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Design Optimization of Space Structures With Nonperiodic Geometries for Vibration Suppression

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Abstract

This paper presents a computational framework for the design of large flexible space structures with nonperiodic geometries to achieve vibration suppression. The present system combines the use of an approximation model management framework (AMMF) developed for evolutionary optimization algorithms (EAs) with a reduced basis approximate dynamic reanalysis technique. A coevolutionary genetic search strategy is developed here to ensure that design changes during the optimization iterations lead to low-rank perturbations in the structural system matrices. The k-means algorithm is employed for cluster analysis of the population of designs to determine design points at which exact analysis should be carried out. Results are presented for optimal design of a 2D cantilevered space structure to achieve passive vibration suppression. It is shown that vibration isolation of the order of 30 dB over a 150 Hz bandwidth can be achieved. Further, it is demonstrated that the AMMF can potentially arrive at a better design compared to a conventional approach when optimization is constrained by a limited computational budget.

Introduction

Design optimization of large flexible space structures to meet stringent performance specifications is an enabling technology for cost effectiveness and success of future space missions. For example, Earth science platforms in both low earth and geostationary orbits requires the ability to acquire simultaneous and continuous observation of the earth with minimum interference, vibrational or otherwise[1]. Further, with the desire of placing remote sensing systems in geostationary orbits, the problem of minimization of payload mass and control energy requirements becomes crucial to the mission cost effectiveness. Integrated controls-structures design procedures utilizing formal optimization techniques have emerged as a rational methodology to design these class of spacecraft which require precise attitude pointing and vibration suppression. This methodology allows for simultaneous improvement of the controlled system performance in terms of pointing performance and controller energy requirements, and also minimization of the payload mass.

The research programs supported by the NASA Controls-Structures-Interaction (CSI) technology program have demonstrated both analytically and experimentally, the potential benefits of using multidisciplinary design optimization (MDO) methodology to simultaneously achieve weight savings and improved control system performance of large flexible space structures; see, for example, references[5–8]. These studies mainly approached the design problem by sizing periodic structures and the controller gain parameters to optimize the system performance in the low frequency region. It is to be noted here that one of the important reasons for parameterizing the design as a periodic structure is to reduce the number of design variables in the optimization procedure, and hence allow for the reduction of computational cost.

Recently, Keane and his colleagues[2,9,10] have explored different directions, wherein space structures with unusual (i.e., nonperiodic or irregular) geometries are designed to achieve passive vibration isolation in the medium frequency regime. The motivation for this comes from earlier theoretical investigations into the effect of disorder on the vibration transmission characteristics of periodic structures. These studies indicate that the mechanism of constructive interference of energy waves in nonperiodic structures can be exploited to design structures which behave as passive vibration filters. Vibration isolation is achieved here via energy confinement as opposed to dissipation. It has been demonstrated using computational optimization studies that significant vibration isolation can be achieved by departing from conventional periodic structural configurations. The theoretical predictions for the optimized designs

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were validated by laboratory experiments on model aerospace structures[10]. Theoretical and numerical studies related to the design of near-periodic structures to minimize vibration transmission levels have been presented by Langley[11] and Langley et al.[12].

This design approach holds great potential since significant vibration isolation can be achieved via passive means alone. It is however, noted here that the inclusion of geometry as design variables leads to a large-scale nonconvex design space[13] and hence, evolutionary optimization techniques such as genetic algorithms (GAs) are required to ensure convergence to good designs.

It is known that stochastic design space search techniques such as GAs tend to be profligate in terms of the number of function evaluations required to converge to an optimal solution. Hence, the design methodology is computationally expensive, and may require supercomputing power for application to practical structures. Moreover, the repeated evaluation of the dynamic response leads to a considerable increase in the computational cost, especially in the medium frequency region when large-scale finite element models are used.

