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SHIFTING RESONANCES FROM A FREQUENCY BAND BY APPLYING CONCENTRATED MASSES TO A THIN RECTANGULAR PLATE

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The vibration of a thin, simply supported rectangular plate carrying a number of concentrated masses is considered. It is shown that the addition of the masses has the effect of reducing the eigenvalues within certain bounds. Two numerical methods are developed for calculating the eigenfunctions, eigenvalues and frequency response of the mass loaded plate. These are employed in an attempt to drive a gap in the eigenvalues directly below 110 Hz for a particular test case of a plate carrying five concentrated masses. A small gap is achieved, but to produce a larger gap many more masses would be necessary. The frequency response and eigenvalues are relatively easily obtained and compare well with those obtained by using the finite element method.

1. INTRODUCTION

The aim of the work reported here is to investigate the possibility of using concentrated masses to modify the resonant behaviour of a thin rectangular plate. In some sense, it is desirable to choose, *a priori*, a set of eigenvalues and from this calculate the plate and mass configuration which would produce it. This may seem an unlikely proposition; however, a similar idea has been put forward by Colin de Verdiere [1] for a finite set of eigenvalues of the Laplacian equation on a domain with an unspecified boundary contour. He was able to show—subject to certain conditions on the eigenvalues—that, theoretically, a boundary contour could be found which would give rise to those eigenvalues. Although this mathematical analysis is encouraging, it does not provide any help as to how to construct a system with given eigenvalues. Here, the vibration of a rectangular simply supported plate is considered (the biharmonic equation) and is modified by the addition of small concentrated masses. Rather than specifying the eigenvalues exactly, the analysis aims to introduce a gap in the eigenvalues over a given frequency range. This problem is relatively simple in that the eigenfunctions of the plate are well known [2] and the eigenfunctions of the modified plate with masses system may be built from them. It also has the attraction of being intuitively simple and possibly of direct use in passive noise control.

To examine the problem, three different analytical approaches have been developed, which are useful in different ways. In addition, the finite element method (FEM) has been used as a means of testing the results achieved.

The first analytical approach, the inductive modal sum (IMS) approach, is based on the solution given by Amba-Rao [3] for the vibration of a plate carrying a single mass.

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By using this method, it is possible to deduce upper and lower bounds for each eigenvalue and also to provide bounds for the eigenvalues of a plate carrying a number of masses. However, as a numerical method, this approach is not very useful. The second method, the direct modal sum (DMS) method, was hinted at by Amba-Rao, and is developed here in more detail. This method leads to a numerically viable technique for determining the eigenvalues, and from these the corresponding eigenfunctions, of a mass loaded plate. It would be possible to construct Green functions from these to obtain the frequency response of the plate plus masses system, but this is rather cumbersome. In the third method one treats the point masses as frequency dependent forces (the frequency dependent forces (FDF) method) and from this determines the frequency response of the plate in terms of the unloaded plate Green function, which is well known. With this approach, the loaded plate eigenvalues and eigenfunctions are not determined explicitly, but the method is computationally quite efficient.

2. THE INDUCTIVE MODAL SUM APPROACH

2.1. EIGENFUNCTIONS AND EIGENVALUES FOR ONE CONCENTRATED MASS

First consider the free vibration of a rectangular plate of length a and breadth b, simply supported on all four sides, and carrying a concentrated mass. The unforced equation of motion is

$$D\nabla^4 w + \{\rho h + M\delta(x-u)\delta(y-v)\}\frac{\partial^2 w}{\partial t^2} = 0, \tag{2.1}$$

where $w = w(x, y, t)$ is the transverse deflection of the plate, $D = Eh^3/12(1 - \nu^2)$ is its flexural rigidity, ρ is the density of the plate material, h is the plate thickness, and the concentrated mass has a mass of M and co-ordinates (u, v). The solution of equation (2.1) can be given in the form of a summation over normal modes,

$$w(x, y, t) = \sum_{r=1}^{\infty} a_r \Psi_r(x, y)\, \mathrm{e}^{\mathrm{i}\omega_r t}, \tag{2.2}$$

where the $\Psi_r(x, y)$ and ω_r are eigenfunctions and eigenvalues, respectively. The eigenfunctions, as derived by Amba-Rao [3], are

$$\Psi_r(x, y) = \frac{4M\omega_r^2}{Dab}\Psi_r(u, v)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin(m\pi u/a)\sin(n\pi v/b)\sin(m\pi x/a)\sin(n\pi y/b)}{[(m\pi/a)^2 + (n\pi/b)^2]^2 - \omega_r^2\rho h/D} \tag{2.3}$$

and the eigenvalues ω_r are the roots of the expression

$$f(\omega_r) = \frac{4M\omega_r^2}{Dab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin^2(m\pi u/a)\sin^2(n\pi v/b)}{[(m\pi/a)^2 + (n\pi/b)^2]^2 - \omega_r^2\rho h/D} - 1. \tag{2.4}$$

From this it may be shown that, if the eigenvalues of the un-mass loaded plate are sorted into ascending order, then the nth eigenvalue of the single mass loaded plate lies between the $(n-1)$th and the nth eigenvalues of the un-mass loaded plate.

