.

A firm who meets a worker in submarket x during the matching stage offers the worker a contract which delivers expected lifetime utility x to the worker. If the worker accepts the offer, he starts working at the new job in the following production stage. Otherwise, the worker stays in his previous employment position (which could be unemployment). Firms and workers cannot coordinate their actions because of search frictions in the labor market: not all workers succeed in finding a job, and not all firms succeed in hiring a worker. A worker searching in a submarket characterized by tightness ratio θ finds a job with probability $p(\theta)$, where $p : \mathbb{R}_+ \to [0,1]$ is strictly increasing, twice-continuously differentiable, and strictly concave with $p(0) = 0$ and $p(\infty) = 1$. Similarly, firms fill vacancies with probability $q(\theta)$, where $q : \mathbb{R}_+ \to [0,1]$ is strictly decreasing, twice-continuously differentiable, and strictly convex with $q(0) = 1$ and $q(\infty) = 0$. Naturally, $q(\theta) = p(\theta)/\theta$.

During the production stage, an employed worker produces $y+z$, and consumes after-tax wage $(1-\tau)\omega$. Therefore, the firm's profits equal $y + z - \omega$ in that period. Unemployed workers receive and consume fixed unemployment benefit b. At the end of the production stage, nature draws next period's aggregate productivity y' and idiosyncratic productivity z' from the probability distributions $\Phi_y(y'|y)$ and $\Phi_z(z'|z)$ respectively.

[7]Intuitively, since search is directed, each individual targets his preferred submarket regardless of the distribution of individuals. Similarly, because of free entry, vacancies are created in any submarket until firms make zero profits.

2.1 Worker's Problem

Consider an individual who's employment status can be summarized by his lifetime utility V. During the search stage, he chooses in which submarket to search in order to maximize his value of search. If the worker searches in submarket x, he finds a job with probability $p(\theta(x, y))$ and the job provides lifetime utility x. If he fails to find a job, which occurs with probability $(1 - p(\theta(x, y)))$, he retains his current employment status in the production stage. Accordingly, an individual with current lifetime utility V who has the opportunity to search chooses the submarket which maximizes his lifetime utility at the beginning of the search stage, $V + R(V, y)$, where the second term is the value of search at the beginning of the search stage. The worker's problem at the search stage can thus be written as

$$R(V, y) = \max_{x \in X} p(\theta(x, y))(x - V), \tag{1}$$

and the solution to the above maximization is the optimal submarket in which to search, given by

$$m(V, y) = \arg\max_{x \in X} p(\theta(x, y))(x - V).$$

To ease notation, let $\tilde{p}(V, y)$ denote the probability of finding a job in the optimal submarket, i.e. $\tilde{p}(V, y) = p(\theta(m(V, y), y), y)$.[8]

Let $U(y)$ denote the value function of an unemployed worker at the beginning of the production stage. This lifetime utility consists of the current value of consuming unemployment benefit, and the value of being unemployed and searching tomorrow:

$$U(y) = \nu(b) + \beta E\{U(y') + R(U(y'), y')\} \tag{2}$$

where the value of a variable next period is denoted by a prime symbol $(')$.

2.2 Firm's Problem

During the matching stage, firms offer contracts $c \in C$ to workers. A contract specifies the current wage ω, the probability that the match will be destroyed in the next separation stage d', and the worker's lifetime utility V' at the beginning of the next period. This future utility will be attained by an implicit sequence of future wages. The firm chooses the contract to maximize its lifetime profits $J(V, s)$, where $s = (y, z)$, while delivering lifetime utility associated with the submarket in which they are (promise-keeping constraint) and also compatible with the worker's option to go to unemployment (individual rationality constraint).[9] The problem of a firm who meets

[8] Menzio and Shi (2010) show that $R(V, y)$ is continuous, differentiable, and decreasing in V; that $m(V, y)$ is continuous, differentiable (over the relevant range), and increasing in V; and that $\tilde{p}(V, y)$ is continuous, differentiable, and decreasing in V.

[9] Note that it is in the best interest of the firm to endogenously separate the match when the individual rationality constraint binds. In essence, workers and firms agree on when to endogenously separate.

a worker in submarket V is therefore given by

$$\begin{aligned} J(V,s) &= \max_{\omega,d',V'} \Big\{ y+z-\omega+\beta E\big[(1-d')(1-\lambda_e\tilde{p}(V',y'))J(V',s')\big]\Big\} \\ V &= \nu((1-\tau)\omega)+\beta E\Big\{d'U(y')+(1-d')\big[V'+\lambda_e R(V',y')\big]\Big\} \\ d' &= \begin{cases} \delta & \text{if } U(y') \leq V'+\lambda_e R(V',y') \\ 1 & \text{otherwise} \end{cases} \end{aligned} \tag{3}$$

where $\omega \in \mathbb{R}_+$. Let $c = (\omega, d', V')$ denote the optimal contract. Then the associated policy functions are the wage $\omega = \omega(V,s)$, next period's probability of separation $d' = d'(V,s,s')$, and the worker's lifetime utility in next period $V' = V'(V,s,s')$.[10]

2.3 Market Tightness

During the search stage, firms choose how many vacancies to open and in which submarket to open them. Free entry ensures that vacancies open until the expected value of opening a vacancy is no more than the cost of creating it, denoted κ. Given the firm's value of a match in submarket x, $J(x,y,\tilde{z})$, where $\tilde{z} \in Z$ is the idiosyncratic productivity for all new matches, and the probability of filling a vacancy in that submarket, $q(\theta(x,y))$, free entry implies that

$$\kappa \geq q(\theta(x,y))J(x,y,\tilde{z}). \tag{4}$$

While the free entry condition must hold with equality for submarkets which are open in equilibrium (i.e. submarkets in which some individuals search), such need not be the case for unvisited submarkets. Following Acemoglu and Shimer (1999) and the subsequent literature, we assume that (4) holds with equality in all submarkets in a relevant range, that is, from the lowest submarket to the submarket where firms would just cover the cost of posting a vacancy with a job filling probability equal to one. Under this assumption, market tightness is decreasing and continuous in x over the relevant range.

2.4 Laws of Motion

In principle, we can compute the probability that a worker transits from one employment state to any other state using the optimal policy functions and the exogenous transition functions of the idiosyncratic productivity shock. However, since the shocks y and z take on a finite number of values, a finite number of submarket will be open (in the long run) in equilibrium. To simplify the exposition of the laws of motion, and slightly abusing notation, we redefine X to be the set of open submarkets in the long run and denote N_x the cardinality of that set. Accordingly, let $e : X \times Z \to [0,1]$ and $u \in [0,1]$ denote the probability distribution over employed

[10]Menzio and Shi (2010) show that $J(V,s)$ is concave, continuous, and differentiable in V.

and unemployed workers at the end of the period, respectively. To compute the transitions between employment states, fix aggregate productivity today at some state y and tomorrow at some state y'.

The measure of unemployed workers at the end of the period tomorrow, u', corresponds to the sum of unemployed workers whose search was unsuccessful plus employed workers who lost their job during the period. An unemployed worker remains unemployed with probability $(1 - \tilde{p}(U(y'), y'))$. As specified by the optimal contract, an employed worker in state (x, s) today will becomes unemployed tomorrow with probability $d'(x, s, s')$ if state $s' = (y', z')$ occurs. Given a mass $e(x, z)$ of workers today, $\Phi_z(z'|z)$ gives the fraction of workers who transit to any state z' tomorrow. The measure of unemployed workers next period can thus be written as

$$u' = (1 - \tilde{p}(U(y'), y'))u + \sum_{x \in X} \sum_{z \in Z} \sum_{z' \in Z} \Phi_z(z'|z) d'(x, s, s') e(x, z).$$

Since all workers in new matches have idiosyncratic productivity $\tilde{z}$, the transition of workers to state (x', z') when $z' \neq \tilde{z}$ is different from when $z' = \tilde{z}$. Let's start with the case where $z' \neq \tilde{z}$, which is simpler since no one can end up with idiosyncratic shock z' though successful search. For these states, the only way for a worker to end up in state (x', z') is for his current match to survive $(1 - d'(x, s, s'))$, his search to be unsuccessful $(1 - \lambda_e \tilde{p}(V'(x, s, s'), y'))$, and for today's contract to specify that tomorrow's promised utility will be x' if state z' occurs. In other words, the contract must specify that $V'(x, s, s') = x'$. Accordingly, let $\mathbb{I}[V'(x, s, s') = x']$ be equal to 1 if that is the case, and 0 otherwise. Then we have

$$e'(x', z') = \sum_{x \in X} \sum_{z \in Z} \Phi_z(z'|z)(1 - d'(x, s, s'))(1 - \lambda_e \tilde{p}(V'(x, s, s'), y'))\mathbb{I}[V'(x, s, s') = x'] e(x, z).$$

Finally, for state $z' = \tilde{z}$, we have to take care of individuals whose search is successful in addition to those who transit to that state though their contract with the firm. First, x' could be the submarket in which unemployed workers search. Let $\mathbb{I}[m(U(y'), y') = x']$ be equal to 1 if that is the case, and 0 otherwise. Similarly, an individual who's contract specifies promised utility V' tomorrow if state z' occurs can search in market x' (note that this occur for any z'). Let $\mathbb{I}[m(V'(x, s, s')) = x']$ be equal to 1 if that is the case, and 0 otherwise. Putting it all together, we have

$$\begin{aligned} e'(x', \tilde{z}) &= \sum_{x \in X} \sum_{z \in Z} \Phi_z(\tilde{z}|z)(1 - d'(x, s, y', \tilde{z}))(1 - \lambda_e \tilde{p}(V'(x, s, y', \tilde{z}), y'))\mathbb{I}_1 e(x, z) \\ &+ \sum_{x \in X} \sum_{z \in Z} \sum_{z' \in Z} \Phi_z(z'|z)(1 - d'(x, s, s'))\lambda_e \tilde{p}(V'(x, s, s'), y')\mathbb{I}_2 e(x, z) \\ &+ \tilde{p}(U(y'), y')\mathbb{I}[m(U(y'), y') = x'] u, \end{aligned}$$

where $\mathbb{I}_1 = \mathbb{I}[V'(x, s, y', \tilde{z}) = x']$ and $\mathbb{I}_2 = \mathbb{I}[m(V'(x, s, s')) = x']$. The measures defined above take the aggregate states as given. We can use these measures and combine them with the transition function of the aggregate state to obtain a transition matrix

$\Phi : X \times Y \times Z \to [0,1]$.[11] Below we use $\phi^e : X \times Y \times Z \to [0,1]$ and $\phi^u : Y \to [0,1]$ to denote the stationary probability distribution over employed and unemployed workers, respectively, associated with Φ.

2.5 Government

The government follows a fiscal policy which guaranties budget balance in the long run:

$$b \sum_{y \in Y} \phi^u(y) = \tau \sum_{x \in X} \sum_{y \in Y} \sum_{z \in Z} \omega(x, y, z) \phi^e(x, y, z). \tag{5}$$

The left-hand side represents transfers to unemployed workers, and the right-hand side corresponds to revenues from the payroll tax. Notice that without aggregate uncertainty, the above budget constraint reduces to period-by-period balanced-budget.

2.6 Block Recursive Equilibrium

In a Recursive Equilibrium, the functions $\{\theta, R, m, U, J, c\}$ all depend on the aggregate state of the economy. Solving for an equilibrium outside of the steady state thus requires solving a system of equations for unknowns which depend on the entire distribution of worker across employment states. Hence, solving a recursive equilibrium outside the steady state is analytically and numerically difficult because the dimension of the state space is very large.

Menzio and Shi (2010) prove existence and uniqueness of a recursive equilibrium in which the functions depend on the aggregate state of the economy only through the aggregate productivity component y and not through the distribution of workers across employment states (u, e), which they refer to as a *block recursive equilibrium*.

Definition A Block Recursive Equilibrium consists of a fiscal policy (b, τ), a market tightness function $\theta : X \times Y \to \mathbb{R}_+$, a search value function $R : X \times Y \to \mathbb{R}_+$, a policy function $m : X \times Y \to X$, an unemployment value function $U : Y \to \mathbb{R}$, a value function for firms $J : X \times Y \times Z \to \mathbb{R}$, a contract policy function $c : X \times Y \times Z \to C$, a transition probability function $\Phi : X \times Y \times Z \to [0,1]$, a stationary probability function for employed workers $\phi^e : X \times Y \times Z \to [0,1]$, and a stationary probability function for unemployed workers $\phi^u : Y \to [0,1]$. These functions satisfy the following requirements:

1. R satisfies (1) for all $(V, y) \in X \times Y$, and m is the associated policy function;
2. U satisfies (2) for all $y \in Y$;
3. J satisfies (3) for all $(V, y, z) \in X \times Y \times Z$, and c is the associated policy function;

[11] Given the discrete nature of shocks, as well as open submarkets, Φ is a Markov matrix.

4. θ satisfies (4) for all $(x, y) \in X \times Y$;
5. Φ is derived from the policy functions (m, c) and the exogenous probability distribution for y and z, and ϕ^e and ϕ^u are the associated stationary probability distributions;
6. the unemployment insurance system (b, τ) satisfies (5).

3 Optimal Unemployment Insurance: Analytics

This section establishes some results regarding the impact of unemployment insurance. Although some of the results hold more generally, we abstract from both idiosyncratic and aggregate uncertainty throughout this section. Furthermore, we assume that workers cannot search on the job, i.e. $\lambda_e = 0$. Under these assumptions, there is only one open submarket in equilibrium: all unemployed individuals search in that submarket. From the firm's problem, it is easy to show that: $i)$ the state variable (lifetime utility of the worker) will remain constant throughout the life of the match, $ii)$ separation will only occur exogenously $(d = \delta)$, and $iii)$ the wage will be constant throughout the life of the match. The latter result implies that there is a one to one relationship between x and ω, which we loosely use interchangeably below. Accordingly, the value of the firm simplifies to

$$J(\omega) = \frac{y - \omega}{1 - \beta(1 - \delta)}. \tag{6}$$

In turn, the wage in submarket x must be given by

$$x = V(\omega) = \frac{\nu((1 - \tau)\omega) + \beta\delta U}{1 - \beta(1 - \delta)}, \tag{7}$$

where the value of unemployment is

$$U = \frac{\nu(b) + \beta R(U)}{1 - \beta}. \tag{8}$$

The value of search is given by

$$R(U) = p(\theta)(x - U), \tag{9}$$

where $x = m(U)$, and θ satisfies the free entry condition

$$\kappa = q(\theta)J(\omega), \tag{10}$$

and the unemployment rate is $u = \delta/(\delta + \tilde{p}(U))$. Moreover, the welfare of an unborn worker in this economy can be defined as $W = uU + (1 - u)x$. It is easy to show that the former welfare is equivalence to the weighted average of lifetime utility from consumption. The welfare can be rewritten as

$$\begin{aligned} W &= u\frac{\nu(b)+\beta R(U)}{1-\beta} + (1-u)\frac{\nu((1-\tau)\omega)+\beta\delta(U-x)}{1-\beta} \\ &= u\frac{\nu(b)}{1-\beta} + (1-u)\frac{\nu((1-\tau)\omega)}{1-\beta} + \frac{\beta}{1-\beta}[uR(U)+(1-u)\delta(U-x)]. \end{aligned}$$

The last term of above equation is zero,

$$[uR(U)+(1-u)\delta(U-x)] = \frac{\delta}{\delta+\tilde{p}(U)}\tilde{p}(U)(x-U) + \frac{\tilde{p}(U)}{\delta+\tilde{p}(U)}\delta(U-x) = 0,$$

so, the welfare is

$$W = u\frac{\nu(b)}{1-\beta} + (1-u)\frac{\nu((1-\tau)\omega)}{1-\beta}. \tag{11}$$

From (11), one can see the trade-off between unemployment benefit and tax rate. If unemployment is fixed, the optimal unemployment benefit implies full insurance, $b=(1-\tau)\omega$, i.e. marginal utility for employed and unemployed workers are equal.[12]

Given the properties of the periodic utility function, it is quite trivial to show that there exists an optimal unemployment insurance system which provides positive unemployment benefits without providing full insurance to individuals: while zero benefits can be ruled out by Inada conditions, full insurance can be ruled out because it can only be achieved at zero consumption as all individuals would be unemployed. We nevertheless state that result in the following proposition.

Proposition 1 *Assume that the periodic utility function ν is strictly increasing, strictly concave, continuously differentiable, and satisfies Inada conditions. Then there exists an optimal unemployment insurance system which provides positive benefits to unemployed workers without providing full insurance:* $0<b<(1-\tau)\omega$.

Proof. By assumption, $\lim_{c\to 0}\nu'(c)=\infty$. It follows that a small increase in b away from zero improves welfare. To see that full insurance is not optimal, assume that the unemployment insurance system does provide full insurance, i.e. $b=(1-\tau)w$. This implies that individuals must choose to search in the submarket which provides exactly the same utility as unemployment, that is, $m(U)=U$ and thus $R(U)=0$. For that to be the case, it must be that $p(\theta(x))=0$ for all $x>U$. Since J in equation (6) is a continuous function of x (or ω), the tightness ratio is also continuous (see equation (10)). By continuity of p, the job finding probability is near zero at $m(x)=U$, and thus the unemployment rate, $u=\delta/(\delta+\tilde{p}(U))\approx 1$, and so consumption is again arbitrarily close to zero. It must therefore be the case that moving away from full risk sharing, i.e. $b<(1-\tau)\omega$, improves welfare.[13] ■

[12] Substituting (10) in (6), then multiplying the equation by $(1-u)$, implies $(1-u)y=(1-u)\omega+\frac{1-\beta+\beta\delta}{\delta}\kappa v$, where v is the the number of vacancies. That is, resource constraint does not hold unless $\delta=1$.

[13] It is worth noting that under a linear periodic utility function, unemployment insurance has no value. To see this, consider the weighted utility of unemployed and employed individuals: $W=ub+(1-u)(1-\tau)\omega$. Since the budget constraint of the government imposes that $ub=(1-u)\tau\omega$, $W=(1-u)w$ is independent of unemployment insurance.

While we cannot fully characterize the optimal unemployment insurance system in terms of parameters of the model, we can nevertheless analyze the impact of a budget-balanced change in unemployment benefits. Before doing so, we establish that a rightward shift of the value of the firm implies higher job finding rates and higher search value for individuals.

Lemma 1 *Assume that* $J_2(x) > J_1(x)$ *for all* $x \in X$*. Then* $\theta_2(x) \geq \theta_1(x)$*,* $p(\theta_2(x)) \geq p(\theta_1(x))$*, and* $R_2(x) \geq R_1(x)$ *for all* $x \in X$*.*

Proof. Assume that $J_2(x) > J_1(x)$ for all $x \in X$. From (10), $q(\theta_2(x)) \leq q(\theta_1(x))$, and $\theta_2(x) \geq \theta_1(x)$ since $q(\theta)$ is decreasing in θ. Similarly, since $p(\theta)$ is increasing in θ, $p(\theta_2(x)) \geq p(\theta_1(x))$. Finally,

$$\begin{aligned} R_2(x) - R_1(x) &= p(\theta_2(m_2(x)))(m_2(x) - x) - p(\theta_1(m_1(x)))(m_1(x) - x) \\ &\geq p(\theta_2(m_1(x)))(m_1(x) - x) - p(\theta_1(m_1(x)))(m_1(x) - x) \\ &\geq p(\theta_1(m_1(x)))(m_1(x) - x) - p(\theta_1(m_1(x)))(m_1(x) - x) \geq 0. \end{aligned}$$

where the second line follows from the fact that individuals cannot do worse by optimally choosing the submarket in which to search. ■

The next proposition shows that while a rise in the benefit shifts the firm's value function to the right, an increase in the tax rate shifts it to the left.

Proposition 2 *If* $b_2 > b_1$ *then* $J_2(x) > J_1(x)$ *for all* $x \in X$*, and if* $\tau_2 > \tau_1$ *then* $J_2(x) < J_1(x)$ *for all* $x \in X$*.*

Proof. In section 6.1. ■

From proposition 2, an increase in b and τ shift the value of a firm to right and left respectively. Therefore, a higher b is beneficial for consumption smoothing, but because it raises the tax rate, it discourages firm entry.

4 Optimal Unemployment Insurance: Numerical Results

In this section we use a parameterized version of the model to study unemployment insurance in the presence of various types of risk. We first set a benchmark in an economy without idiosyncratic nor aggregate uncertainty, and successively study the impact of adding each type of risk. We also discuss the desirability for unemployment benefits to be pro- or counter-cyclical. Before doing so, we parameterize the economy.[14]

[14] The algorithm used to solve the model numerically appears in the section 6.2.

4.1 Parameterization

We take a period to be a quarter and set the discount factor to 0.987, which implies an interest rate of 5 percent annually. The worker's periodic utility function is given by $(c^{(1-\sigma)}-1)/(1-\sigma)$, where σ, the risk aversion coefficient, is equal to 2. The matching technology $p(\theta)$ has the functional form of $\theta(1+\theta^{\gamma})^{-1/\gamma}$, with $\gamma=0.5$, which implies an elasticity of substitution between vacancies and applicants of 2/3.[15]

Following Menzio and Shi (2010), we set κ, δ, and λ_e equal to 0.001, 0.045, and 0.3 respectively. The aggregate productivity follows a two-state Markov process with $Y=\{0.95,1.05\}$ and transition matrix $\Phi_y=[0.75\ 0.25;0.25\ 0.75]$, with unconditional mean equal to 1.[16] We set the unconditional mean of idiosyncratic productivity to zero, and assume that z follows a three-state Markov process with $Z=\{-0.4,0,0.4\}$ and transition matrix $\Phi_z=[0.75\ 0\ 0.25;0.5\ 0\ 0.5;0.25\ 0\ 0.75]$. Note that idiosyncratic productivity is assume to be zero ($\tilde{z}=0$) in all newly formed matches.

