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THE INTEGRATION OF ADVANCED ACTIVE AND PASSIVE STRUCTURAL VIBRATION CONTROL

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ABSTRACT

In this paper the superiority of integrated and optimized passive and active vibration control methods in the design of lightweight structures is investigated. The structure studied here is a 4.5meter long satellite boom consisting of 10 identical bays with equilateral triangular cross sections. The main points that are presented here are:

i) The results from a Fortran code that is based on a receptance analysis are fully validated against detailed FE models of the structure.
ii) The experimental forced response of a test model in the shape of a tetrahedron is compared with these codes.
iii) The above is repeated for a regular boom structure.
iv) The response of the passively optimized boom structure.

Nomenclature

$G_{(A)}$	Green function for the substructure (A)
$\{F_{(A)}\}^e$	external force vector on beam (A) due to loading at *e*
$\{x_{(A)}\}^f$	six-dimensional local displacement vector
$\phi_n(x)$	mode shape of the *n*th mode at the point x
ω_n	natural frequency
ζ_n	modal damping ratio
$[\overline{\Phi}]$	normalized modal matrix

INTRODUCTION

Vibration and noise control problems that arise in many engineering projects are particularly severe when lightweight structures are employed. This is an area where many traditional techniques have been tried with relatively little success. Perhaps the most challenging vibration control issues arise in the design of space missions that involve satellites with highly sensitive instrumentation packages. To function correctly these packages must be supported on structures where the vibration levels have been reduced to extremely low levels (micro vibrations). This need becomes more severe when the instrument concerned forms one of the individual sensors of a multi sensor interferometric telescope or synthetic aperture radar [Melody and Briggs, 1993]. In such cases there is a need to support instruments spaced tens of meters apart using structural booms, with the relative motions between their ends being restricted to microns over wide ranges of excitation frequency [Sirlin and Laskin, 1990]. A number of design approaches have been proposed to try to meet these demanding requirements but it is still not clear how best to proceed in this field [Melody, *et al.*, 1996]. The most common treatment for such problems is to use anti-vibration mountings or to coat the structural elements with heavy viscoelastic damping materials with consequent weight and cost penalties. Moreover, the effectiveness of such treatments diminishes with the vibration levels, which makes continuously improving noise and vibration targets difficult to meet. Clearly, if the vibrational energy could be contained near the points of excitation there would be a reduced need for damping treatments and, additionally, they could be concentrated in regions where they were most effective. This is precisely the aim of the vibration isolators used between most pieces of equipment and their supporting structure. However, such isolators cannot deliver the desired behaviour in all situations, particularly for sensitive equipment. The upshot of this problem is the need for some kind of widely applicable, generic structural filter design capability that can be used to build desirable characteristics into a structure, retaining its ability to carry static loads while blocking higher frequency motions. To gain maximum benefit from the available technologies such a capability would ideally be based on an integrated active / passive approach, with these two techniques being used in tandem and together tackling the widest possible range of excitation frequencies. This paper reflects part of the research program currently being undertaken into the development of a vibration control approach for such systems. The work combines active vibration control using robust control techniques together with passive vibration isolation that geometrically optimizes the boom's truss structure. The passive optimization is based on energy flow analysis models combined with Genetic Algorithm optimization methods. This approach relies on the wavelengths of the vibrations being controlled being of a similar order to the changes introduced in the geometry of the structure and hence is not most effective at very low

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frequencies. This is a fortunate condition as it complements active vibration control and therefore, a mixed active / passive approach is potentially beneficial. Although the ideas of combined active-passive vibration control [Maghami, *et al.*, 1993] or the use of evolutionary methods in control system design [Chipperfield and Flemming, 1996] are not new, still this paper contains new ideas. First, we do not know of any other work mixing all three of these ideas together in one system. Secondly, the kind of passive vibration control that is used here is based on energy reflection rather that absorption, this is an idea that is very much in its infancy and one that was originally proposed by one of the authors [Keane, 1995]. Finally, we intend to build a working prototype system whereas almost all previous studies in this area have been essentially computational in nature. In this paper, the focus is mainly on optimizing a full 3-dimensional truss structure by using a parallel, multi-workstation GA. The structure consists of 93 individual beams and allows for bending, axial and torsional vibrations. The analysis is fully validated against detailed finite element models of the structures carried out with commercial FE codes. In order to ratify the usual inherent modeling uncertainties that arise when working in this field, and based on the theoretical designs produced, a series of experiments are to be performed. An initial base-line structure of 4.5meter length with 10 identical bays (representative of a satellite boom) is constructed. The response of this structure, when suspended in the laboratory, is measured and the experimental forced responses are presented. The paper also presents various designs for truss joints together with the experimental results on a tetrahedral structure that is representative of the system. Although the investigation presented in this paper is based on satellite structures, the overall results may be of interest in all fields where reduced vibration transmission is important, such as aircraft and car design.