2.2. INDUCTIVE STEP

Suppose that the solution of the partial differential equation (PDE) for n added masses is known, and the solution for the PDE for these n masses plus one further mass is desired. It is possible to say anything about where the eigenvalues will lie? The partial differential equation governing the plate plus $n+1$ masses is

$$D\nabla^4 w + \left\{\rho h + \sum_{k=1}^{n+1} M_k \delta(x-u_k)\delta(y-v_k)\right\}\frac{\partial^2 w}{\partial t^2} = 0,$$

where $w(x, y, t)$ may be expressed in terms of a sum over normal modes,

$$w(x, y, t) = \sum_{r=1}^{\infty} A_r^{[n+1]}\Psi_r^{[n+1]}(x, y)\, e^{i\omega_r^{[n+1]}t}, \tag{2.5}$$

so that

$$D\nabla^2\Psi_r^{[n+1]} - \omega_r^{[n+1]2}\left\{\rho h + \sum_{k=1}^{n+1} M_k \delta(x-u_k)\delta(y-v_k)\right\}\Psi_r^{[n+1]} = 0. \tag{2.6}$$

Letting

$$\Omega_r = \int_0^a \int_0^b \Psi_s^{[n+1]}\Psi_r^{[n]}\left\{\rho h + \sum_{k=1}^{n} M_k \delta(x-u_k)\delta(y-v_k)\right\} \mathrm{d}y\, \mathrm{d}x \tag{2.7}$$

and taking the $\Psi^{[n]}$ finite transform of equation (2.6), viz.:

$$\int_0^a \int_0^b \Psi_r^{[n]}\left[\nabla^4 - \frac{\omega_s^{[n+1]2}}{D}\left\{\rho h + \sum_{k=1}^{n+1} M_k \delta(x-u_k)\delta(y-v_k)\right\}\right]\Psi_s^{[n+1]}\, \mathrm{d}y\, \mathrm{d}x = 0, \tag{2.8}$$

it is possible to obtain

$$\Omega_r = \omega_s^{[n+1]2}\frac{M_{n+1}}{\rho h}\frac{\Psi_s^{[n+1]}(u_{n+1}, v_{n+1})\Psi_r^{[n]}(u_{n+1}, v_{n+1})}{\omega_r^{[n]2} - \omega_s^{[n+1]2}}. \tag{2.9}$$

The inverse $\Psi^{[n]}$ finite transform is then given by

$$\Psi_s^{[n+1]}(x, y) = \sum_{r=1}^{\infty} a_r \Omega_r \Psi_r^{[n]}(x, y), \tag{2.10}$$

where

$$\frac{1}{a_r} = \int_0^a \int_0^b \Psi_r^{[n]2}(x, y)\left\{\rho h + \sum_{k=1}^{n} M_k \delta(x-u_k)\delta(y-v_k)\right\} \mathrm{d}y\, \mathrm{d}x. \tag{2.11}$$

Therefore, upon re-inverting equation (2.9) this becomes

$$\Psi_s^{[n+1]}(x, y) = \omega_s^{[n+1]2}\frac{M_{n+1}}{\rho h}\Psi_s^{[n+1]}(u_{n+1}, v_{n+1})\sum_{r=1}^{\infty}\frac{a_r \Psi_r^{[n]}(u_{n+1}, v_{n+1})\Psi_r^{[n]}(x, y)}{\omega_r^{[n]2} - \omega_s^{[n+1]2}}. \tag{2.12}$$

Substituting $x = u_{n+1}$ and $y = v_{n+1}$ and cancelling $\Psi_s^{[n+1]}(u_{n+1}, v_{n+1})$ yields

$$\omega_s^{[n+1]2}\frac{M_{n+1}}{\rho h}\sum_{r=1}^{\infty}\frac{a_r \Psi_r^{[n]2}(u_{n+1}, v_{n+1})}{\omega_r^{[n]2} - \omega_s^{[n]2}} = 1, \tag{2.13}$$

for each eigenvalue $\omega_s^{[n+1]}$ and eigenfunction $\Psi_s^{[n+1]}$. In theory, it is possible to obtain the eigenvalues by finding the roots of the expression

$$f(\omega) = \omega^2\frac{M_{n+1}}{\rho h}\sum_{r=1}^{\infty}\frac{a_r \Psi_r^{[n]2}(u_{n+1}, v_{n+1})}{\omega_r^{[n]2} - \omega^2} - 1, \tag{2.14}$$

and the corresponding eigenfunctions are then given by substituting the eigenvalues back into equation (2.12) and normalizing. Furthermore, their positions are predictable:

note that

$$f'(\omega) = 2\omega \frac{M_{n+1}}{\rho h} \sum_{r=1}^{\infty} \frac{a_r \Psi_r^{[n]2}(u_{n+1}, v_{n+1}) \omega_r^{[n]2}}{(\omega_r^{[n]2} - \omega^2)^2} > 0. \tag{2.15}$$

Since $f(\omega)$ is undefined for, and changes sign at, $\omega_r^{[n]} = \omega$, and is strictly increasing between $\omega = \omega_r^{[n]}$ and $\omega = \omega_{r+1}^{[n]}$, it is clear that

$$\omega_r^{[n]} < \omega_{r+1}^{[n+1]} \leqslant \omega_{r+1}^{[n]}, \tag{2.16}$$

where there is equality with the upper limit if, and only if, the $(n+1)$th mass lies on a node line of the eigenfunction $\Psi_{r+1}^{[n]}$. Since this is true for the eigenvalues of the plate carrying a single mass compared with the eigenvalues of the plate without a mass, i.e., $\omega_r^{[0]} < \omega_{r+1}^{[1]} \leqslant \omega_{r+1}^{[0]}$, it is then clear that

$$\omega_r^{[0]} < \omega_{r+n}^{[n]} \leqslant \omega_{r+n}^{[0]}, \tag{2.17}$$

or, in other words, to shift n resonances from a giving frequency band, at least n masses must be used. This will be possible, in general, if there is no practical limitation on the size of the masses and where they can be placed.