This paper focuses on design optimization of such large flexible space structures for vibration suppression. A methodology proposed earlier by Keane[2], which exploits the intrinsic vibration filtering capabilities of nonperiodic structural systems to achieve passive vibration isolation, is the underlying design philosophy used in the present approach. The primary objective of this paper is to develop a framework for arriving at good designs on a limited computational budget (or reduced number of exact analysis). The proposed design framework combines a new approximate dynamic reanalysis technique presented by the authors in reference[3] with a form of the AMMF for evolutionary algorithms (EAs) introduced in Nair et al[4].

A major challenge faced in this research is due to the highly nonlinear nature of the dynamic response as a function of the geometrical design variables. This characteristic inevitably leads to a large magnitude of approximation error for moderate design changes. In order to control the approximation error, a coevolutionary genetic search framework is proposed. This framework ensures that design changes during the optimization iterations lead to low-rank perturbations of the structural system matrices, for which the approximation method used here gives high-quality approximations. In order to determine the design points at which exact analysis should be carried out, the k-means algorithm is used to divide the EA population into several clusters. Exact analysis is carried out for the center of each cluster, and the fitness of the designs within this cluster is predicted using an approximation model constructed around the cluster center. The overall aim of the adaptive procedure developed here is to improve the possibility of predicting design improvements using the approximation technique.

Results are presented for the optimal design of a 2D space structure to achieve passive vibration suppression over a 150 Hz bandwidth. It is shown that vibration isolation of the order of 30 dB can be achieved using the present design methodology. It is also demonstrated that the AMMF gives superior designs as compared to conventional approaches, when a limit is imposed on the computational budget for optimization.

Periodic and Nonperiodic Structures

Many modern aerospace structures are composed of a number of identical substructures which are uniformly connected in a repetitive pattern, e.g., stiffened shell structures, turbine assembly and large flexible space structures. Such structures tend to have very particular vibration transmission behavior characterized by a series of overlapping passbands and stopbands (where energy is either transmitted very easily or hardly at all). This behavior has been the focus of much theoretical research which has focused on the performance of perfectly periodic structures (see, for example, Mead[14]) and also those with disordered features (see, for example, Langley[11]). However, the mean behavior of ensembles of randomly disordered systems can sometimes hide the dramatically different behavior of individual members with unusual properties, i.e., some sets of irregularities could result in the ability to block vibration transmission across wide frequency ranges.

In traditional design practice, the aim is to exploit this behavior of periodic structures so that the frequencies of vibration excitation lie within the stop bands. Theoretical and experimental studies have well demonstrated that, when a structure deviates from ideal periodicity, the vibration transmission characteristics can change dramatically due to mode localization phenomena.

Hence, the idea of designing periodic structures has not met with much practical success. In summary, manufacturing uncertainties or structural faults occurring in service could easily lead to *vibration localization*, which could cause potentially destabilizing controls-structures interactions. As shown in an earlier study by Benediksen[15], localization is mostly likely to occur in space structures which have high modal density and many weakly coupled substructures. See reference[16] for an overview of the challenges to structural control which arise due to deviation from periodicity.

The idea of intentionally designing disorder into

periodic structures to reduce vibration levels has been recently examined by Castanier and Pierre[17] for turbomachinery rotors. It was reported that, for this application, the maximum blade forced response amplitude is often largest at a relatively small level of mistuning. However, increasing the level of mistuning beyond this critical point actually leads to a decrease in the maximum response level. It was shown that intentional mistuning can greatly reduce the rotor's sensitivity to random mistuning.

Approximate Dynamic Reanalysis

The approximation method used in this study was recently presented by the authors in reference[3]. This method makes use of the baseline eigenvector and its first-order approximation term as basis vectors for Ritz analysis of the perturbed eigenvalue problem. An approximation for the eigenvalue and eigenvector of mode i can hence be computed by solving a 2×2 eigensystem given below.

$$[\mathbf{K}_T^i]\{\mathrm{Z}\}^i = \lambda_i[\mathbf{M}_T^i]\{\mathrm{Z}\}^i \tag{1}$$

where

$$[\mathbf{K}_T^i] = [\phi_i^o, \Delta\phi_i]^T[\mathbf{K}][\phi_i^o, \Delta\phi_i] \in \Re^{2\times 2} \tag{2}$$

and

$$[\mathbf{M}_T^i] = [\phi_i^o, \Delta\phi_i]^T[\mathbf{M}][\phi_i^o, \Delta\phi_i] \in \Re^{2\times 2}. \tag{3}$$