4.2 Benchmark Model without Uncertainty

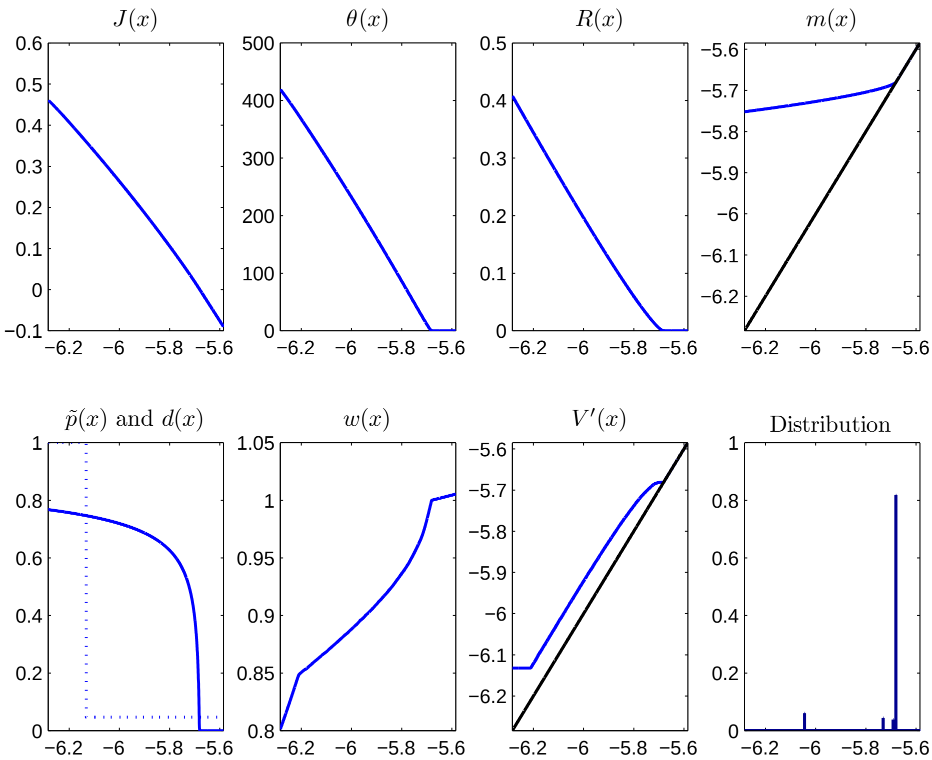
We now assume that both aggregate and idiosyncratic productivities are constant at their unconditional means. Figure 1.1 displays various value and policy functions, the submarket tightness function, as well as the stationary distribution over open submarkets. Since the value of the firm ($J(x)$) is decreasing in promised utility to the worker, firms need to be compensated by a high job filling probability ($q(x)$) to open vacancies in high submarkets. This translates into a tightness ratio ($\theta(x)$) and a job finding probability ($\tilde{p}(x)$) which decrease as promised utility increases. Notice that unemployed individuals search in relatively high submarkets: this is because of the fairly generous unemployment benefit. As a result, the optimal submarket in which to search is fairly flat, and the distance between the current lifetime utility x and the lifetime utility associated with the submarket in which individuals search ($m(x)$) narrows down quickly. It also follows that the value of search ($R(x)$) decreases monotonically with current lifetime utility.

The wage ($\omega(x)$), future utility ($V'(x)$), and the separation probability ($d'(x)$) are all chosen by the firm as part of the contract. As one would expect, wages and future promised utility both increase with the submarket value. Because of curvature in the periodic utility function, the wage is a convex function of the submarket value over the relevant range: increasingly higher wages are necessary to provide the extra lifetime utility. Also note that the contract is designed to backload wage payments: by promising higher future utility, firms can keep current wages low while at the same

[15]The underlying matching function, as first introduced by den Haan et al. (2000), is given by $(v^{-\gamma}+a^{-\gamma})^{-1/\gamma}$, where v and a are the number of vacancies and applicants respectively, and $\theta=v/a$. The elasticity of substitution between vacancies and applicants is $\frac{1}{1-\gamma}$.

[16]We also experimented with a Markov process with $Y=\{0.95,1.02\}$ and transition matrix $\Phi_y=[0.875\ 0.125;0.05\ 0.95]$, thereby making recessions harsher, less frequent, and more persistent. We omit presenting these results as the different with our benchmark results are negligible.

FIGURE 1.1: Economy without Uncertainty



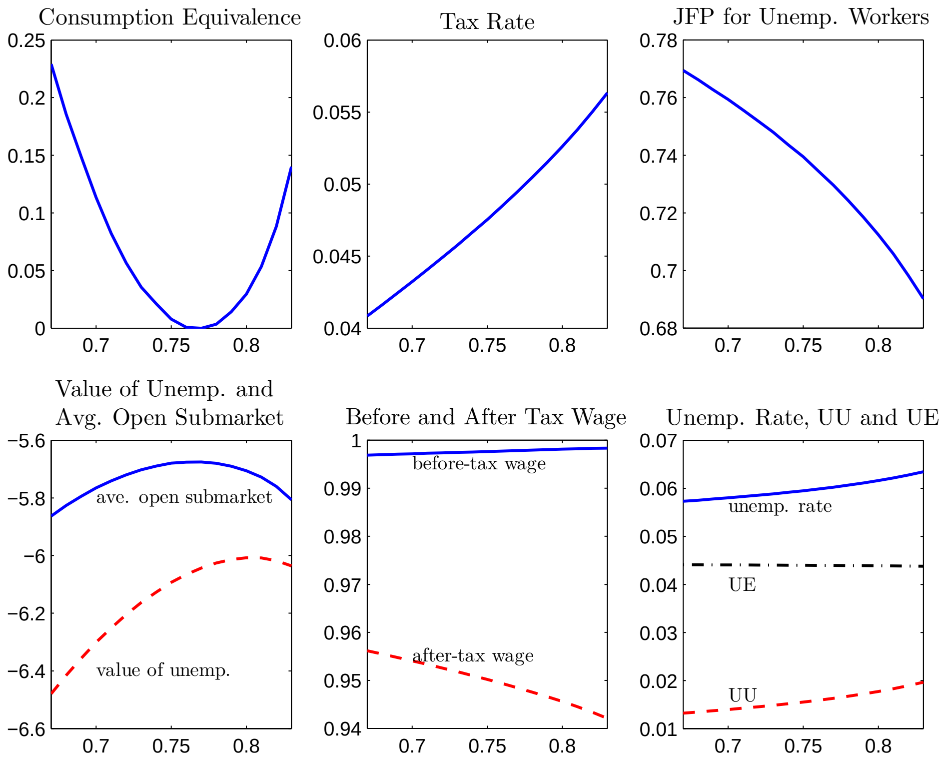
Notes: In all panels the horizontal axis is the value of submarkets x. This figure was generated under unemployment benefit $b = 0.77$, for which the replacement ratio is on average 0.812, and a labor tax rate $\tau = 0.0495$. Other parameter values are described in section 4.1.

time lowering the worker's job finding probability in the future. The separation probability is such that no match is ever endogenously destroyed. Finally, note that the model generates some heterogeneity among homogenous workers, as shown in the last panel of Figure 1.1. The first spike corresponds to the measure of unemployed individuals in the population. As dictated by the policy function $m(x)$, these individuals search in a higher submarket, which corresponds to the second spike, and so on. Of course, a limited number of submarkets can emerge without idiosyncratic nor aggregate uncertainty. Naturally, all open submarkets are located between the value of unemployment and the submarket for which firms would just cover the cost of posting a vacancy with a job filling probability equal to one.

Figure 1.2 displays how the economy behaves under various levels of unemployment benefits, each financed through a labor income tax which balances the government's budget on a period-by-period basis. The top left panel shows that there exists a unique optimum unemployment insurance configuration.[17] As discussed earlier, the intuition for this result is straightforward: while higher unemployment benefits provide better consumption smoothing, it also reduces production and thus average con-

[17]Consumption Equivalence refers to the amount of extra consumption per period, in percentage terms, which individuals would require in order to be indifferent between a given unemployment insurance system and the optimal one.

FIGURE 1.2: Optimal Unemployment Insurance without Uncertainty



Notes: In all panels the horizontal axis is the value of unemployment benefits b. The tax rate is such that the government budget constraint holds on a period-by-period basis in the stationary equilibrium given b. Wages refer to average before- and after-tax wages of employed workers. 'UU' refers to the flow of individuals who remain unemployed, and 'UE' the flow of unemployed who transit to employment. Consumption Equivalence refers to the amount of extra consumption per period, in percentage terms, which individuals would require in order to be indifferent between a given unemployment insurance system and the optimal one. The calibration is outlined in Section 4.1.

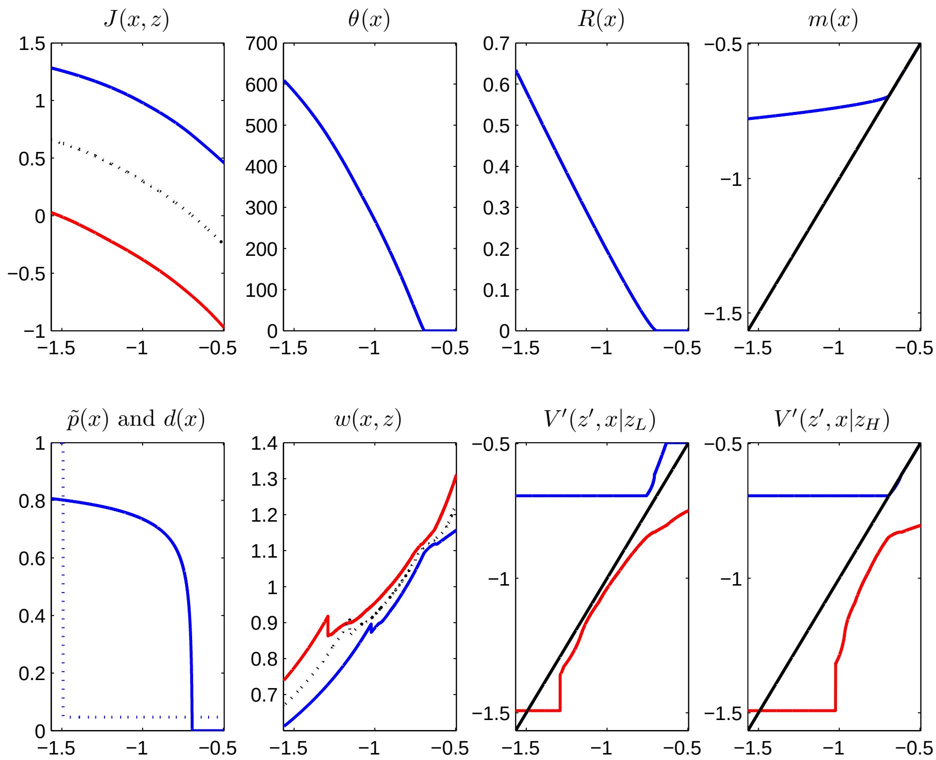
sumption as the unemployment rate increases.[18]

Moreover, a more generous unemployment insurance system provide better consumption smoothing not only because benefits are higher, but also because after-tax wages are lower. The lower-left panel of Figure 1.2 shows that despite lower consumption, employed workers work on average in higher submarkets as unemployment benefits increase, at least at relatively low unemployment benefit levels.[19] The extra utility, of course, comes from the possibility of becoming unemployed in the future, which has a higher value. But because unemployed workers search in higher submarkets, for which the job finding probability is lower, individuals spend more time in the unemployment state as unemployment insurance becomes more generous. Eventually, even the value of unemployment falls as the prospect of becoming employed not only becomes less likely but also more dire.

[18] Production need not decrease in an economy in which firms choose the amount of capital to put in place prior to filling vacancies, as shown by Acemoglu and Shimer (1999). In their environment, firms put in place a larger amount of capital when this capital is expected to remain idle for shorter periods of time, as happens when individuals direct their search to higher submarkets.

[19] Note that the value of the average open submarket peaks earlier than average welfare: the value of unemployment is still rising, which, over a narrow range, dominates the rise of the weight on the value of unemployment (the unemployment rate).

FIGURE 1.3: Economy with Idiosyncratic Uncertainty



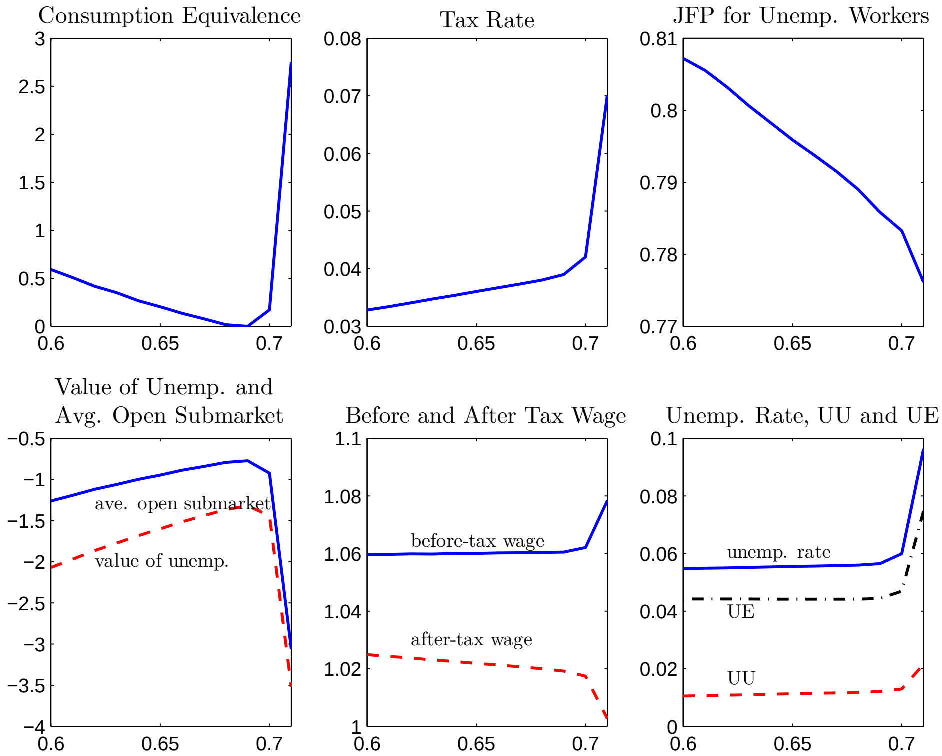
Notes: In all panels the horizontal axis is the value of submarkets x. Red lines represent low productivity, and blue lines represent high productivity. This figure was generated under unemployment benefit $b = 0.69$, for which the replacement ratio is on average 0.677, and a labor tax rate $\tau = 0.039$. Other parameter values are described in section 4.1.

4.3 Introducing Idiosyncratic Uncertainty

In this section, we keep aggregate productivity fixed at its unconditional mean, but introduce idiosyncratic uncertainty in the form of a three-state Markov chain for individual productivity z. Recall that idiosyncratic productivity is assumed to be the same in all new matches, equal to its unconditional mean ($\tilde{z} = 0$). Thereafter, idiosyncratic productivity can either be low or hight, and evolves according to Φ_z.

Figure 1.3 illustrates the properties of the economy. Most functions look familiar following the discussion around Figure 1.1. The contract, however, has some new properties. First, only the dashed-line value of the firm ($J(x, \tilde{z})$) in the top panel is relevant for job creation. No new match are created for submarkets $x > \hat{x}$, where $\hat{x}$ satisfies $J(\hat{x}, \tilde{z}) = \kappa$—i.e. submarkets for which $\theta(x) = 0$. Accordingly, firms design contracts such that high productivity individuals have no incentive to search: they search in markets where the job finding probability is zero. On the flip side, firms design the contract in such a way that matches with some low productivity individuals are destroyed. Again, because firms cannot offer less than the value of unemployment, this limits the firm's ability to backload wages. Finally, notice that wages are higher for low productivity individuals than for high productivity individuals given a submarket x. This is due to the persistence of the shock and is most easily seen at low values of x, for which the contract specifies the same future utility for both low

FIGURE 1.4: Optimal Unemployment Insurance with Idiosyncratic Uncertainty

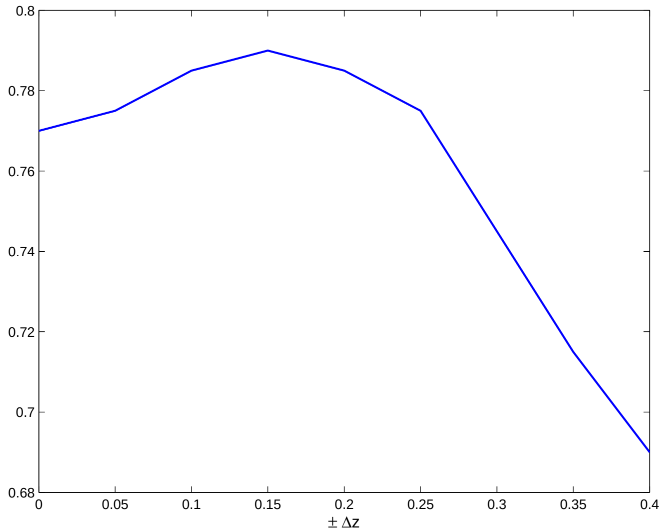


Notes: In all panels the horizontal axis is the value of unemployment benefits b. The tax rate is such that the government budget constraint holds on a period-by-period basis in the stationary equilibrium given b. Wages refer to average before- and after-tax wages of employed workers. 'UU' refers to the flow of individuals who remain unemployed, and 'UE' the flow of unemployed who transit to employment. Consumption Equivalence refers to the amount of extra consumption per period, in percentage terms, which individuals would require in order to be indifferent between a given unemployment insurance system and the optimal one. The calibration is outlined in Section 4.1.

and high productivity individuals: since the weight on high future utility is higher for the high productivity individual $(V'(z' = z_H, x|z_H))$, a smaller wage is required to meet the promised utility x. Accordingly, the value of the firm is higher when their worker has high productivity.

Figure 1.4 displays how the economy with idiosyncratic risk behaves under various levels of unemployment benefits, each financed through a labor income tax which balances the government's budget on a period-by-period basis. As was the case in the economy without risk, the top left panel indicates that there exists a unique optimum unemployment insurance configuration. The same mechanism as in the benchmark model is at work: a more generous system provides better risk sharing at the cost of higher unemployment and thus lower average consumption. However, a separate mechanism is responsible for the generosity of the system to be so much lower than in the benchmark model—the average replacement ratio falls from 81 to 68 percent. The new mechanism revolves around endogenous match separation, which plays a crucial role especially at high benefit levels. This can be seen in the lower right panel, which shows that the flow from unemployment to employment rises sharply at high

FIGURE 1.5: Optimal Unemployment Benefit and Idiosyncratic Risk



Notes: The idiosyncratic shock takes on values $z \in \{-\Delta z, 0, \Delta z\}$. For any Δz, we compute the optimal unemployment insurance system, indexed by the unemployment benefit on the vertical axis. The calibration is outlined in Section 4.1.

benefit levels.[20] This is in contrast to the benchmark model, where the rise in the unemployment rate was essentially all due to the lower job finding probability, and hence longer unemployment duration. Together, these effect translate into very high tax rates, so much so that high unemployment benefits are simply unsustainable. For the same reason, the welfare cost of running 'too generous' an unemployment insurance system is sizable.

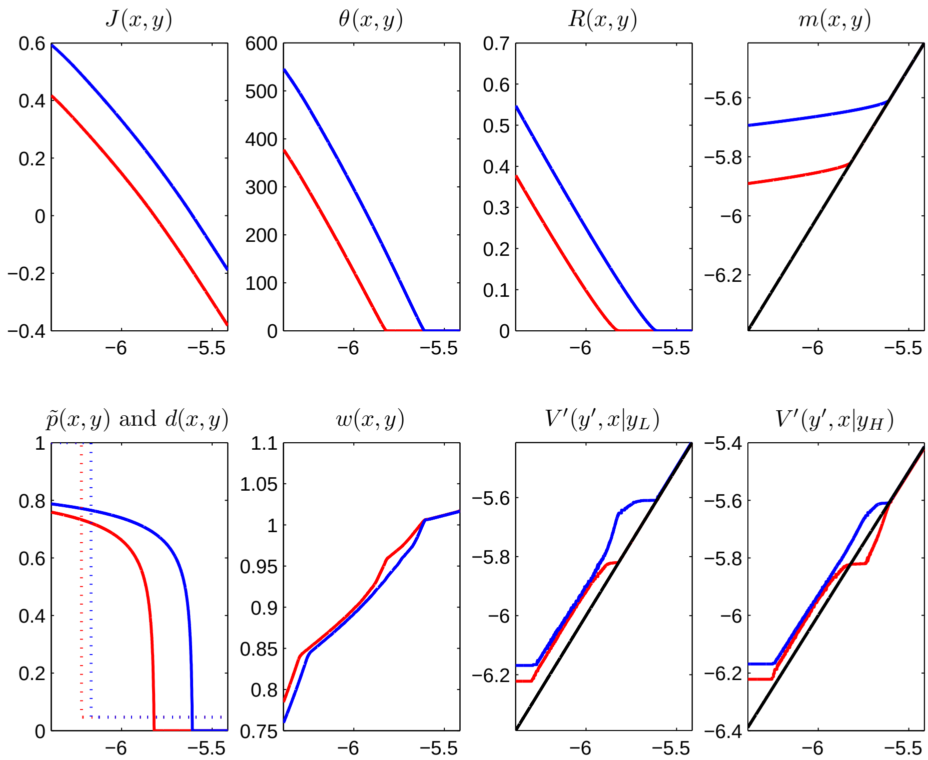
Figure 1.5 illustrates how the optimal unemployment insurance system—as indexed by the level of the benefit—changes with the amount of idiosyncratic risk that individuals face. As idiosyncratic risk rises, the optimal unemployment benefit initially rises smoothly but then falls rather sharply. While the initial rise allows individuals to better smooth consumption, the subsequent fall is due to endogenous separation. As the amount of risk rises, more matches with low productivity workers are endogenously destroyed. As a result, the unemployment rate rises faster, and so does the wage tax on the shirking pool of workers. These findings offer a similar interpretation of the 'European unemployment dilemma' to that of Ljungqvist and Sargent (1998): when income risk rises, generous welfare states become costly as they produce high unemployment rates and longer unemployment duration.

4.4 Introducing Aggregate Uncertainty

We now remove idiosyncratic uncertainty but introduce aggregate productivity in the form of a two-state Markov chain for aggregate productivity y. From Figure 1.6, the same patterns as in the economy without uncertainty emerge here as well. Under

[20] Recall that in a stationary environment, this flow must be equal to the flow from employment to unemployment.

FIGURE 1.6: Economy with Aggregate Uncertainty

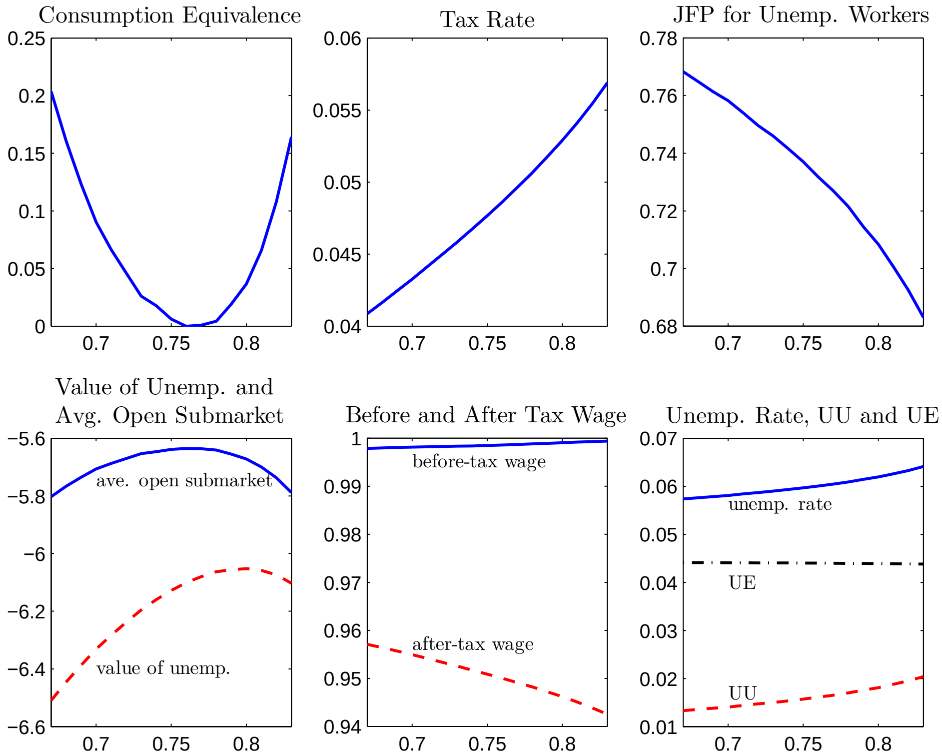


Notes: In all panels the horizontal axis is the value of submarkets x. Red lines represent low productivity, and blue lines represent high productivity. This figure was generated under unemployment benefit $b = 0.76$, for which the replacement ratio is on average 0.80, and a labor tax rate $\tau = 0.049$. Other parameter values are described in section 4.1.

high productivity, individuals search in higher submarkets. But because the value of the firm is higher, their job finding probability of unemployed workers is nevertheless higher in good times. Aggregate shocks, on their own, aren't sufficiently large for any match to be destroyed endogenously. Interestingly, a lot of consumption smoothing is built in to the contract designed by the firms: promised utility in good and bad times are fairly close to one another, at least at relatively low submarket values. This also allows the firm to backload wages. However, promised utilities in higher submarkets offer less insurance, and most open submarkets end up being in this region. This is because individuals' promise utility always rises in good time, but does not necessarily go down in bad times except in the period of the shock. But because of the downgrade in the period of the shock, wages are lower in bad times. Furthermore, the average wage goes up in good times because new matches occur in higher submarkets. As a result, with a constant unemployment insurance system, the government runs a surplus in good times and deficits in bad times.

