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LOCAL SHAPE OPTIMISATION OF TURBINE DISC FIRTREES USING NURBS

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Abstract

This paper describes a general shape optimisation method for a structural optimisation problem using a combined feature-based and B-spline based free form geometry modelling approach. The geometry is defined by a number of features for the global shape and by Non-Uniform Rational B-splines (NURBS), via the use of control points for the local shape. The coordinates of the control points along with other dimensions are chosen as design variables. A knot insertion/removal process is introduced to increase flexibility and to obtain a compact definition of the final optimised shape. Here the geometry is modelled using an intelligent CAD system (ICAD), which enables complete automation of the design-to-analysis process. Stresses are obtained from finite element analysis. A two-stage hybrid strategy is used to solve the formulated optimisation problem. Its application to the design of a turbine disc firtree root is demonstrated. This new approach is compared to a more traditional optimisation analysis using a geometry model based on straight-lines and circular-arcs. Improvements in the objective function and flexibility in geometry definition are achieved compared to designs obtained using the traditional approach.

1. Introduction

The use of computer aided design tools and computational analysis techniques such as finite element analysis and computational fluid dynamics has become common practice in the engineering design community. It is also known that utilization of various numerical optimisation techniques can bring benefits in terms of improvement in product quality and reduction in design cycle times. In general, there are two scenarios in which optimisation techniques can be used: 1) during the preliminary design phase, where many design configurations need to be considered to identify a design which meets the requirements as well as to accommodate innovation, 2) once the configuration has been determined, at which point the detailed dimensions of the component can be optimised against a set of detailed constraints like stresses and cost. Optimisation techniques have typically been used in structural optimisation where weight reduction is chosen as the objective, subject to a number of stress constraints at points within the structure. A comprehensive survey on structural optimisation can be found in the literature[1].

A typical structural optimisation problem commonly involves sizing, topology and shape optimisation[2]. Sizing optimisation is used to find the size related variables, such as cross-sectional area for bars and trusses. Topology optimisation mainly involves the layout of the materials while shape optimisation considers the optimum shape of the component boundaries. In general, shape optimisation problems can be further classified into two types. The first type involves the determination of dimensions of pre-defined shapes (geometric features) such as the radius of circular holes, etc. This type of design method can be referred to as feature-based design. The second type involves the determination of the shape of an arbitrary open or closed boundary.

Within these classes there are generally three types of shape parameterisation approach currently being used in the engineering community. The first is based on using the coordinates of the boundary grid points in a discretized domain as design variables; this is relatively easy to implement but makes it difficult to maintain a smooth geometry, resulting in some designs which are impractical to manufacture. The second is based on using CAD systems. Although the advantages of using a CAD system are obvious: smooth geometry, relatively small number of design variables, etc., it is difficult for traditional CAD systems to handle the variations of topology due to large perturbations in some dimensions. In addition, the underlying mathematical representations describing the geometry are not normally available to calculate sensitivities analytically. The third approach is similar to the morphing techniques used in computer science, and is termed free-form deformation (FFD); it is based on the idea of

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modelling the deformation rather than the base geometry. A comprehensive survey on shape parameterisation techniques can be found in the paper[2].

Shape optimisation of open boundaries has been studied for some time using various shape representation methods, for example, in a study carried out by Braibant and Fleury[3], three shape optimisation problems were examined. These were the determination of the optimal shape of a beam in bending, the optimal shape of a hole and the optimal shape of a fillet. They were solved using the concept of design elements which were defined using B-splines composed of a number of elements. In another work[4], the shape was parameterised using a parametric cubic representation of primitives supported in PATRAN[5]. Equidistant mapping in parametric space of the curve was used to create the finite element discretization. This is also referred to as an iso-parametric mapping in which the curve or surface is discretized evenly in the parametric space. Using these methods, attention must be paid to the creation of internal nodes during FE analysis to avoid the generation of distorted elements. A geometry based approach for coupling CAD with finite element methods was presented[6] where NURBS were used to model the shape of a cross section in a torsion problem. A more recent work is reported by Waldman *et al.*[7] in which an optimum free-form shape for a shoulder fillet was obtained but presented in tabular form rather than in a way that could be easily used in a CAD-based environment. The use of NURBS can also be seen in CFD-based shape optimisation, for example, the work carried out by Lépine *et al.*[8], where it is mainly used to reduce the number of design variables. One of the major drawbacks in the above applications is that the optimised shape is not CAD-ready and additional effort is required to utilize the results in the design process.