$[\mathbf{K}_T^i]$ and $[\mathbf{M}_T^i]$ are the reduced stiffness and mass matrix, respectively; $\{\mathrm{Z}\}^i = \{\zeta_1, \zeta_2\}^T$ and λ_i are the eigenvector and eigenvalue of mode i, respectively; $[\mathbf{K}]$ and $[\mathbf{M}]$ are the structural stiffness and mass matrix, respectively; ϕ_i^o is the eigenvector corresponding to the stiffness and mass matrices evaluated at a nominal value of the structural parameters; $\Delta\phi_i$ is the first-order approximation for the perturbation in the eigenvector, which is given below as

$$\Delta\phi_i = \sum_{j=1}^{p} \frac{\partial\phi_i}{\partial x_j}\Delta x_j \tag{4}$$

where $\frac{\partial\phi_i}{\partial x_j}$ is the sensitivity of the eigenvector with respect to the structural parameters, which is evaluated at the nominal values of the structural parameters.

Solution of equation (1) leads to two possible values for the approximate eigenvalue. Corresponding to these values, $\{\mathrm{Z}\}^i$ can be calculated as $\{\mathrm{Z}\}^i = \{\zeta_1, \zeta_2\}^T$. The eigenvalue of equation (1) which is closest to the higher-order eigenvalue perturbation derived in reference[18] is chosen as the best approximation for that mode. The expression for the higher-order eigenvalue perturbation is given below[18]

$$\hat{\lambda_i} = \lambda_i^o + \frac{\phi_i^{oT}\left(\Delta\mathbf{K} - \lambda_i^o\Delta\mathbf{M}\right)\left(\phi_i^o + \Delta\phi_i\right)}{\phi_i^{oT}\left(\mathbf{M}^o + \Delta\mathbf{M}\right)\left(\phi_i^o + \Delta\phi_i\right)} \tag{5}$$

where λ_i^o is the eigenvalue of mode i at the nominal values of the structural parameters.

After the best approximation for the eigenvalue has been chosen, the corresponding eigenvector approximation can be written as

$$\hat{\phi}_i = \zeta_1\phi_i^o + \zeta_2\Delta\phi_i \tag{6}$$

It has been shown that this method gives good quality results for moderate to large magnitudes of perturbation in the system matrices. In particular, high-quality approximations can be obtained for low-rank perturbations in the stiffness and mass matrices. It was also shown in reference[19] that the method is robust to approximations in computation of the basis vectors. This is an important characteristic which allows for the possibility of using computationally efficient approximate eigenvector derivative/perturbation analysis formulations in the reduced basis approximation procedure.

There exists a wealth of methods in the literature for computing first-order approximations of the eigenvectors of perturbed linear algebraic eigenvalue problems. For the problems considered in this research, the number of design variables under consideration are of the order of 40 - 200. Hence, from the computational perspective it would be expensive to make use of the eigenvector derivatives to compute the basis vectors. A formulation based on the family of modal methods introduced by Akgun[20] is used here to directly compute a first-order approximation of the eigenvector perturbations. This formulation allows for ease of implementation and is computationally more efficient for problems with large numbers of design variables and eigenmodes such as that under consideration here.

The approximation for the eigenvector could be further improved by doing one step of the inverse iteration procedure as

$$\phi_i = [\mathbf{K}]^{-1}[\mathbf{M}]\hat{\phi}_i \tag{7}$$

This improved eigenvector approximation could then be used to compute a better approximation for the eigenvalues. However, this would involve solving a reduced eigenproblem of size equal to the number of eigenmodes of interest. Hence, the attendant computational cost would be quite high for cases involving large number of eigenmodes, which are of interest in this research. In the present work, equation (7) is, however, used to improve the eigenvector approximation.

