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MULTIOBJECTIVE OPTIMISATION APPROACH TO ROBUST CONTROLLER DESIGN

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Abstract: A control system design procedure based on the optimisation of multiple objectives is used to address the control design requirements of a simulated gasification plant. The non-linear gasifier is represented by three linear models relating to three separate operating points. The H_∞ LSDP is used to guarantee the stability of the linear controller at it's design point while the multiobjective (MO) search method is used in order to optimise the robustness of the controller in terms of both stability and performance. A linear controller emerges that provides closed loop stability at all three operating points and is capable of meeting all the performance requirements except for those of the gas pressure at the 0% load operating point.

Keywords: Genetic algorithms, H-infinity control, Multiobjective optimisation, Multivariable control, Robust control.

1. INTRODUCTION

Modern awareness of environmental issues has led to the desire for low pollution power generation techniques. One such technique, studied by ALSTOM, involves the combustion of pulverised coal using an Integrated Gasification Combined Cycle (IGCC) power plant. The operation of this Pilot Integrated Plant (PIP) is based on the Air Blown Gasification Cycle (ABGC). Limestone, which is required in order to minimise sulphur emission, is added to the pulverised coal and this mixture is then fluidised in a stream of air and steam and conveyed into the gasification plant (gasifier). A low calorific value fuel gas is produced by the reaction between the air and steam and the volatile constituents of the coal. The limestone, ash and residual carbon are extracted as bed material from the base of the gasifier or elutriated from the product gas which is then cleaned and used to power a gas turbine.

The specified objective of the work summarised in this paper was to perform a controller design procedure for the gasifier. The nature of the input and output constraints categorise the gasifier as a critical system (Zakian, 1989; Whidbourne and Lui, 1993). The loop-shaping design procedure (LSDP) (McFarlane and Glover, 1990) used here is suitable for multiple input multiple output (MIMO) systems. Weighting functions are employed to shape the open-loop transfer function followed by H_∞ optimisation of a normalised coprime factorisation of the nominal plant description. This procedure results in the state space realisation of a linear controller.

The gasifier is a non-linear, multivariable system. The non-linear operating envelope is represented by three linear state space models which represent the open loop plant at 100%, 50% & 0% load operating points. The control system design procedure was performed using the 100% load linear model as this is the operating condition at which the gasifier spends the majority of its operational life. In order to ensure that the resulting linear controller was closed loop stable and met performance requirements at all three operating points, a multiobjective search method was employed to select suitable weighting functions. Hence, the robustness required for the linear controller to be applicable to a non-linear operating range was achieved through multiobjective optimisation.

2. H_∞ LOOP-SHAPING DESIGN PROCEDURE

The normalised left coprime factorisation (NLCF) of a plant G is given by $G = M^{-1}N$. A perturbed plant model G_p is then given by,

$$G_p = (M + \Delta M)^{-1} (N + \Delta N) \qquad (1)$$

To maximise this class of perturbed models such that the configuration shown in Fig. 1 is stable, controllers K_s that stabilise the nominal closed-loop system and minimise γ must be found where

$$\gamma = \left\| \begin{bmatrix} K_s \\ I \end{bmatrix} (I - GK_s)^{-1} M^{-1} \right\|_\infty \qquad (2)$$

This is the problem of robust stabilisation of normalised coprime factor plant descriptions (Glover and McFarlane, 1989). From the small gain theorem (Skogestad and Postlethwaite, 1996), the closed-loop plant will remain stable if,

$$\left\| \begin{bmatrix} \Delta N \\ \Delta M \end{bmatrix} \right\|_\infty < \gamma^{-1} \qquad (3)$$

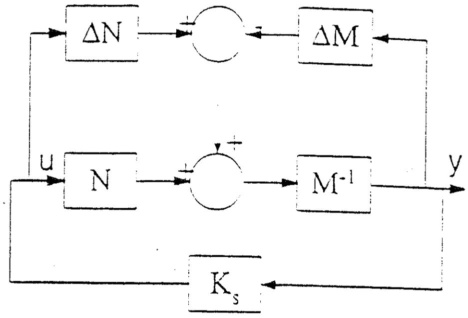

Fig. 1 Robust Stabilisation with respect to coprime factor uncertainty.