The development of parametric feature-based solid modelling capabilities in CAD systems has enabled the implementation of more complex shape optimisation approaches to be developed which output the optimised geometry in a CAD-ready form. Such capabilities have also enabled designers to focus on design innovation by allowing them to quickly develop design concepts utilising the available geometric features. The limitation of this approach is that it is difficult to generate shapes which do not exist in the library of pre-defined features provided by the system. Although free form geometric modelling can alleviate this problem to some extent, it is still very difficult to design a product purely using free-form geometry, due to the prohibitively large number of control points required. The effect of movement of the control points then becomes very difficult to control. This is particularly important when the design is embedded in an optimisation loop. Here the second CAD-based approach is adopted but the parametric modelling is performed using the design automation tool ICAD® from KTI[9].

In this paper, a combined approach using feature-based design and free-form geometry is adopted in the design optimisation of a turbine blade firtree root. The notch stresses of a firtree root have been identified in a previous study as being the critical factors affecting the final design but traditional modelling methods using arcs and straight lines provide little flexibility to further optimise the design. Non-uniform rational B-splines (NURBS) are therefore introduced here to define the two-dimensional local tooth profile of the root. A general approach for defining the position of control points is described using non-dimensional quantities that are dependent on the geometric features. One of the advantages of this approach is that flexibility can be achieved without loss of the geometric significance of the design variables used to formulate the optimisation problem. It also provides good control over the range of design variables and the resultant geometry.

The use of the ICAD system from KTI (Knowledge Technologies International) to construct the geometry makes this method applicable to a broad range of problems. A direct link has been established between ICAD and the finite element package to enable the modified geometry to be passed directly to the analysis code along with the associated boundary conditions and mesh control properties. Each time the geometry is modified, a new mesh is generated on the geometry using automatic meshing tools. This is preferable to utilising the internal parametric description of the geometry within the analysis code, due to the difficulties of extending the latter method to solve more complex problems than those dealt with in most previous papers[2,3,5,6].

This paper is organized as follows. The traditional shape modelling and optimisation of a firtree root is described in section 2. Section 3 deals with the parameterisation of local shape in the notch region using NURBS. Finite element modelling of the firtree root is detailed in section 4. The formulation of the optimisation problem and optimisation strategy used is described in section 5. Comparisons are made in section 6 between different local shape definitions and conclusions are drawn in section 7.

2. Modelling and optimisation of traditional firtree root shapes

A multi-lobe firtree root is commonly used in turbine structures to attach blades to the rotating disc. This feature is usually identified as a critical component in an aero-engine and is subject to high mechanical loads. The loading on the root is mainly due to centrifugal load, which is dependent on the mass of the whole blade.

Today the design of such a structure cannot be confidently carried out without extensive use of finite element methods and judgments based on both analysis results and industrial experience gained in the design and operation of such structures.

Although a 2D model of such a structure cannot reveal the significance of the effect of skew angle and stress variation along the axial direction, it is a useful simplification that can be used in the preliminary design stage (an estimate of the effect of the skew angle can be included as a scalar factor, called the skew factor, in the stress calculation). 3D detailed analysis can then be carried out at the component design stage to verify the choice of design candidates.

The geometry of turbine disc firtrees can be described using a number of dimension-based features and modelled using a feature based CAD system. The inclusion of a CAD system in the optimisation cycle offers a number of benefits, the most obvious of which is that it is consistent with the overall engineering design process and results can be directly used by the design team.

The two-dimensional shape of this type of structure can be described using a set of straight lines and circular arcs. This is referred to as a traditional design in this paper. A typical firtree root shape, which consists of three teeth, is illustrated in Figure 1 (left). Other forms of notch representations, such as elliptical fillets and compound arcs, could also be used in the notch shape modelling, as mentioned in the literature[10], an example of these using elliptical fillet was reported in an effort to reduce the notch peak stress[11].