Figure 1.7 displays how the economy with aggregate uncertainty behaves under various levels of unemployment benefits, each financed through a labor income tax which balances the government's budget on average in the long run. Comparing this figure to its counterpart without uncertainty (Figure 1.2), we conclude that aggregate uncertainty does not alter how unemployment insurance affects the economy in general. Indeed even the optimal level of benefits is very similar, albeit a little smaller.

FIGURE 1.7: Optimal Unemployment Insurance with Aggregate Uncertainty



Notes: In all panels the horizontal axis is the value of unemployment benefits b. The tax rate is such that the government budget constraint holds on a period-by-period basis in the stationary equilibrium given b. Wages refer to average before- and after-tax wages of employed workers. 'UU' refers to the flow of individuals who remain unemployed, and 'UE' the flow of unemployed who transit to employment. Consumption Equivalence refers to the amount of extra consumption per period, in percentage terms, which individuals would require in order to be indifferent between a given unemployment insurance system and the optimal one. The calibration is outlined in Section 4.1.

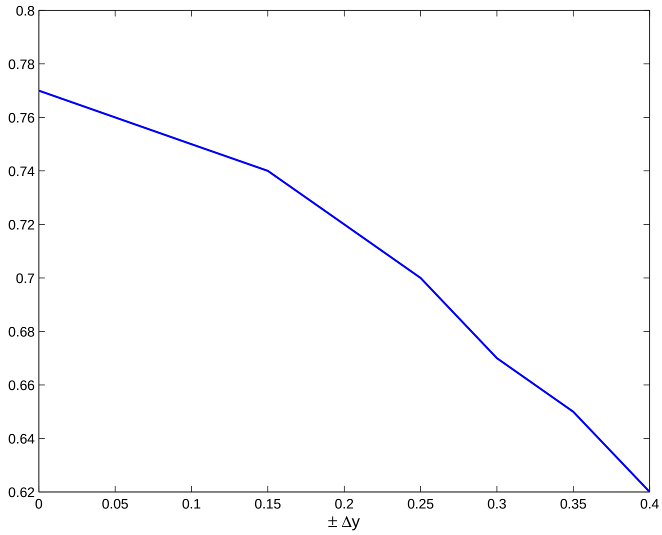
It follows that the cost of deviating from the optimal system are rather small.

Figure 1.8 illustrates how the optimal unemployment insurance system—as indexed by the level of the benefit—changes with the amount of aggregate risk in the economy.[21] This figure shows that the generosity of unemployment insurance should decline monotonically as the economy becomes riskier. Our interpretation of this finding is that because the unemployment benefit is the same in good and bad times, it has a larger detrimental effect on unemployment duration during bad times, when wages are low, than a beneficial on in good times, when wages are high. On average, then, the unemployment rate is higher, and so the tax rate necessary to finance any level of benefit is higher.

The reasoning above is that a flat benefit provides 'too much' consumption smoothing in bad times, and 'too little' in good times. As such, it points to the idea that the unemployment benefit should be pro-cyclical, that is, benefits should be reduced in recessions. We investigated this issue to find that this is indeed the case. The optimal benefits in bad and good times turn out to be 0.76 and 0.80, respectively, which translate into replacement ratios of 0.80 and 0.84.

[21]Note that the relevant range of aggregate risk is below 0.05.

FIGURE 1.8: Optimal Unemployment Benefit and Aggregate Risk



Notes: The aggregate shock takes on values $y \in \{-\Delta y, \Delta y\}$. For any Δy, we compute the optimal unemployment insurance system, indexed by the unemployment benefit on the vertical axis. The calibration is outlined in Section 4.1.

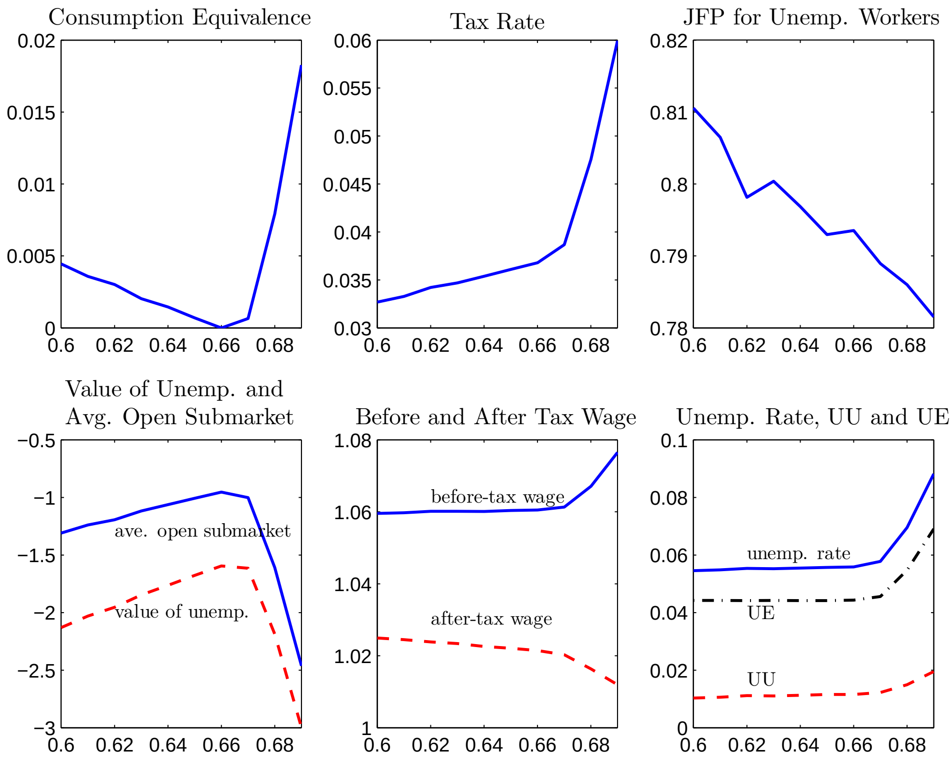
4.5 Aggregate and Idiosyncratic Uncertainty Together

We now investigate the full model, with both aggregate and idiosyncratic uncertainty. Since idiosyncratic uncertainty is much more important than aggregate uncertainty, this economy behaves essentially in the same way as the economy with only idiosyncratic uncertainty (Figure 1.9). The most interesting aspect of combining the two types of uncertainty is that the amount of endogenous destruction increases in recessions. This effect is strong enough to essentially double the reduction in the optimal benefit from adding aggregate uncertainty to an economy with idiosyncratic risk relative to one with no risk. For similar reasons, the highest level of benefit that is sustainable is smaller in the full model.

5 Conclusion

Over the last few decades, several economies have experienced fundamental changes, i.e. a reduction in the volatility of output growth and an increase in wage dispersion. This chapter investigates the appropriate labor market policy response to these changes. I introduce unemployment benefits financed by a proportional payroll tax within a model of directed search on the job. I show that there exists a unique positive level of unemployment benefit which maximizes *ex ante* welfare of individuals. The optimal unemployment benefit level is hump-shaped as a function of the level of idiosyncratic risk. At empirically relevant levels of idiosyncratic risk, a much less generous system than in the economy without uncertainty emerges. Furthermore, the welfare costs of deviating from the optimal level are substantial, and accompanied by high unemployment rates. I also find that while the optimal generosity of the

FIGURE 1.9: Optimal Unemployment Insurance with Aggregate and Idiosyncratic Uncertainty



Notes: In all panels the horizontal axis is the value of unemployment benefits b. The tax rate is such that the government budget constraint holds on a period-by-period basis in the stationary equilibrium given b. Wages refer to average before- and after-tax wages of employed workers. 'UU' refers to the flow of individuals who remain unemployed, and 'UE' the flow of unemployed who transit to employment. Consumption Equivalence refers to the amount of extra consumption per period, in percentage terms, which individuals would require in order to be indifferent between a given unemployment insurance system and the optimal one. The calibration is outlined in Section 4.1.

unemployment insurance system declines monotonically with the amount of aggregate risk in the economy, the welfare costs of deviating from the optimal system are rather small.

The alternative formulation of the unemployment insurance system where unemployment benefits are a function of past employment experience could be subject for further research. Moreover, one also may study whether unemployment benefits should be extended for longer periods of time in recessions, a policy that was implemented in the U.S. during the last recession.

6 Appendix

6.1 Proof of Proposition 2

Proof. Assume that $J_2(x) > J_1(x)$ for all $x \in X$—we will show below that this is indeed the case. The difference $R_2(U_2) - R_1(U_1)$ is such that

$$\begin{aligned} R_2(U_2) - R_1(U_1) &= p(\theta_2(m_2(U_2)))(m_2(U_2) - U_2) - p(\theta_1(m_1(U_1)))(m_1(U_1) - U_1) \\ &\geq p(\theta_2(m_1(U_1)))(m_1(U_1) - U_2) - p(\theta_1(m_1(U_1)))(m_1(U_1) - U_1) \\ &\geq p(\theta_1(m_1(U_1)))(m_1(U_1) - U_2) - p(\theta_1(m_1(U_1)))(m_1(U_1) - U_1) \\ &= -p(\theta_1(m_1(U_1)))(U_2 - U_1), \end{aligned}$$

where the second line follows from the fact that individuals cannot do worse by optimally choosing the submarket in which to search, and the third line follows from Lemma 1. From (8), the difference $U_2 - U_1$ is

$$\begin{aligned} U_2 - U_1 &= (\nu(b_2) - \nu(b_1) + \beta(R_2(U_2) - R_1(U_1)))/(1-\beta) \\ &\geq \frac{\nu(b_2) - \nu(b_1)}{(1 - \beta + \beta p(\theta_1(m_1(U_1))))}, \end{aligned}$$

where $(1 - \beta + \beta p(\theta_1(m_1(U_1)))) > 0$. It follows that if $b_2 > b_1$ then $U_2 - U_1 > 0$. Now in order to deliver a given utility V, from (7), the firm offers a lower wage under b_2 than under b_1, i.e. $\omega_2 < \omega_1$, since

$$\nu((1-\tau)\omega_2) - \nu((1-\tau)\omega_1) = -\beta\delta(U_2 - U_1) < 0.$$

From (6), the difference $J_2(V) - J_1(V)$ is

$$J_2(V) - J_1(V) = \frac{-(\omega_2 - \omega_1)}{1 - \beta(1-\delta)} > 0.$$

Since this is true for any $V \in X$, this verifies that indeed $J_2(x) > J_1(x)$ for all $x \in X$.

Now let's turn to second part of the proposition. Let $\tau_2 > \tau_1$, then from (7), the difference $\nu((1-\tau_2)\omega_2) - \nu((1-\tau_1)\omega_1)$ given some fix point $V \in X$ is given by

$$\nu((1-\tau_2)\omega_2) - \nu((1-\tau_1)\omega_1) = -\beta\delta(U_2 - U_1).$$

If $U_2 - U_1 = 0$, then $\omega_2 > \omega_1$. From (6), the difference $J_2(V) - J_1(V)$ is

$$J_2(V) - J_1(V) = \frac{-(\omega_2 - \omega_1)}{1 - \beta(1-\delta)} < 0.$$

Now the difference $R_2(U_2) - R_1(U_1)$ is

$$\begin{aligned} R_2(U_2) - R_1(U_1) &= p(\theta_2(m_2(U_2)))(m_2(U_2) - U_2) - p(\theta_1(m_1(U_1)))(m_1(U_1) - U_1) \\ &\leq p(\theta_2(m_2(U_2)))(m_2(U_2) - U_2) - p(\theta_1(m_2(U_2)))(m_2(U_2) - U_1) \\ &\leq p(\theta_1(m_2(U_2)))(m_2(U_2) - U_2) - p(\theta_1(m_2(U_2)))(m_2(U_2) - U_1) \\ &= -p(\theta_1(m_2(U_2)))(U_2 - U_1), \end{aligned}$$

where we again used Lemma 1. From (8), the difference $U_2 - U_1$ is

$$\begin{aligned} U_2 - U_1 &= (\beta(R_2(U_2) - R_1(U_1)))/(1-\beta) \\ &\leq -\beta(p(\theta_1(m_2(U_2)))(U_2 - U_1))/(1-\beta) \leq 0. \end{aligned}$$

This sharps the result $U_2 - U_1 \leq 0$, then $\omega_2 > \omega_1$, and $J_2(V) - J_1(V) < 0$. ■

6.2 Algorithm

For any given level of unemployment benefit b:

1. Set an initial guess for tax rate τ;
2. Compute the firm's value function J, submarket tightness function θ, the policy function m and the optimal contract c by iterating on the following steps:
   (a) Set an initial guess for J, and U;
   (b) Compute the market tightness θ using (4) and the implied job finding probability p from the matching function;
   (c) Compute the value of search R, and associated policy function m using (1);
   (d) Compute the new firm's value function J' and associated optimal contract $c(\omega, d', V')$ using (3);
   (e) Construct the new value of unemployment U' using (2).
   (f) If $J \approx J'$, done, otherwise use the J' and U' as new guesses and go back to step 2b;
3. Calculate the stationary distributions ϕ^e and ϕ^u associated with the policy function m the optimal contract c, and the tightness ratio θ;
4. Compute the government's budget deficit: if the budget deficit is zero, stop; otherwise set a new guess of the tax rate and go back to step 2.

The optimal unemployment insurance system is found by repeating these steps for several values of unemployment benefits.

Chapter II

Monetary Policy in a Small Open Economy with Nominal Rigidities

1 Introduction

It seems an agreement has been reached about the optimality of inflation-targeting among monetary authorities. This is mainly the case because the models supporting the idea are built on staggered price setting. It means agents cannot adjust prices of their products whenever they want. On the other hand, rigidity in wages – at least downward rigidity – is also reasonable and firmly supported by both macro and micro data, see Gali (2010) and Nickell and Quintini (2003). In this chapter, I study the performance of different monetary policy rules in terms of their effects on the welfare of a representative agent who lives in a small open economy. The open economy is an interesting research topic because money affects the real economy not only through nominal interest rate (same as in a closed economy), but also through its effects on the exchange rate. The existence of this additional transmission channel consequently raises the question of whether monetary policy should be different from a closed or an open economy.

In the present chapter, I develop a small open-economy model with both nominal prices and wages rigidities, which subjects to domestic and global technological uncertainty. In order to model staggered prices and wages, I assume intermediate goods and labor markets have monopolistic competition; thus, households and firms face downward sloping demand for their labor services and outputs. Similar to Erceg et al. (2000), I show that there is no policy that can ensure Pareto efficiency.

Then, the performance of some alternative simple policy rules are compared by using a welfare loss criterion. I find that, firstly, the performance of simple rules depends on the degree of stickiness in prices and wages. However, the performance of pegged exchange rate and CPI inflation-targeting are lower than domestic inflation-targeting or wage inflation-targeting. Therefore the decision about policy rule is only between wage and domestic price inflation-targetings, without any regard to the openness of the economy. Second, for a wide combination of stickiness in wage and price, wage inflation-targeting perform better than domestic inflation targeting. These results are in line with some other research.[1]

Literature review: Recently, Woodford (2003) showed that, in a closed economy with staggered price, a monetary rule that places a very high weight on inflation-targeting is close to the optimal policy rule. In the same way, but in an open economy with staggered price-setting, Gali and Monacelli (2005) and Clarida et al. (2001) argue that domestic inflation-targeting constitutes the optimal policy. Smets and Wouters (2002) also uses an open economy framework with both staggered domestic and import price-setting and shows that optimal monetary policy minimizes a weighted average of domestic and import price inflation. Corsetti and Pesenti (2005) finds that inward policies that focus on stabilizing domestic prices and output gap, while there is limited pass-through, may result in suboptimal welfare levels for domestic consumers. It is worth noting that the common feature of these studies is the price rigidity, while wages are completely flexible.

Although wage rigidity is ignored in lots of papers, there are pieces of research that emphasize its importance. For example, Christiano et al. (2005) argues that nominal wage stickiness is more crucial than the stickiness in prices, and Gali (2010) discusses whether, with nominal wage rigidity, the model could explain the negative correlation between wage inflation and unemployment. Furthermore, Kollmann (2001) showed that a model of a small open economy with staggered wages and prices exhibits exchange rate overshooting (large exchange rate movements) in response to money supply shocks.

The present chapter is different from other studies on open economies. First, nominal wage rigidity is a new feature, whereas most of the studies allow only for price rigidity, see Gali and Monacelli (2005), Kollmann (2002), Clarida et al. (2001), and McCallum and Nelson (1999). Second, welfare loss is derived from utility function in order to study the welfare effects of alternative rules, while similar studies only examine the transmission mechanism of monetary policy, see Kollmann (2001).

The rest of the chapter is as follows. Section 2 discusses the model. In section 3, I study the conditions under which flexible prices and wages equilibrium allocation is optimal. In section 4, performances of alternative simple policy rules are studied. Finally, section 5 concludes.

2 Model

The world consists of a continuum of small economies indexed by $i \in [0, 1]$, such that any decision in a typical country has no effect on the rest of the world. In each country,

[1] Erceg et al. (2000) and Gali (2008) developed a model of a closed economy with both rigidities and argue that the strict inflation-targeting is not optimal. Besides, Faia (2008) achieves the same result in a model with labor market frictions.

there is one distortion: monopolistic competition in goods markets. In one of the countries, which I model as home or domestic economy, besides the distortion in goods markets, there is also distortion in the labor market as monopolistic competition. Therefore, in home economy the equilibrium output and labor supply are lower than their counterparts in competitive equilibrium with fully flexible prices and wages. I assume that fiscal policy is responsible for offsetting the real distortion caused by imperfect competition in goods and labor markets, while the central bank's objective is to mitigate the effects of nominal rigidities by conducting a simple monetary rule.[2]

2.1 Household

The home economy is populated by a continuum of monopolistically competitive households which value consumption and leisure, indexed by $h \in [0,1]$. Each household supplies a differentiated labor service by signing a contract with a fixed wage. Each period only a fraction of households $(1-\theta_w)$, selected randomly from the population, have the opportunity to renew their contracts. The utility function of household h is

$$E_k \sum_{t=k}^{\infty} (\beta\theta_w)^{t-k} \left(\frac{C_{t|k}(h)^{1-\sigma}}{1-\sigma} - \frac{N_{t|k}(h)^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

where $C_{t|k}(h)$ and $N_{t|k}(h)$ denote the consumption and labor supply in period t of household h that last reset its wage in period k, $0 < \beta < 1$ is discount factor, $\sigma > 0$ is measure of risk aversion, and $\frac{1}{\varphi} > 0$ is elasticity of labor supply. E_k is expectation operator conditional on information available at time k. Household h's budget constraint in period t is

$$P_t C_{t|k}(h) + E_t\{\Omega_{t,t+1} D_{t+1|k}(h)\} \leq (1+\tau_h)\bar{W}_k(h) N_{t|k}(h) + D_{t|k}(h) + \Pi_t - T_t \tag{2}$$

where P_t is the price of a final good at time t, $\bar{W}_k(h)$ is the wage last reset at time k, T_t is lump-sum tax paid to government, and τ_h is subsidy rate. Each household owns an equal share of all firms and collects the dividend Π_t in period t. Each household also has access to complete international finance market.[3] $D_{t+1|k}(h)$ is the nominal pay-off received in period $t+1$ from the portfolio which consists of state contingent securities, and bonds. $\Omega_{t,t+1}$ is the stochastic discount factor which discounts the pay-off of period $t+1$ into value of period t. Assuming complete financial market implies $E\{\Omega_{t,t+1}\} = \frac{1}{R_t}$ where $\Omega_{t,t+1}$ could be interpreted as price of risk-free bond in time t which gives one unit of domestic currency in the next period, and R_t is the gross

[2] Gali and Monacelli (2005) shows that the optimal monetary policy in a small open model with staggered prices is domestic inflation-targeting. However, I will explain later that there is no monetary policy which can achieve Pareto optimum in this model like what is shown by Erceg et al. (2000) for a close economy. The flexible wage in all foreign countries will guarantee that the world economy follows optimal policy, and the home country cannot implement any allocation which is better than flexible prices and wages allocation.

[3] I may assume that each household consists of measure one family members with differentiated labor, and total household income is divided equally between members. This assumption guarantees that each member of the family has the same consumption. Instead, I can assume that financial markets are complete; that is, households can trade state-contingent securities for all possible state of nature to guarantee smooth consumption.

return on the risk-free domestic bond.[4] In addition, the complete financial markets also imply that households have access to complete contingent securities market, and securities' payoff is contingent on whether household can reset its wage or not. It implies that consumption is identical across households in every period. Therefore specific index of household h would disappear thereafter, $C_t(h) = C_t$.

2.2 Final-Goods Firm

The final good is non-tradable and is produced in a competitive market by firms that use intermediate domestic and imported goods as inputs and supply their homogeneous outputs directly to the domestic final consumers. F-firms produce final goods by using a constant elasticity of substitution technology

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where $\eta > 0$ is the elasticity of substitution between domestic and foreign goods, C_t is the final good, $C_{H,t}$ and $C_{F,t}$ are respectively quantity indices of domestic and imported intermediate goods. Furthermore, $C_{H,t}$ and $C_{F,t}$ are equivalent to consumption of domestic and imported goods because F-firms' output is consumed entirely by the domestic households; hence, in the steady state α is the share of foreign goods in aggregate consumption i.e. α is a measure of openness of the economy.

Foreign composite goods are produced using technology

$$C_{F,t} = \left(\int_0^1 \left(C_{i,t}\right)^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$$

where $C_{i,t}$ is the quantity index of imported goods from country i and used by F-firms in the home economy, and γ is measure of substitutability of goods imported from different countries. Cost minimization implies the demand for goods imported from country i is

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t}$$

with foreign price index,

$$P_{F,t} = \left(\int_0^1 {P_{i,t}}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$$

where $P_{i,t}$ is the price index of imported goods from country i expressed in home currency.