The lowest possible value of γ and hence the highest achievable stability margin is given by $\gamma_{min} = (1 + \rho(ZX))^{\frac{1}{2}}$, where Z and X are the solutions to the following algebraic Ricatti equations,

$$(A - BS^{-1}D^TC)Z + Z(A - BS^{-1}D^TC)^T - ZC^TR^{-1}CZ + BS^{-1}B^T = 0 \qquad (4)$$

$$(A - BS^{-1}D^TC)^TX + X(A - BS^{-1}D^TC) - XBS^{-1}B^TX + C^TR^{-1}C = 0 \qquad (5)$$

where A, B, C, and D are the matrices of the state space representation of G and,

$$R = I + DD^T, \qquad S = I + D^TD. \qquad (6)$$

By solving these equations the state space controller, K, can be generated explicitly (Skogestad and Postlethwaite, 1996). This controller gives no guarantee of the system's performance, only that it is stable with reasonable robustness.

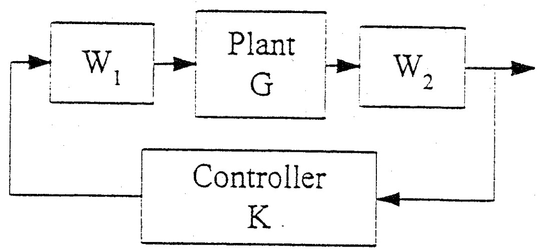

Fig. 2 Loop Shaping Controller Structure

It is therefore necessary to shape the system's response with both pre- and post-plant weighting function matrices W_1 and W_2, as shown in Fig. 2. This will ensure that the closed-loop performance meets the specifications. It is throught adjusting these weighting functions that the dsigner influences the procedure.

3. PROBLEM DESCRIPTION

The gasifier is a highly non-linear multivariable component. The plant has four manipulated inputs and four measured outputs.

Manipulated inputs:

- WCHR – char extraction flow (kg/s).
- WAIR – air mass flow (kg/s).
- WCOL – coal flow (kg/s).
- WSTM – steam mass flow (kg/s).

Measured outputs:

- CVGAS – calorific value of fuel gas (J/kg).
- MASS – Bed mass (kg)
- PGAS – Pressure of fuel gas (N/m^2)
- TGAS – Temperature of fuel gas (^{0}K)

Limestone is a dependent input, being introduced at a ratio of 10:1 coal:limestone. Disturbances to the plant take the form of changes in pressure (PSINK) upstream of the turbine that the gasifier is powering. These are due to adjustments to the gas turbine fuel valve. The system is required to maintain the fluctuations of the measured outputs within certain limits during prescribed disturbance conditions, namely step and sinusoidal changes in the value of PSINK. Actuator limitations are represented by saturation and rate limits applied to the manipulated inputs.

The gasifier is simulated in the MATLAB/Simulink environment using three linear models. These represent the plant at the 100%, 50% and 0% load operating points. The gasifier is run at 100% load for