RECEPTANCE METHOD

Receptance theory provides a method for predicting the behaviour of a composite system based on a proper combination of the component receptances. In the receptance method, the aim is to obtain the equations relating the coupling forces at either end of a beam to the displacements at the ends [Shankar and Keane, 1995]. Solution of an assembly of such equations yields the velocities and coupling forces at the joints of a complex arrangement of beams. Hence, one of the advantages of the receptance approach is its ability to calculate directly from these quantities the substructure energy flow and further, it can allow for substructure specific damping. It should be noted that other substructuring approaches such as dynamic stiffness method can also yield interface forces, but the receptance method is perhaps the most natural approach to adopt. Shankar and Keane [Shankar and Keane, 1995] have already presented a detailed receptance based formulation for studying the behavior of coupled, two and three-dimensional substructures. Without repeating the whole method here, consider a structure made up of beams A, B, C, ..., N, each executing Euler-Bernoulli transverse modes and purely longitudinal axial ones and torsional rotations. Each beam vibrates under excitation from point harmonic forcing on itself or on other beams in the system. The external forcing vector on beam (A) due to loading at point *e*, $\{F_{(A)}\}^e$, is constituted by six component forces (and torques) corresponding to the local x, y, z, θ_x, θ_y and θ_z co-ordinates, *i.e.*, $F^e_{(A).x}$, $F^e_{(A).y}$, $F^e_{(A).z}$, $F^e_{(A).\theta x}$, $F^e_{(A).\theta y}$, and $F^e_{(A).\theta z}$. The coupling force vectors at the I, J, K, ... coupling points of each substructure, such as $\{F_{(A)}\}^I$, $\{F_{(A)}\}^J$, $\{F_{(A)}\}^K$, ..., are likewise each constituted by six component forces, presented in similar notation. A six-dimensional local displacement vector, such as $\{x_{(A)}\}^I$, may then be expressed by:

$$\sum_{\hat{N}=\hat{I}}^{\hat{Z}}\left[G_{(A)}\right]^{I\hat{N}e}\left\{F_{(A)}\right\}^{\hat{N}e}+\sum_{N=I}^{Z}\left[G_{(A)}\right]^{IN}\left\{F_{(A)}\right\}^{N}=\left\{\mathrm{x}_{(A)}\right\}^{I} \tag{1}$$

where $[G_{(A)}]$ represent all the matrices of Green functions for the substructure (A) that are of order 6×6. For example:

$$\left[\mathrm{G}_{(A)}\right]^{IJ} =$$

$$\begin{bmatrix}
G_{(A)}(I.x,J.x) & G_{(A)}(I.x,J.y) & G_{(A)}(I.x,J.z) & G_{(A)}(I.x,J.\theta_x) & G_{(A)}(I.x,J.\theta_y) & G_{(A)}(I.x,J.\theta_z) \\
G_{(A)}(I.y,J.x) & G_{(A)}(I.y,J.y) & G_{(A)}(I.y,J.z) & G_{(A)}(I.y,J.\theta_x) & G_{(A)}(I.y,J.\theta_y) & G_{(A)}(I.y,J.\theta_z) \\
G_{(A)}(I.z,J.x) & G_{(A)}(I.z,J.y) & G_{(A)}(I.z,J.z) & G_{(A)}(I.z,J.\theta_x) & G_{(A)}(I.z,J.\theta_y) & G_{(A)}(I.z,J.\theta_z) \\
G_{(A)}(I.\theta_x,J.x) & G_{(A)}(I.\theta_x,J.y) & G_{(A)}(I.\theta_x,J.z) & G_{(A)}(I.\theta_x,J.\theta_x) & G_{(A)}(I.\theta_x,J.\theta_y) & G_{(A)}(I.\theta_x,J.\theta_z) \\
G_{(A)}(I.\theta_y,J.x) & G_{(A)}(I.\theta_y,J.y) & G_{(A)}(I.\theta_y,J.z) & G_{(A)}(I.\theta_y,J.\theta_x) & G_{(A)}(I.\theta_y,J.\theta_y) & G_{(A)}(I.\theta_y,J.\theta_z) \\
G_{(A)}(I.\theta_z,J.x) & G_{(A)}(I.\theta_z,J.y) & G_{(A)}(I.\theta_z,J.z) & G_{(A)}(I.\theta_z,J.\theta_x) & G_{(A)}(I.\theta_z,J.\theta_y) & G_{(A)}(I.\theta_z,J.\theta_z)
\end{bmatrix}$$