Each country produces a continuum of intermediate goods indexed by $j \in [0, 1]$. Bundles of intermediate goods manufactured in country i and used by F-firms in the home

[4] Complete financial market assumption implies that returns on all assets are equal (no arbitrage condition). Then the price of risk-free bond equals ratio of an asset price $V_{t,t+1}$ to probability of occurrence of the state that the asset pays $\zeta_{t,t+1}$, $\Omega_{t,t+1} = V_{t,t+1}/\zeta_{t,t+1}$.

economy $C_{i,t}$ are produced using technology

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $C_{i,t}(j)$ is the intermediate good j which is made in country i, and $\varepsilon > 1$ is elasticity of substitution between varieties. F-firm's problem is to minimize the cost of producing a fixed amount of country i composite goods while taking the price of intermediate goods as given. The minimization implies intermediate goods demand function

$$C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\varepsilon} C_{i,t} \tag{3}$$

with price index

$$P_{i,t} = \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$$

where $P_{i,t}(j)$ is the price of intermediate good j produced in country i. This means, for example, that the demand for intermediate goods produced domestically depends directly on the demand for domestic composite goods and domestic price index and inversely on its price.

F-firm's problem also is to minimize its cost for producing a fixed amount of final goods, while taking the prices of domestic and foreign composite goods as given. The optimization implies domestic and foreign composite goods demand function

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t, \quad \text{and} \quad C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t,$$

with price index

$$P_t = \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

where P_t is the consumer price index (CPI) in the home economy. It is worth noting that all price indices are reported in the domestic currency.

The final good is produced in a competitive market; consequently, each firm earns zero profit. This can be shown by using the definition of price indices and demand functions.

2.3 Intermediate-Goods Firm

The intermediate goods are produced in the monopolistic competitive market; moreover, each intermediate good is produced by a specific I-firm while intermediate goods are a close substitute. The technology of the I-firm that produces domestic intermediate good j is

$$Y_t(j) = A_t N_t(j) \tag{4}$$

where A_t is economy-wide productivity, and $N_t(j)$ is aggregate labor used by firm j. Productivity follows an $AR(1)$ process

$$a_t = \rho_a a_{t-1} + \varepsilon_t, \qquad 0 < \rho_a < 1$$

where $a_t = \log A_t$, and $\varepsilon_t \sim N(0, \sigma_a^2)$.

I-firms only use labor as input, so the real marginal cost is the real wage divided by marginal product of labor, MP_t,

$$MC_t = (1-\tau_f)\frac{W_t/P_{H,t}}{MP_t} = (1-\tau_f)\frac{1}{A_t}\frac{W_t}{P_{H,t}} \tag{5}$$

where τ_f is the subsidy rate and is set such that the firm produces the competitive amount of output in steady state.

Aggregate domestic production Y_t is given by

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where ε is correspondent with elasticity of substitution between intermediate goods. Aggregate labor supply N_t is

$$N_t = \int_0^1 N_t(j)dj = \frac{Y_t}{A_t}\int_0^1 \frac{Y_t(j)}{Y_t}dj = Z_t\frac{Y_t}{A_t},$$

and aggregate production function is

$$Y_t = \frac{A_t N_t}{Z_t}. \tag{6}$$

I-firms sell their outputs to F-firms by signing contracts with fixed prices, and they cannot negotiate over prices each period. Only a portion of I-firms $(1-\theta_p)$ have the opportunity to renew their contracts which are selected randomly.

2.4 Job-Finding Firms

Labor market is a monopolistic competition, because each household supplies a specific labor service while the services are close substitutes. I-firms need all types of labor; therefore, J-firms buy households labor services $N_t(h)$ as input and supply final labor as output N_t through a competitive market to I-firm. J-firm's production function is

$$N_t = \left(\int N_t(h)^{\frac{\varsigma - 1}{\varsigma}} dh \right)^{\frac{\varsigma}{\varsigma - 1}}$$

where $\varsigma > 1$ is the measure of substitutability between differentiated labor services. J-firm's problem is to minimize the cost of producing a fixed amount of final labor subject to a given wage for each kind of labor service. Expenditure minimization implies demand for type-h labor service

$$N_t(h) = \left(\frac{W_t(h)}{W_t} \right)^{-\varsigma} N_t \tag{7}$$

with

$$W_t = \left(\int_0^1 W_t(h)^{1-\varsigma} dh \right)^{\frac{1}{1-\varsigma}}$$

where W_t is the aggregate wage index.

2.5 Competitive Equilibrium

It is worth noting that in a special case – which is presented in section 3 – the model predicts balanced trade in all time. Therefore, individuals in home economy do not hold foreign bonds in equilibrium.

Definition A competitive equilibrium in home economy consists of price $\{R_t, P_t, W_t, \{P_{H,t}(j)\}_{j=0}^1, \{W_t(h)\}_{h=0}^1\}$, allocation for home individual $\{C_t(h), N_t(h), D_t(h)\}$ for all h, allocation for J-firm $\{N_t, \{N_t(h)\}_{h=0}^1\}$, allocation for F-firm $\{C_t, \{C_{i,t}(j)\}_{j=0,i=0}^1\}$, allocation for I-firms $\{Y_t(j), N_t(j)\}$ for all j, and aggregates $\{C_t, N_t, D_t\}$ such that:

1. given prices $\{R_t, W_t, P_t\}$, home individual maximizes utility (1) subject to their budget constraint (2) and labor demands (7).
2. given prices $\{P_t, \{P_{i,t}(j)\}_{j=0,i=0}^1\}$, F-firm's allocation maximizes profit.
3. given prices $\{P_{H,t}, W_t\}$, I-firm maximizes profit

$$\max_{\bar{P}_{H,t}(j)} \; E_t \sum_{k=0}^{\infty} (\beta\theta_p)^k \left[\bar{P}_{H,t}(j) Y_{t+k}(j) - (1-\tau_f) W_{t+k} N_{t+k}(j) \right]$$

   subject to technology constraint (4) and demand for its output (3).

4. given prices $\{W_t, \{W_t\,(h)\}_{h=0}^1\}$, J-firm's allocation maximizes profit.
5. government sets the tax $\{T_t\}$ such that the budget constraint is balanced

$$T_t = \tau_h \int W_t(h) N_t(h) dh + \tau_f W_t \int N_t(j) dj.$$

6. Markets clear:

- final goods market.

$$C_t = \int C_t(h)dh.$$

- intermediate goods markets

$$Y_t(j) = C_{H,t}(j) + \int C^i_{H,t}(j)di.$$

- labor market

$$\int N_t(j)dj = \left(\int N_t(h)^{\frac{\varsigma-1}{\varsigma}}dh\right)^{\frac{\varsigma}{\varsigma-1}}.$$

- financial market

$$\int D_t(h)dh = 0.$$

2.6 Optimal Decisions

2.6.1 Household decisions

Each period, a constant fraction $(1-\theta_w)$ of households renew their contracts with J-firm. Then with fixed probability θ_w households are paid last period wages, and this probability is independent across households and time. The household's problem is to maximize its expected lifetime utility (1) subject to budget constraint (2) and labor demand (7). The following equations show the first order conditions of this decision problem

$$\beta R_t E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \right\} = 1 \tag{8}$$

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left\{ \left(\frac{(1+\tau_h)\bar{W}_t(h)}{P_{t+k}} C^{-\sigma}_{t+k} - \frac{\varsigma}{(\varsigma-1)} N^{\varphi}_{t+k|t}(h) \right) N_{t+k|t}(h) = 0 \right\}. \tag{9}$$

Equations (8) and (9) are Euler and wage-setting equations respectively. Thus, the household's optimal wage is a constant markup over the ratio of weighted marginal utilities of leisure to marginal utilities of consumption over the period that the contract is enforced.

I assume that employment is subsidized to eliminate the monopolistic distortion associated with a positive mark-up; that is $\tau_h = \frac{1}{\varsigma-1}$. In the absence of wage rigidity $(\theta_w = 0)$, and subsidy in effect, (9) reduces to optimal labor supply under competitive market

$$MRS = \frac{N_t^{\varphi}(h)}{C_t^{-\sigma}} = \frac{W_t(h)}{P_t}$$

where the marginal rate of substitution equals real wage.

2.6.2 Intermediate-goods firm decisions

Each period a fraction of I-firms change their prices and renew their contracts with F-firm.[5] Every I-firm resets its price with the constant probability of $(1-\theta_p)$ that is independent of time and other I-firms. Therefore, when it can reset its price, the firm maximizes the discounted profit of present and future periods subject to demand for its output. The following first order condition must be met by I-firms

$$E_t \sum_{k=0}^{\infty} (\beta\theta_p)^k \left\{ \left[\frac{\bar{P}_{H,t}(j)}{P_{H,t+k}} - \frac{\varepsilon}{\varepsilon-1} MC_{t+k} \right] Y_{t+k}(j) \right\} = 0 \tag{10}$$

where $\bar{P}_{H,t}(j)$ is the price of product j that is reset in period t. From (10), I-firms target a constant mark-up $\frac{\varepsilon}{\varepsilon-1} > 1$ over real marginal cost for the period during which the price contracts are in effect. It is worth noting that all I-firms resetting prices at time t will choose the same $\bar{P}_{H,t}(j)$ because the above condition depends on aggregate variable, not firm-specific price history.

2.6.3 Uncovered interest rate parity

In the domestic economy, the gross return of a one-period risk-free bond which pays one unit of domestic currency next period is equal to the inverse of its price

$$(R_t)^{-1} = E_t\{\Omega_{t,t+1}\}. \tag{11}$$

In addition, households have access to the international financial market. As with the above, if the household buys foreign bonds it pays $(R_t^i)^{-1}$ and receives one unit of foreign currency in period $t+1$. Since I assume complete financial markets, it implies that

$$\xi_{i,t} \left(R_t^i\right)^{-1} = E_t \left\{\Omega_{t,t+1}\xi_{i,t+1}\right\} \tag{12}$$

where $\xi_{i,t}$ is the bilateral nominal exchange rate between domestic economy and country i. Integrating (11) and (12) obtains uncovered interest rate parity (UIR) condition

[5]Different methods are suggested for modeling staggered prices, e.g. Taylor (1980) and Calvo (1983). Taylor assume that the price contracts are in effect in two periods. However, Calvo's is more flexible about the average period of contract. Moreover, latter can be solved without explicitly tracking the distribution of prices across firms, so it has been used widely in the research.

$$R_t - R_t^i E_t \left\{ \frac{\xi_{i,t+1}}{\xi_{i,t}} \right\} = 0. \tag{13}$$

2.6.4 International risk sharing

Since I assume symmetry between countries, Euler equation (8) must hold for the representative household in every country. Euler equation for a household residing in country i is

$$\beta E_t \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left(\frac{P_t^i}{P_{t+1}^i} \right) \right\} = (R_t^i)^{-1} \tag{14}$$

where C_t^i and P_t^i are respectively household consumption and the price index in local currency in country i. Substituting R_t^i from (12), (14) can be written as follows

$$\beta E_t \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left(\frac{P_t^i}{P_{t+1}^i} \right) \left(\frac{\xi_{i,t}}{\xi_{i,t+1}} \right) \right\} = E_t\{\Omega_{t,t+1}\}. \tag{15}$$

Bilateral real exchange rate could be defined as ratio of CPI in foreign and home countries, both expressed in domestic currency, $Q_{i,t} = \frac{\xi_{i,t} P_t^i}{P_t}$. Then, integrating (8) and (15) for one period ago gives

$$\begin{aligned} C_t &= \left(\frac{C_{t-1}}{C_{t-1}^i} Q_{i,t-1}^{-\frac{1}{\sigma}} \right) C_t^i Q_{i,t}^{\frac{1}{\sigma}} \\ &= \vartheta_i C_t^i Q_{i,t}^{\frac{1}{\sigma}} \end{aligned}$$

where ϑ_i is a constant which depends on the relative net assets possessed by households in two countries initially. Moreover, I assume initial assets position to be the same among countries, and ϑ_i equals to one. Finally, domestic consumption is related to the consumption in country i and the real exchange rate between them.

2.6.5 Goods market

Output of I-firm j is consumed by domestic or foreign F-firms. Therefore, intermediate goods markets clearing in the domestic economy requires

$$Y_t(j) = C_{H,t}(j) + \int C_{H,t}^i(j) di$$

where $C_{H,t}^i(j)$ is the intermediate good j produced in the home economy and used by F-firm in country i. Because I assume symmetry between countries, home export of good j is

$$C_{H,t}^i(j) = \alpha \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left(\frac{P_{H,t}}{\xi_{i,t} P_{F,t}^i} \right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i. \tag{16}$$

Then,

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int \left(\frac{P_{H,t}}{\xi_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di\right]$$

and using the definition of aggregate output, we end up with

$$\begin{aligned} Y_t &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int \left(\frac{P_{H,t}}{\xi_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di \\ &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left[(1-\alpha) + \alpha \int (S^i_t S_{i,t})^{\gamma-\eta} Q_{i,t}^{\eta-\frac{1}{\sigma}} di\right] \end{aligned} \tag{17}$$

where $S_{i,t}$ is bilateral terms of trade defined as $S_{i,t} = \frac{P_{i,t}}{P_{H,t}}$, and S_t is effective terms of trade defined as $S_t = \frac{P_{F,t}}{P_{H,t}}$. In the special case of $\sigma = \eta = \gamma = 1$, the condition (17) can be written as $Y_t = S_t^{\alpha} C_t$, which implies $P_{H,t} Y_t = P_t C_t$.[6] It implies the value of domestic output equals the value of consumption in home economy; hence, the value of imports is equal to the value of exports.

2.7 Linearized Model

2.7.1 Definitions and identities

Effective terms of trade is defined as price of foreign goods in terms of home produced goods both expressed in domestic currency. Log of terms of trade s_t is

$$s_t = p_{F,t} - p_{H,t}. \tag{18}$$

From now on, all log variables are indicated with small letters.

Let's normalize the price level to one in steady state. Log-linearization of the CPI expression around a steady state with zero inflation yields

$$p_t = (1-\alpha) p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha s_t \tag{19}$$

where p_t is the log of CPI.[7] Substituting (18) in (19) with the definition of CPI inflation, $\pi^p_t = p_t - p_{t-1}$, and domestic inflation, $\pi^p_{H,t} = p_{H,t} - p_{H,t-1}$, gives

$$\pi^p_t = \pi^p_{H,t} + \alpha \triangle s_t \tag{20}$$

that is, CPI inflation depends on domestic inflation and the change in terms of trade.

I assume that the law of one price holds for all intermediate goods. It implies that the price of an intermediate good in home country equals the price of that good in the producing country multiplied by the bilateral nominal exchange rate, $P_{i,t}(j) = \xi_{i,t} P^i_{i,t}(j)$ where $P^i_{i,t}(j)$ is the local price of good j produced in country i. Using the definition of

[6] The CPI takes the form $P_t = P_{H,t}^{1-\alpha} P_{F,t}^{\alpha}$ when $\eta = 1$.

[7] By assumption the price level equals one in steady state, which implies zero inflation.

price index for imported good from country i implies $P_{i,t} = \xi_{i,t} P^i_{i,t}$. Substituting in the foreign price index and log-linearizing around the symmetric steady state gives

$$p_{F,t} = \int_0^1 \left(e_{i,t} + p^i_{i,t}\right) di = e_t + p^*_t \tag{21}$$

where $e_t = \int e_{i,t} di$ is the log effective nominal exchange rate, and $p^*_t = \int p^i_{i,t} di$ is the log world price index. Combining the previous result with the definition of terms of trade (18) leads to

$$s_t = e_t + p^*_t - p_{H,t}.$$

Log of bilateral real exchange rate is $q_{i,t} = e_{i,t} + p^i_t - p_t$, and integrating over all countries gives

$$q_t = e_t + p^*_t - p_t \simeq (1-\alpha) s_t. \tag{22}$$

This implies that any improvement in terms of trade is the same as real exchange rate depreciation. It is worth noting that the previous equations are identities and not behavioral ones.

Log-linearizing uncovered interest rate parity (13) around steady state and aggregating over all i gives

$$i_t = i^*_t + E_t\{\triangle e_{t+1}\}$$

and this implies that, if domestic rate is larger than world interest rate, nominal exchange rate is expected to depreciate in the next period.

2.7.2 Goods markets

First-order log-linear approximation of goods market clearing condition (17) around the symmetric steady state implies

$$y_t = c_t + \frac{\alpha\omega}{\sigma} s_t \tag{23}$$

where $\omega = \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$, and I make use of $\int_0^1 s^i_t di = 0$. It is worth noting that in the global goods market total production is consumed each period, so global goods market clearing condition is $y^*_t = c^*_t$. Log of international risk sharing condition yields

$$c_t = c^*_t + \frac{1}{\sigma} q_t = y^*_t + \frac{1-\alpha}{\sigma} s_t, \tag{24}$$

where in second equality I make use of the relationship between real exchange rate and terms of trade (22). Substituting (24), and global market clearing condition in

(23) gives

$$y_t = y_t^* + \frac{1}{\sigma_\alpha} s_t \tag{25}$$

where $\sigma_\alpha = \frac{\sigma}{1-\alpha+\alpha\omega} > 0$. Actual production relates positively to world output and terms of trade.

2.7.3 Aggregate demand and supply

First order approximation of aggregate production function (6) gives

$$y_t = a_t + n_t + z_t \tag{26}$$

and it is worth noting that $z_t = \log(Z_t)$ is of the second order, thus it is zero in the previous equation.

In the short run, the actual production fluctuates around its natural level.[8] Therefore, I should first determine the natural level of production, and ensure that it is optimal. In section 3, I discuss the conditions under which the natural level of production is optimal.

Natural production y_t^n is the level of output under flexible prices and wages when log of marginal cost is equal to frictionless mark-up $mc = -\mu = -\log \frac{\varepsilon}{\varepsilon-1}$, and marginal rate of substitution is equal to natural real wage. Log-linearizing marginal cost (5) yields

$$-v + (\sigma - \sigma_\alpha) y_t^* + (\varphi + \sigma_\alpha) y_t^n - (1+\varphi) a_t = -\mu$$

where $v = -\log(1 - \tau_f) > 0$. Rearranging the previous equation for y_t^n gives

$$y_t^n = \Gamma_0 + \Gamma_1 a_t + \Gamma_2 y_t^* \tag{27}$$

where $\Gamma_0 = \frac{v-\mu}{\sigma_\alpha+\varphi}$, $\Gamma_1 = \frac{1+\varphi}{\sigma_\alpha+\varphi} > 0$, and $\Gamma_2 = \frac{\sigma_\alpha-\sigma}{\sigma_\alpha+\varphi}$. Therefore, natural output is only a function of real variables: technology shock and world output – it is not a function of nominal variables. Positive technology shock increases the natural production, but the effect of world output is ambiguous.[9] In addition, I frequently use natural production to calculate the output gap which is deviation of actual output from its natural level, $x_t = y_t - y_t^n$.

Log of Euler equation (8) implies

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - b) \tag{28}$$

[8] In long run when prices and wages are completely flexible, the model is a Real Business Cycle model in which real shocks (technology and world output shocks) are the only forces that change the real variables (production, real interest rate, real wage rate). The levels of variables under flexible prices and wages are called natural in order to be distinguished from actual level of variables which are induced by staggered prices and wages.

[9] Under special parameterization $(\sigma = \eta = \gamma = 1)$, $\Gamma_2 = 0$.

where $b = -\log\beta$. As shown in section 6.1, linearized Euler equation in terms of output gap x_t and natural real interest rate r_t^n gives

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma_\alpha}(i_t - E_t\{\pi^p_{H,t+1}\} - r_t^n) \tag{29}$$

$$r_t^n = b - \sigma_\alpha\Gamma_a(1-\rho)a_t + \sigma_\alpha(\alpha\Theta + \Gamma^*)E_t\{\triangle y^*_{t+1}\} \tag{30}$$

where $\Theta = \omega - 1$. From (29) output gap relates to real interest rate gap (deviation of actual real interest rate, $r_t = i_t - E_t\{\pi^p_{H,t+1}\}$, from its natural level) and expectation about future output gap. Moreover, from (30) natural real interest rate is a function of domestic productivity and world output changes. The increase in productivity causes a decrease in natural real interest rate. Consequently, from (29), a decrease in natural real interest rate creates a positive real interest gap which causes a decrease in output gap.

In the absence of both price and wage rigidities and presence of labor subsidy, condition (9) reduces to the condition that the natural real wage, ω_t^n, equals marginal rate of substitution, and in log-linear format

$$\omega_t^n = w_t - p_t = \sigma c_t + \varphi n_t, \tag{31}$$

then rearranging (31) yields

$$\omega_t^n = \Upsilon_0 + \Upsilon_1 a_t + \Upsilon_2 y_t^*$$

where $\Upsilon_0 = (\varphi + (1-\alpha)\,\sigma_\alpha)\,\Gamma_0$, $\Upsilon_1 = 1 - \alpha\sigma_\alpha\Gamma_1$ and $\Upsilon_2 = \frac{\alpha\sigma_\alpha(\sigma+\varphi)}{\sigma_\alpha+\varphi} > 0$, which makes use of (24), (26), (25), (27) and global goods market clearing condition. From (31) an increase in world output and positive technology shock, provided that $\Upsilon_a > 0$, increases the natural real wage.

As shown in section 6.2, domestic price inflation equation, derived from log-linearized price-setting condition (10), is

$$\pi^p_{H,t} = \beta E_t\{\pi^p_{H,t+1}\} + \kappa^p x_t + \lambda^p\tilde{\omega}_t \tag{32}$$

where $\lambda^p = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} > 0$, $\kappa^p = \lambda^p\alpha\sigma_\alpha > 0$, and $\tilde{\omega}_t$ is the gap between actual real wage and natural real wage, $\tilde{\omega}_t = \omega_t - \omega_t^n$. Therefore, domestic price inflation positively relates to output gap, real wage gap, and expectations about future domestic inflation. Positive technology shock, for example, indirectly reduces domestic price inflation, through decreasing output and wage gaps.

In section 6.3, I derive the wage inflation (changes in the nominal wage rate) equation (33) from the log-linearized wage-setting condition

$$\pi_t^w = \beta E_t\{\pi^w_{t+1}\} + \kappa^w x_t - \lambda^w\tilde{\omega}_t \tag{33}$$

where $\lambda^w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varphi\varsigma)} > 0$, $\kappa^w = \lambda^w(\varphi + (1-\alpha)\sigma_\alpha) > 0$, and π_t^w is the wage inflation. So, wage inflation in the current period relates not only to the output gap and real wage gap, but also to the expectation about wage inflation in the next period. Moreover, output gap has a positive effect on wage inflation rate, while real wage gap has a negative effect on it. Finally, as shown in section 6.4, the law of motion of real wage gap is

$$\tilde{\omega}_t = \tilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \triangle\omega_t^n. \tag{34}$$

Analytically, a positive technology shock raises natural output and real wage, and decreases output and real wage gaps. Consequently, domestic price inflation decreases. However, the exact effect of technology shock on wage inflation is not clear and it depends on the relative size of output and wage gaps in (33).