The tooth profile used in the traditional firtree root geometry is shown in Figure 1 (right); it is defined using a set of geometric dimensions including lengths, angles and arc radii. There are 10 parameters used here in defining the basic tooth geometry: root-wedge-angle, tooth-pitch, top-flank-angle, under-flank-angle, next-top-flank-angle, blade-crest-radius, blade-trough-radius, disc-crest-radius, disc-trough-radius, and non-contact-face-clearance, see Figure 1 (right). When used in the optimisation process, these geometric features are treated as design variables and are varied by the optimisers in an effort to minimize the rim load subject to geometric and mechanical constraints drawn from both the structural analysis domain and industrial experience. The area outside of the last continuous radius of the turbine disc, referred to as the firtree frontal area, is chosen as the objective since it is proportional to the rim load by virtue of a constant axial depth, assuming that the blade load remains constant.

This geometric definition method has been used in a previous paper[12] in an effort to improve the design by first automating the repetitive design modification process and then applying various search techniques. However, it suffers from two major drawbacks: first, it becomes more difficult to characterize the effect of different parameters as the total number of parameters increases. Although some global search techniques such as GAs can be used to explore large design spaces, the computational cost related to such methods may prove impractical against the requirements for a reduced design time. Also, understanding the characteristics of a problem can be best achieved if attention is focused on local changes. In practice, almost all new designs can be traced back to earlier models and often a large

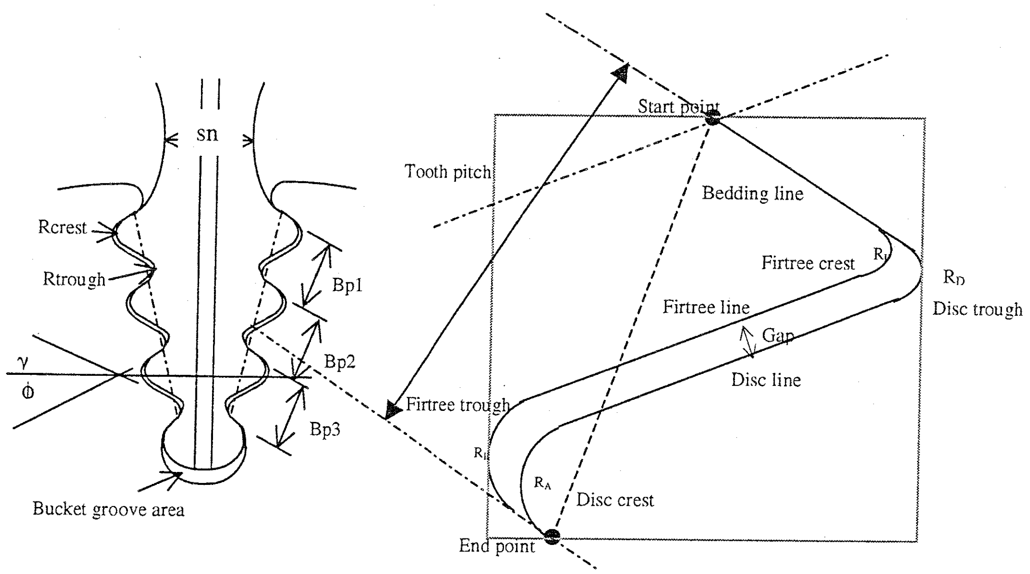

Figure 1: Typical geometry definition of firtree root (left) and single-arc fillet tooth (right)

amount of experience exists for the heuristic determination of some parameters. This can be used to reduce the number of design variables and makes it possible to focus on local areas where improvements can be critical to the quality of the whole product. At the same time, the designer may have more confidence in applying such local changes to the actual product. In addition, although minimising rim-load will reduce overall weight, the life of blade/disc is highly dependent on the notch stresses, and therefore the notch stress should ultimately be minimised for the required life target. An example of minimising notch peak stress using elliptical fillets can be found in the study[11].

In the previous study[12], this traditional geometry definition was optimised using various optimisation approaches including gradient-based methods, population-based genetic algorithms, and a two-stage hybrid method. It was observed that the notch stresses remain the critical factor affecting the final design. In addition, the notch stress minimization studied in the previous work further revealed that any reduction of notch stress almost inevitably resulted in an increase in the firtree frontal area and an increase in the crushing stresses due to a reduction in the bedding length resulting from an increase in the notch radii. It is this observation that motivated the use of a free-form shape in the definition of a tooth profile in an effort to further minimize the maximum notch stress within the notch region without increasing the firtree frontal area or the magnitude of the crushing stresses.