Approximation Model Management Framework (AMMF)

The AMMF used here was first introduced in Nair et al.[4] for integrating general single-point approximation models with evolutionary optimization procedures. The computational cost of approximate analysis is generally a fraction of that required for exact analysis. Hence, the approximation model can be used in lieu of the exact model during the optimization iterations at a considerably lower computational cost. The underlying idea of the design framework developed here is to use the approximation model for efficiently sampling larger regions of the design space. This is expected to enable the possibility of obtaining superior designs on a limited computational budget.

The main component of the AMMF involves the procedure used for the selection of *anchor* points at which exact analysis is carried out. The baseline eigenparameters computed at this point can then be used to approximate the dynamic response data at new design points using the approximation method described earlier. In general, approximation errors will increase as the new design point moves away from the anchor point. Since evolutionary optimization algorithms make use of a population of designs and stochastic search operators, the design points in a generation may span the entire design space, i.e., the control of step size of the design changes is not a straight forward task as compared to line search based optimization algorithms. Hence, advanced strategies are required to ensure that the errors involved in approximate fitness evaluations are controlled so as to enhance the capability to predict design improvements.

A useful way to control the approximation error is to use a domain decomposition strategy for grouping the design points in a generation into clusters. The *k-means* algorithm[21] is used here for cluster analysis of the population in a given generation. The anchor point is chosen as the mean vector of the individuals in a cluster. Exact analysis is carried out for this anchor point, and the dynamic response of the other designs in the cluster is approximated based on the results of this exact analysis. The fitness of all the designs in the population is approximated using this procedure. It is of interest to note that this step also allows for the possibility of efficient parallelization of the AMMF, since the fitness evaluations for designs in each cluster can be carried out concurrently.

It was observed during numerical experiments that varying all the geometrical variables of a structure simultaneously, leads to large magnitude of approximation errors. These errors are quite high so as to make the approximate fitness predictions useless for the purposes of optimization. In order to reduce the approximation error, a large number of anchor points/clusters are required. However, the computational cost incurred by the AMMF approach goes up substantially with increase in the number of anchor points at which exact analysis is carried out. Fortunately, as observed earlier in reference[3], the approximation errors are lower for low rank perturbations in the system matrices. In order to exploit this characteristic of the approximation method, a *Coevolutionary Genetic Algorithm* (CGA) is employed here in conjunction with the AMMF. A CGA by construction varies only a subset of the design variables at a time. Hence, the design changes during the optimization iterations lead to low rank perturbations in the system matrices.

A CGA models an ecosystem consisting of two or more sympatric species having an ecological relationship of mutualism. In the current context, the design variables are grouped into sets corresponding to each substructure. A species is set up to control the design variables for each substructure, i.e., each species contain a population of alternative values for the substructure design variables. Collaboration between the various species involves selection of representative values from all the species and combining them into a vector which is then used to compute the objective function. Note here that only the optimization is carried out at a substructure level, not the analysis. An individual in a species is hence rewarded based on how well it maximizes the objective function within the context of the representatives selected from the other species.

The proposed AMMF for CGA-based search is summarized below :

Step 1 : Initialize a population of individuals for each species randomly.

Step 2 : Evaluate the fitness of the members of each species. Fitness evaluation involves the use of the following algorithm :

Choose *representatives* from all the other species.

Decompose the the design subspace into clusters and compute the anchor point for each cluster. Do exact analysis for each anchor point and construct an approximation model using this data around this point.

FOR each individual i in a species being evaluated **DO**

- Form collaboration between i and representatives from other species to form the design vector
- Approximate the fitness of collaboration using the approximation model constructed using the anchor point closest to i
- Assign fitness of collaboration to i

ENDDO

Evaluate the best design as predicted by the approximation model using exact analysis. If the exact fit-

ness is greater than that of the best design so far (if it exists), replace the elite individual of this species with the new design.

Step 3 : If the termination criteria is not met, then apply a canonical GA involving the operators of reproduction and genetic recombination to arrive at a new population for each species. Go to Step 2.