This analysis has demonstrated that n pre-chosen resonances may be shifted downwards, but has not yet addressed the problem of the higher resonances also shifting down into the frequency band being cleared. It is obvious from the reasoning above that, if further masses are not positioned on node lines, then the order of the eigenvalues will remain the same. If all the masses lie very close to the node lines of a given eigenfunction, then the masses will have little effect on that eigenfunction or the corresponding eigenvalue. This eigenvalue then is reduced very gradually, and all the higher numbered eigenvalues must be greater, by equation (2.17). Inevitably, as more or larger masses are added, the effect on that eigenfunction will become more dramatic, until it suddenly changes form and the eigenvalue drops sharply. It would seem that the best that can be obtained is that the masses are placed exactly on the node lines, although the theorem then does not apply and the eigenvalues may re-order.

3. THE DIRECT MODAL SUM METHOD

The derivation of the direct modal sum method for a number of point masses on a plate is exactly analogous to the method given by Amba-Rao [3] for a single point mass. As above, consider a plate carrying n point masses, where the kth masses is given by M_k, and its location is given by (u_k, v_k). The partial differential equation is

$$D\nabla^4 w + \left\{\rho h + \sum_{k=1}^{n} M_k \delta(x - u_k)\delta(y - v_k)\right\} \frac{\partial^2 w}{\partial t^2} = 0. \tag{3.1}$$

Writing $w(x, y, t)$ as a sum over normal modes,

$$w(x, y, t) = \sum_{r=1}^{\infty} A_r \Psi_r(x, y)\, \mathrm{e}^{\mathrm{i}\omega_r t}, \tag{3.2}$$

leads to

$$D\nabla^4 \Psi_r - \omega_r^2 \left\{\rho h + \sum_{k=1}^{n} M_k \delta(x - u_k)\delta(y - v_k)\right\} \Psi_r = 0. \tag{3.3}$$

TABLE 1

Plate parameters

Young's modulus E (GPa)	206·8
Poisson's ratio ν	0·29
Density ρ (kg/m^3)	7820
Plate thickness, h (m)	0·01
Length, a (m)	7
Breadth, b (m)	2
Forcing point, (x_i, y_i) (m)	(0·1, 0·1)
Response point, (x_0, y_0) (m)	(6·9, 1·9)

By taking the finite sine transform of equation (3.3), it may be shown that

$$\int_0^b \int_0^a \Psi_r(x, y) \sin(m\pi x/a) \sin(n\pi y/b)\, \mathrm{d}x\, \mathrm{d}y$$

$$= \frac{(\omega_r^2/D) \sum_{k=1}^{n} M_k \Psi_r(u_k, v_k) \sin(m\pi u_k/a) \sin(n\pi v_k/b)}{[(m\pi/a)^2 + (n\pi/b)^2]^2 - \omega_r^2 \rho h/D}. \quad (3.4)$$

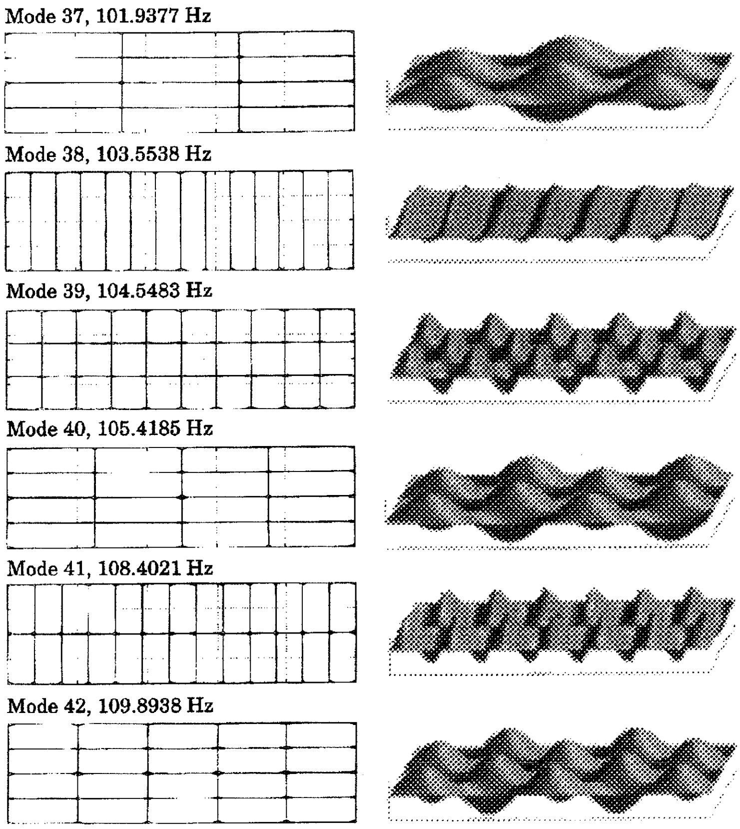

Figure 1. Eigenvalues and functions for the unloaded plate.