3. Local shape parameterisation using NURBS

Geometry modelling using polynomial and spline representations has been incorporated into most CAD packages, and these methods provide a universal mathematical approach to represent and exchange geometry in engineering applications.

The curves and surfaces represented in the ICAD system are, at their lowest level, represented by piecewise parametric polynomial functions. Many mathematical forms can be used to represent curves and surfaces. The most commonly used mathematical representation of curves and surfaces include Bezier and B-Spline functions, often referred to as B-forms. The Bezier representation of a curve has the following form

$$N(u) = \sum_{i-0}^{n} B_{i,p}(u)P_i \tag{1}$$

where $\{P_i\}$ are control points and $B_{i,p}(u)$ are Bernstein polynomials of degree p. The $(n+1)$ control points form a control polygon and they are usually used as design variables. The definition of a Bezier curve requires only information on the control points. Although Bezier curves have many characteristics that are desired when designing a curve, they become more oscillatory when the degree increases as the number of control points increases. In practice, several low-order Bezier curves are usually used to form a multi-segment curve, which is called a B-spline. A B-spline can be represented by

$$N(u) = \sum_{i=0}^{n} N_{i,p}(u)P_i \tag{2}$$

where $N_{i,p}(u)$ is the i-th B-spline basis function of degree p. Unsurprisingly, B-splines have all the desirable properties of the Bezier representation. The definition of a B-spline requires not only the control points, but also a knot vector of length $m+1$ $(m = n + p + 1)$. The only drawback of the B-spline is its inability to represent the class of geometry represented by implicit conic sections. This results in the extension of B-spline to non-uniform rational B-spline (NURBS), which can be used to represent almost any shape.

A NURBS curve is defined from a set of control points, a weight vector and Bernstein basis functions as[13]

$$N(u) = \frac{\sum_{i-0}^{n} N_{i,p}(u) w_i P_i}{\sum_{i-0}^{n} N_{i,p}(u) w_i} \tag{3}$$

where w_i is the weight correspondent to the control point P_i. For the design of a NURBS curve, these additional control parameters can be used as design variables along with the coordinates of the control points.

There are a number of properties that have resulted in NURBS being considered as a desirable mathematical representation in many CAD systems, so that most CAD systems now support NURBS not only as their internal shape representation, but also provide an interface for NURBS modelling. In terms of shape optimisation, perhaps the most useful two characteristics are their strong convex hull property and their local control property. (1) The strong convex hull property means that the curve is contained in the convex hull of its control points; in fact, the control polygon constitutes its approximation, therefore, the choice of the control points as design variables has clear geometric meaning. (2) The local control property means that moving P_i only changes the curve in a limited interval; in contrast the change of the position of one control point affects the whole of a Bezier curve.

NURBS thus promise to be the future geometry standard for free-form curve and surface representations for geometry processing applications within CAD/CAM and scientific visualisation, solid modelling, numerically controlled machining and contouring. For example, a cubic NURBS curve defined using 4 control points is illustrated in Figure 2. The control of the curve, achieved by changing the control point positions and weights, are of interest for structural shape optimisation. The effect of modifying the weight of point P_2 is also shown in Figure 2.

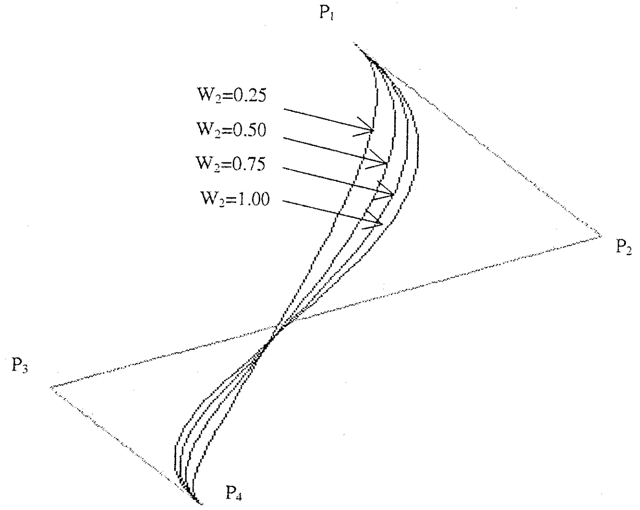

Figure 2: A NURBS curve and effect of changing the weight w_2 of point P_2 on the shape.