If n_{clus} clusters are used for predicting the fitness of m species, the total number of exact analysis at each generation of the CGA is $mn_{clus}+m$. The first term is due to the requirement of carrying out exact analysis at the cluster centers for each species. The second term results due to exact analysis of the best design (as predicted by the approximation model) to implement elitism and prevent loss of the best design to date.

The underlying premise of the CGA approach is that, when variable interdependencies are low, faster progress in the search can be made by decomposing the design space. This assumption can generally be justified for static structural analysis, wherein changes in the elemental properties of a substructure will tend to have little influence on the response of substructures spatially far away from it.

In the present design framework, the CGA is allocated the task of finding design improvements in the face of uncertainty in fitness predictions. Moreover, since each species independently coevolves a subset of the design variables, effective collaboration between the species is of crucial importance. Each species must constantly adapt just to remain in parity with the others. Hence, the evolution of each species is constantly driven by evolutionary changes in the species it interacts with as well as the dynamics of the fitness evaluation scheme. However, the effect of fitness uncertainties on the convergence behavior of CGAs is a subject area which is not yet fully investigated. Preliminary investigations by Potter[22] appear to indicate that CGAs are more sensitive to noise as compared to a single species GA.

Collaboration between the species involves exchange of representative individuals as mentioned earlier. In this research, the best/elite design in a species is chosen as its representative. *Elitism* is implemented here to prevent loss of the fittest design in each species due to both the stochastic nature of the search operators, and the dynamic nature of the fitness evaluation scheme.

For a more detailed theoretical overview of CGAs, the reader is referred to reference[22]. See also the dissertation of Serafini[23] for a detailed exposition of general AMMF for pattern search algorithms.

Demonstration Example, Results and Discussion

The present methodology is applied to the optimal design of a two dimensional beam network shown in Figure 1. The structure is subjected to transverse excitation at the node marked 'F'. The objective of the design problem considered here is to achieve vibration isolation at node 'R' over the frequency bandwidth of 100 to 250 Hz. Results are presented here for the case when only passive vibration isolation characteristics are considered. The objective function is defined as the integral of the frequency response at node 'R' from 100 to 250 Hz.

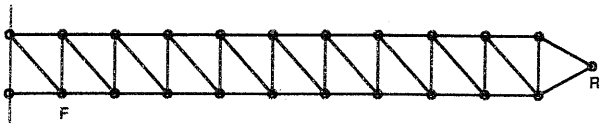

Figure 1 : Example Problem

The structure is modeled using 84 beam elements (two elements per structural member), with flexural rigidity $EI = 1.286 \times 10^3 N/m^2$, axial rigidity $EA = 6.987 \times 10^6 N/m^2$ and mass per unit length $m = 2.74 kg/m$. The first 80 modes are used to compute the dynamic response of the structure. The design is parameterized in terms of the nodal coordinates which are allowed to vary between $\pm$ 25 % from the baseline values, with the coordinates of node 'F' being kept fixed. This leads to a total of 40 geometrical design variables. The design variables are grouped in to 5 sets of 8 variables each. Five species are set up control each set of design variables.

Results are presented for both the conventional approach which makes use of exact dynamic analysis throughout the GA search, and when the AMMF is used in conjunction with the approximation technique outlined earlier.

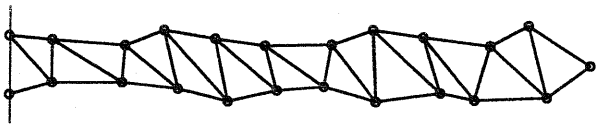

Figure 2 : Optimized Structure Using Conventional Approach

The GA employed for the evolution of each species uses uniform crossover and bit mutation at a probability of 0.5 and 0.001, respectively. Each design variable is represented using a binary string of 10 bits. Creep mutation is applied at a probability of 0.2. A population size of 100 is used for the conventional approach which makes use of exact fitness predictions throughout the search. For the CGA approach combined with the AMMF, a population

size of 40 is used for each species. Further, for each species, the design space is decomposed into two clusters for computing the anchor points, i.e., three exact analysis were carried out for each species at a generation. The third exact analysis is needed to implement elitism. In summary the conventional approach uses 100 exact analysis at each generation, whereas the CGA/AMMF approach uses 15 exact analysis at each generation. Three runs were carried for each case to compare the averaged search performance.