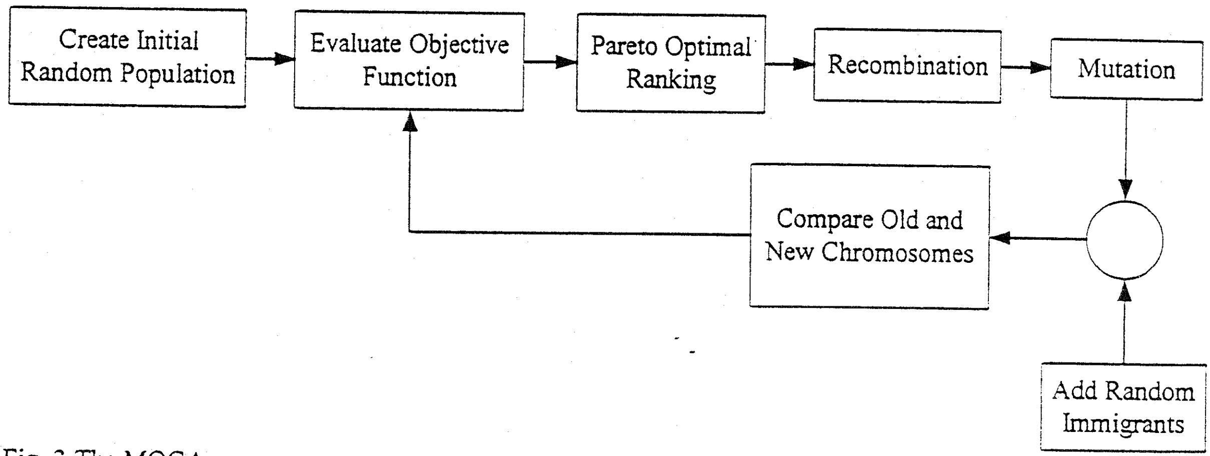

Fig. 3 The MOGA

the vast majority of its operating life. Therefore, the H_∞ controller is to be designed for the 100% load operating point but with sufficient robustness to meet the performance requirements at the other two operating points.

4. MULTIOBJECTIVE GA STRUCTURE

A multiobjective genetic algorithm (MOGA) is used to find a set of optimal weighting functions. It is implemented using the GA Toolbox for MATLAB (Chipperfield, *et al.*, 1994), developed in house, with additional extensions to accommodate multiobjective ranking, sharing and mating restrictions (Fonseca and Fleming, 1993). The salient features of this MOGA are shown in Fig. 3. In order for the MOGA to rank the prospective controllers, an objective function is required to evaluate the controller's performance against a number of objectives. The objective function first has to construct the H_∞ controller by solving the algebraic Ricatti equations (4) & (5). This was done using a state space linear model of the gasifier at the 100% load operating point and the weighting functions formed from the individual under evaluation. The weighting function structures used were those of a diagonal matrix of first order lags for W_1 and a diagonal matrix of gains for W_2. The first order lag structure of W_1 was considered necessary to break any algebraic loop which may appear in simulation due to the non-zero D matrices in the linear models. The terms in W_2 were specified as stateless in order to minimise the order of the resultant controllers. The chromosome is structured as a binary string in which the binary representations of the decision variables are right-concatenated. The controller's performance was then evaluated by running closed loop simulations using the linear models representing the 100%, 50% and 0% load operating points of the gasifier. Each simulation involved subjecting the closed-loop system to a step and a sine wave disturbance in turn.

As the optimisation philosophy of MOGA is to minimise objective function values, the linear plant models were not off-set, relative values about the operating point being preferred to absolute input/output values. This allowed the peak deviation of each output from its operating point value to be assessed by taking the maximum absolute value of each output vector.

Table 1. The Objectives

Objective No.	Objective Description
1	Peak fluctuation of CVGAS from 100% operating point
2	Peak fluctuation of MASS from 100% operating point
3	Peak fluctuation of PGAS from 100% operating point
4	Peak fluctuation of TGAS from 100% operating point
5	Peak fluctuation of CVGAS from 50% operating point
6	Peak fluctuation of MASS from 50% operating point
7	Peak fluctuation of PGAS from 50% operating point
8	Peak fluctuation of TGAS from 50% operating point
9	Peak fluctuation of CVGAS from 0% operating point
10	Peak fluctuation of MASS from 0% operating point
11	Peak fluctuation of PGAS from 0% operating point
12	Peak fluctuation of TGAS from 0% operating point
13	Maximum continuous eigenvalue of closed loop system
14	H_∞ robustness measure γ

This was done for each disturbance condition and each candidate controller in turn. Input constraints, representing actuator limitations, were adhered to by placing saturation and rate-limit blocks on the input lines of the closed loop Simulink system representation. These blocks contained relative values appropriate to the operating point under investigation.