Although the dynamic price-setting (32) and the dynamic wage-setting (33) equations are apparently identical, they have different coefficients which linked to parameters in preferences and technologies in different ways, and bring different characteristics for each equation.[10]

3 Monetary Policy: a Special Case

As mentioned above, it is important to be sure that natural production and employment are optimal. Even if prices and wages are fully flexible, there are still two sources of distortion in this economy: monopolistic competition in labor and goods markets. These distortions cause households to supply less labor and firms to produce less output than the perfect competitive equilibrium. Therefore, fiscal policy should subsidize employment and production (τ_h, τ_f) to offset mark-ups. The social planner's problem is to maximize the utility subject to the production function, labor demand, risk-sharing condition (24), and market clearing condition (17).[11]

I show that, if employment subsidy is $\tau_h = \frac{1}{\varsigma - 1}$, then households supply labor as in a competitive market, and the social planner only needs to choose remaining subsidy, τ_f. For a special case $(\sigma = \eta = \gamma = 1)$, Gali and Monacelli (2005) show that by setting the production subsidy such that $v = \mu + \log(1 - \alpha)$ is satisfied, flexible price equilibrium allocation is optimal. Moreover, households supply constant labor, $N_t = (1 - \alpha)^{\frac{1}{1+\varphi}}$, under the assumed preferences. I assume that government subsidizes according to the above conditions, which ensures optimality of flexible equilibrium allocation. However, the cost of imposing this condition is to be restricted in particular parameters. The rest of this chapter is based on the holding of the above condition.

As is the case for a closed economy, it is impossible for monetary policy to attain the Pareto optimum under staggered wages and prices. Domestic price inflation and wage

[10] Huang and Liu (2002) show that the staggered price-setting mechanism by itself is incapable of generating persistence in output, while the staggered wage-setting mechanism helps in producing output persistence.

[11] For a formal proof, see Gali and Monacelli (2005).

inflation are at their desired level (zero) if there is no real wage gap, or if real wage was at its natural level forever. However, this is impossible because natural wage rate is a function of exogenous shocks in domestic technology and world output.[12]

3.1 The Welfare Loss

To evaluate the performance of alternative policy rules, I need a quantitative criterion. While some suggested criteria are exogenous, one that was introduced by Rotemberg and Woodford (1999) which is based on consumer utility, seems to be more appropriate. In this case, welfare loss is the second-order approximation to the utility experienced by the consumers as a result of deviation from efficient allocation. As shown in section 6.5, the welfare loss function is

$$W = -\frac{1-\alpha}{2}\sum_{t=0}^{\infty}\beta^t\left[(1+\varphi)(x_t)^2 + \frac{\varepsilon}{\lambda^p}(\pi^p_{H,t})^2 + \frac{\varsigma}{\lambda^w}(\pi^w_t)^2\right] \tag{35}$$

and taking expectation and letting $\beta \to 1$, (35) can be represented as

$$V = -\frac{1-\alpha}{2}\left[(1+\varphi)var\,(x_t) + \frac{\varepsilon}{\lambda^p}var(\pi^p_{H,t}) + \frac{\varsigma}{\lambda^w}var(\pi^w_t)\right]. \tag{36}$$

It is clear that the loss function is the weighted average of variance of three variables: output gap, domestic price inflation, and wage inflation. The weight of output gap depends on the parameter of labor in utility, and the weight of both domestic price and wage inflations depend on elasticity of substitution and level of stickiness. For example, the higher degree of wage stickiness ($\theta_w \to 1$), the higher is the penalty of variance in wage inflation ($\frac{\varsigma}{\lambda^w} \to \infty$). The intuition is straightforward: when wages are sticky, wage inflation results in an inefficient dispersion of supply of labor. The dispersion of wages changes the relative price of differentiated labor; hence, firms employ more of the relatively cheaper labor. Because of curvature in utility function, wage dispersion reduces utility.

3.2 Optimal Monetary Policy

Monetary policy cannot achieve Pareto optimum equilibrium, but I can define the optimal monetary policy as the policy which maximizes household welfare. Under the optimal monetary policy, the central bank minimizes welfare loss (35) subject to domestic price-setting (32), wage-setting (33), and low of motion of real wage gap (34)

[12]This is shown by Erceg et al. (2000) for a closed economy.

for $t = 1, 2, \dots$. The first order conditions for the optimal monetary policy are

$$
\begin{aligned}
&-(1-\alpha)(1+\varphi)\, x_t + \kappa^p \chi_{1t} + \kappa^w \chi_{2t} + \alpha\sigma_\alpha(\chi_{3t} - \beta\chi_{3t-1}) = 0 \\
&-(1-\alpha)\frac{\varepsilon}{\lambda^p}\pi^p_{H,t} - \chi_{1t} + \chi_{1t-1} + \chi_{3t} = 0 \\
&-(1-\alpha)\frac{\varsigma}{\lambda^w}\pi^w_t - \chi_{2t} + \chi_{2t-1} - \chi_{3t} = 0 \\
&\lambda^p\chi_{1t} - \lambda^w\chi_{2t} + \chi_{3t} - \beta E\{\chi_{3t+1}\} = 0
\end{aligned}
$$

where χ_{1t}, χ_{2t} and χ_{3t} are Lagrange multipliers associated with constraints (32), (33), and (34) respectively. In order to implement the *optimal policy*, the output gap and real wage gap should be observed by the central bank; that is, the central bank has the right information at the right time, which is not the case in the real world. However, the welfare under the optimal monetary policy provides a benchmark for comparing the performance of alternative policy rules which are explained in the next section.

3.3 Alternative Monetary Policy Rules

The main objective of this chapter is to use the above model to evaluate the performance of different policy rules. Henceforth, I use the term "Taylor-type" to indicate an interest rate rule in which the interest rate is a policy instrument that is completely controlled by the central bank. In theory the central bank could adjust nominal interest rate if inflations, output gap, or natural real interest rate change; however, in reality output gap is not observed, or it is known with lag in the best case. Therefore, I limit the alternative rules to ones that are simple to implement.[13] The general form of the next four Taylor-type interest rate rules is

$$i_t = b + \phi\pi_t \tag{37}$$

while real interest rate is fixed in the unconditional mean of natural real interest rate, b. The nominal interest rate is changed in response to a change in

- CPI inflation under CPI inflation-targeting (CIT), $\pi_t = \pi^p_t$,
- domestic price inflation under domestic price inflation-targeting (DIT), $\pi_t = \pi^p_{H,t}$,
- domestic wage inflation under wage inflation-targeting (WIT), $\pi_t = \pi^w_t$,
- both domestic price and wage inflations under hybrid inflation-targeting (HIT), $\pi_t = \delta\pi^p_{H,t} + (1-\delta)\pi^w_t$ where $\delta \in [0, 1]$.[14]

[13] There is an implicit assumption that the central bank has insufficient information about the shocks in the current period to change the nominal interest rate alongside the changes in the natural real interest rate and output gap.

[14] HIT is a linear combination of DIT and WIT, and choosing the weight of each policy, δ, is important. Therefore the weight are going to be chosen such that HIT has its best performance in regard to representative welfare.In present model, optimal choice of δ is 0 in benchmark parameterization. That means HIT is the same as WIT; therefore, performance of HIT is not reported separately.

The final rule is the pegged exchange rate (PER) which the central bank fixes nominal exchange rate in advance, $e_t = 0$.

3.3.1 Equilibrium

Dynamic properties of the model can be studied by using the Euler equation (29), domestic price inflation (32), wage inflation (33), wage gap motion (34), and policy rule (37). Let's replace (37) with a more general format of the Taylor-type rule that the central bank adjusts the nominal interest rate in response to the deviation of domestic price inflation, wage inflation, and output gap from their steady-state levels,

$$i_t = b + \phi_p \pi^p_{H,t} + \phi_w \pi^w_t + \phi_x x_t \tag{38}$$

where ϕ_p, ϕ_w, $\phi_x \geq 0$. The above rule covers DIT, WIT and HIT. These equations can be rearranged in matrix format

$$AX_{t+1} = BX_t + CZ_t \tag{39}$$

where $X_t = [\tilde{\omega}_{t-1}, x_{t-1}, x_t, \pi^p_{H,t}, \pi^w_t]'$ and $Z_t = [a_t, y^*_t]'$. Blanchard and Kahn (1980) show that a necessary and sufficient condition for uniqueness of a solution to a system with two predetermined variables and three non-predetermined variables like (39) is that three eigenvalues of $A^{-1}B$ lie outside the unit circle.

It is not possible to derive the eigenvalues of $A^{-1}B$ analytically; hence, I use the benchmark parameters – from the next section – to examine the uniqueness of equilibrium under different parameterization of monetary policy (38). Figure 2.1 indicates the combinations of ϕ_p and ϕ_w associated with unique equilibrium under the condition $\phi_x = 0$. The region of indeterminacy is shaded, and corresponds to any pair of ϕ_p and ϕ_w such that $\phi_p + \phi_w \leq 1$. If, for instance, the monetary authorities decide to adjust nominal interest rate only in response to changes in domestic price inflation, the coefficient of inflation in policy rule should be more than one to guarantee equilibrium.

Figure 2.2 indicates how an increase in ϕ_x affects the determinacy region. Now, if ϕ_x is positive, the sum of domestic price and wage inflations could fall below one, and the system still has a unique solution. It can be shown that these conditions apply for CIT, so the coefficient of CPI inflation in the Taylor-type rule must be larger than one in order to guarantee equilibrium. Likewise, if nominal interest rate is also adjusted to changes in output gap, the coefficient of CPI inflation can be less than one, and the system still has a unique equilibrium.

FIGURE 2.1: Determinacy and Indeterminacy Regions when $\phi_x = 0$

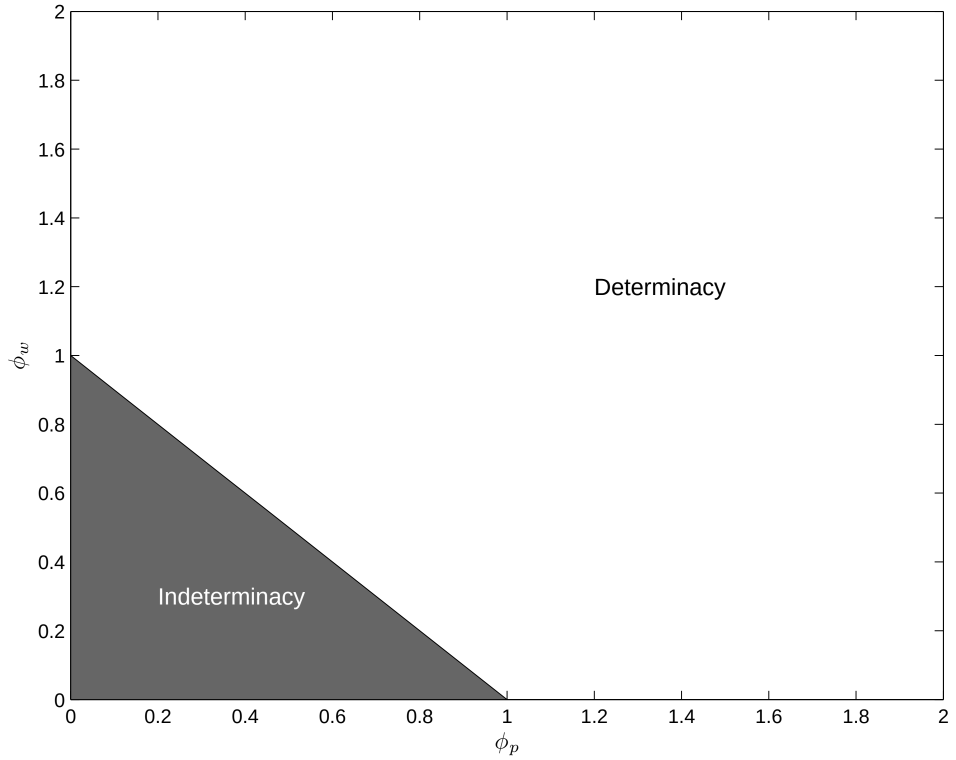
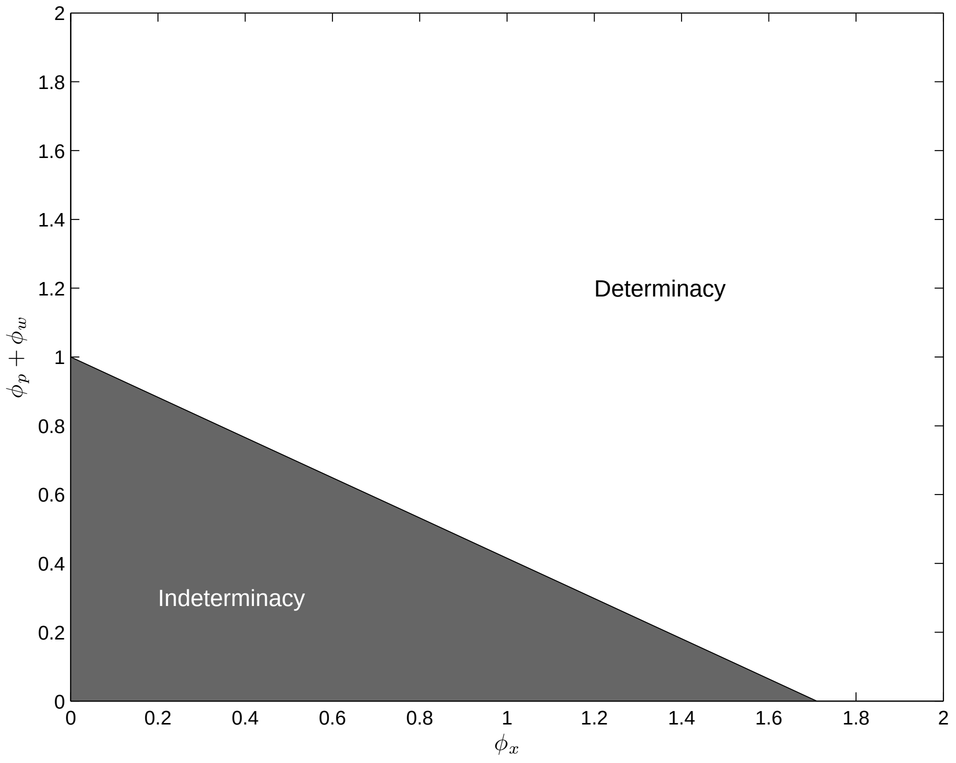


FIGURE 2.2: Determinacy and Indeterminacy Regions



4 Numerical Analyses

4.1 Calibration

I take a period to be a quarter and set the discount factor to $\beta = 0.99$, which implies 1 percent interest rate in one quarter (4 percent real interest rate annually). Consumption utility is logarithmic, $\sigma = 1$, and labor utility parameter is $\varphi = 3$, which implies a labor supply elasticity of 1/3.

Moreover, substitutability of bundles of goods with different origins are equal, $\eta = \gamma = 1$, while elasticity of substitution between differentiated goods is $\varepsilon = 6$. The latter implies a mark-up of 1.2 in steady state; similarly I assume that $\varsigma = 6$ and a markup 1.2. Nominal rigidity parameters are $\theta_p = \theta_w = 0.75$, which imply a one-year duration of price and wage contracts on average ($\frac{1}{1-0.75} = 4$ quarters).[15] In order to ensure equilibrium existence, the coefficient of domestic inflation, wage inflation, and CPI inflation in the Taylor-type rules (37) is 1.5.

In steady state, the share of foreign goods in total domestic consumption is $\alpha = 0.4$. I also assume that both domestic and world technology follow $AR(1)$ process

$$u_{t+1} = \rho_u u_t + \varepsilon_{u,t+1}$$

where $\rho_u \in [0, 1)$ and $\varepsilon_{u,t}$ is a random variable with zero mean and constant variance. I use $\rho_a = 0.66$, $\rho_{a^*} = 0.86$, $\sigma_a = 0.0071$, $\sigma_{a^*} = 0.0078$, and $corr(\varepsilon_a, \varepsilon_{a^*}) = 0.3$. All above numbers are from Gali and Monacelli (2005), except θ_w and ς.

4.2 Impulse Responses

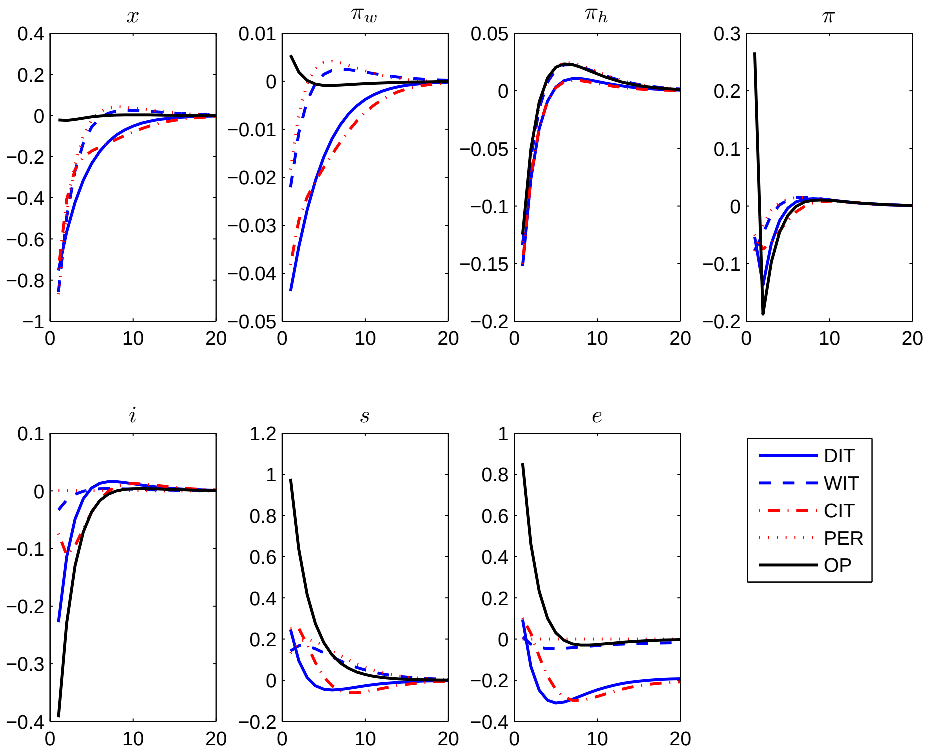
In this section, I study the dynamic behavior of alternative rules when reacting to domestic technology shocks. Figure 2.3 indicates impulse responses to a one percentage point increase in domestic productivity. The figure shows that patterns of impulse response are similar for alternative rules, especially for output gap, domestic inflation, and wage inflation. However, these rules imply different responses for CPI inflation, terms of trade, the interest rate, and the exchange rate.

A positive technology shock decreases the natural real interest rate and increases natural output and natural real wage. Therefore, negative output and wage gaps cause reduction in domestic prices. In contrast, these gaps have opposite impacts on wage inflation, and the change in wage inflation is ambiguous. Moreover, the reaction of interest rate, exchange rate, terms of trade, and CPI inflation depend on monetary policy in effect.

Under the *optimal policy*, nominal interest rate decreases severely, and it causes a large depreciation in exchange rate and improvement in terms of trade. In spite of

[15] Gali (2010) uses $\varsigma = 4.52$ and estimates $\theta_w = 0.81$ for US.

FIGURE 2.3: Impulse Response to a Rise in Domestic Productivity



fall in domestic prices, CPI inflation increases because foreign prices sky rocket due to strong depreciation of exchange rate. Moreover, the effect of decrease in wage gap gets dominant over effect of decrease in output gap, and lead to an increase in wage inflation.

Under DIT and WIT, the central bank reacts by decreasing nominal interest rate in order to encourage consumption. The large nominal interest rate fall under DIT is the source of exchange rate depreciation; however, uncovered interest parity implies formation of expectation for exchange rate appreciation in future. Under DIT, CPI inflation decreases initially because the large appreciation of exchange rate dominates the increase in domestic prices, and then it rises sharply. The pattern under WIT is quite similar to under DIT, except the fall in interest rate is smaller under WIT, because domestic inflation decreases more than wage inflation.

Uncover interest rate implies that the interest rate is fixed under PER. The initial terms of trade improvement under PER is less than under DIT and CIT because exchange rate deprecation in the latter cases causes a larger increase in terms of trade. Hence, under PER, interest rate and exchange rate are constant and terms of trade has a hump-shape pattern, which makes CPI and wage inflation more stable.

Under CIT, the central bank adjusts interest rate in response to CPI inflation, and since foreign prices are constant, the decrease in CPI inflation is less than domestic inflation; hence, the decrease in interest is less than under DIT. Interest rate continues to decrease slightly and then rises slowly to its steady state. Slow adjustment of interest rate causes persistent appreciation in exchange rate and improvement in terms of trade, which keeps CPI inflation more stable under CIT.

4.3 Welfare Analysis

Table 2.1 reports business cycle properties of several key variables under alternative monetary policy regimes. In the table, all figures are reported in percentage terms, except for the exchange rate which is expressed in first difference, because the level of exchange rate is not stationary. Under *optimal policy*, wage and domestic inflations are stable, while output and exchange rate are very volatile.

In terms of volatility of variables, DIT and WIT are quite similar to each other. The very small differences are that under DIT wage inflation, domestic inflation, and interest rate volatility and under WIT output volatility are higher relative to each other. These rules generate less volatility in output, while *optimal policy* provides less volatility in wage and domestic inflations.

Under CIT, the variability of wage inflation, domestic price inflation, output, and interest rate are higher than DIT and WIT. Since CPI inflation depends on both domestic price inflation and exchange rate, under CIT interest rate is very volatile and it makes other variables more volatile too.

Under PER, exchange rate, terms of trade, and CPI inflation are more stable, but interest rate is very volatile because uncover interest parity implies that exchange rate is fixed if domestic interest rate changes one to one as world interest rate changes. Then high volatile interest rate causes output and wage inflation to be more volatile under PER. Therefore, the patterns under PER and CIT are different from the pattern under *optimal policy*.

We can conclude that DIT and WIT roughly perform as well as the *optimal policy*. However, by using the information in Table 2.1, it is impossible to rank alternative rules based on their performances, so it is necessary to use the welfare criterion (36) in order to evaluate rules' performances. Table 2.2 summarizes the performance of alternative policy rules. For the benchmark parameterization, WIT performs better among simple rules. Besides, CIT and PER have the worst performances, and DIT

TABLE 2.1: Cyclical Properties of Alternative Policy Rules

	Optimal Policy	DIT	WIT	CIT	PER
Output	0.92	0.20	0.28	0.85	1.40
Wage inflation	0.00	0.05	0.02	0.13	0.08
Domestic inflation	0.10	0.12	0.11	0.17	0.11
CPI inflation	0.35	0.35	0.35	0.32	0.07
Interest rate	0.34	0.19	0.03	0.48	0.21
Terms of trade	1.56	1.52	1.47	2.05	0.53
Exchange rate	0.91	0.80	0.81	0.65	0.00

Notes: Figures are standard deviation reported in percentages.

TABLE 2.2: Contribution to Welfare Losses

	optimal policy	*DIT*	*WIT*	*CIT*	*PER*
$\varepsilon = 6, \varsigma = 6, \varphi = 3$					
Var(output gap)	0.0000	0.0077	0.0065	0.0140	0.0204
Var(domestic inflation)	0.0022	0.0032	0.0026	0.0062	0.0028
Var(wage inflation)	0.0001	0.0097	0.0013	0.0722	0.0246
Total loss	0.0023	0.0206	0.0104	0.0924	0.0477

performs moderately.