Re-inverting gives

$$\Psi_r(x, y) = \frac{4\omega_r^2}{Dab} \sum_{k=1}^{n} M_k \Psi_r(u_k, v_k)$$

$$\times \frac{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin(m\pi x/a) \sin(n\pi y/b) \sin(m\pi u_k/a) \sin(n\pi v_k/b)}{[(m\pi/a)^2 + (n\pi/b)^2]^2 - \omega_r^2 \rho h/D}. \quad (3.5)$$

In equation (3.5) (u_l, v_l) is then substituted for (x, y) to obtain n equations. These can be written as a matrix eigenvalue equation of the form

$$(\mathbf{A} - \lambda \mathbf{I})\mathbf{x} = \mathbf{0}, \quad (3.6)$$

where

$$A_{k,l} = \frac{M_k \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin(m\pi u_l/a) \sin(n\pi v_l/b) \sin(m\pi u_k/a) \sin(n\pi v_k/b)}{[(m\pi/a)^2 + (n\pi/b)^2]^2 - \omega_r^2 \rho h/D}, \quad (3.7a)$$

$$x_l = \Psi_r(u_l, v_l) \quad \text{and} \quad \lambda = Dab/4\omega_r^2. \quad (3.7b, c)$$

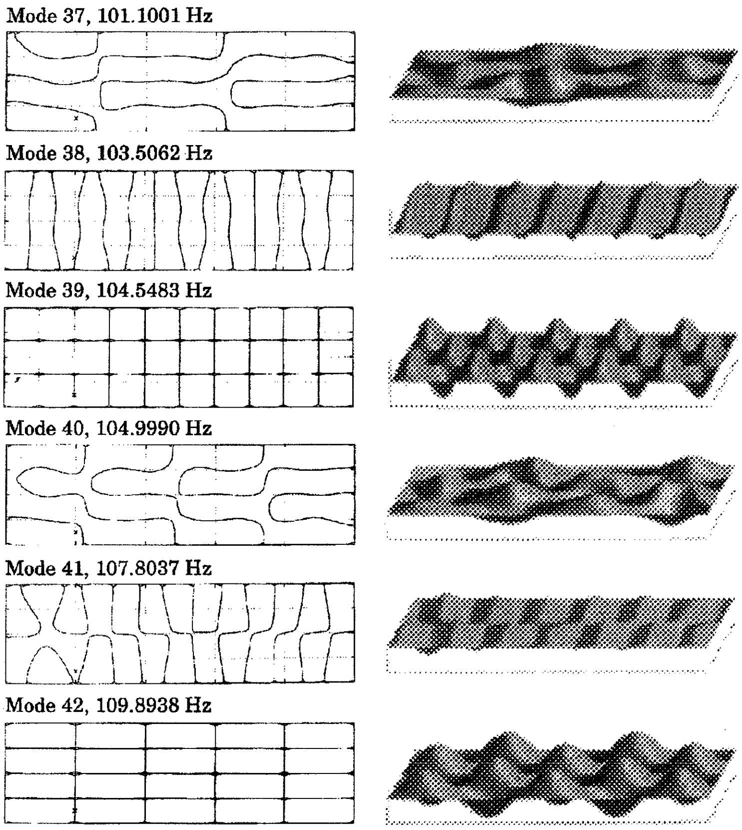

Figure 2. Eigenvalues and functions for the plate carrying one mass.

Since equation (3.6) is true if, and only if,

$$|\mathbf{A} - \lambda \mathbf{I}| = 0, \tag{3.8}$$

it is possible to solve for ω_r by evaluating the determinant for a range of values and use those either side of a sign change as a starting point for a bisection root search. Note that the reader should not confuse the eigenvalues of the matrix equation, λ, with those of the plate and masses system, ω_r. In this context λ and the matrix $\mathbf{A}$ both contain the eigenvalue ω_r.

4. THE FREQUENCY DEPENDENT FORCES METHOD

The purpose of this analysis is to find the response of the plate due to harmonic forcing. The masses are considered as equivalent frequency dependent point forces, so that the partial differential equation is now

$$D\nabla^4 w + \rho h\, \partial^2 w/\partial t^2 = F(x, y, t), \tag{4.1}$$

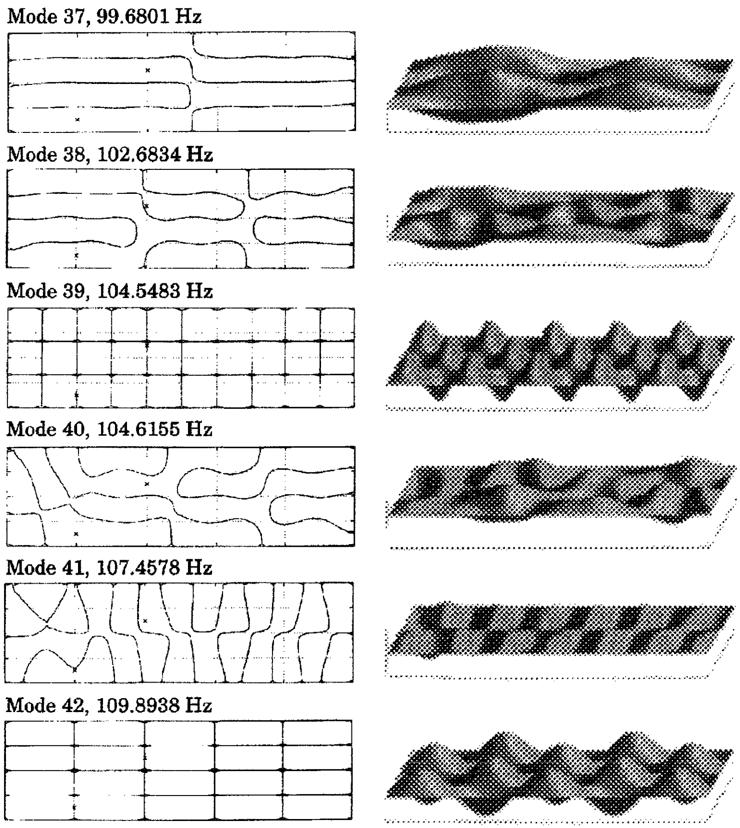

Figure 3. Eigenvalues and functions for the plate carrying two masses.

where $F(x, y, t)$ is the sum of all forces applied to the plate. This includes both the real force applied at (x_i, y_i),

$$F_i(x, y, t) = f_i \delta(x - x_i)\delta(y - y_i)\,e^{i\omega t} \tag{4.2}$$

and the forces associated with the point masses.