For this problem, exact analysis requires around 7.2 seconds as compared to the approximation model which requires 0.5 seconds on a SGI R10000 processor. In order to compare the design framework developed here with a conventional approach, the termination criteria chosen here is based on the idea of a fixed computational budget. The termination criteria for this problem is taken to be 5 hours of wall clock time.

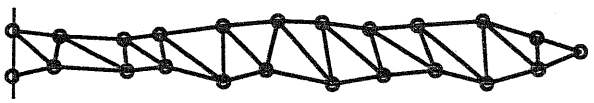

Figure 3 : Optimized Structure Using CGA/AMMF Approach

Figures 2 and 3 compares the configuration of the optimal design obtained using the conventional and the CGA/AMMF approach. It can be seen that both approaches have converged to different designs using the termination criteria taken up. However, the vibration isolation characteristics of both the designs are not very different. Figures 4 compares the total vibration response levels at node 'R' for the optimized designs with the baseline structure. It can be seen that vibration isolation of the order of 30 dB has been obtained over the frequency bandwidth of interest. It is also interesting to note that the optimized designs do not show any significant increase in the response at excitation frequencies which are not considered in the optimization formulation.

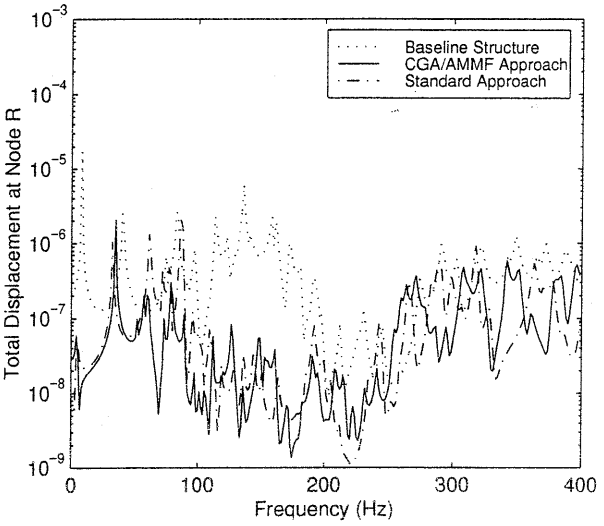

Figure 4 : Comparison of Total Displacement Levels at 'R' for the Optimal Designs

Figure 5 compares the optimization convergence history as a function of computational time. Note that this trend is a function of the ratio of computational cost between the exact and approximate analysis for the problem under consideration.

Figure 6 compares the performance of the approaches as a function of the number of exact evaluations. It can be seen that the AMMF uses a smaller number of calls to the exact analysis routine. Ideally, the approximate method would have negligible cost compared to the exact. Were this so both searches would have used nearly the same number of exact analysis in the alloted time. The fact that the CGA/AMMF uses only some 65 % as many arises because the approximation here takes nearly 7 % of the time needed for an exact calculation. As the problem size increases, this ratio can be expected to fall. Even so some 17,400 approximate analysis have used by the method.

It is important to note that, since the problem taken up was modeled using a low-fidelity finite element model with 189 degrees of freedom (dof), the difference between the computational cost of the exact analysis and the approximation method (around 7 % of exact) is not very significant. However, for larger finite element models with a couple of thousands of dof, the computational cost difference will be more pronounced. It is expected that in such scenarios, the performance of the AMMF will be significantly better than that of a conventional approach using exact analysis throughout the search.

From a theoretical viewpoint, it is also of interest to consider what happens when the computational budget is increased beyond what is considered here. If the termination criteria for this problem is doubled to 10 hours, it was observed that a further reduction

of 5-7dB is possible in the objective function. However, for this scenario it was noted that the AMMF approach is unable to find such a solution with accuracy as compared to the conventional approach. This indicates that the AMMF would generally fail to find a high quality design when a substantial computational budget is available for optimization. This is primarily due to the inability of the AMMF to fine tune the design when it reaches a near optimal solution. It is expected that the use of a local search technique at the final stages of the search would alleviate this difficulty.

