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Approximate static and dynamic reanalysis techniques for structural optimization

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ABSTRACT

This paper presents an approach based on reduced basis approximation concepts for static and dynamic reanalysis of structural systems. In the presented approach, scaling parameters are introduced to increase the range of applicability of local approximation techniques based on Taylor or matrix perturbation series. The terms of the local approximation series are used as basis vectors for constructing an approximation of the perturbed response quantities. The undetermined scalar quantities are then estimated by solving the perturbed equilibrium equations in the reduced basis. This approach was earlier proposed in the context of statics by Kirsch (1991). This paper presents in brief the reanalysis procedure for statics and a new method based on a similar line of approach is proposed for approximate dynamic reanalysis. The method is applied to approximate dynamic reanalysis of a cantilevered beam structure. Preliminary results for this example problem indicate that high quality approximation of the natural frequencies and mode shapes can be obtained for moderate perturbations in the stiffness matrix elements of the order of ±40%.

1. INTRODUCTION

The development of approximate response models of structural systems has been a topic of extensive research over the last few decades. This has been mainly due to the ever increasing requirement of efficiently designing large scale structural systems using variable complexity analytical models. Applications also exist in the area of structural identification involving the reconciliation of finite element models with experimental data.

A detailed review of static reanalysis techniques can be found in Topping (1987). More recent reviews of the field have been presented by Barthelemy and Haftka (1991) and Grandhi (1993). It is important to mention here that many of the approximation concepts reported in the literature are valid only for small perturbations in the structural parameters. The literature review included in this paper is restricted to reanalysis techniques which are valid for moderate to large perturbations in the structural parameters.

Approximate reanalysis techniques can be broadly classified in to global and local methods. Global approximation methods either make use of polynomial regression (response surface analysis) or reduced basis approaches. The reduced basis approaches (see, for example, Kapania and Byun, 1993) are also commonly referred to as model reduction in the literature. These methods essentially seek to approximate the response throughout the design space of interest. In contrast, the local methods are based on Taylor or matrix perturbation series around a nominal design point. Hence the range of applicability is local in nature since the series may not converge for moderate to large perturbations in the structural parameters.

Kirsch (1991, 1995) proposed a method based on reduced basis approximation concepts to static reanalysis. This method can be thought of as an combined approximation technique since it attempts to give global characteristics to the conventional local approximation. It was shown that this approach can be used to compute high quality approximation of the static response quantities for very large perturbations in the structural parameters. More recently, Kirsch and Liu (1997) presented a formulation based on a similar line of approach to static reanalysis of structures undergoing topological modifications. The approach developed in this paper has been primarily motivated by the research of Kirsch.

Very few studies in the literature have approached the structural dynamic reanalysis problem for large perturbations in the structural parameters. High (1990) proposed an iterative modal method to compute the perturbations in the frequencies and mode shapes. This method was implemented in version 66 of MSC NASTRAN. Later studies by Eldred *et. al.* (1992) indicated that difficulties may arise in the convergence of High's method for moderate perturbations. They developed an improved scheme for normalization of the eigenvector perturbations in order to improve the convergence properties.

An interesting approach based on interpreting the eigen parameter perturbation equations as differential equations in terms of the perturbation parameters was proposed by Inamura (1988). It was shown via demonstration examples that this procedure could lead to improvements over the conventional local approximation. Pritchard and Adelman (1991) developed a similar procedure using the sensitivity equations of the eigenvalues and eigenvectors.

An iterative procedure using the nonlinear form of the eigenproblem perturbation equations was developed by Eldred et al. (1992). It was shown that this method converges to the exact solution for moderate to large perturbations in the stiffness matrix of the order of 150%. An exact method based on the block Lanczos algorithm was proposed by Carey *et. al.* (1994). Even though both these procedures can provide exact results, the computational effort involved is substantial compared to conventional techniques based on first order Taylor or matrix perturbation series approximation.

More recently, Balmes (1996) presented a novel approach in which the finite element model is represented as a parametric family of reduced order. The full order order finite element model is reduced using a transformation matrix composed of Ritz vectors evaluated at different points in the design space. Excellent results were obtained for approximate static and dynamic reanalysis of a cantilevered box beam structure.