The motivation behind the introduction of NURBS into the definition of the tooth profile comes from the idea that it may reduce the stress concentration around the notch regions of the firtree root while maintaining a constant contact length between the blade and disc. The first model considered is simply an extension of an existing traditional single-arc model, referred to as the conic fillet model. It is derived from a NURBS representation of the single-arc fillet, using the simplest NURBS-arc of degree 2. A second model, using a NURBS curve of degree 3, is derived from a double-arc fillet model; this model is referred to as a cubic fillet model.

3.1. NURBS fillet of degree 2 – conic fillet

The simplest NURBS-arc of degree 2 can be defined using three control points, as shown in Figure 3 (defined by points P_0^0, P_1^0, P_2^0). The complete single tooth profile defined using this 3-point NURBS is shown in Figure 4. Instead of using absolute coordinates, the non-dimensional coordinates are used here to define the position of the control points. This definition of the tooth profile is used as a starting point. Now moving the middle control points will generate a series of cubic curves, while modifying the weight associated with the middle control point will generate a different series of curves. Initial comparison results showed that the benefit in terms of stress reduction from this model is not as good as a double-radii fillet, which is the normal alternative to the single-arc fillet. Therefore, instead of using the simplest NURBS arc, a more general form of degree 3 involving four control points was adopted in this study; see again Figure 3 (defined by $P_0^1, P_1^1, P_2^1, P_3^1$).

3.2. NURBS fillet of degree 3 – cubic fillet

A double-arc fillet can be described using 7 control points if the component arc is defined by a

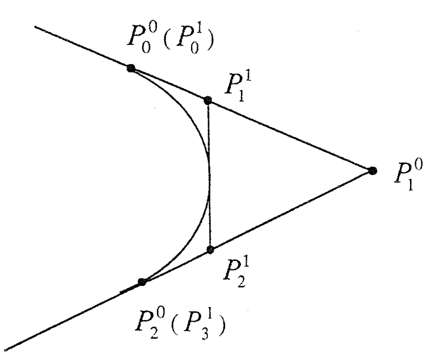

Figure 3: NURBS representation of single-arc fillet using three or four control points.

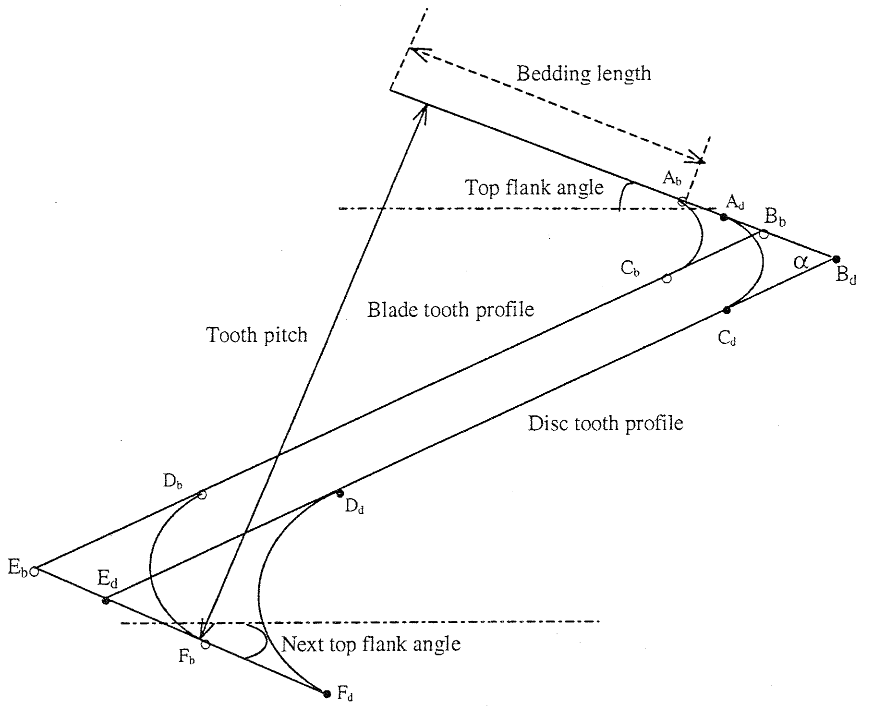

Figure 4: Notch conic fillet design using three control points.

cubic NURBS, and the points are properly positioned, as illustrated in Figure 5, Describing the double-arc using NURBS instead of two radii brings much greater flexibility. The choice of cubic curve is also consistent with the fact the most CAD systems use cubic curves to represent various geometry.