The deflection of the plate at (x_0, y_0) due to a single force of magnitude f_i applied at (x_i, y_i) is given by

$$w(x_0, y_0, \omega) = f_i g(x_i, y_i, x_0, y_0; \omega), \tag{4.3}$$

where

$$g(x_i, y_i, x_0, y_0; \omega) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\Psi_{mn}(x_i, y_i)\Psi_{mn}(x_0, y_0)}{b_{mn}(\omega_{mn}^2 - \omega^2)} \tag{4.4}$$

is the Green function for any two-dimensional system in terms of its uncoupled modes. The eigenfunctions are given by

$$\Psi_{mn}(x, y) = \sin(m\pi x/a)\sin(n\pi y/b), \tag{4.4a}$$

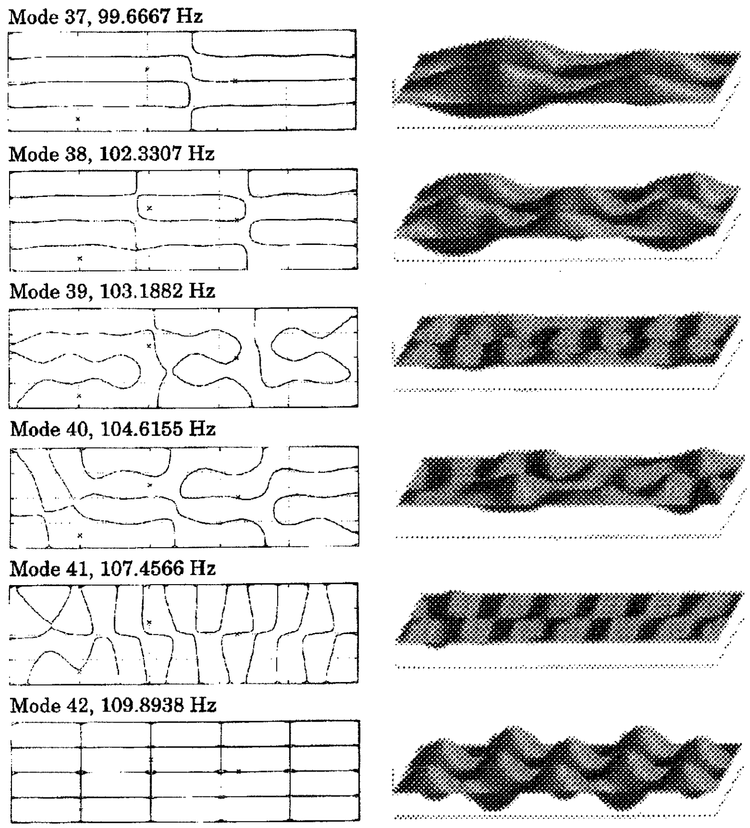

Figure 4. Eigenvalues and functions for the plate carrying three masses.

the eigenvalues by

$$\omega_{mn} = \sqrt{\frac{D}{\rho h}}\,[(m\pi/a)^2 + (n\pi/b)^2] \tag{4.4b}$$

and the orthogonality constant by $b_{mn} = \rho hab/4$.

Now, if there are n masses with positions and values of (x_k, y_k) and M_k, respectively, then each mass provides a point force on the plate of

$$f_k(x, y) = -M_k\, \partial^2 w/\partial t^2|_{(x = x_k, y = y_k)} = M_k \omega^2 w(x_k, y_k). \tag{4.5}$$

The deflection of the plate at any point (x_0, x_0) due to the applied force f_i and those due to the n masses is then

$$w(x_0, y_0; \omega) = f_i g(x_i, y_i, x_0, y_0; \omega) + \sum_{k=1}^{n} f_k g(x_k, y_k, x_0, y_0; \omega). \tag{4.6}$$

In particular, by using equation (4.5), n equations of the form

$$f_j = M_j \omega^2 \left[f_i g(x_i, y_i, x_j, y_j; \omega) + \sum_{k=1}^{n} f_k g(x_k, y_k, x_j, y_j; \omega) \right], \tag{4.7}$$

are obtained for the n positions (x_j, y_j) on the plate corresponding to the n masses. These simultaneous equations can be solved for the n unknown frequency dependent forces f_k.