The approach developed in this paper is in spirit similar to the approach of Balmes (1996). However the present method differs in the choice of basis vectors used to construct the transformation matrix. In the present approach, the basis vectors are chosen to be an implicit function of the parametric perturbations. In particular, the terms of the conventional Taylor or matrix perturbation series are chosen as basis vectors. Hence the present approach can also be viewed as an improved local approximation procedure. In contrast, Balmes's method uses a constant transformation matrix which is invariant with the parametric perturbations. Furthermore, the approach presented in this paper aims at building a reduced basis approximation of each eigenmode independently.

The long term objective of the present study is to develop techniques which can be used for obtaining high quality approximation of the dynamic response quantities for moderate to large perturbations in the structural parameters.

This paper is organized as follows. Section 2 briefly describes Kirsch's formulation for approximate static reanalysis. Based on Kirsch's method for statics, the extension of the theory to approximate dynamic reanalysis is proposed. Some comments on the computational aspects of the proposed procedure and analogies with existing approximation concepts are briefly discussed in section 3. Section 4 presents results for approximate dynamic reanalysis of a cantilevered beam. Approximations are sought for the natural frequencies and mode shapes for perturbations in the flexural rigidities. The effects of both global and local perturbations in the structural parameters on the proposed procedure is studied. Results indicate that a reliable approximation of the natural frequencies and mode shapes can be obtained for moderate perturbations in the stiffness matrix of the order of ±40%. Comparison studies with the conventional first order Taylor series approximation shows that the accuracy has been considerably improved with a relatively small computational effort. Section 5 summarizes the present work and future areas of investigation are outlined.

2. THEORETICAL DEVELOPMENT

The equations of motion of a multi-degree of freedom linear structural system can be written as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F} \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K} \in \Re^{n\times n}$ are the structural mass, damping and stiffness matrices respectively, $\mathbf{x}$ is the vector of displacements corresponding to the analytical degrees of freedom and $\mathbf{F}$ is the vector of external forces.

2.1 Approximate Static Reanalysis

Consider the case of static analysis, wherein the derivatives of the displacement vector $\mathbf{x}$ with respect to time are zero. For this case, if the stiffness matrix is perturbed by $\Delta\mathbf{K}$, the displacement at the perturbed design point can be expressed as a matrix series of the form

$$\mathbf{x} = (\mathbf{I} - \mathbf{B} + \mathbf{B}^2 - \ldots)\mathbf{x}^o \quad (2)$$

where $\mathbf{B} = \mathbf{K}^{o-1}\Delta\mathbf{K}$. $\mathbf{K}^o$ and $\mathbf{x}^o$ are the stiffness matrix and displacement vector at the nominal design point respectively. Note here that the above series will converge to the exact value of the perturbed displacement vector only when $(\mathbf{K}^{o-1}\Delta\mathbf{K})^k \rightarrow 0$ as $k \rightarrow \infty$. In general, for moderate to large perturbations in the stiffness matrix, the series will not converge.

Kirsch (1991) proposed the use of terms of the above series as high quality basis vectors for constructing an approximation to the perturbed displacement vector. It was shown via various demonstration examples that high quality approximation of the static response quantities can be obtained for very large perturbations in the stiffness matrix of the order of 900%. In a sequel paper, Kirsch (1995) showed that the use of Taylor series terms can lead to similar results. More implementation specific details including results for various benchmark structures can be found in the afore mentioned references.

In the next section, extension of Kirsch's method to approximate dynamic reanalysis is described. In particular, it is sought to develop a procedure for approximation of the eigenvalues and eigenvectors of perturbed eigensystems on a similar line of approach.

2.2 Approximate Dynamic Reanalysis

Typically, modal analysis is used for computing the dynamic response of large scale finite element models of structural systems. This procedure essentially involves computing the first few natural frequencies and the corresponding mode shapes and transforming the original equations of motion to the modal coordinates. This permits dynamic response analysis of the finite element model in a computationally efficient fashion. This paper is restricted to approximate reanalysis of the eigen parameters for perturbations in the stiffness and mass matrices. Once the eigen parameters have been approximated for the perturbed system, computation of the dynamic response for arbitrary time varying loading conditions is relatively straight forward.

The free vibration undamped natural frequencies and mode shapes of a structural system, whose equilibrium equations can be expressed in the form of equation (1), can be computed by solving the algebraic eigenvalue problem posed below.

$$\mathbf{K}\phi = \lambda\mathbf{M}\phi \quad (3)$$

where ϕ denotes the mode shape of the structural response and λ is the eigenvalue which is the square of the natural frequency.