Describing the positions of the control points becomes difficult if the actual coordinates of the points are used, especially when each tooth profile is varied independently. Even for a uniform tooth profile, defining one tooth will involve 14 coordinates for these 7 points, and it is very difficult to manipulate these coordinates to ensure that an acceptable geometry is produced. Considering the convex property of NURBS curves and the fact that the tooth profile should always lie within the polygon prescribed by the points P_0^0, P_1^0, P_2^0 shown in Figure 3, a number of non-dimensional coordinates are used to define the position of the control points, as described next.

The point P_0 will be defined by the bedding-length (i.e., the end of the straight line segment) as illustrated in Figure 4, which is the main quantity affecting the crushing stress. The point P_6 is the end-point of the curve, essentially the point C_d in Figure 4 in the case of the disc profile. The point P_3 is defined by two non-dimensional quantities h, d as shown in Figure 5 (right) using points P_1 and P_5 as reference points. The points P_2 and P_4 are defined in a similar manner using P_1, P_3 and P_3, P_5 as reference points, respectively. This approach of defining control points provides better control as well

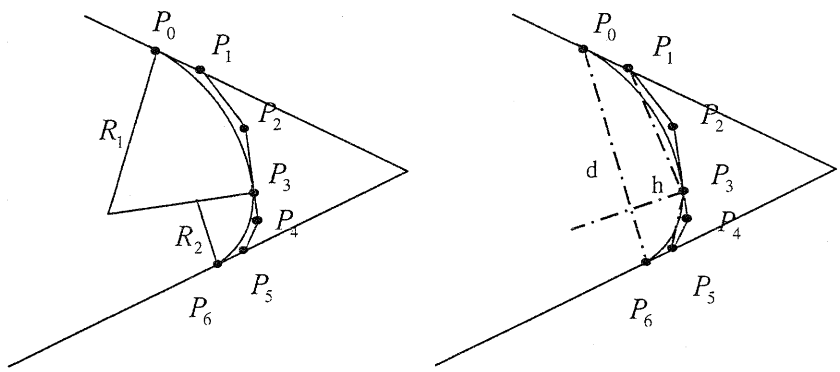

Figure 5: NURBS representation of double-arc fillet using seven control points and its defining coordinates.

as sufficient flexibility. Continuity requirements can then be built in to reduce the number of independent coordinates; in this case, the three points P_2, P_3, P_4 should form a straight line to maintain at least G^2 continuity of the curve at point P_3

To define a tooth profile using these 7 control points thus requires 9 independent variables. The total number to be optimised will be several times more if each tooth has a different profile of course. However, from initial analyses, the stress distributions along each tooth notch follow a similar pattern and, therefore, the same non-dimensional values are used for all the teeth. Although this could be further relaxed to consider different tooth profiles, as a first step it is useful to focus on a constant profile to obtain the optimum shape in terms of a smooth distribution of the stress.

4. Finite element analysis of the firtree root

The Rolls-Royce plc. in-house finite element analysis program SC03 is used here to perform the stress analyses. An automatic and smooth coupling between the ICAD system and finite element analysis package SC03 has been previously established for use in optimisation. The geometry and related information such as boundary conditions and loads are defined in the CAD system and transferred automatically into SC03. For further details about how these two systems are coupled and information exchanged, see a previous paper by the authors[12].