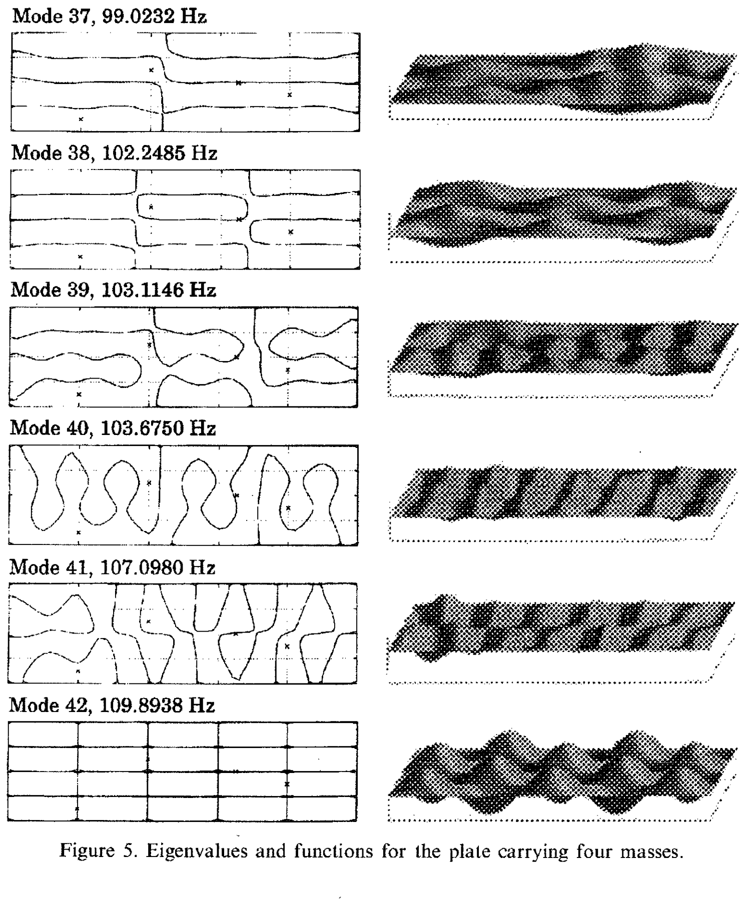

Figure 5. Eigenvalues and functions for the plate carrying four masses.

Then, by using equation (4.6), it is possible to calculate the deflection of the plate at any given point (x_0, y_0).

Additionally, it is worth noting that the eigenvalues of the plate and mass system may also be determined by finding the values of ω which give infinite deflections.

5. NUMERICAL RESULTS

5.1. SHIFTING THE EIGENVALUES USING DMS

As a test case, a plate with the dimensions and material properties given in Table 1 was considered. Five 10 kg masses were individually added to the plate and at each stage the eigenvalues lying between 100 Hz and 110 Hz and the corresponding eigenfunctions determined. The masses were constrained to lie on the node lines of the 42nd eigenfunction, which has an eigenvalue of about 110 Hz, so that this eigenfunction would be unaffected by the addition of the masses. For the purpose of choosing the $(n + 1)$th mass position, the eigenfunctions for the plate carrying n masses were inspected, and the mass placed near the point of greatest deflection occurring on the constraint lines (node lines) taken from the 42nd eigenfunction.

The eigenfunctions for the unloaded plate, which are illustrated in Figure 1, exhibit a regular grid structure as expected. The effects on the eigenfunctions of the addition of further masses are then shown in Figures 2–6 (the crosses indicate the positions of the

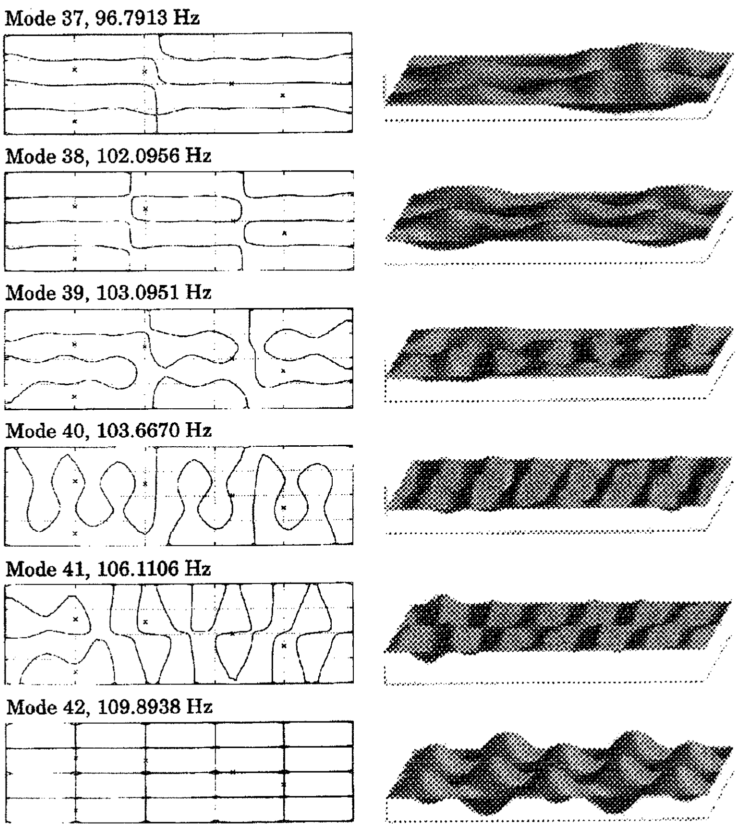

Figure 6. Eigenvalues and functions for the plate carrying five masses.

added masses). Note that for each additional mass the corresponding eigenvalues are reduced or remain constant, as predicted by the IMS method.

With the addition of five masses there is an encouraging degree of reduction in eigenvalues 37–41, although it is not really sufficient to be useful. Despite the results from the IMS method, it is clear that significantly more masses must be applied to the plate to have the desired effect. The evaluation of the determinant in equation (3.8) for ω between 99 and 110 Hz, for each quantity of masses is shown in Figure 7. The vertical lines at around 102, 103·6, 104·5, 105·4 and 108·4 Hz represent eigenvalues 37–41 of the unloaded plate. These verticals all form asymptotes for the determinants, with the exception of the line at the 39th eigenvalue, for the plate carrying one or two masses. This is because the positions of these two masses coincided with the node lines for the 39th eigenfunction and so do not affect it. In the case of the plate with one mass, the determinant increases from $-\infty$ at one asymptote to $+\infty$ at the next. This is to be expected, as the determinant is, in fact, the function of equation (2.4), which has been shown to be monotonically increasing. A more interesting observation is that, in the case of the plate with two masses, the determinant seems to be monotonically decreasing, while for the systems with more masses there is a mixture of increasing, decreasing, u-shaped and n-shaped curves. However, none of these curves crosses any of the asymptotes at a finite value, as the IMS method would suggest is possible.