The original/baseline system matrices and the response quantities are denoted by the superscript o. The set of structural parameters or design variables are denoted by the vector $\mathbf{X}$. Let $\Delta\mathbf{X}$ denote the perturbation in the structural parameters. Then the corresponding perturbation in the system matrices and the response quantities can be expressed as

$$\begin{aligned} \lambda_i &= \lambda_i{}^o + \Delta\lambda_i \\ \phi_i &= \phi_i^o + \Delta\phi_i \\ \mathbf{K} &= \mathbf{K}^o + \Delta\mathbf{K} \\ \mathbf{M} &= \mathbf{M}^o + \Delta\mathbf{M} \end{aligned}$$

The index i is used to denote the eigenmode number. Typically, the eigenvalue and eigenvector perturbations are calculated using first order sensitivity information, i.e., the perturbation in the eigenvalue and eigenvector for mode i can be expressed as

$$\Delta\lambda_i = \sum_{j=1}^{p} \frac{\partial\lambda_i}{\partial x_j}\Delta x_j \quad (4)$$

$$\Delta\phi_i = \sum_{j=1}^{p} \frac{\partial\phi_i}{\partial x_j}\Delta x_j \quad (5)$$

where $\frac{\partial\lambda_i}{\partial x_j}$ and $\frac{\partial\phi_i}{\partial x_j}$ are the sensitivities of the eigenvalues and eigenvectors with respect to the structural parameters denoted by $\mathbf{X} = \{x_1, x_2, \ldots, x_p\}$. For many cases of practical interest, the eigenvalue and eigenvector sensitivities may not be easy to evaluate. Hence it may be more convenient to compute the eigenvalue and eigenvector perturbations using a first order matrix perturbation series. Note here that the first order approximation of eigen parameters using matrix perturbation series are equivalent to the first order Taylor series terms. A first order approximation of the eigenvalues and eigenvectors using a matrix perturbation approach (Brandon, 1990) can be written as

$$\Delta\lambda_i = \frac{\phi_i^{oT}[\Delta\mathbf{K} - \lambda_i{}^o\Delta\mathbf{M}]\phi_i^o}{\phi_i^{oT}\mathbf{M}\phi_i^o} \quad (6)$$

$$\Delta\phi_i = \sum_{j=1, j\neq i}^{n} \alpha_{ij}\phi_j^o \quad (7)$$

where

$$\alpha_{ij} = \frac{\phi_j^{oT}[\Delta\mathbf{K} - \lambda_i{}^o\Delta\mathbf{M}]\phi_i^o}{(\lambda_i{}^o - \lambda_j{}^o)\phi_i^{oT}\mathbf{M}\phi_i^o} \quad (8)$$

Using these equations, a first order approximation of the eigenvalues and eigenvectors of the perturbed system can be calculated. Note here that in order to compute the perturbation in the eigenvector, a modal summation approach is used which requires all the eigenvectors of the original/baseline system to be evaluated. This can prove to be computationally prohibitive for large scale systems. In order to circumvent this requirement, a truncated set of baseline eigenvectors could be used. This could potentially lead to reduced accuracy for large perturbations in the structural parameters. The other alternative could be to make use of iterative schemes (see for example; Zhang and Zerva, 1997) or Nelson's method (Nelson, 1976) in order to compute the eigenvector perturbations in a computationally efficient fashion.

The approximation procedure developed in the present study is based on the proposition stated below.

Proposition : The eigenvector of the perturbed system can be approximated in the subspace spanned by $\phi_i{}^o$ and $\Delta\phi_i$. i.e., an approximation to the perturbed eigenvector can be written as

$$\hat{\phi}_i = \zeta_1\phi_i^o + \zeta_2\Delta\phi_i \quad (9)$$

where ζ_1 and ζ_2 are the undetermined scalar quantities in the approximate representation of the perturbed eigenvector. The assumption implicit in the proposition is that even for moderate to large perturbations in the structural parameters, the first order approximation yields a $\Delta\phi_i$ vector which gives a reasonable indication of direction of change of the baseline eigenvector, although the magnitude of change may be erroneous. It can also be seen that for $\zeta_1 = \zeta_2 = 1$, the proposition reduces to the conventional first order approximation. Equation (9) can be expressed in matrix form as

$$\hat{\phi}_i = \mathbf{T}\mathrm{Z} \quad (10)$$

where $\mathbf{T} = \{\phi_i^o, \Delta\phi_i\} \in \Re^{n\times 2}$ and $\mathrm{Z}^T = \{\zeta_1, \zeta_2\} \in \Re^{1\times 2}$