$$= \left[\left[\mathrm{G}_{(A)}\right]^{JI}\right]^{T} \tag{2}$$

The Green function $G_N(x,y)$ is evaluated as summations over the uncoupled modes of vibration of the beam N as:

$$G_N(x,y)=\sum_{n=0}^{\infty}\frac{\phi_n(x)\,\phi_n(y)}{m_n l_n\left(\omega_n^2-\omega^2+i2\zeta_n\omega\omega_n\right)} \tag{3}$$

where $\phi_n(x)$ is the mode shape of the *n*th mode of beam N at the point x, ω_n is the natural frequency and ζ_n is the modal damping ratio of N. The *n* modes of the substructure are normalized to have unit modal masses, *i.e.*,

$$[\overline{\Phi}]^T[\mathrm{M}][\overline{\Phi}]=[\mathrm{I}] \tag{4}$$

where the normalized modal matrix $[\overline{\Phi}]$ is obtained by dividing the *r*th column of $[\Phi]$ by the square root of the modal mass M_r. Such a scheme can be helpful for assigning modal damping, since the separate modal masses then need not be taken into account, making it easier to assign damping parameters relative to other substructures. Adoption of the mass normalized mode shapes simplifies the equation for the Green function summation to:

$$G_N(x,y)=\sum_{n=0}^{\infty}\frac{\overline{\phi}_n(x)\,\overline{\phi}_n(y)}{\left(\omega_n^2-\omega^2+i2\zeta_n\omega\omega_n\right)} \tag{5}$$

For free-free substructures, six rigid body modes have to be included in these modal summations, consisting of three purely translatory modes and three purely rotary modes, or six linear combinations involving them with the corresponding natural frequencies set to zero. The scheme for the assembly of equations, solution of the joint coupling forces and power calculations, once they have been transformed into a consistent global co-ordinate scheme, is relatively straightforward [Shankar and Keane, 1995].

FINITE ELEMENT MODEL

In order to verify and confirm the accuracy of the receptance method, the frequency response curves obtained from this method can be compared with finite element results. Before defining and generating the finite element mesh, the boom was modeled as a wire frame. In other words, each of the 93 beams in the structure was drawn as a simple line in 3-dimensional space. When defining the mesh, the geometric cross-sections of the beams were input along with their properties per unit length. The three joints at the left-hand end were restrained in all degrees of freedom and a point transverse excitation force in the positive Y direction at a point halfway along the bottom left-hand beam was applied (see figure 1). The frequency response (*i.e.*, displacement versus frequency) was calculated at the right-hand end of the boom at joint 28, having coordinates (4.5, 0.3897, -0.225). In the following section, the comparison of the frequency response of the boom structure obtained from the receptance theory and finite element analysis is made.

THE BOOM STRUCTURE

The initial structure that is to be optimized, as figure 1 shows, consists of 93 Euler-Bernoulli beams all having the same properties per unit length. Bearing in mind that it is intended that physical models be build of this three-dimensional structure, the overall length of the structure had to be kept within reasonable limits. Furthermore, the satellite launch vehicle dimensions also restrict such booms to approximately this chosen size.