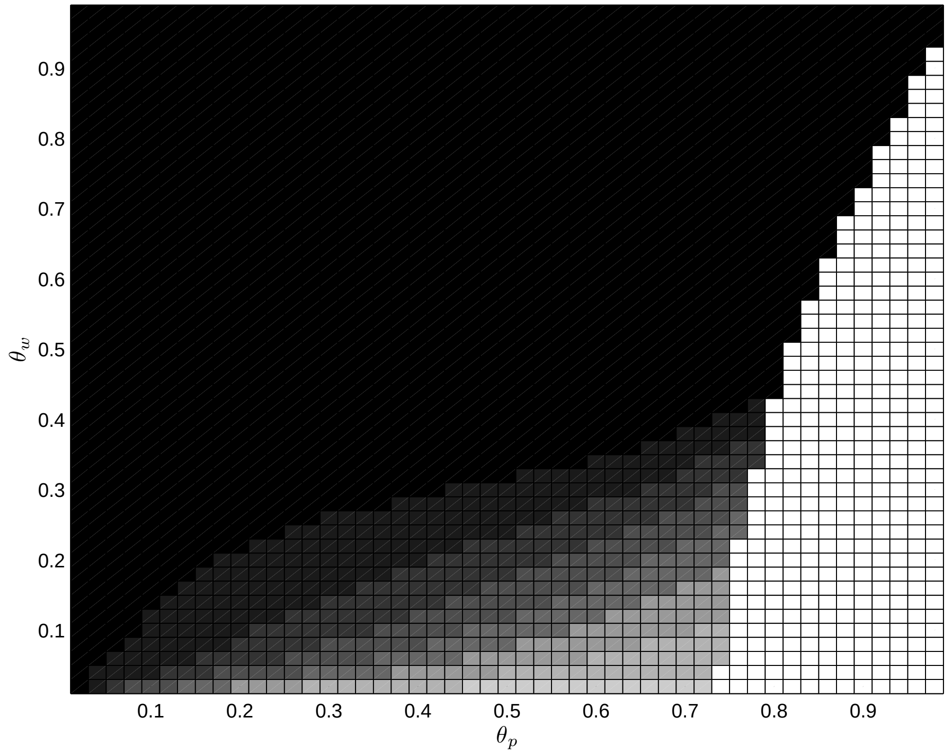
Table 2.3 reports results from robustness analysis, and it has four panels, each of which shows welfare loss under different parameterizations. In panel A, the firm mark-up decreases to 1.1, while household mark-up and labor elasticity are the same as in benchmark. In panel B only household mark-up decreases to 1.1. Next, the labor supply elasticity decreases to 0.1 in panel C. Finally, all three parameters change in panel D. Table 2.3 displays that the pattern obtained under the benchmark parameterization holds under different parameterizations.

TABLE 2.3: Contribution to Welfare Losses: Sensitivity Analysis

	optimal policy	*DIT*	*WIT*	*CIT*	*PER*
A: $\varepsilon = 11, \varsigma = 6, \varphi = 3$					
Var(output gap)	0.0000	0.0078	0.0065	0.0139	0.0201
Var(domestic inflation)	0.0041	0.0060	0.0048	0.0112	0.0050
Var(wage inflation)	0.0001	0.0098	0.0013	0.0722	0.0243
Total loss	0.0042	0.0236	0.0126	0.0973	0.0494
B: $\varepsilon = 6, \varsigma = 11, \varphi = 3$					
Var(output gap)	0.0000	0.0089	0.0065	0.0218	0.0224
Var(domestic inflation)	0.0023	0.0028	0.0024	0.0047	0.0025
Var(wage inflation)	0.0001	0.0126	0.0012	0.1414	0.0355
Total loss	0.0023	0.0243	0.0102	0.1679	0.0604
C: $\varepsilon = 6, \varsigma = 6, \varphi = 10$					
Var(output gap)	0.0000	0.0222	0.0179	0.0420	0.0572
Var(domestic inflation)	0.0023	0.0032	0.0026	0.0058	0.0027
Var(wage inflation)	0.0000	0.0280	0.0046	0.2094	0.0651
Total loss	0.0023	0.0534	0.0251	0.2572	0.1250
D: $\varepsilon = 11, \varsigma = 11, \varphi = 10$					
Var(output gap)	0.0000	0.0255	0.0179	0.0684	0.0620
Var(domestic inflation)	0.0042	0.0051	0.0045	0.0082	0.0046
Var(wage inflation)	0.0000	0.0344	0.0042	0.3990	0.0903
Total loss	0.0042	0.0650	0.0266	0.4756	0.1568

Although WIT performs the best, the performance of DIT is better than CIT and PER and not far from WIT. In order to examine how much these results depend on the degree of price and wage stickinesses, we introduce HIT with $\delta \in [0, 1]$ where δ determines the weight of domestic inflation in the rule. In all possible combinations of stickiness, there is a DIT or WIT such that its performance is better than both CIT and PER. Therefore, the choice of simple monetary rule is only between DIT

FIGURE 2.4: Optimal Weight of Domestic Inflation in HIT



Notes: The optimal weight of domestic inflation is zero in black region, and it is one in wight region.

and WIT. Figure 2.4 indicates the best simple rule under different combinations of wage and domestic price stickiness.

Figure 2.4 shows that for a wide combinations of wage and price stickiness, WIT is the best simple policy rule. For cases where the degree of wage stickiness is higher than that of price stickiness, WIT is the best policy rules. The intuition is that coefficients on the output gap and the wage gap in the wage-setting equations are lower than their counterparts in the price-setting equation; therefore, deviations in wage inflation will be more persistent and cause more inefficiency in comparison to domestic price inflation. As the price stickiness increases, the weight of DIT in HIT goes up, and for high enough price stickiness pure DIT is the best. In the area that DIT performs well, the coefficients on output and real wage gaps in the price-setting equation are lower than their counterparts in the wage-setting equation; hence, DIT provide less welfare loss. In the area that a combination of DIT and WIT is a desirable policy, the coefficient on the output gap in the wage-setting equation is larger than its counterpart in the price-setting equation; in contrast, the coefficient on the wage gap in the wage-setting equation is smaller than its counterpart in the price-setting equation.

Therefore, it could be argued that the desired simple policy rule depends on the relative rigidity of wages and prices. However, wage inflation-targeting is the desirable simple rule for a plausible calibration of the model.

5 Conclusion

This chapter has analyzed monetary policy in an open economy with both staggered wages and prices. It has been shown that domestic inflation-targeting – which is optimal in a model with staggered nominal prices – is not optimal anymore. In an open economy, the central bank has an opportunity to follow a pegged exchange rate or CPI inflation-targeting, but I show that the latter policies are not better than domestic inflation-targeting or wage inflation-targeting. The performance of domestic inflation-targeting and wage inflation-targeting depends on the degree of stickiness in domestic prices and wages. However, wage inflation-targeting is the desirable simple rule for a plausible calibration of the model.

There are some other open-economy issues that are important to consider: incomplete pass-through, and imported capital. In this chapter, we assume that exchange rate pass-through is complete and that production is the only function of labor. Introducing imperfect pass-through and imported capital causes the central bank to leverage some weight on imported price inflation. These could be subjects for further research.

6 Appendix

6.1 Euler Equation

Combining (28), (20), and (25) gives

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha}(i_t - E_t\{\pi^p_{H,t+1}\} - b) + \alpha\Theta E_t\{\triangle y^*_{t+1}\}$$

where $\Theta = \omega - 1$. Using $x_t = y_t - y_t^n$ and natural output (27), the above equation can be written as (29) and (30) in the text.

6.2 Optimal Price-setting

Log of price-setting condition (10) implies

$$\bar{p}_{H,t} = (1-\beta\theta_p)\sum_{k=0}^{\infty}(\beta\theta_p)^k E_t\{p_{H,t}\} + (1-\beta\theta_p)\sum_{k=0}^{\infty}(\beta\theta_p)^k E_t\{mc_t + \mu\},$$

and the above equation can be written

$$\bar{p}_{H,t} = (1-\beta\theta_p)p_{H,t} + \beta\theta_p E_t\{\bar{p}_{H,t+1}\} + (1-\beta\theta_p)(mc_t + \mu). \tag{40}$$

In each period, $(1-\theta_p)$ percent of firms change their prices while the rest post previous prices, so aggregate price is the weighted average of re-optimized price and previous price level which in linear form is

$$p_{H,t} = (1-\theta_p)\bar{p}_{H,t} + \theta_p p_{H,t-1}. \tag{41}$$

Thus, combining (40) and (41) gives

$$\pi^p_{H,t} = \beta E_t\{\pi^p_{H,t+1}\} + \lambda^p(mc_t + \mu) \tag{42}$$

where $\lambda^p = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}$. Log-linearizing the marginal cost is

$$mc_t = -v + w_t - p_{H,t} - a_t. \tag{43}$$

Rearranging (43) with (19) and (25) gives

$$mc_t = -v + \omega_t + \alpha\sigma_\alpha(y_t - y_t^*) - a_t \tag{44}$$

where ω_t is real wage rate. Natural level of output is defined as output corresponding with flexible wages and prices. The real marginal cost in frictionless situation is equal to μ,

$$\mu = -v + \omega_t^n + \alpha\sigma_\alpha(y_t^n - y_t^*) - a_t. \tag{45}$$

Subtracting (45) from (44) gives

$$mc_t - \mu = \alpha\sigma_\alpha x_t + \tilde{\omega}_t \tag{46}$$

where $\tilde{\omega}_t = \omega_t - \omega_t^n$ is the deviation of real wage from its natural level. Equations (42) and (46) together give the domestic price-setting

$$\pi_{H,t}^p = \beta E_t\{\pi_{H,t+1}^p\} + \kappa^p x_t + \lambda^p \tilde{\omega}_t \tag{47}$$

where $\kappa^p = \lambda^p \alpha\sigma_\alpha > 0$. It implies that the domestic inflation in time t is not only the function of output gap and real wage gap, but is the function of expectation of domestic inflation.

6.3 Optimal Wage-setting

Log-linearizing wage-setting condition (9) obtains

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \{(1+\varphi\varsigma)\bar{w}_t - p_{t+k} - \sigma c_{t+k} - \varphi n_{t+k} - \varphi\varsigma w_{t+k}\} = 0$$

where $\bar{w}_t$ is (log) wage which is set in time t, and it can be rewritten as

$$\bar{w}_t = \frac{1-\beta\theta_w}{1+\varphi\varsigma}(p_t + \sigma c_t + \varphi n_t + \varphi\varsigma w_t) + \beta\theta_w E_t\{\bar{w}_{t+1}\}. \tag{48}$$

Equation (48) with linearized form of aggregate wage definition, $w_t = (1-\theta_w)\bar{w}_t + \theta_w w_{t-1}$, give

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \lambda^w(\sigma c_t + \varphi n_t - \omega_t) \tag{49}$$

where $\lambda^w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varphi\varsigma)} > 0$.

Using (31), (24), and (25), marginal rate of substitution can be represented as the function of output gap and real wage gap

$$\varphi n_t + \sigma c_t = (\varphi + (1-\alpha)\sigma_\alpha)x_t + \omega_t^n. \tag{50}$$

Finally, using (49) and (50), wage inflation equation yields as a function of output and real wage gaps,

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \kappa^w x_t - \lambda^w \tilde{\omega}_t$$

where $\kappa^w = \lambda^w(\varphi + (1-\alpha)\sigma_\alpha) > 0$.

6.4 Real Wage Gap Motion

Changes in wage gap are given by following identity

$$\tilde{\omega}_t = \tilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \triangle\omega_t^n.$$

That is, changes in wage gap relate to wage inflation, price inflation, and natural wage difference. Combining (14) and (27), natural wage difference can be written as a function of exogenous state variables

$$\triangle\omega_t^n = \Upsilon_1 \triangle a_t + \Upsilon_2 \triangle y_t^*.$$

6.5 Welfare Loss

In this section I derive the welfare loss for the special case of $\sigma = \eta = \gamma = 1$. In this case, the consumption utility is logarithmic form, and goods from different origins are complete substitute. Using a second-order Taylor expansion to household h utility around steady state, combined with complete financial market assumption and integrating across households, yields

$$\begin{aligned}
\int_0^1 U(h)dh &= \int_0^1 \left(\log\left(C_t\right) - \frac{N_t\left(h\right)^{(1+\varphi)}}{1+\varphi} \right) dh \\
&= c_t - \frac{N^{(1+\varphi)}}{1+\varphi} - N^{(1+\varphi)} \left(\int_0^1 \tilde{n}_t(h)dh + \frac{1+\varphi}{2} \int_0^1 \tilde{n}_t(h)^2 dh \right) \qquad (51)
\end{aligned}$$

where N is the efficient employment when subsidies are in effect. In the next steps, I should rewrite the above equation in terms of (log) aggregate employment and then output gap. Consumption utility is logarithmic, so it could be written as $c_t^n + \tilde{c}_t$, in which the second element could be replaced by using the equation $c_t = (1-\alpha)\, y_t + \alpha y_t^*$.

The next lemmas help to derive the welfare loss function easily.

Lemma 1 $E_h\{\tilde{w}_t(h)\} = \frac{\varsigma-1}{2} E_h\{\tilde{w}_t(h)^2\}$ *and* $E_j\{\tilde{p}_{H,t}(j)\} = \frac{\varepsilon-1}{2} E_j\{\tilde{p}_{H,t}(j)^2\}$.

Proof. Let $\tilde{w}_t(h) = w_t(h) - w_t$. Second-order expansion of $\left(\frac{W_t(h)}{W_t}\right)^{1-\varsigma}$ gives

$$\begin{aligned}
\left(\frac{W_t(h)}{W_t}\right)^{1-\varsigma} &= \exp\{(1-\varsigma)\tilde{w}_t(h)\} \\
&= 1 + (1-\varsigma)\tilde{w}_t(h) + \frac{(1-\varsigma)^2}{2}\tilde{w}_t(h)^2
\end{aligned}$$

and from the definition of W_t, we have $1 = \int_0^1 \left(\frac{W_t(h)}{W_t}\right)^{1-\varsigma} dh$. Thus, integrating across households yields

$$E_h\{\tilde{w}_t(h)\} = \frac{\varsigma-1}{2} E_h\{\tilde{w}_t(h)^2\}.$$

It is worth noting that the measure of each household is zero, so integrating $\tilde{w}_t(h)^2$

over all households is the same as the variance of nominal wage across households.

In the same way, it can be shown that $E_j\{\tilde{p}_{H,t}(j)\} = \frac{\varepsilon-1}{2}E_j\{\tilde{p}_{H,t}(j)^2\}$. ■

Lemma 2 $\int_0^1 \tilde{n}_t(h)dh + \frac{1+\varphi}{2}\int_0^1 \tilde{n}_t(h)^2 dh = \tilde{n}_t + \frac{1+\varphi}{2}\tilde{n}_t^2 + \frac{\varphi\varsigma^2}{2}var_h\{w_t(h)\}$.

Proof. Define aggregate employment as $N_t = \int_0^1 N_t(h)dh$. In terms of log deviation from steady state and up to the second-order approximation, the latter equation can be expressed as

$$\tilde{n}_t + \frac{1}{2}\tilde{n}_t^2 \simeq \int_0^1 \tilde{n}_t(h)dh + \frac{1}{2}\int_0^1 \tilde{n}_t(h)^2 dh.$$

Moreover, $\int_0^1 \tilde{n}_t(h)^2 dh$ can be rewritten as

$$\begin{aligned}\int_0^1 \tilde{n}_t(h)^2 dh &= \int_0^1 \left[\tilde{n}_t(h) - \tilde{n}_t + \tilde{n}_t\right]^2 dh \\ &= \tilde{n}_t^2 + 2\tilde{n}_t \int_0^1 \left[\tilde{n}_t(h) - \tilde{n}_t\right] dh + \int_0^1 \left[\tilde{n}_t(h) - \tilde{n}_t\right]^2 dh,\end{aligned}$$

and using (log) labor demand, $\tilde{n}_t(h) - \tilde{n}_t = -\varsigma\tilde{w}_t(h)$, the above equation is expressed as

$$\begin{aligned}\int_0^1 \tilde{n}_t(h)^2 dh &= \tilde{n}_t^2 - 2\varsigma\tilde{n}_t \int_0^1 \tilde{w}_t(h)dh + \varsigma^2 \int_0^1 \tilde{w}_t(h)^2 dh \\ &\simeq \tilde{n}_t^2 + \varsigma^2 var_h\{w_t(h)\}.\end{aligned}$$

where in deriving the second equality, I have made use of Lemma 1. ■

Lemma 3 $\tilde{n}_t = x_t + \frac{\varsigma}{2}var_h\left\{w_t(h)\right\} + \frac{\varepsilon}{2}var_j\left\{p_{H,t}(j)\right\}$

Proof. Aggregate labor supply can be expressed as

$$N_t = \int_0^1 N_t(h)dh = N_t \int_0^1 \left(\frac{W_t(h)}{W_t}\right)^{-\varsigma} dh = \delta_{w,t}N_t,$$

and also aggregate labor demand can rewritten as

$$N_t = \int_0^1 N_t(j)dj = \frac{Y_t}{A_t}\int_0^1 \frac{Y_t(j)}{Y_t}dj = \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj = \delta_{p,t}\frac{Y_t}{A_t}.$$

Collecting the previous results, we have $N_t = \delta_{w,t}\delta_{p,t}\frac{Y_t}{A_t}$. Log-linearizing the latter equation gives

$$n_t = y_t - a_t + \log(\delta_{w,t}) + \log(\delta_{p,t})$$

where $\delta_{w,t}$ and $\delta_{p,t}$ are respectively wage and price dispersion. Subtracting the above equation from its counterpart in natural state yields

$$\tilde{n}_t = x_t + \log(\delta_{w,t}) + \log(\delta_{p,t}).$$

Notice that

$$
\begin{aligned}
\left(\frac{W_t(h)}{W_t}\right)^{-\varsigma} &= \exp\{-\varsigma \tilde{w}_t(h)\} \\
&= 1 - \varsigma \tilde{w}_t(h) + \frac{\varsigma^2}{2} \tilde{w}_t(h)^2.
\end{aligned}
$$

Then, integrating across all households and using Lemma 2, log of wage dispersion is

$$
\begin{aligned}
\log(\delta_{w,t}) &= \log(1 + \frac{\varsigma}{2} E_h\{w_t(h)^2\}) \\
&\simeq \frac{\varsigma}{2} var_h\{w_t(h)\}.
\end{aligned}
$$

It can be shown, in the same way, that log of price dispersion approximately is $\log(\delta_{p,t}) \simeq \frac{\varepsilon}{2} var_j\{p_{H,t}(j)\}$. Therefore, collecting the three previous results, we have $n_t = y_t - a_t + \frac{\varsigma}{2} var_h\{w_t(h)\} + \frac{\varepsilon}{2} var_j\{p_{H,t}(j)\}$. ■

Using Lemma 1-3, the sum of household utility in period t, (51), is

$$
\int U(h)dh = \frac{\alpha - 1}{2}[(1 + \varphi)x_t^2 + \varepsilon var_j\{p_{H,t}(j)\} + (\varsigma + \varphi\varsigma^2) var_h\{w_t(h)\}] + t.i.p.
$$

where in deriving the above equation, we use the efficient employment (under the optimal subsidy, it is $N^{(1+\varphi)} = (1 - \alpha)$, and $\tilde{c}_t = (1 - \alpha)x_t$. Furthermore, t.i.p. stands for *term independent of policy*.

Lemma 4 $\sum \beta^t var_h\{w_t(h)\} = \frac{1}{\lambda_w(1+\varphi\varsigma)} \sum \beta^t (\pi_t^w)^2$.

Proof. See Walsh (2003) chapter 11, or Woodford (2003) chapter 3. ■

Lemma 5 $\sum \beta^t var_j\{p_{H,t}(j)\} = \frac{1}{\lambda^p} \sum \beta^t (\pi_{H,t}^p)^2$.

Proof. See Walsh (2003) chapter 11, or Woodford (2003) chapter 3. ■

Collecting the previous results, one can write the second-order approximation to the small open economy's consumer utility function as equation (35) in the text.

Chapter III

Can Investment Risk Help in Solving Equity Premium Puzzle?

1 Introduction

Over the past three decades, economic theory has had difficulties in explaining the behavior of historical equity premium (the return on a risky asset in excess of that earned by a risk-free Treasury bill) in U.S. and some other industrial economies. Prominent features of the equity premium are; *i*) level: the size of equity premium is larger than predicted in standard business cycle models, *ii*) variability: the standard deviation of the return on stocks is larger than that of the return on T-bill, and *iii*) cyclicality: the equity premium is countercyclical.[1] Mehra and Prescott (1985) were the first to declare the equity premium a puzzle.

In order to provide a rational for the equity premium behavior, several explanations have been proposed e.g. habit formation (Campbell and Cochrane (1999)), limited participation (Guvenen (2009)), and uninsurable capital income or investment risk (Angeletos (2007) and Angeletos and Calvet (2006)).[2] Angeletos (2007) shows that a neoclassical growth model with uninsurable capital income introduces a premium on private equity, or in other words, investment risk can explain the high level of equity premium.[3] In this chapter, I examine the ability of this model to account for other aspects of equity premium puzzle, namely the cyclicality and the variability as discussed above.

The present model builds on Angeletos (2007) by adding a labor supply decision and

[1]The average annual real return on the U.S. stocks for the period 1890-1991 is 8.64 percent, and the real return on risk-free asset is 1.94 percent. Then, average annual equity premium has been 6.17 percent. The standard deviation of equity premium is about 20 percent, which in comparison with standard deviation of risk-free rate, 5 percent, is very high. Furthermore, the correlation between equity premium and output is -0.55. For more details see Guvenen (2009).

[2]Despite some efforts, Mehra (2003) concludes that the puzzle still remains. See Mehra and Prescott (2008) for a brief review of the puzzle and the literature arising from it.

[3]Recent microdata on household finance provides strong support for heterogeneity in household portfolios of risky assets, see Calvet et al. (2007). Then households are heterogeneous in their income and wealth because their incomes depend on their holding portfolio.

by considering the movement in aggregate productivity. Two important features of the model are idiosyncratic capital risk and incomplete markets. Market incompleteness arises because of capital trading restriction and the absence of state contingent securities to insure against idiosyncratic shocks to capital, causing uncertainty in capital income. Each individual consumes a final good, supplies labor, and holds a portfolio of risky and risk-free assets. While individuals are risk-averse and have access to risk-free bonds, they hold capital provided that its return is higher than the risk-free interest rate. Therefore, a wedge appears between the return on capital and the risk-free interest rate.

As shown in Angeletos (2007), the mechanism that generates the equity premium is quite simple. In steady state, capital per worker and interest rate are lower than under complete markets because of precautionary saving and investment risk. Precautionary motive encourages individuals to save more when they face uninsurable income, so the interest rate is lower because of higher bonds demand.[4] On the other hand, investment risk decreases the demand for capital and reduces the capital-labor ratio in equilibrium. Working hour is also lower under incomplete markets. As the investment risk increases in the economy, the risk-free interest rate, the capital stock, and hours worked fall, and the return on capital as well as the equity premium rise.

In order to investigate the cyclical properties of the equity premium, I introduce movement in total factor productivity in the model.[5] Through a calibrated exercise, I show that the model can generate a countercyclical equity premium. In response to a rise in aggregate productivity, the risk-free interest rate and the return on capital increase, but the rise in the former is larger then the latter, so the equity premium decreases. Interestingly with more idiosyncratic risk in the economy, the model predicts lower correlation between the equity premium and output. Moreover, higher idiosyncratic risk is associated with relatively more volatile consumption, investment, and equity premium.