Substituting equation (10) in to equation (3) and premultiplying by $\mathbf{T}^T$, the resulting set of equations can be expressed as

$$\mathbf{K}_T\mathrm{Z} = \lambda\mathbf{M}_T\mathrm{Z} \quad (11)$$

where $\mathbf{K}_T = \mathbf{T}^T\mathbf{K}\mathbf{T} \in \Re^{2\times 2}$ and $\mathbf{M}_T = \mathbf{T}^T\mathbf{M}\mathbf{T} \in \Re^{2\times 2}$ are the reduced stiffness and mass matrices. Hence using the present approach, the original $n \times n$ eigensystem is represented by a reduced 2×2 eigensystem for each eigenmode to be approximated.

A non-trivial solution to Z can be obtained only when λ is an eigenvalue of the matrix pair $(\mathbf{K}_T, \mathbf{M}_T)$. Hence an approximation to the eigenvalue of the perturbed system $(\hat{\lambda}_i)$ can be computed by solving for the roots of the quadratic given below.

$$a\hat{\lambda}_i^2 + b\hat{\lambda}_i + c = 0 \quad (12)$$

where $a = m_{11}m_{22} - m_{12}^2$, $b = 2k_{12}m_{12} - k_{11}m_{22} - m_{11}k_{22}$ and $c = k_{11}k_{22} - k_{12}^2$, k_{ij} and m_{ij} are the elements of the reduced stiffness and mass matrices ($\mathbf{K}_T$ and $\mathbf{M}_T$) respectively. The elements of $\mathbf{K}_T$ and $\mathbf{M}_T$ are given below as

$$k_{11} = \phi_i^{oT}\mathbf{K}\phi_i^o \qquad m_{11} = \phi_i^{oT}\mathbf{M}\phi_i^o$$
$$k_{12} = \phi_i^{oT}\mathbf{K}\Delta\phi_i \qquad m_{12} = \phi_i^{oT}\mathbf{M}\Delta\phi_i$$
$$k_{22} = \Delta\phi_i^T\mathbf{K}\Delta\phi_i \qquad m_{22} = \Delta\phi_i^T\mathbf{M}\Delta\phi_i$$

Solution of the above quadratic give two values for the perturbed eigenvalue. Since the transformed matrices $\mathbf{K}_T$ and $\mathbf{M}_T$ are real and symmetric, the roots of equation (12) will be real. Now the question arises regarding which root to choose as the best approximation for the perturbed eigenvector. For the demonstration example considered, it was confirmed via numerical experiments that the root with the lowest magnitude gives the best approximation. The mathematical proof of this is involved and is beyond the scope of this paper. Once an approximation to the eigenvalue has been computed, the approximate eigenvector can be evaluated by calculating the values of c_1 and c_2 (i.e., the eigenvectors of the reduced eigensystem given by equation (11)) and using equation (9).

3. COMMENTS

It can be observed from the formulation that an approximation to the eigenvalues and eigenvectors of the perturbed system can be calculated by solving for the roots of an quadratic for each eigenmode of interest. The coefficients of the quadratic equation can be easily calculated after the first order approximation of the perturbed eigenvector is computed. Hence the proposed procedure involves only a few additional computations when compared to the conventional first order local approximation.

For many large scale structures, the response is dominated by the first few eigenmodes (typically 10-20 even for a structure with 10,000 degrees of freedom). Hence using the proposed procedure, the perturbations in the eigenvalues and eigenvectors can be approximated by solving a few number of quadratic equations which could lead to substantial savings in the computational time required for dynamic response synthesis.

In order to improve the accuracy of the present procedure, it may be desirable to make use of second order approximation terms (i.e, three basis vectors) at the cost of increased computations. It will be shown in the subsequent section that using the first order terms alone, a reliable approximation of the perturbed frequencies and mode shapes can be computed for moderate perturbations in the structural parameters.

Variants of the proposed approach, for example; using a common set of basis vectors for each eigenmode and solving a single reduced eigensystem also merits consideration. This could potentially lead to better accuracy in the approximation of eigen parameters for many modally complex structural systems. However, due to constraints on data presentation, results for these cases will not be presented in the present paper.