The quality of approximations is another fundamental characteristic influencing the trends reported here. Even though a large number of approximate calculations were used by the CGA/AMMF, the improvements achieved over the conventional approach are not very significant. This suggests that, most of the time, the quality of the approximations available during the search were not very useful for predicting design improvements. In contrast, in reference[4], much better results were obtained when a high-quality approximation model was used for a structural statics problem. Hence, further research on improving the accuracy of the approximation method is expected to lead to performance improvements.

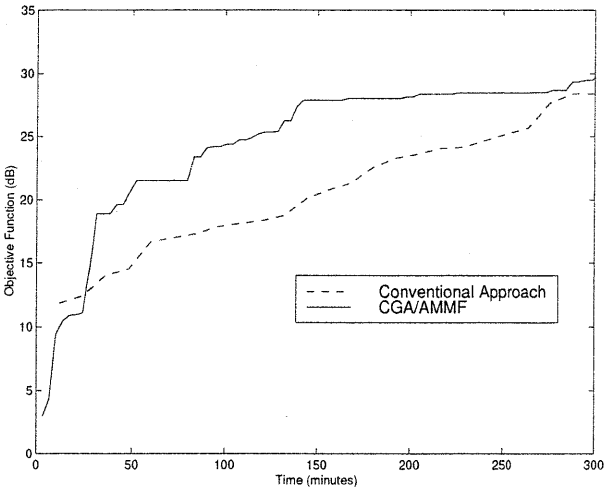

Figure 5 : Comparison of Optimization History as a Function of Computational Time

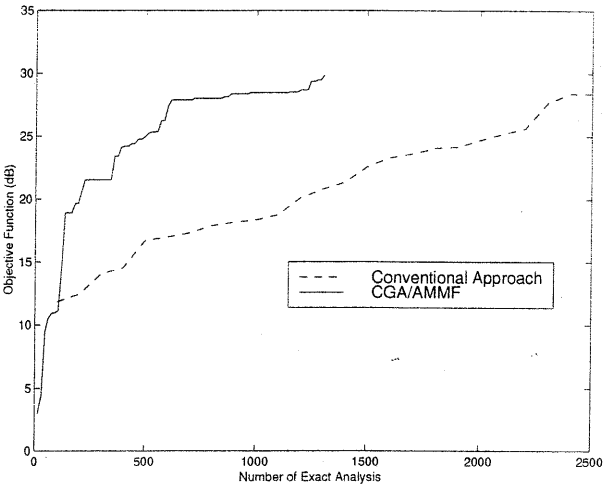

Figure 6 : Comparison of Optimization History as a Function of the Number of Exact Analysis

Concluding Remarks

A computational framework for the design of nonperiodic structural systems to achieve vibration suppression is presented. This framework combines the use of a new approximate dynamic reanalysis technique with an AMMF for CGAs. The proposed AMMF uses an adaptive procedure for ensuring that the approximation model can be reliably used for predicting design improvements. Results obtained for the demonstration example indicate that vibration isolation of the order of 30 dB can be achieved over a 150 Hz bandwidth via passive means alone. It is also important to note that these reductions in vibration levels were obtained without any significant weight penalty.

It is also shown that the proposed design framework can potentially arrive at good designs when optimization is carried out within a limited computational budget. It is expected that for structures modeled using a large-scale finite element model, the performance of the proposed design framework will improve considerably as compared to the conventional approach. This can be primarily attributed to the mathematical characteristics of the approximation technique, the computational cost of which is roughly an order less than that required for exact eigen solution.

Currently, work is underway to incorporate active control strategies within the design framework. The key idea is to use nonperiodic geometries for vibration suppression in the medium frequency region, and use active controllers to suppress the low frequency vibration response levels. The long term objective of this research is to develop an integrated controls-structures design framework for optimal synthesis of high performance space structures

with enhanced vibration rejection capability.

Acknowledgments

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