Closed- loop stability at all three operating points was guaranteed by calculating the maximum closed-loop continuous eigenvalue and discarding any individual in the population which did not result in this value being less than zero across all the three operating points. One further objective attempted to minimise the H_∞ norm, γ, in order to maximise the robustness of the closed loop control system. The objectives are shown in Table 1 above.

5. RESULTS

Figure 4 shows a typical trade off graph for the gasifier. The x-axis shows the design objectives from Table 1 and the y-axis shows the objective domain performance of the controllers.

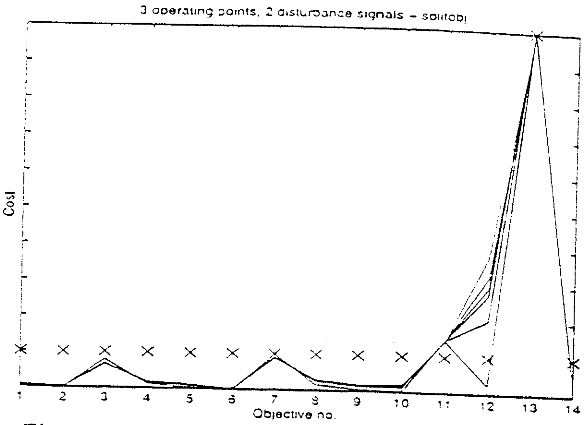

Figure 4. Trade off graph

With the exception of objective 13, the displayed ranges of each objective are normalised to leave the 'x' marks representing the optimisation goals one tenth of the way up the cost axis. Trade-offs between adjacent objectives result in the crossing of lines whereas concurrent lines represent non-competing objectives. Here, the goals relating to maximum output fluctuation (objectives 1-12) are set to the maximum allowed control errors for the associated output. These targets are specified as constraints in order to guarantee that controllers represented on the trade off graph satisfy the output constraints over the run-time of the simulation. From fig. 4 it can be seen that all the controllers represented here offer excellent control over peak bed mass fluctuation (objectives. 2, 6 & 10). Therefore the bed mass fluctuation, as an objective, does not compete heavily with any other objective.

Figures 5 & 6 show output time responses for the closed loop system with a selected controller at the 100% and 0% load operating points respectively. The performance requirements for the gasifier which these time responses are required to conform to are as follows (figures relate to the relative deviation of the output value from the operating point during disturbance conditions).

Performance Requirements:

$$-10000 < \text{CVGAS (J/kg)} < 10000$$
$$-500 < \text{MASS (kg)} < 500$$
$$-10000 < \text{PGAS (N/m}^2\text{)} < 10000$$
$$-1 < \text{TGAS (}^0\text{K)} < 1$$

6. DISCUSSION OF RESULTS

Results show a robustly stable controller design capable of exerting effective control and meeting all constraints over all the outputs at the 100% and 50% load operating points. For the case of the 100% load operating point, the step disturbance results are shown in Fig. 5. These show all four outputs remaining within their maximum deviation limits but with varying degrees of steady state error. The fuel gas calorific value and bed mass are controlled very effectively with deviation of less than 2% of the values allowed by the performance specifications. Control of the gas temperature is also achieved reasonably tightly with a maximum deviation of around 15%. However, extended run-time simulation beyond 300 seconds show that the responses are not returning to the set point. Whilst control of the gas pressure is not achieved as tightly as for the other three outputs, the maximum deviation from the operating point is well within the performance specifications. Again, however, the steady state error persists for extended simulations. The response at the 100% operating point to a sine wave disturbance shows similar characteristics to those of the step disturbance shown in Fig. 5. The tightest control was achieved over the calorific value and bed mass. Again, whilst remaining within the constraints, the gas pressure proves the most challenging control problem.

At the 0% load operating point, the shapes of the responses are similar to the previous operating points (see Fig. 6). Tightest control was again achieved for the calorific value and bed mass and for the step disturbance, steady-state errors were present on all the outputs which did not return to the set point during extended simulations. However, for the 0% operating point, it did not prove possible to keep the peak fluctuation of the gas pressure within the maximum deviation limits. For the step responses shown in Fig. 6 there is a constraint breach which could not be contained. However, in each case the breach is less than 50% of the size of the limits specified in the performance requirements. The settled value of the step response was within the specified limits. The same characteristics were present in the sinusoidal disturbance responses.