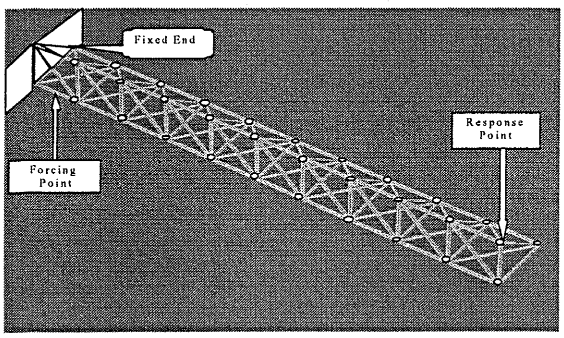

Figure 1- the geometry of regular three-dimensional satellite boom structure.

Aluminum 6063 rods of 0.25 inch diameter were used to construct the boom and the length of the non-diagonal beams were chosen to be 45cm, giving an overall structure length of 4.5 meters. All the rods were connected together at 30 joints through aluminum spheres (type 6063) each having a diameter of 25mm. In the finite element model, these spheres were modeled as lumped masses whereas in the receptance method, they were modeled as beams of 0.1μm length with very high mass per unit length, giving the overall required masses. All the rods have a flexural rigidity EA of 0.821 MN, bending rigidity EI of 2.069 kNm^2 and mass per unit length of 0.0855 kg/m. For the two models, a viscous damping ratio of 0.001 was assumed.

THEORETICAL RESULTS

Figure 2 shows the displacement in the global Y-direction of joint 29 with coordinates (4.5,0.3897,-0.225) meters as a function of driving frequency in the range 0-1500 Hz, using both FEA and the receptance theory.

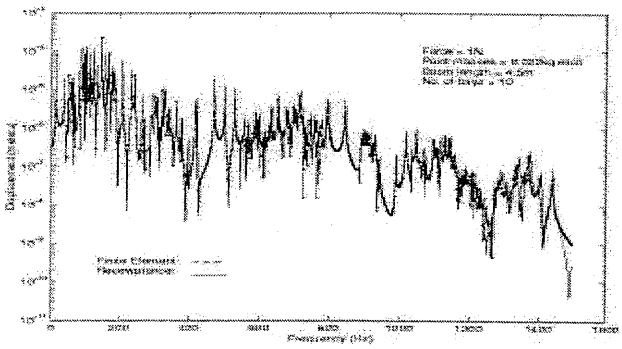

Figure 2- frequency response of the three-dimensional boom structure calculated using FEA, shown dotted, with response found using receptance theory.

As can be seen from the figure, there is almost a perfect match between the two results particularly in regions with lower modal densities.

THE TETRAHEDRAL EXPERIMENT

The tetrahedral structure, as shown in figure 3, consists of six 30cm aluminum rods all having the same properties per unit length that are joined together by four aluminum spheres of diameter 25mm each.

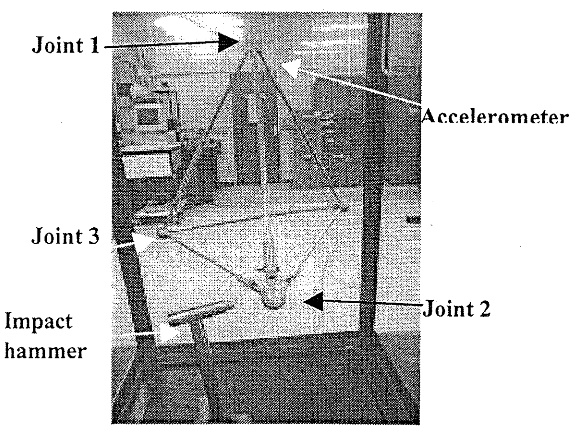

Figure 3- the tetrahedral structure consisting of three different rod-sphere joint designs.

Basically, this prototype model was initially manufactured for the following reasons:

a) it provides a quick and easier (compared to the actual boom structure) means of verifying the theory,
b) three different rod-sphere joint designs can be compared and the most suitable and practical one can be selected, and
c) it is a convenient way of estimating the damping ratio for the model of the boom structure.