Literature review: The present study relates to two rich branches of macroeconomic literature: incomplete markets and equity premium puzzle. Huggett (1993) is a pioneer paper in incomplete market literature. Huggett (1993) shows how uninsurable endowment shocks and borrowing constraint encourage individuals to save more (precautionary saving) and, as a result, the risk-free interest rate is lower than under complete markets. Aiyagari (1994) shows that the model with uninsurable labor-income predicts over accumulation of capital due to low level of return on safe assets. Some other researchers add aggregate uncertainty and generalize the model; for example, Krusell and Smith (1998) argue that the business cycle properties of the model do not change while allowing for uninsurable labor-income and borrowing

[4] In complete market, individuals save because they like to smooth consumption over time. If they save less than the amount required to keep capital stock fixed, capital stock falls and the return on it goes up. Then, current consumption is more expensive relative to the future, and saving will increase. In the present model, there is another saving motivation - called precautionary - besides consumption-smoothing motivation.

[5] It is worth noting that the model no longer has a reduced form solution if the aggregate productivity follows a stochastic process.

constraint, which suggests that the wealth distribution has small effects on business cycles.

Although there is an extensive literature on uninsurable labor-income, there are only a few studies on the effects of uninsurable capital-income. Angeletos (2007) and Angeletos and Calvet (2006) are the earlier and pioneer research in the latter area. In the presence of uninsurable capital-income not only is risk-free interest rate lower because of precautionary saving, but capital stock is also lower because of reduction in investment demand caused by investment risk. Therefore, the return on private equity increases because of the fall in the capital stock, and a positive premium appears on private equity. The differences in a model with incomplete and complete markets are not only about the levels, but it may change the policy implications. Angeletos and Panousi (2009), for example, show that in a neoclassical growth model with capital-income risk, an increase in government consumption causes a reduction in risk-free rate and capital-labor ratio, while these variables are not affected under complete markets.

Covas (2006) considers a model that includes both uninsurable investment risk and borrowing constraint, and he uses it to study capital accumulation. Similar to Aiyagari (1994), Covas (2006) show that the precautionary saving is strong enough to cause over-accumulation of capital. Covas and Fujita (2007) extend the model by adding aggregate uncertainty and find that it fails to generate plausible volatility in equity premium, and it lacks a mechanism to magnify aggregate shock through that.

Another branch of related thought is the equity premium puzzle. Mehra and Prescott (1985) show that a plausibly calibrated business cycle model fails to reproduce the equity premium.[6] In order to rationalize the equity premium, Campbell and Cochrane (1999) argue that most of the puzzles related to equity premium can be understood by incorporating habit formation in the utility function. So, under new preferences individuals are more risk-averse while consumption is low. Therefore, the model can generate a high equity premium because the risk aversion of investors increases sharply when the chance of a recession rises. Guvenen (2009) rationalizes the equity premium puzzle by developing a model with limited stock market participation and heterogeneity in the elasticity of intertemporal substitution in consumption (EIS). In this model, non-stockholders (who have low EIS) smooth the fluctuations in their labor income by saving in risk-free assets. This process concentrates non-stockholders' aggregate labor income risk among a small group of stockholders, who then demand a high premium for bearing the aggregate equity risk. Although Campbell and Cochrane (1999) and Guvenen (2009) are successful in accounting for some aspects of the equity premium puzzle, these models fail to generate the realistic volatile in the equity premium and risk-free rate.

The chapter is organized as follows. Section 2 describes the model. Section 3 charac-

[6]Mehra and Prescott (1985) argue that representative-agent model reproduces the equity premium (6 pps) if coefficient of risk aversion in utility function is 48 and discount factor is 0.55, which are not supported by other studies.

terizes the equilibrium and defines general equilibrium in the economy. Numerical analysis are presented in section 4. Section 5 concludes.

2 Model

The economy is populated by a large number of infinitely-lived individuals indexed by i over the unit interval. All individuals have the same preferences, and they are running individual-owned firms using the same production technology. Individuals have the opportunity to save in risk-free bonds or invest in non-tradable uninsurable capital. Since the returns on the assets in individual's portfolio may be different, they have to decide about the optimal combination of these two assets in their portfolios. Moreover, households supply labor in a competitive market.

Each individual has its own firm which uses labor and individual-owned capital to produce a single final good. Then the final good could be allocated between investment and consumption. Investment causes an increase in next period capital stock. Firms' production technology depends on a firm-specific shock and aggregate productivity. Aggregate productivity is common across all firms and reflect the general position of the economy; however, firm-specific shocks are idiosyncratic, and in the absence of state contingent securities, it causes risk on capital return. Time is discrete and indexed by $t = 0, 1, 2, ...$

Preferences. The economy is populated by *ex ante* identical individuals who maximize their lifetime utility defined over a sequence of consumption c_t^i and hours worked $n_{s,t}^i$;

$$U_0^i = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^i, n_{s,t}^i\right) \tag{1}$$

where E_t is the expectation operator over the information available at time t, and $0 < \beta < 1$ is the discount factor. The functional form of per-period utility is

$$u(c_t, n_{s,t}) = \frac{\left(c_t^{\mu}(1 - n_{s,t})^{1-\mu}\right)^{1-\gamma}}{1-\gamma}, \quad 0 < \mu < 1 \text{ and } 0 < \gamma. \tag{2}$$

Budget. An individuals supplies labor in a competitive labor market and receives a wage. The wage rate at time t is denoted W_t and is determined in competitive markets. Individuals also earn income on holding physical capital and bonds. The risk-free interest rate at time t is R_t. Individual i's budget constraint in period t is

$$c_t^i + k_t^i + b_t^i = \pi_t^i + k_{t-1}^i + (1 + R_{t-1})b_{t-1}^i + W_t n_{s,t}^i \tag{3}$$

where k_t^i is physical capital, b_t^i is saving in risk-free bonds, and π_t^i is the capital income from running the individual's own business. Each individual has three sources of income (capital income, saving, and labor income) and allocates all its resources to consumption, investment, and risk-free saving.

Technology and Idiosyncratic Risk. Each individual manages its own business by utilizing its own capital and so earns income. In order to run their businesses, individuals hire labor from the competitive labor market; therefore, the capital income of individuals is firm output y_t^i minus the wage bill

$$\pi_t^i = y_t^i - W_t n_{d,t}^i \tag{4}$$

where $n_{d,t}^i$ is the demand for labor in firm i, and y_t^i is output produced at time t. Output is produced by the following technology

$$y_t^i = F(k_{t-1}^i, n_{d,t}^i, \delta_t^i, z_t) \tag{5}$$

where δ_t^i is firm-specific shock which is identically and independently distributed across individuals and time. The shock is important because it generates risk in the return on capital. I assume that the source of idiosyncratic shock is stochastic depreciation. This means that the amount by which capital depreciates in the production process is not deterministic. Moreover, z_t is the aggregate productivity. Output is produced with the CRS production function

$$y_t^i = z_t(k_{t-1}^i)^\alpha (n_{d,t}^i)^{1-\alpha} - \delta_t^i k_{t-1}^i \tag{6}$$

where $0 < \alpha < 1$ is the capital share of output, and idiosyncratic shocks is $\delta_t^i \sim N(\delta, \sigma_\delta^2)$. The aggregate productivity is deterministic and follows an $AR(1)$ process

$$z_{t+1} = (1 - \rho_z) + \rho_z z_t, \tag{7}$$

where $0 < \rho_z < 1$ measures the persistence of the aggregate shocks, and aggregate productivity has the unconditional mean of one. Here I assume that idiosyncratic shocks and aggregate productivity are independent. Let's define a competitive equilibrium for this economy.

Definition A competitive equilibrium is sequence of price $\{R_t, W_t\}_{t=0}^\infty$ and individual contingent plans $\left\{c_t^i, n_t^i, k_t^i, y_t^i, b_t^i\right\}_{t=0}^\infty$ for $i \in [0,1]$ such that the following conditions are satisfied; $i)$ optimality: contingent plan maximizes the utility of every individual i, $ii)$ labor and bonds markets clear: $\int_i n_{s,t}^i = \int_i n_{d,t}^i$ and $\int_i b_t^i = 0$, and $iii)$ aggregation: $C_t = \int_i c_t^i$, $N_t = \int_i n_t^i$, $K_t = \int_i k_t^i$, $Y_t = \int_i y_t^i + \delta \int_i k_{t-1}^i$.

There are three markets in the economy: goods, labor, and bonds. If the last two markets clear, then the goods market clears too.

3 Equilibrium Characterization

At the beginning of each period individuals know the stock of capital and bonds they possess. They realize aggregate productivity, and then they hire labor with wages determined in the competitive market. After producing output they realize how much their capital has depreciated. Then individuals allocate their wealth to consumption, investment, and saving in bonds. Because of our assumption about stochastic depreciation, labor demand only depends on economy wide wage rate and not on firm-specific depreciation rate. If I assume idiosyncratic stochastic total factor productivity, then idiosyncratic shock is important in demand for labor (see Angeletos (2007) and Covas (2006)).

3.1 Individual Behavior

Each individual's problem can be represented by dynamic programming. In (8), the subscript i is removed from individual variables because the following dynamic program applies to all agents. The value of an individual variable at current period is denoted by prime symbol ($'$). However, the time subscript on an aggregate variable is kept because it is given to individuals. The dynamic program is

$$\begin{aligned} V(k,b,\delta,z_t) &= \max_{\{c',n_s',n_d',k',b'\}} \left\{u(c',n_s') + \beta EV(k',b',\delta',z_{t+1})\right\} \\ s.t.\quad & c' + k' + b' = \pi' + k + (1+R_{t-1})b + W_t n_s' \\ & \pi' = F(k,n_d',\delta',z_t) - W_t n_d' \\ & c' \geq 0,\ k' \geq 0,\ b' \geq -h' \end{aligned} \tag{8}$$

where h_t is the present value of future labor income defined as

$$h_t = \sum_{j=0}^{\infty} \frac{W_{t+j} n_{s,t+j}}{(1+R_t)...(1+R_{t+j})} \quad \text{or} \quad h_t = W_t n_{s,t} + \frac{h_{t+1}}{1+R_t}, \tag{9}$$

and the last inequality is the natural solvency constraint. This assumption rules out Ponzi schemes. In other words, individuals are not allowed to borrow more than the present value of their labor incomes.[7]

I am going to solve the individuals' problem in two stages. First, I characterize the two static optimizations about running business and labor supply. Individuals have to decide about hiring workers and supplying labor, both of which decisions are intratemporal. Second, I characterize the optimal intertemporal decisions about consumption, investment in physical capital, and saving in bonds. Lemmas 1 and 2 present the individuals' intratemporal decisions.

Lemma 1 *Individual labor demand and capital income are linear in capital stock*

$$n_{d,t}^i = n(W_t,z_t)k_{t-1}^i = \left(\frac{(1-\alpha)z_t}{W_t}\right)^{\frac{1}{\alpha}} k_{t-1}^i \tag{10}$$

$$\pi_t^i = r(\delta_t^i,W_t,z_t)k_{t-1}^i = [z_t n(W_t,z_t)^{1-\alpha} - \delta_t^i - W_t n(W_t,z_t)]k_{t-1}^i \tag{11}$$

[7] There is an implicit assumption that individual can sell their lifetime labor force in market, and h_t is the value of labor force from time t.

where $r_t^i = r(\delta_t^i, W_t, z_t)$ *is the return on capital in time* t, *and* $n(W_t, z_t)$ *is the optimal labor-capital ratio.*

The individual chooses labor in order to maximize the profit. The optimal labor-capital ratio depends on the wage rate and aggregate productivity, and $n(W_t, z_t)$ is decreasing in W_t and increasing in z_t. Rearranging (10) gives

$$W_t = (1-\alpha)z_t \left(k_{t-1}^i\right)^\alpha \left(n_{d,t}^i\right)^{-\alpha} \tag{12}$$

which means firm hires worker to the extent that marginal product of the last worker equals marginal cost of hiring.

Hence, from (11), capital return is increasing in z_t, and decreasing in W_t and δ_t^i. Rearranging (11) yields

$$r_t^i = r(\delta_t^i, W_t, z_t) = \theta z_t^{\frac{1}{\alpha}} W_t^{\frac{\alpha-1}{\alpha}} - \delta_t^i \tag{13}$$

where $\theta = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}$, and $\theta \in (0,1)$. From (13), the return on capital is function of wage, aggregate productivity, and idiosyncratic depreciation rate. Moreover, substituting (10) in (6) gives

$$y_t^i = y(\delta_t^i, k_{t-1}^i, W_t, z_t) = \frac{\theta}{\alpha}[z_t^{\frac{1}{\alpha}} W_t^{\frac{\alpha-1}{\alpha}} - \delta_t^i]k_{t-1}^i,$$

so individual output is linear in capital.

Lemma 2 *Optimal labor supply for individual* i *depends on wage rate and consumption*

$$-\frac{U_n}{U_c} = \frac{1-\mu}{\mu}\frac{c_t^i}{(1-n_{s,t}^i)} = W_t, \tag{14}$$

or $(1-n_{s,t}^i) = \frac{1-\mu}{\mu}\frac{c_t^i}{W_t}$.

Lemma 2 implies that consumption-leisure ratio is constant across individuals, because it depends on the wage rate which is given to them.

To make the characterization of individual intertemporal optimization simple, I define wealth as the summation of all resources, $w_t^i = \pi_t^i + k_{t-1}^i + (1+R_{t-1})b_{t-1}^i + h_t$. Using Lemma 2 and writing capital income as a function of capital stock, total wealth can be represented by

$$w_t^i = (1+r_t^i)k_{t-1}^i + (1+R_{t-1})b_{t-1}^i + h_t. \tag{15}$$

Hence, combining (9) with (15) reduces individual budget constraint (3) to $c_t^i + k_t^i + b_t^i + \frac{h_{t+1}}{1+R_t} = w_t^i$.

Now the dynamic program (8) can be represented as

$$V(w;t) = \max_{\{c,k,b\}} \left\{u(c,n_s) + \beta EV(w';t+1)\right\} \tag{16}$$
$$s.t. \quad c+k+b+\frac{h'}{1+R_t} = w$$
$$w' = (1+r')k + (1+R_t)b + h',$$

where individual variables at time $t+1$ are indicated by prime.

Now, let's back to characterizing the individual's intertemporal decisions. The next proposition indicates that the individuals' policy functions are linear in wealth. Then, in the next section, I explain how linear policy functions make aggregating simple.

Proposition 3 *Given prices $\{W_t, R_t\}$, optimal consumption, investment, and bond-holding are linear in wealth:*

$$c_t^i = (1-\varsigma_t)w_t^i \tag{17}$$
$$k_t^i = \varphi_t \varsigma_t w_t^i \tag{18}$$
$$b_t^i = (1-\varphi_t)\varsigma_t w_t^i - \frac{h_{t+1}}{1+R_t} \tag{19}$$

where w_t^i and h_t are defined by (15) *and* (9)*, and where*

$$A_t(1-\varsigma_t)^{-\gamma} = \beta\varsigma_t^{-\gamma}E(A_{t+1}(1-\varsigma_{t+1})^{-\gamma}\rho_{t+1}^{1-\gamma}) \tag{20}$$
$$\rho_{t+1} = [\varphi_t(1+r_{t+1}^i) + (1-\varphi_t)(1+R_t)] \tag{21}$$
$$\varphi_t = \varphi(W_{t+1}, R_t, z_{t+1}) \equiv \arg\max_{\varphi\in[0,1]} E[A_{t+1}(1-\varsigma_{t+1})^{-\gamma}\rho_{t+1}^{1-\gamma}] \tag{22}$$

where $A_t = A(W_t) = (W_t\mu/(1-\mu))^{(1-\mu)(\gamma-1)}$.

Proof. In section 6.1. ■

All policy functions are linear in wealth. Linearity of consumption implies all individuals consume the same proportion of their wealth - without any regard to wealth position. Saving rate is ς_t, which is the saving out of wealth not income. Savings are allocated between capital investment and bonds. The fraction of saving invested in physical capital is φ_t and the rest is invested in bonds.

Equation (20) is the Euler equation; its left-hand side is the reduction in utility because of one unit decrease in current consumption, and the right-hand side is the gain from investing in optimal portfolio in terms of utility from increase in consumption in the next period.

The individual's portfolios consist of two assets; risky and risk-free assets. Expected return on portfolio depends on the risk-free interest rate, TFP, and the wage rate. Equation (21) is the return on portfolio with optimal combination of risky and risk-free assets. The optimal portfolio is characterized by the optimal allocation of resources between the two available assets. The optimal combination is given by φ_t in

(22).

Indeed, individuals choose φ_t in order to smooth consumptions over time. Using a second-order Taylor approximation, the solution to maximization problem (22) gives

$$\varphi_t \simeq \frac{E_t r^i_{t+1} - R_t}{\gamma \sigma_r^2} \tag{23}$$

where σ_r^2 is the conditional variance of r^i_{t+1} using information in t (proof in section 6.2). The individual invests in a risky asset provided that the risky asset has, on average, a higher return. The difference between returns on risky and risk-free asset is the equity premium. Moreover, since individuals are risk-averse, the investment in risky assets depends on the measure of risk aversion and the risk level. The higher the risk, the higher the investment in the risk-free assets. Similarly, the more risk-averse the individual, the more investment in risk-free asset. Therefore, φ_t is decreasing in σ_r^2, R_t, and γ, while it is increasing in $E_t r^i_{t+1}$.

3.2 General Equilibrium

By assumption, shocks are iid across individuals and time. Aggregating (10) and (14) imply aggregate labor demand, $N_t = n(W_t, Z_t)K_{t-1}$, and aggregate labor supply, $(1 - N_t) = \frac{1-\mu}{\mu}\frac{C_t}{W_t}$, which determine the equilibrium in the labor market. Similarly, aggregate capital income is $\Pi = r(W_t, z_t)K_{t-1}$, where $r_t = r(W_t, z_t)$ is the average return on capital, and aggregate output is $Y_t = \frac{\theta}{\alpha}[z_t^{\frac{1}{\alpha}} W_t^{\frac{\alpha-1}{\alpha}}]K_{t-1}$.

By Proposition 3, consumption, investment, and saving are linear in individual wealth; therefore, their aggregates do not depend on the distribution of wealth across individuals.[8] The closed-form recursive characterization of the general equilibrium is provided in the following proposition.

Proposition 4 *In equilibrium the aggregate dynamics satisfy*

$$Y_t = z_t K_{t-1}^{\alpha} N_t^{1-\alpha} \tag{24}$$

$$C_t + K_t = Y_t + (1-\delta)K_{t-1} \tag{25}$$

$$C_t = (1-\varsigma_t)(\alpha Y_t + (1-\delta)K_{t-1} + H_t) \tag{26}$$

$$A_t(1-\varsigma_t)^{-\gamma} = \beta \varsigma_t^{-\gamma} E(A_{t+1}(1-\varsigma_{t+1})^{-\gamma} \rho_{t+1}^{1-\gamma}) \tag{27}$$

$$K_t = \varphi_t \varsigma_t (\alpha Y_t + (1-\delta)K_{t-1} + H_t) \tag{28}$$

[8] Why is wealth distribution not in state set? In Aiyagari-type incomplete market models, distribution of wealth is part of state set; otherwise, aggregate variables could not be calculated. This is the case because, in that type of model, individuals are restricted by borrowing constraints. So, in this world, the individual policy functions depend on the wealth position. Therefore, it is essential to keep track of the history of shocks for each individual. However, in the present model borrowing constraint does not exist. Therefore the policy functions are linear in wealth and the sum of individual policy functions give the aggregate macro variables.

$$1 - N_t = \frac{1-\mu}{\mu}\frac{C_t}{W_t} \tag{29}$$

$$W_t = (1-\alpha) z_t K_{t-1}^{\alpha} N_t^{-\alpha} \tag{30}$$

$$r_t = \alpha z_t K_{t-1}^{\alpha-1} N_t^{1-\alpha} - \delta \tag{31}$$

$$H_t = W_t N_t + \frac{H_{t+1}}{1+R_t} \tag{32}$$

where z_t *is* (7), $A_t = A(W_t)$, $\rho_{t+1} = \rho(W_{t+1}, R_t, z_{t+1})$, *and* $\varphi_t = \varphi(W_{t+1}, R_t, z_{t+1})$.

Equation (24) is aggregate output, and condition (25) is the resource constraint. Equations (26), (27), and (28) are respectively aggregate consumption, recursive Euler equation, and aggregate capital demand. Given aggregate capital and consumption, (29) and (30) determine equilibrium wage rate and labor in labor market. Finally (31) is the average net return on capital, and (32) is recursive human wealth. The equity premium can be define as the difference between average return on risky and risk-free assets, $r_t - R_t$.

3.3 Steady State

In this section the steady state of the economy will be examined. Steady state here is the fixed point of the system of dynamic equations which is described in proposition 5. The fixed point is the solution of two equations in two unknowns, as shown in the next proposition.

Proposition 5 *In steady state the capital-labor ratio,* $\frac{K}{N}$*, and the risk-free interest rate,* R*, solve*

$$\beta^{\frac{1}{\gamma}} \rho^{\frac{1-\gamma}{\gamma}} \left[(1-\delta) + \frac{1+R-\alpha}{R} \left(\frac{K}{N}\right)^{\alpha-1} \right] = \frac{1-\alpha}{R} \left(\frac{K}{N}\right)^{\alpha-1} + 1 \tag{33}$$

$$\varphi \beta^{\frac{1}{\gamma}} \rho^{\frac{1-\gamma}{\gamma}} \left[(1-\delta) + \frac{1+R-\alpha}{R} \left(\frac{K}{N}\right)^{\alpha-1} \right] = 1 \tag{34}$$

where $\varphi = \varphi(\frac{K}{N}, R)$ *and* $\rho = \rho(\frac{K}{N}, R)$*; and given the capital-labor ratio, labor* N *solves*

$$\frac{1}{N} - 1 = \frac{1-\mu}{\mu(1-\alpha)} \left[1 - \delta \left(\frac{K}{N}\right)^{1-\alpha} \right]. \tag{35}$$

Proof. In section 6.3. ■

Equation (33) and (34) are respectively the resource constraint and capital demand, and the two unknowns are the capital-labor ratio and the risk-free interest rate. Knowing the capital-labor ratio from the first two equations, equation (35) gives the labor market equilibrium in steady state. After calculating labor, aggregate capital can be calculated and similarly all other macro variables.

What happens to steady state values if idiosyncratic risk increases? It is not possible to answer the question analytically, but it could be done numerically, which is the subject of the next section.

4 Numerical Analysis

I take a period to be a quarter, and set the discount factor to 0.99. The coefficient of risk aversion, γ, and share of consumption, μ, in periodic utility function are respectively equal to 2 and 0.25. The share of capital, α, in production function is 0.36, and the aggregate capital stock depreciates at average rate of 0.025. The aggregate productivity follows an $AR(1)$ process with $\rho_z = 0.95$, and unconditional mean equals to one. These values are common in business cycle literature. There is no evidence about the exact variance of idiosyncratic shocks. Therefore, following Angeletos and Panousi (2009) and Angeletos (2007), I use 0.2 and 0.4 for standard deviation of idiosyncratic shocks.