It is well known that the use of Rayleigh's quotient yields a better approximation to the lowest natural frequency as compared to the conventional first order approximation. The lowest eigenvalue is typically approximated using the equation

$$\lambda_{rqa} = \frac{\phi^{oT}\mathbf{K}\phi^{o}}{\phi^{oT}\mathbf{M}\phi^{o}} \tag{13}$$

The assumption made here is that the mode shapes are invariant to the parametric perturbations. Earlier studies (see for example; Canfield, 1990) have conclusively shown that the modal strain and kinetic energy (i.e., the numerator and denominator of Rayleigh's quotient) can be used as intervening variables to approximate the natural frequency with better accuracy as compared to the local first order approximation.

It can be checked that if $\Delta\phi_i$ is considered to be a very small quantity, the approximation for the first eigenmode using the present procedure tends to the Rayleigh quotient approximation.

4. DEMONSTRATION EXAMPLE, RESULTS AND DISCUSSION

The proposed procedure is applied to approximate structural dynamic reanalysis of a cantilevered beam structure. The baseline values of the structural parameters are taken as : flexural rigidity $EI = 1.286 \times 10^4 Nm^2$, mass per unit length $m = 2.73kg/m$ and length of the beam $L = 2.5m$. The beam is modeled using five finite elements with each node constrained to have only two degrees of freedom (translational and rotational). For this example, the structural parameters which are perturbed correspond to the flexural rigidities of the five elements, i.e, only perturbation in the stiffness matrix is considered.

Results are presented for two cases. In the first case, the approximate natural frequencies and mode shapes are evaluated for simultaneous (global) perturbations in the structural parameters. In the second case, the effect of local perturbations on the approximation procedure is studied.

In order to evaluate the accuracy of the approximation, two error indices are defined. The first error index is the Frobenius norm of the difference between the exact mode shape (obtained using exact reanalysis) and the approximate mode shape using the present approach. The mode shape error index for eigenmode i is defined below as

$$MSE_i = ||\hat{\phi}_i - (\phi_i)_{exact}||_f \tag{14}$$

where $||.||_f$ denotes the Frobenius norm of the vector $\{.\}$. The second index, henceforth referred to as FE_i is the percentage error in approximation of the natural frequency for eigenmode i.

Case 1 : For this case five different parameter sets were considered with perturbations in the flexural rigidities ranging from ±10% to ±50%. The percentage perturbations in the flexural rigidities for the five parameter sets are given in table 1.

Table 1: **Percentage Perturbation in Flexural Rigidities of the Elements for the Parameter Sets**

Parameter	Perturbation in Parameters(%)				
Set	1	2	3	4	5
PS1	+5	+8	-9	+10	-5
PS2	+15	-18	-19	+20	-12
PS3	+15	-25	+30	-27	+22
PS4	-35	-25	+25	-40	+37
PS5	+40	-50	+45	-30	+47

For each case, the flexural rigidities of the five elements were perturbed from the baseline values and the approximate frequencies and mode shapes were evaluated using the proposed procedure. The approximate results are compared with results obtained via exact eigensolution of the perturbed system. Table 2 summarizes the results for all the parameter sets. The results using the present approach is compared with those obtained a first order Taylor series approximation denoted by TS1. Results are presented only for approximation of the first three eigenmodes. The accuracy of the approximation is evaluated using the two error indices defined earlier.

It can be seen from the results that excellent improvements have been obtained over the conventional first order approximation for moderate perturbations in the structural parameters of the order of $\pm 30\%$. Deterioration in the approximation can be observed only when the perturbations approach the order of $\pm 50\%$. It can also be seen that for the range of perturbations considered in this demonstration example, the errors in approximation using first order Taylor series are substantial.

The mode shape error index defined in this study does not give a good indication of the accuracy of the approximation. In general, it was found that the mode shape can be approximated with better precision as compared to the natural frequency. Figure 1 compares the approximate and exact mode shape of the third eigenmode for parameter set PS5. Note here that the transverse displacement and rotation quantities alternate in the mode shape vector. It can be seen from the figure that using the present approach, the first order approximation of the perturbed eigenvector has been improved substantially.

Case 2 : The effect of perturbations in the flexural rigidity of element 3 on approximation of the natural frequencies and the mode shapes are studied. Perturbations in the flexural rigidity of this element is studied in the range of $10 - 100\%$.