Hence, three different types of rod-sphere joint designs were used and as a result only type 3 (shown in figure 3) was chosen as the most suitable connection in assembling the boom structure. The material properties of all the rods and spheres are identical to that assumed in the theoretical calculations. The damping of the structure for the theoretical analysis was fixed so that the normal modes of the uncoupled beam elements all had a constant damping ratio of 0.001. In order to create the required free-free boundary conditions, the structure was then suspended from a metal frame using soft elastic strings. The structure was excited at a joint by using a B&K impact hammer type 8202 and the point response was measured by using a B&K 4374 miniature accelerometer attached to a sphere by a small dab of wax. The signals from both the accelerometer and the impact hammer were taken to a Hewlett Packard Spectrum analyzer through charge amplifiers. The whole force/response measuring chain was calibrated by being applied to a 4.41 kg ballistic pendulum. The calibration constants produced in this way were within 0.1% of those predicted by using the manufacturer's calibration data for the equipment used. Investigation of the coherence plots seen when studying the structure showed that good results could be achieved by using 10 averages over a 2 kHz bandwidth, each weighted with a hanning window and no overlap. To obtain the point response curve for the range of interest, the data were downloaded from the analyzer and processed off-line.

RESULTS

The theoretical and experimental receptances (*i.e.*, displacement over input force) of the tetrahedral structure are shown in figure 4

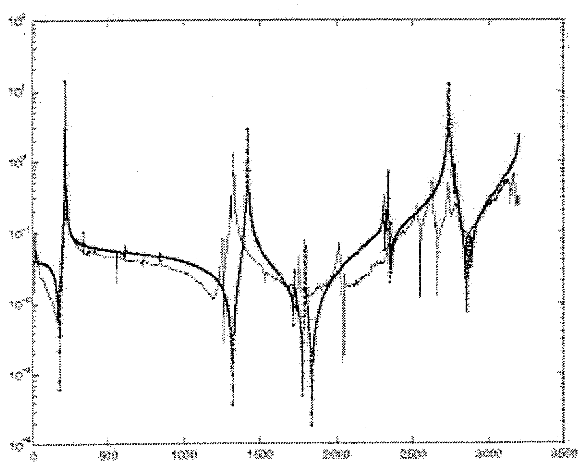

Figure 4 experimental (solid line) and theoretical (dotted line) results of the tetrahedral structure's point response.

The coherence function for the experimental data (not shown here) had values close to unity up to around 2 kHz before gradually deteriorating. The first interesting point to notice about the response curves is that at low frequencies, the whole structure seems to behave like a first order isolator. The discrepancy between the two results is mainly due to the fact that in the theoretical model, all the joints are assume to be identical whereas, as can be seen in figure 3, the test piece has different joint configurations. Despite these differences, the match between the two curves is still acceptable which confirms the reliability of the theoretical model. More importantly, the measured damping ratio of the structure was found to be about 0.005, a value that is used in the receptance model of the next experiment.

THE BOOM STRUCTURE EXPERIMENT

The experimental setup for the boom structure is show in figure 5 below:

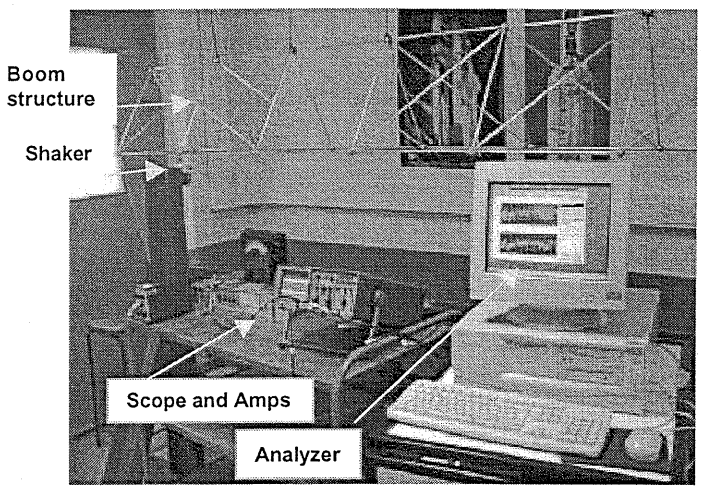

Figure 5 the experimental setup for the forced response of the satellite boom structure

The structure is connected to a wooden panel via several elastic strings and the wooden panel is subsequently suspended from the ceiling by two equal length strings. Free-free boundary conditions were chosen here (as compared to the fixed-free conditions in the theoretical study in previous sections) as they are much easier to achieve in practice and the results are as valid as the real life fixed-free condition. Another difference in the configuration with the earlier theoretical case is the change in the force location. In this experiment a force transducer (B&K 8200) was used in conjunction with a shaker (LDS V201) that had to be screwed to the structure. The fact that the threaded end of the transducer has a diameter equal to the diameter of a rod meant that it was more appropriate to screw it to a sphere instead. Therefore, the joint with coordinates (0.45,0.0,0.0) was chosen as the input force location. The response was measured at the previous location of coordinates (4.5,0.3897,-0.225) meters, as shown in figure 1.