7. ADVANTAGES AND LIMITATIONS OF LSDP

The evolutionary algorithm approach to controller design employed here is advantageous in that it results in the designer having a choice of controllers rather than one specific design. The final selection between varying performance characteristics can be made in the knowledge that all comply with the restrictions imposed on the system. The trade off graph informs the designer of the implications that his choice will have on all the explicit design objectives. The approach greatly aided the selection of suitable weighting functions, searching a far greater space than would be readily achieved by conventional techniques. A particularly strong feature of the GA approach is the ability to use a mixture if discrete and continuous parameters in the problem formulation. This leads to an intuitive and natural representation for the problem being considered and simplifies the process of efficiently searching a large space.

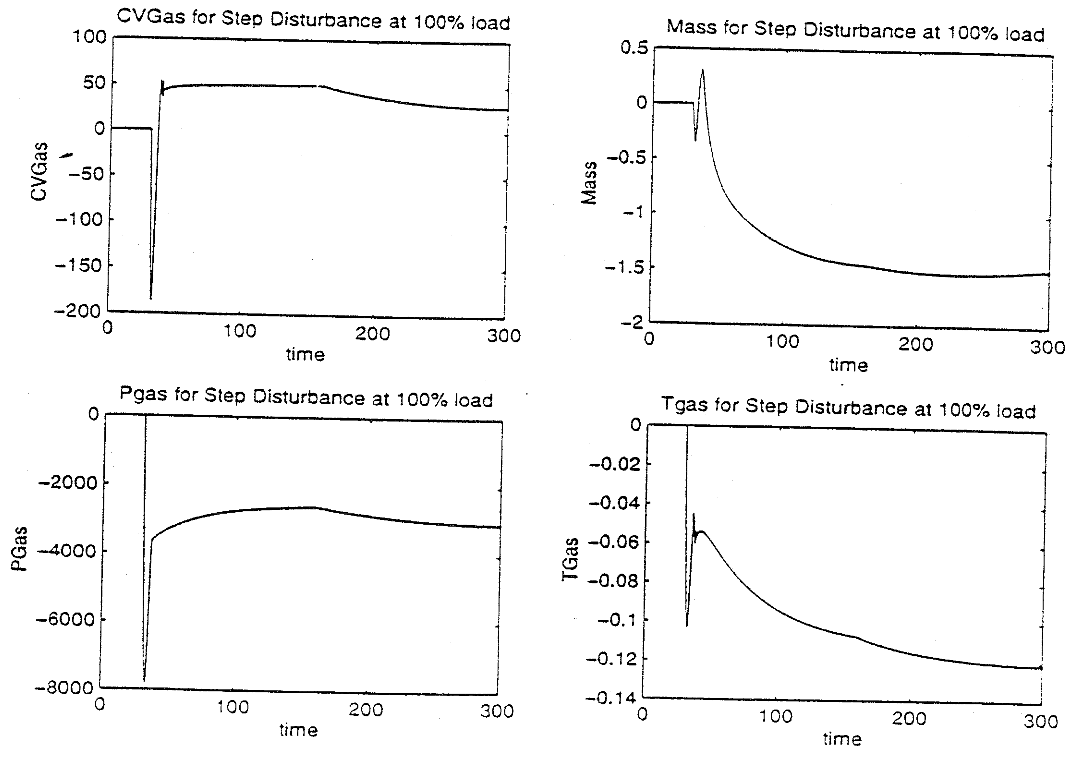

Figure 5. Output time responses at 100% load for a step disturbance applied at 30 secs.