Figure 2 depicts the variation of errors in approximation of the first three natural frequencies for increasing perturbation in the flexural rigidity of element 3. It can be seen from the figure that even for large local perturbation in the flexural rigidity of the order of 100%, the maximum error in approximation of the first three frequencies are of the order of 5%. It can also be noted that the maximum error occurs in approximation of the second eigenmode. Numerical experiments on cases involving perturbation of other local parameters

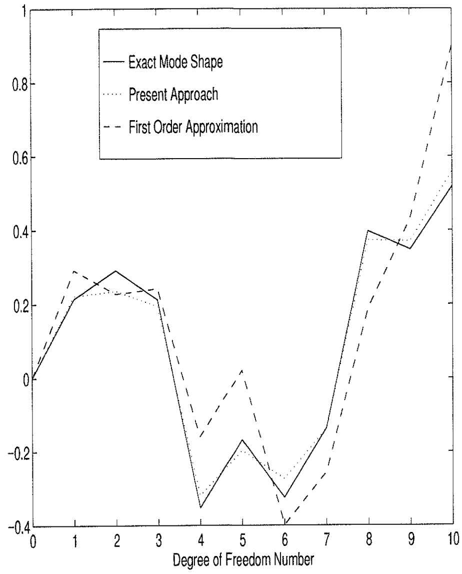

Figure 1: **Comparison Between the Approximate and Exact Mode Shape of the Third Eigenmode for Parameter Set PS5**

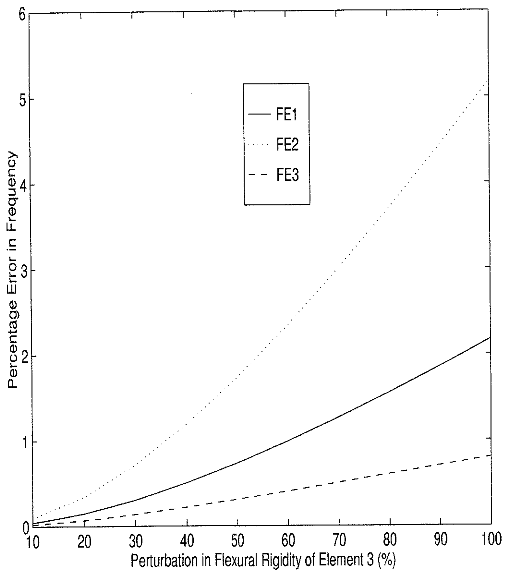

Figure 2: **Variation of Frequency Errors with Local Perturbation in the Structural Parameters**

indicated that the eigenmode which is difficult to approximate may change. It was found that when

Table 2: **Comparison of Results using the Present Approach with Taylor Series Approximation for the Five Parameter Sets**
*Negative Eigenvalue Approximation

Parameter Set	Method	FE1 (%)	FE2 (%)	FE3 (%)	MSE1	MSE2	MSE3
PS1	Present Approach	0.11	0.31	0.17	2.9×10^{-5}	4.9×10^{-4}	6.7×10^{-4}
	TS1	4.27	1.39	-3.16	1.2×10^{-4}	1.6×10^{-3}	8.8×10^{-3}
PS2	Present Approach	1.09	1.05	0.9	1.6×10^{-4}	1.9×10^{-3}	9.5×10^{-3}
	TS1	34.3	16.8	1.4	1.2×10^{-2}	6.1×10^{-3}	9.6×10^{-2}
PS3	Present Approach	2.01	2.86	2.03	2×10^{-4}	5.3×10^{-3}	5.7×10^{-3}
	TS1	28.5	10.86	19.8	1.2×10^{-2}	1.6×10^{-2}	0.12
PS4	Present Approach	1.85	4.74	3.51	4.7×10^{-4}	6.9×10^{-3}	9×10^{3}
	TS1	*	-3.92	3.7	5.2×10^{-3}	0.05	0.26
PS5	Present Approach	9.78	5.82	5.15	1×10^{-3}	8×10^{-3}	0.01
	TS1	69.8	27.97	33.23	0.09	0.05	0.3

the flexural rigidity of element 3 is perturbed by 150%, the percentage errors in approximation of the first three natural frequencies were 3.9, 9.2 and 1.3 respectively. Hence it can be concluded that for this particular example, high quality approximation of the first eigenmode can be obtained for large local perturbations in the structural parameters.