A random input force of 1 to 1.6kHz, generated by the analyzer was supplied to the shaker via an amplifier (type TPA100-D). An ammeter was then used in series with the amplifier to avoid burning the shaker's coil. The force transducer's output was fed back to the analyzer's first acquisition channel via a charge amplifier (B&K 2635) whose gain was recorded for later data calibrations. The output of the B&K 4374 miniature accelerometer (attached to a sphere by a small dab of wax) was also fed back to the analyzer's second acquisition channel via a charge amplifier. In order to monitor the quality of input and output signals, they were both displayed on an oscilloscope before reaching the analyzer.

RESULTS

Figure 6 shows the experimental forced response of the structure for the frequency band of 1 to 1.6kHz compared with the same result obtained by using the receptance theory.

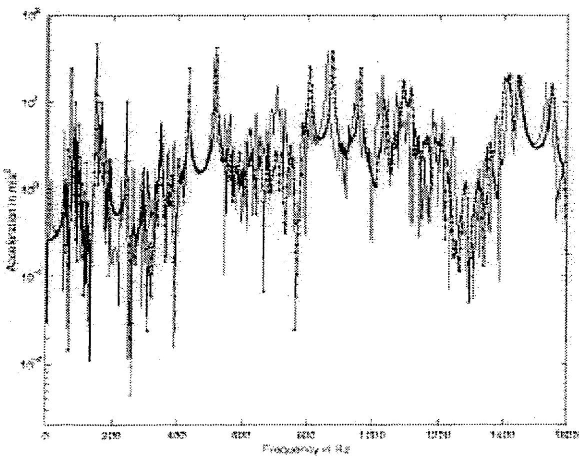

Figure 6 Comparison of the experimental and theoretical (receptance method) response of the boom structure.

In this plot the experimental data, downloaded from the analyzer, were transferred to Matlab environment where the two charge amplifier gains where imposed on them. Also, to demonstrate the quality of the acquired experimental data, the recorded coherence function between the input and output data is shown in figure 7. From the figures, it may be seen that there is a poor coherence at anti-resonances due to very low output at these frequencies. Also, the big discrepancy at very low frequencies is mainly due to the fact that the miniature accelerometer does not really start picking up the structural vibration accurately before around 30 Hz. This problem can be overcome by using a bigger accelerometer to measure very low frequencies. It is worth mentioning at this point that the above frequency bandwidth was later on divided into 25Hz bands and the same experiment was repeated until the whole range was covered.

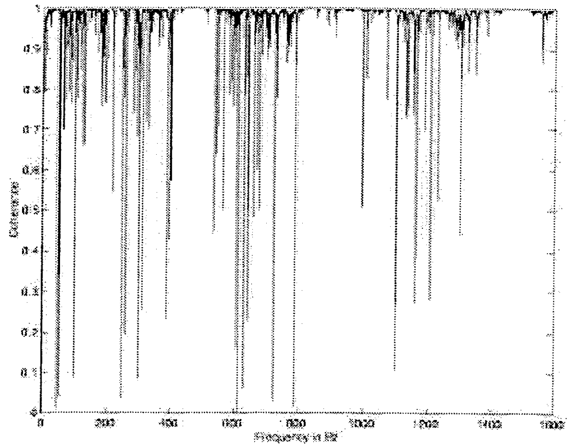

Figure 7 the Coherence function for the experimental response of the boom structure.

In order to improve the quality of the data around peaks and troughs within each frequency band, frequency lines and the input gain were increased, respectively. The obtained results that are not shown here revealed a much better response with an improved coherence function. However, as can clearly be seen, the result shown in figure 6 is very encouraging and there is a convincing agreement between the experimental results and the theoretical curve. Obviously, this agreement implies that the theoretical model is perfectly capable of predicting the vibrational behaviour of the satellite boom structure. This gives us the confidence to go ahead and search for a geometrically optimized structure to reduce its vibration transmission levels.