As a further experiment, the values of all five masses were increased to 11 kg, then to 15 kg and finally to 20 kg. The results of these changes are shown in Figure 8. The crossing of an eigenvalue over an asymptote then becomes clear. Consider the roots of the 10 kg and 11 kg mass systems near 102 Hz. As the mass is increased, the roots are squeezed towards the asymptote, until at a mass between 11 kg and 15 kg, the roots are exactly

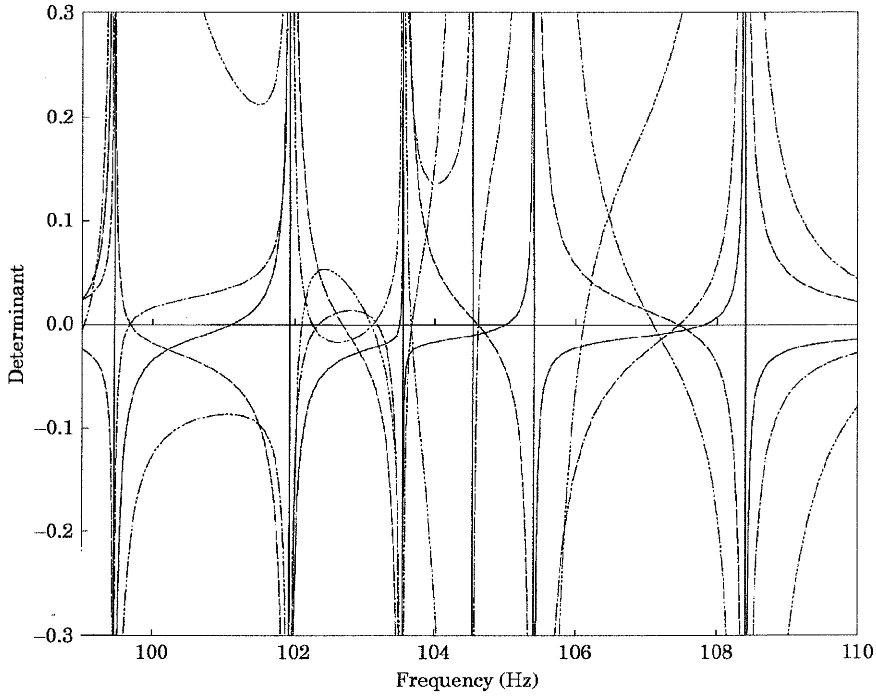

Figure 7. The eigenvalue search. ——, 1; – – –, 2; - - - -, 3; - - - - - -, 4; - - - - - - - -, 5.

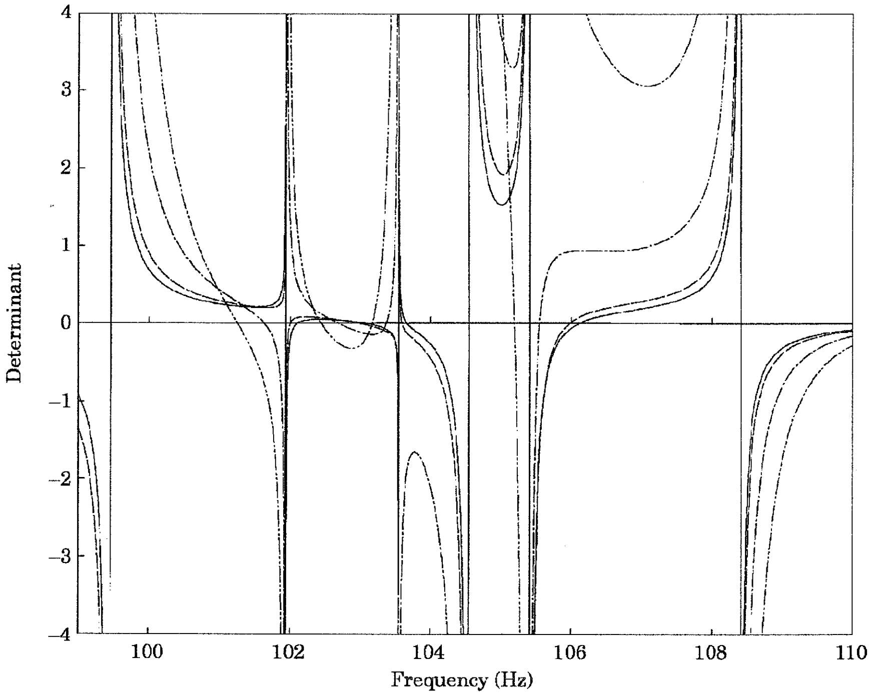

Figure 8. The eigenvalue search for the plate loaded with five masses of various sizes. Mass size: ——, 10 kg; – – –, 11 kg; – – – –, 15 kg; – – – – – –, 20 kg.

equal. Where there is no root between any given pair of asymptotes, there is an n- or u-shaped curve which does not cross the axis. Surprisingly, there do not seem to be any curves with more than one turning point.