5. CONCLUDING REMARKS

An improved first order approximation procedure for reanalysis of eigenvalues and eigenvectors of modified structural systems was proposed. It has been shown via a simple demonstration example that the present approach can be used to estimate a reliable approximation of the natural frequencies and mode shapes for simultaneous perturbation in the structural parameters of the order of $\pm 40\%$. It was also demonstrated that the proposed procedure can be used for arriving at high quality approximation of the eigen parameters for large local perturbations in the structural parameters.

It is expected that the present formulation may find applications in the area of structural optimization and identification. It is important to note here that extension of the present approach to approximate eigensensitivity is relatively straight forward and could lead to computationally efficient procedures for structural design with dynamic response constraints.

In the form presented in this paper, the formulations lack mathematical rigor. It would be more useful if a formal theoretical background could be established which would enable one to study the numerical characteristics of the approximation procedure and possibly arrive at bounds on the estimates of the perturbed eigen parameters.

Further studies are also required to examine issues related to the effect of mode swapping on the proposed approximation procedure. It is expected that for modally complex structures, moderate perturbations in the structural parameters could potentially lead to switching of the eigenmodes. It may be required to make use of more basis vectors in order to capture the mode switch accurately.

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REFERENCES

Abu Kassim, A. M., and Topping, B. H. V., 1987, "Static Reanalysis of Structures : A Review," *Journal of Structural Engineering*, ASCE, Vol. 113, pp. 1029-1045.

Balmes, E., 1996, "Parametric Families of Reduced Finite Element Models. Theory and Applications," *Mechanical Systems and Signal Processing*, Vol. 10, pp. 381-394.

Barthelemy, J. -F. M., and Haftka, R. T., 1991, "Recent Advances in Approximation Concepts for Optimum Structural Design," *Proceedings of NATO/DFG ASI on Optimization of Large Structural Systems*, Berchtesgaden, Germany, pp. 235-256.

Brandon, J.A., 1990, "*Strategies for Structural Dynamic Modification*," Wiley, New York.

Canfield, R. A., 1990, "High Quality Approximation of Eigenvalues in Structural Optimization," *AIAA Journal*, Vol. 28, pp. 1116-1122.

Carey, C. M. M., and Golub, G. H., and Law, K. H., 1994, "A Lanczos-Based Method for Structural Dynamic Reanalysis Problems," *International Journal of Numerical Methods in Engineering*, Vol. 37, pp. 2857-2883.

Eldred, M. S., Lerner, P. B., and Anderson, W. J., 1992, "Higher Order Eigenpair Perturbations," *AIAA Journal*, Vol. 30, pp. 1870-1876.

Grandhi, R.V., 1993, "Structural Optimization with Frequency Constraints - A Review," *AIAA Journal*, Vo. 31, pp. 2296-2303.

High, G. D., 1990, "An Iterative Method for Eigenvector Derivatives," *Proceedings of 1990 MSC World Users Conference*, Paper 17, Los Angeles, CA.

Inamura, T., 1988, "Eigenvalue Reanalysis By Improved Perturbations," *International Journal of Numerical Methods in Engineering*, Vol. 26, pp. 167-181.

Kapania, R. K., and Byun, C., 1993, "Reduction Methods based on Eigenvectors and Ritz Vectors for Nonlinear Transient Analysis," *Computational Mechanics*, Vol.11, pp. 65-82.

Kirsch, U., 1991, "Reduced Basis Approximation of Structural Displacements for Optimal Design," *AIAA Journal*, Vol. 29, pp. 1751-1758.

Kirsch, U., 1995, "Improved Stiffness-Based First-Order Approximations for Structural Optimization," *AIAA Journal*, Vol. 29, pp. 143-150.

Kirsch, U., and Liu, S., 1997, "Structural Reanalysis for General Layout Modifications," *AIAA Journal*, Vol. 35, pp. 382-388.

Nelson, R. B., 1976, "Simplified Calculation of Eigenvector Derivatives," *AIAA Journal*, Vol. 14, pp. 1201-1205.

Pritchard, J. I., and Adelman, H. M., 1991, "Differential Equation Based Method for Accurate Modal Approximations," *AIAA Journal*, Vol. 29, pp. 484-486.

Zhang, O., and Zerva, A., 1997, "Accelerated Iterative Procedure for Calculating Eigenvector Derivatives," *AIAA Journal*, Vol. 35, pp. 340-348.