PASSIVE OPTIMIZATION

Having proved the reliability of the receptance code, it was then interfaced with the optimization package that has been developed by one of the authors [Keane, 1994]. The aim here is to produce a new boom geometry with an improved frequency response curve. More specifically, the aim is to reduce the frequency-averaged magnitude of the square root of the sum of the velocity squares in x, y and z directions that are measured at the three end right hand joints, *i.e.*,

$$\textit{Objective function} = \int_{freq} \sum_{j=31}^{33} \sqrt{V_{xj}^2 + V_{yj}^2 + V_{zj}^2} \quad , \quad \mathbf{j = joint} \qquad (6)$$

The above reduction is carried out over the frequency range of 150-250 Hz. To this end, a GA optimizer was selected from the optimization package to produce the improved design. For this particular run of the GA, the number of generations was set as 10 and the population size as 300. This means that during the first generation, the optimizer generates 300 random new geometries. In the creation of a new geometry, the coordinates of all joints in the structure were varied within a specified maximum deviation from their original positions, here ±20%. After analyzing the frequency response of each of these new boom designs, the GA outputs the design with the best performance and then continues on to the second generation. The designs carried over to the second generation and the creation of the subsequent new designs are governed by the specific *natural selection* algorithm employed by the GA. The geometry producing the best frequency response curve from the 300 random geometries created after the 9th generation is displayed in figure 8. Unfortunately, there was not enough time to do more runs at the time of writing this paper. However, what is presented here, clearly demonstrates the type of geometry that will be produced at the end of passive optimization. Note that the design presented here facilitates the construction of such irregular geometries.

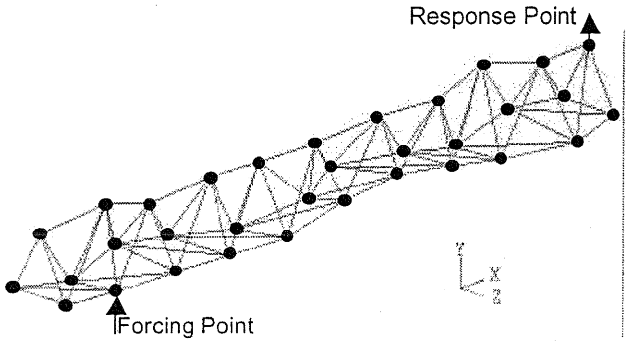

Figure 8 Geometry of the best design after 7 generations

As can be seen, the locations of all the 'interior' joints have been significantly changed. In order to confirm that the optimizer has indeed produced a geometry with an improved frequency response curve, the response of the new structure may be compared with the original regular boom structure. This is shown in Figure 9, where the frequency response of the optimized structure is plotted along with that produced using the regular geometry.

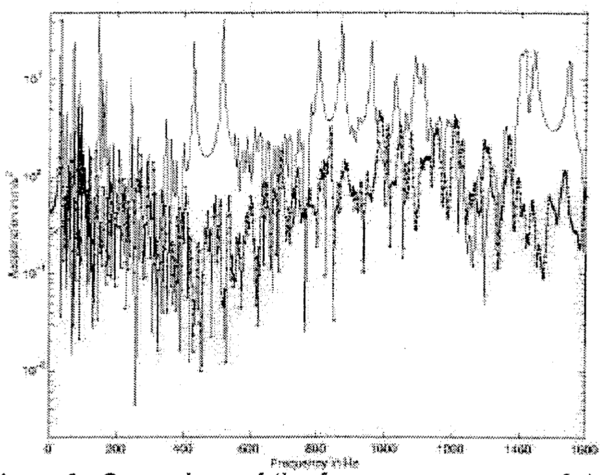

Figure 9 Comparison of the frequency response of the regular boom structure (solid line) with the optimized structure (dotted line)

From this initial observation, it is evident that the new design has greatly improved the frequency response of the structure. The magnitude of acceleration at joint 32 has been decreased across most of the frequency range of 0-1600 Hz. Naturally, the next step in the optimization process would be to run the GA for more generations before going on to build and test the new structure.

CONCLUSIONS

This paper has shown the credibility of a receptance code to successfully model the vibrational behaviour of a satellite boom structure over a wide frequency bandwidth. It also illustrates the kinds of improvements in performance that may be achieved using passive optimization

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