As the firtree geometry is constant along the root centre-line, it is possible to think of the stresses as two-dimensional. However, the loading applied along the root centre-line is not uniform, so strictly speaking, the distribution of stresses will be three-dimensional. Nonetheless, it is still possible to assume that each section behaves essentially as a two dimensional problem with different loadings applied to it. The difference of loading on each section is affected by the existence of skew angle which will increase the peak stresses in the obtuse corners of the blade root and increase the stresses in the acute corners of the disc head. From previous root analysis and research, it is feasible and convenient to use a factor to estimate the peak stresses at each notch of the blade and disc, and this factor takes different values for different teeth.

It is generally assumed that there are two forms of loading which act on the blade, the primary radial centrifugal tensile load resulting from the rotation of the disc, and bending of the blade as a cantilever which is produced by the action of the gas pressure on the aerofoil and forces due to tilting of the aerofoil. The resulting stress distribution in the root attachment area is a function of geometry, material and loading conditions (which are of course related to the speed of rotation).

Different types of mechanical constraints are involved in the design of such a structure, based on industrial experience. Finite element analysis is utilized to obtain these stresses.

- Crushing stress describes the direct tensile stress on the teeth: bedding width is the factor affecting the stress.
- Disc neck creep: the disc posts are subject to direct tensile stress that causes material creep. Too much creep, combined with low cycle fatigue, can dramatically reduce the component life.
- Peak stresses: peak stresses occur at the inner fillet radii of both the blade and the disc. If the fillet radii are too small and produce unacceptable peak stresses, some bedding width has to be sacrificed to make the radii bigger.

A single sector model is analysed to obtain the required stress data: maximum notch stresses, crushing stresses, blade and disc neck mean stress. Moreover, as finite element analysis is computationally expensive, a compromise between accuracy, numerical noise and computation cost is required to enable its practical implementation within an optimisation run. This compromise is made here by an appropriate choice of mesh density parameters.

It has been identified in previous work that the notch stress constraints are the limiting factors in the optimisation and remain active in the final results. It is possible to reduce the notch stress by increasing the notch radius, however in a traditional design, such an increase will inevitably result in a reduction in the bedding length, which then will lead to an increase in crushing stress. This is because the traditional shape parameterisation does not provide sufficient local geometric flexibility in the notch region to reduce both the crushing stress and notch stress to the required levels.

The aim of introducing NURBS in defining the notch fillet is to balance these two conflicting factors in the design. Achieving the minimum notch stress is essentially equivalent to achieving the most uniform possible stress distribution along the notch region. The stress distribution along the notch length may be plotted along with the radius of curvature to compare the effects of different shapes on the stress distribution and the radius of curvature. Such plots for a conic notch and a double-arc notch are given in Figure 6. It can be seen that the peak stress along the conic fillet is higher than a double-arc notch. This is why a cubic fillet derived from a double-arc fillet was chosen as a basis for notch shape parameterisation in the optimisation study.