5.2. COMPARISON WITH FEM

The main reason for comparing the FDF method with FEM here is as a confidence check. A finite element model of the plate carrying five 10 kg masses was created. Since the eigenvalues of interest are reasonably high it was important to use a fine enough mesh and to select Kirchhoff thin plate elements in order to ensure correspondence between the models; some 2800 three noded, right-angled triangular elements were used, with base and height of 0·1 m. In Figure 9 is shown the response of the plate at (6·9, 1·9) subject to a unit forcing at (0·1, 0·1). Overall, the agreement between the FDF method and the FEM is good. Although there is a slight shift in the higher frequencies, this could no doubt be improved by using more elements.

The FDF method and FEM were also used to compare eigenvalues directly with the results of the DMS method. These are shown in Table 2. The FEM values are consistently about 0·1% higher than those given by FDF and those of the DMS are about 0·4% lower. The exception of the 42nd eigenvalue is due to the fact that this is unaffected by the extra masses and is therefore calculated directly by equation (4.4b).

The reason for the comparatively poor performance of the DMS is not at first apparent. The bisection roots search was performed to high precision and is therefore not the cause. In fact, it would seem that it is due to poor convergence of the summations

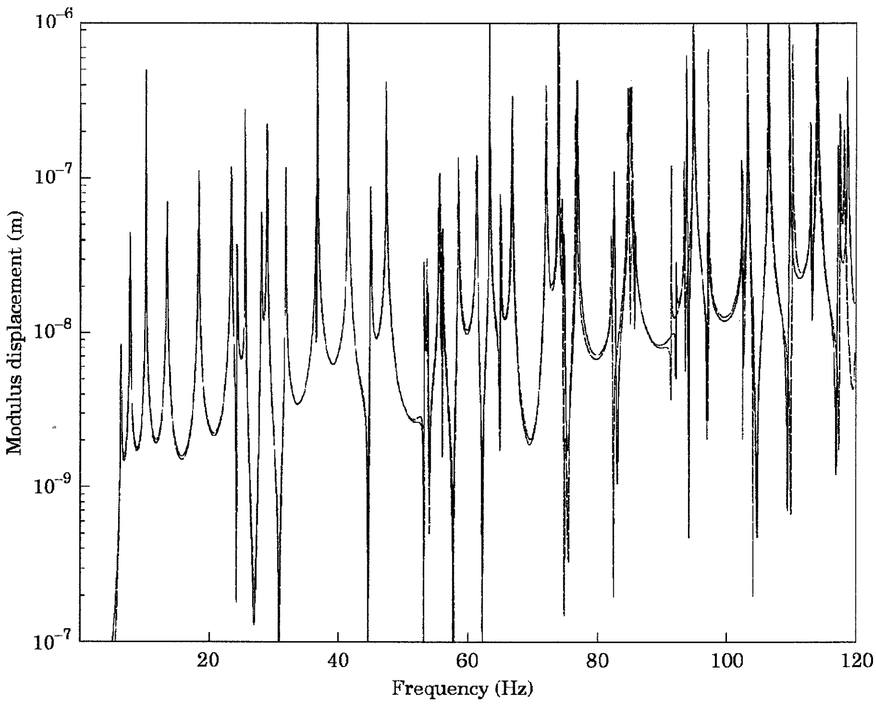

Figure 9. The frequency response of the plate carrying five 10 kg masses. ——, FDF; - - -, FEM.

over m and n in equation (3.7a). This problem of convergence was pointed out by Magrab [4], for the case of a plate carrying a single mass. The unloaded plate eigenvalues were sorted into ascending order and the double summation converted into a single sum of 50 terms. This summation could have been taken further but, even so, for the plate with a five-mass system, the time taken to find each root is about 30 minutes on a Sun Sparc 10 computer. Prior to this, another 2 hours was spent finding the approximate locations for the roots. By comparison, the finite element solution takes about 90 minutes and the FDF around 1 minute to generate the 1200 points comprising the graph in Figure 9. Finding the eigenvalues correct to three decimal places then takes about another 2 minutes each.

TABLE 2

Eigenvalues of the plate carrying five 10 *kg masses*

Eigennumber	DMS eigenvalue (Hz)	FEM eigenvalue (Hz)	FDF eigenvalue (Hz)
37	96·791	97·304	97·157
38	102·096	102·546	102·415
39	103·095	103·403	103·447
40	103·667	104·155	103·928
41	106·111	106·587	106·679
42	109·894	110·410	109·894

6. CONCLUSIONS

The analysis developed here and corroborating numerical results give weight to the argument that special positioning of concentrated masses on a plate may improve its passive noise filtering performance. This is achieved by introducing a frequency band containing a reduced density of eigenvalues and, hence, where vibration transmission is poor. A numerical example has been studied that demonstrates how this might be achieved and such a method could be used practically. However, the direct modal method used is slow and inaccurate compared with FEM for more than one or two masses. The use of the unloaded plate Green function is much faster, but it does not give the eigenvalues directly or the eigenfunctions at all. It does, however, give information about the response of one point on the plate due to a harmonic forcing at another, and by trying many forcing and response points the location of the node lines of the eigenfunctions could be ascertained.

The search for better positions for the masses could be automated by the use of an optimizer such as the Genetic Algorithm. To keep computational times as low as possible, the Green function method would then seem to be the most useful. Perhaps the objective function in such a process might be the integral of the response curve, where a small amount of damping would be required to keep values finite. Since it is possible that an optimizer might then choose forcing and/or response points on or near node lines, and thus not reflect the average behaviour of the plate, it would be necessary to average over three or more forcing and response points during such a search.

Finally, it seems unlikely that five 10 kg masses would be enough to shift all the eigenvalues from the range 100–110 Hz for the 1000 kg plate used here. An increase in the number of masses seems essential, although it is not clear whether a corresponding increase in total added mass would be necessary.

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