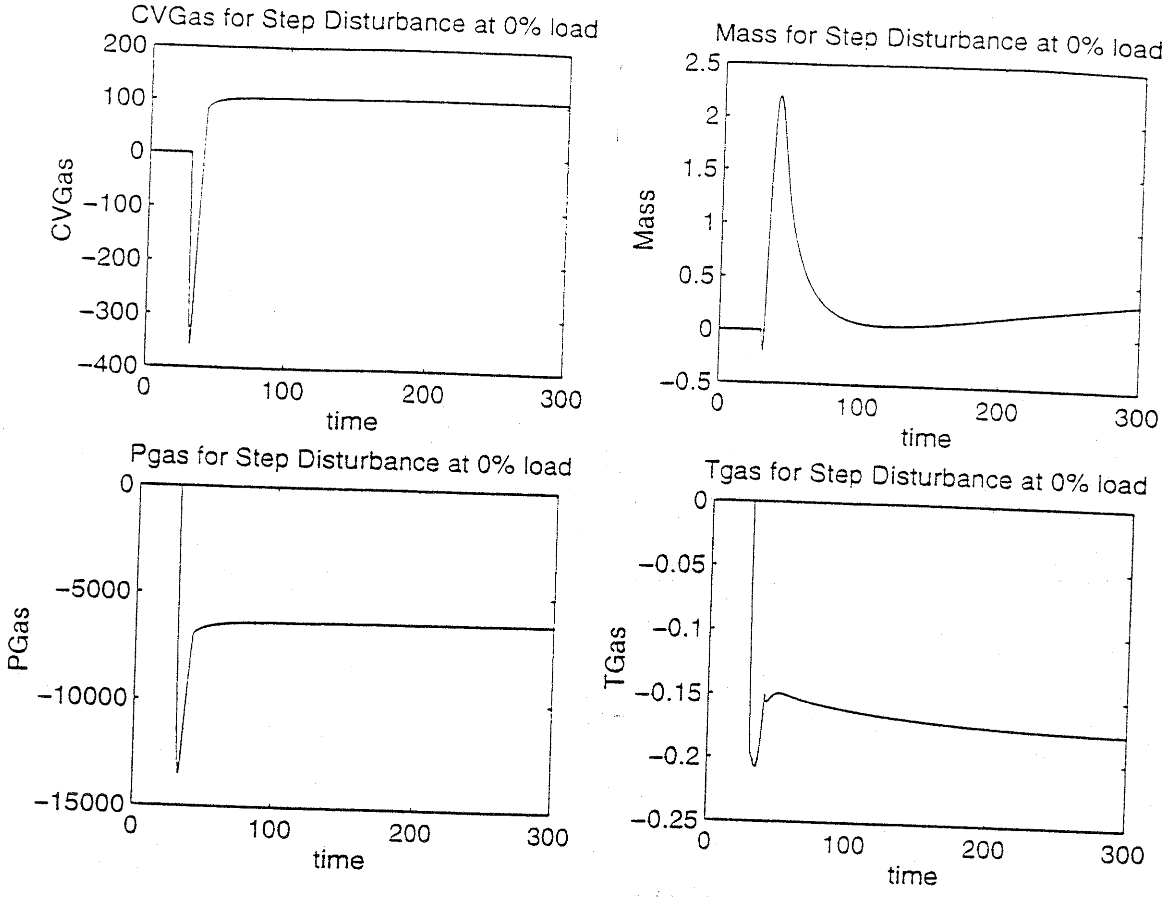

Fig. 6 Output time responses at 0% load for a step disturbance applied at 30 secs.

One limitation of this approach is that it produces solutions to a very specific formulation of the design problem. Whilst the response of the chosen controller may be optimal in terms of the objectives applied in the objective functions, it may not be optimal to similar objectives which the plant could reasonably be expected to face.

8. CONCLUDING REMARKS

An evolutionary algorithm approach has been proposed for the H_∞ controller design for a coal burning gasification plant. The controller design procedure was applied to a primary linear operating point and robustified through optimisation. In this way, a linear controller has been fitted to a non-linear operating envelope. It should be noted that the mixed optimisation approach allows other design parameters to be included in the problem formulation. For example the magnitude of the real part of the largest eigenvalue and the robustness metric, γ, were declared as explicit design criteria.

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