Figure 3.1 and 3.2 numerically explains proposition 5. Blue lines are resource constraint (33), and red lines are capital demand (34). The figure indicates there is a unique steady state which is the intersection of (33) and (34). Increase in idiosyncratic risk decreases the capital demand, and also causes stronger precautionary saving motive; hence, both lines shift downward. That means higher risk is associated with both lower capital per hour and lower risk-free interest rate. Figure 3.2 plots the labor in different levels of capital per hour which is equivalent of (35). The figure reveals that the level of hours worked falls if risk increases.

It is interesting to compare the value of important variables in steady state. Table 3.1 reports the level of aggregate variables in steady state. The first column is the standard deviation of idiosyncratic shocks. Three scenarios are examined: $i)$ complete markets; second row reports the steady state under complete markets, since there is no idiosyncratic risk;[9] $ii)$ incomplete markets with low risk, $\sigma_\delta = 0.2$; $iii)$ incomplete markets with high risk, $\sigma_\delta = 0.4$. It is clear that increase in idiosyncratic risk increases the equity premium, and decreases output, consumption, capital stock, employment, and interest rate.

TABLE 3.1: Impact of Risk on Aggregate Variables

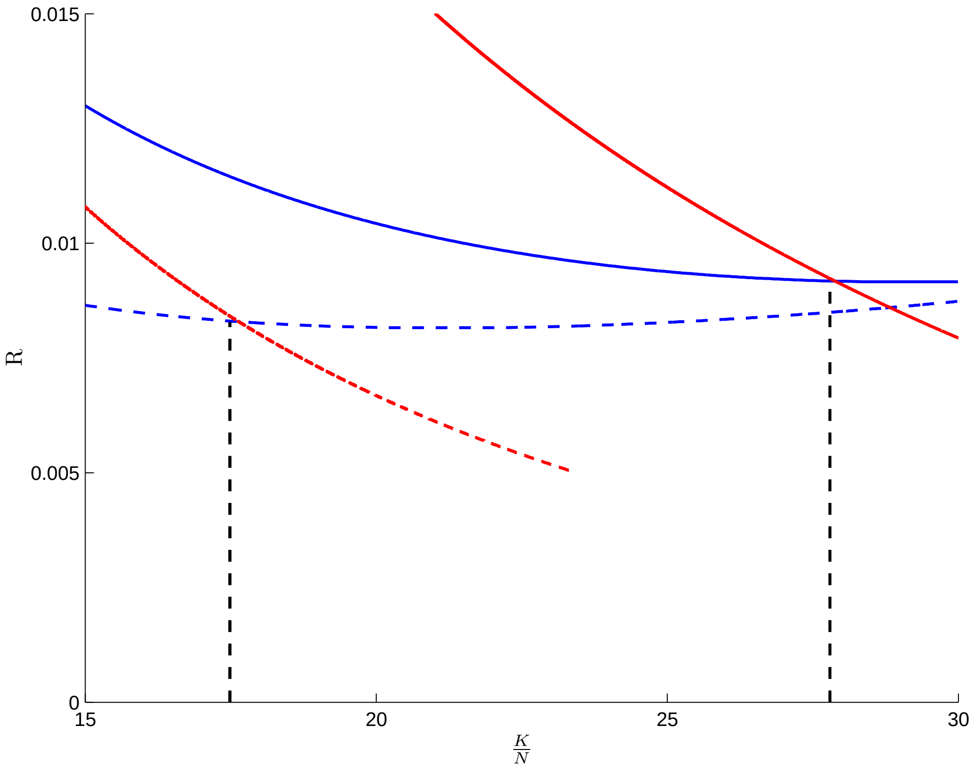
σ_δ	Y	C	K	N	R	$(r-R)$
0.0	0.82	0.61	8.47	0.22	1.0	0.0
0.2	0.70(−15%)	0.56(−8%)	5.91(−30%)	0.21(−4%)	0.9	0.8
0.4	0.56(−32%)	0.48(−21%)	3.52(−58%)	0.20(−9%)	0.8	2.4

Notes: Figures in parenthesis are the percentage change in steady-state value of variables compare to their counterparts in complete market model. The risk-free interest rate, R, and equity premium, $r - R$, are reported in percentage.

In the incomplete market literature with uninsurable labor-income, risk-free interest

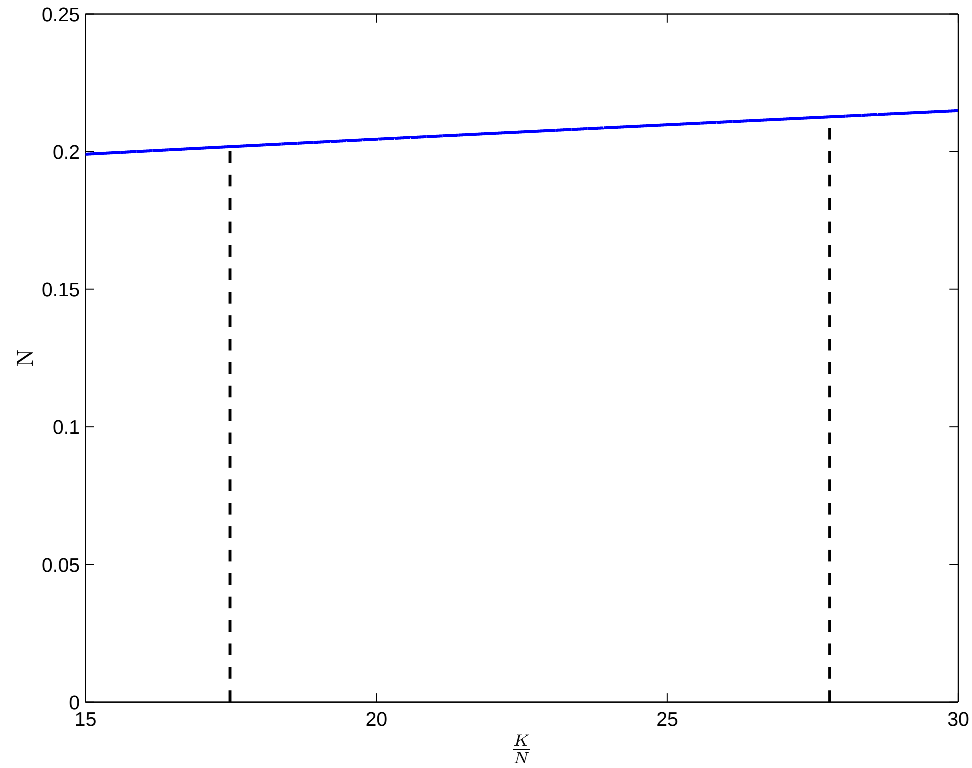
[9] The results are from a classic real business cycle model, where interests on bond and capital are equal in equilibrium.

FIGURE 3.1: Impact of Risk on Interest Rate and Capital-Labor Ratio



Notes: Solid line indicates low-risk, $\sigma_\delta = 0.2$, and dash line shows high-risk, $\sigma_\delta = 0.4$, economies. Red lines are demand for capital (34), and blue lines are resource constraint (33).

FIGURE 3.2: Impact of Risk on Labor



rates are lower than with complete markets, and this causes over-accumulation of capital. However, these are not all true in the model with uninsurable capital-income. Although uninsured capital-income decreases the risk-free interest rate, the capital stock is smaller than under complete markets. In the model with uninsurable labor-income, individuals insure themselves by saving more; therefore, the interest rate is lower, and it encourages more investment in physical capital. In the present model the demand for capital is lower because the return on it is risky, so the capital stock is lower under incomplete markets. Moreover, as risk increases the wedge between returns on risk-free and risky assets grows.

TABLE 3.2: Impact of Risk Aversion on Aggregate Variables

γ	Y	C	K	N	R	$(r-R)$
2	0.70	0.56	5.91	0.21	0.9	0.8
5	0.61(−13%)	0.51(−9%)	4.37(−26%)	0.20(−5%)	0.8	1.7

Notes: Figures in parenthesis are the percentage change in steady-state value of variables. The risk-free interest rate, R, and equity premium, $r - R$, are reported in percentage, and $\sigma_\delta = 0.2$.

Table 3.2 reports the level of aggregate variables under different level of risk aversion. Under higher risk aversion, the precautionary saving is stronger; therefore, individuals hold more risk-free assets and less capital in their portfolios. As risk aversion increases, the capital stock falls and the return on capital rises. Higher return on capital and lower risk-free interest rate – because of stronger precautionary saving– implies higher equity premium.

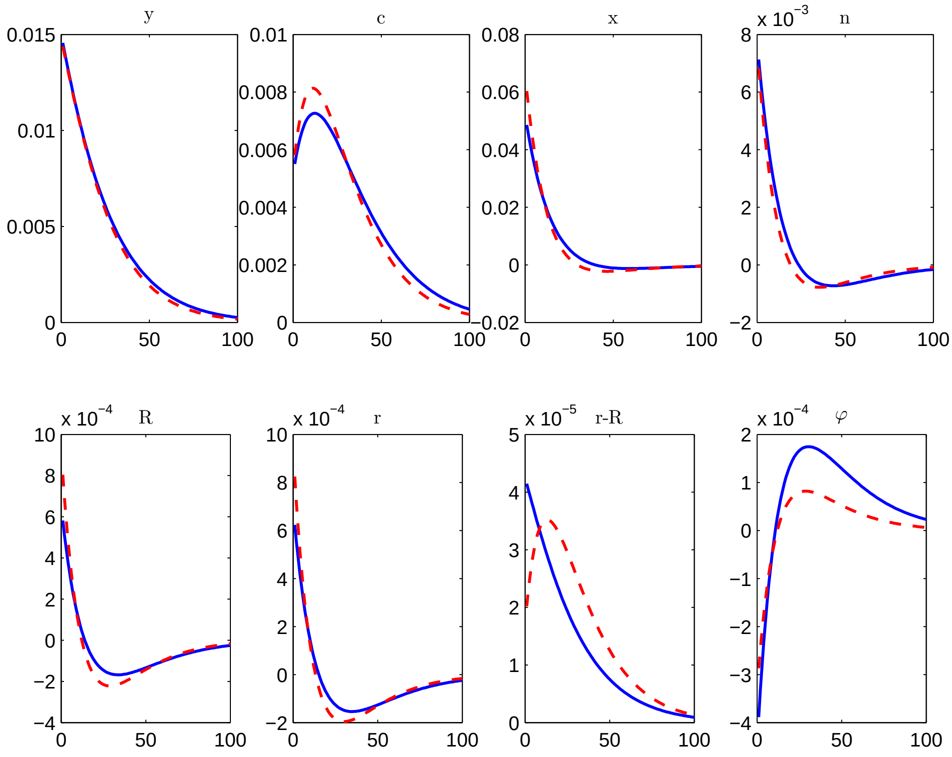
In order to study the prediction of the model about the behavior of the equity premium, I examine the impulse response of the economy to a positive shock in aggregate productivity. This is subject of the next section.

4.1 Impulse Response

Impulse responses are used in this section in order to examine the dynamics of the model. Figure 3.3 displays impulse responses to a one percent increase in aggregate productivity under low and high investment risk. In response to a positive aggregate productivity shock, the marginal product of labor increases; hence firms employ more workers, and output and consumption rise. Since the marginal product of capital increases, the return on capital goes up and induces more investment. Moreover, the return on bonds increases because the supply of bonds rises and the bond price decreases. The risk-free interest rate and return on capital increase, but because the rise in the latter variable is larger than the former, the equity premium increases too. The optimal combination of assets in portfolio implies that the share of capital decreases, because individuals expect lower productivity and average return on capital, they reallocate their resources toward the risk-free asset.

An increase in idiosyncratic risk has two effects. First, as discussed in the previous section, it drives the capital stock per worker down, which means the return on cap-

FIGURE 3.3: Impulse Response to a Rise in Productivity



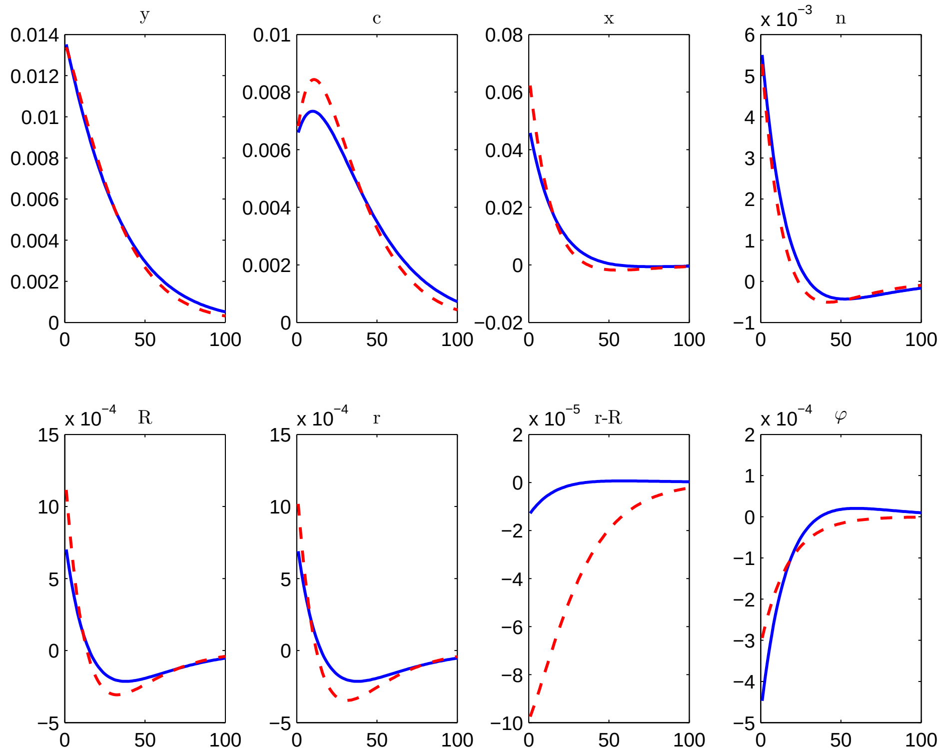
Notes: Blue and red lines indicate low-risk and high-risk economies respectively. Output, y, consumption, c, investment, x, and labor, n, are in log.

ital goes up. Second, it induces stronger precautionary saving which implies a lower risk-free rate. Because the capital stock is lower in the riskier economy, the same change in technology increases the marginal product of capital and capital return more, and investment increases more. Moreover, precautionary saving is stronger and the saving rate is higher in a high-risk economy.

An increase in aggregate productivity has no discernable impact on the dynamic of output and labor. Now consumption, interest rate, and investment deviate more from their steady state level. That means these variables are more volatile relative to output under higher risk. In high-risk economy the marginal product of capital is higher; therefore, the change in return on capital is larger. Moreover, the risk-free rate increases more under high risk, because risk averse individuals have higher precautionary motives. Therefore, the equity premium goes up less than in a low-risk economy, and because of the sharp fall in risk-free rate, the equity premium increases initially and then falls. It is clear that the correlation of equity premium with output decreases. As risk increase, the correlation between output and equity premium decreases and it might fall below zero for a high risk.

Instead of increasing the risk, which is not plausible, one may think about higher risk aversion, since these two parameters play the same rule in (23). Figure 3.4 indicates the impulse responses where the risk aversion coefficient, γ, is 5. Generally the patterns of response for output, consumption, investment, hours, risk-free rate,

FIGURE 3.4: Impulse Response to a Rise in Productivity; $\gamma = 5$



Notes: Blue and red lines indicate low-risk and high-risk economies respectively. Output, y, consumption, c, investment, x, and labor, n, are in log.

and capital return are similar to those under low risk aversion.

Under higher risk aversion, individuals tend to save more in risk-free assets and invest less in physical capital. Hence a wider gap appears between risk-free rate and return on capital. Risk averse individuals save more in bond, so the demand for bond increases, and the risk-free rate decreases. An rise in productivity increase both risk-free and risky returns, but the effect on the former is larger, so the equity premium decreases. Moreover, individuals keep even more bond in their portfolio because of increase in risk-free rate.

As one may predict, a rise in risk decreases the correlation between output and equity premium. Moreover, consumption, investment, risk-free interest rate, and equity premium deviate more from their steady state levels, which implies these variables are more volatile under higher risk.

It worth noting that the present model abstracts from many features of a standard real business cycle model e.g. capital adjustment cost.[10] However, introducing the investment risk in a simple growth model seems to be a step in right direction because with investment risk model predicts more volatile and countercyclical equity premium.

[10]This fact is neglected since adding nonlinear capital adjustment cost undermines proposition 3, and the model no longer has a reduced form solution.

5 Conclusion

The equity premium is an important variable which has consequences for the effectiveness of economic policies. In the present chapter, I have developed a dynamic general equilibrium model in which equity premium is positive and significant. Then, I have used the model to examine the dynamic of equity premium.

I show that the neoclassic growth model with investment risk and incomplete markets not only can introduce a sizable premium on private equity, but it can generate a countercyclical equity premium. In response to a rise in aggregate productivity, the risk-free interest rate and the return on capital increase, but the rise in the former is larger then the latter variable, so the equity premium decreases. Interestingly with more risk in the economy, the model predicts a stronger negative correlation between the equity premium and output. Moreover, the model predicts relatively more volatile consumption, investment, and equity premium as the risk increases in economy.

The present model abstracts from many of the features which are common in business cycles literature e.g. capital adjustment cost and habit formation. However, it seems the model has the potential to answer many interesting questions.

6 Appendix

6.1 Proof of Proposition 3

Proof. It is convenient to derive the indirect utility by substituting labor supply in the utility function (2) which gives

$$u(c_t, W_t) = A_t \frac{c_t^{1-\gamma}}{1-\gamma} \tag{36}$$

where $A_t = \left(\frac{1-\mu}{\mu}\frac{1}{W_t}\right)^{(1-\mu)(1-\gamma)} > 0$.

First-order conditions for the dynamic program (16) are

$$\beta E\left\{V'(w';t+1)(1+r_{t+1})\right\} = u_c \tag{37}$$
$$\beta E\left\{V'(w';t+1)(1+R_t)\right\} = u_c \tag{38}$$

where $r_{t+1} = r(\delta_{t+1}^i, W_{t+1}, z_{t+1})$. Moreover, envelope theorem implies $V'(w;t) = u_c$.

The educated initial guesses for value function and policy functions are

$$\begin{aligned}
V(w;t) &= (1-\gamma)^{-1} a_t w_t^{1-\gamma} \\
c(w;t) &= (1-\varsigma_t) w_t \\
k(w;t) &= \varphi_t \varsigma_t w_t \\
b(w;t) &= (1-\varphi_t)\varsigma_t w_t - \frac{h_{t+1}}{1+R_t}.
\end{aligned}$$

Substituting guesses for k_{t+1} and b_{t+1} in w_{t+1} gives

$$w_{t+1} = [\varphi_t(1+r_{t+1}) + (1-\varphi_t)(1+R_t)]\varsigma_t w_t. \tag{39}$$

That is, individual wealth in $t+1$ is the saving in t plus the return on it. Let's define return on portfolio as $\rho_{t+1} = [\varphi_t(1+r_{t+1}) + (1-\varphi_t)(1+R_t)]$, which is the weighted average of returns on risky and risk-free assets. Subtracting (37) and (38), and using the guess for value function and (39) gives

$$E\left\{a_{t+1}[\varphi_t(1+r_{t+1}) + (1-\varphi_t)(1+R_t)]^{-\gamma}(r_{t+1} - R_t)\right\} = 0$$

which is the first-order condition of maximization (22). It also gives combination of assets in optimal portfolio, $\varphi_t = \varphi(W_{t+1}, R_t, z_{t+1})$.

Now I shall verify the initial guesses. I do that in three stages. First, using (36) in envelope equation implies

$$a_t = A_t(1-\varsigma_t)^{-\gamma}. \tag{40}$$

Second, multiplying (37) and (38) with φ_t and $(1-\varphi_t)$ respectively, and summing them up gives

$$\beta E\left\{V'(w';t+1)[\varphi_t(1+r_{t+1})+(1-\varphi_t)(1+R_t)]\right\}=u_c. \tag{41}$$

Substituting our guesses in (41) and rearranging implies

$$\beta\varsigma_t^{-\gamma}E\left(a_{t+1}\rho_{t+1}^{1-\gamma}\right)=A_t(1-\varsigma_t)^{-\gamma}.$$

Third, it is now time to verify that our guesses solve the Bellman equation. Substituting our guesses in dynamic program (16) gives

$$a_t w_t^{1-\gamma}=A_t c_t^{1-\gamma}+\beta E a_{t+1}w_{t+1}^{1-\gamma}.$$

Using (39), labor supply, and our result in the first stage together gives

$$A_t(1-\varsigma_t)^{-\gamma}=\beta\varsigma_t^{-\gamma}E(a_{t+1}\rho_{t+1}^{1-\gamma}) \tag{42}$$

which is the same as the result in the second stage. Finally, substituting (40) in (42) gets (20) in text. ■

6.2 Optimal Portfolio Combination

I define a new variable named T as

$$T^{1-\gamma}=E\{(a_{t+1}\rho_{t+1}^{1-\gamma}\}.$$

Now a second-order Taylor approximation for $\ln(T)$ gives

$$\ln(T)\simeq\frac{\ln Ea_{t+1}}{1-\gamma}+\varphi_t\ln(1+Er_{t+1})+(1-\varphi_t)\ln(1+R_t)-\frac{\gamma}{2}\varphi_t^2\sigma_r^2$$

where σ_r^2 is conditional variance of r_{t+1} using information in t. Taking derivative with respect to φ_t implies

$$\ln(1+E_t r_{t+1})-\ln(1+R_t)-\gamma\varphi_t\sigma_r^2=0$$

and it gives (23) in text. For the low level of $0<r<1$, applying first-order Taylor expansion implies that $\ln(1+r)\simeq r$. I sometimes use this relation to make the calculation simple.

6.3 Proof of Proposition 5

Proof. Human wealth and average return on capital are $H=\frac{1+R}{R}WN$, and $r=\theta W^{\frac{\alpha-1}{\alpha}}-\delta$. Substituting wage rate from labor demand, $W=(1-\alpha)(Y/N)$, in former equations, wealth can be written $w_t=(1-\delta)K+\frac{1-\alpha+R}{R}(Y)$.

Substituting wealth in consumption $C = (1-\varsigma)\, w$, and replacing it in resource constraint $C = K^{\alpha}N^{1-\alpha} - \delta K$ gives

$$K^{\alpha}N^{1-\alpha} - \delta K = (1-\varsigma)\left((1-\delta)K + \frac{1-\alpha+R}{R}Y\right).$$

Replacing ς by $\beta^{\frac{1}{\gamma}}\rho^{\frac{1-\gamma}{\gamma}}$ and dividing both sides by K gives (33) in text.

Substituting wealth in capital demand $K = \varphi\varsigma w$ yields

$$K = \varphi\varsigma\left((1-\delta)K + \frac{1-\alpha+R}{R}Y\right).$$

Replacing ς by $\beta^{\frac{1}{\gamma}}\rho^{\frac{1-\gamma}{\gamma}}$ and dividing both sides by K gives (34) in text.

Integrating wage rate and consumption in labor supply $1 - N = \frac{1-\mu}{\mu}\frac{C}{W}$ gives

$$(1-N)(1-\alpha)(Y/N) = \frac{1-\mu}{\mu}(K^{\alpha}N^{1-\alpha} - \delta K).$$

Rearranging and dividing both sides by K gives (35) in text. ■

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