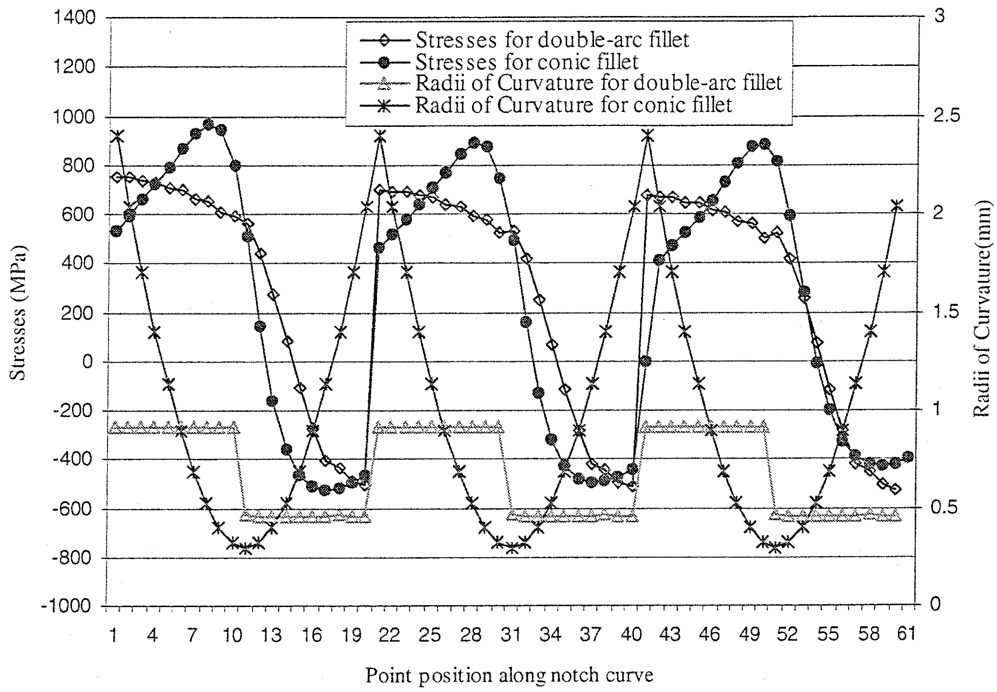

Figure 6: Distribution of notch stresses and radius of curvature along the notch curve.

5. Formulation of the optimisation problem and two-stage search strategy

The formulation of the shape optimisation problem involves the choice of a number of important geometric features as design variables and a set of geometric and mechanical constraints based on industrial experience and previous analysis results. It is known that a number of critical geometric features, such as flank angle, fillet radii and skew angle have a significant effect on the stress distribution. The effect of these quantities has been explored in the previous study[12]. However, in this paper, the notch region is the main area of interest. The parameters used to define the notch local profile were designated as design variables while the parameters describing the global shape were held fixed, based on previous results.

A number of geometrical constraints and mechanical constraints were included in the optimisation problem as shown in table 1. Three types of mechanical constraints are used, as described in the previous section.

Table 1: Normalized constraint vector for the base design.

No	Name of constraint	Numeric values		
		Lower Bound	Value	Upper Bound
1	Ratio of R1 to R2 [R1/R2]	-1.0	-0.8642	-
2	Ratio of H to D [H/D]	-1.0	-0.8955	-
3	Ration of blade bottom neck width to tooth pitch[BNP]	-1.0	-1.1474	-
4	Minimum wall thickness of bottom blade notch[BNMIN]	-1.0	-1.3038	-
5	Ratio of bucket groove region area to cooling passage area[AR]	-1.0	-1.0900	
6-13	Maximum blade notch stress[NBL(R)(2^1)]^2	-1.0	0.9270	1.0
14-17	Maximum disc notch stress[NDL(R)(3)]	-1.0	0.9948	1.0
18-23	Maximum blade section stress[SB(1)]	-1.0	0.6931	1.0
24-30	Maximum disc section stress[SD(4)]	-1.0	0.5623	1.0
31-38	Maximum crushing stress[CS(1)]	-1.0	0.6514	1.0
39-40	Maximum bucket groove stress	-1.0	0.9023	1.0

Notes: The numbers in bracket indicate the no. of the tooth or the section where the maximum stress occurs. For the purpose of compactness, only the maximum stresses are shown in the table.

The optimisation is performed here using the OPTIONS software package[14], which provides designers with a flexible structure for incorporating problem specific code as well as access to more than forty optimisation algorithms. The critical parameters to be optimised, known as the design variables, are stored in a design database, which also includes the objective, constraints and limits. The design variables are transferred to ICAD by means of a property list file which contains a series of pairs of variable names and values. This file is updated during the process of optimisation and reflects the current configuration. The geometry file produced by ICAD is then passed to the FE code SC03, which is executed by a command file. The analysis results are written out to another file, which is read in by the optimisation code. The design variables are then modified according to the optimisation strategy in use until convergence or a specified number of loops has been executed.

A simple two-stage search strategy of combining a Genetic Algorithm (GA) with a gradient search was used in this problem. A typical GA, as discussed by Goldberg[15] but with a clustering algorithm, is first employed in an attempt to give a fairly even coverage on the search space, and then gradient based search methods are applied on promising individuals with the number of teeth held fixed.

The main consideration behind this strategy is that as the GA proceeds, the population tends to saturate with designs close to all the likely optima, including sub-optimal and globally optimum designs, while gradient based methods are efficient at locating the exact position of individual optimum. Moreover, the GA is capable of dealing with discrete design variables such as the number of teeth. Hence the GA results are used to provide good starting points for the gradient search methods.

Several gradient based search methods have been applied to the problem after the GA search, these include the Hooke and Jeeves direct search method plus various other methods discussed in Schwefel's book[15].

6. Optimum notch shape and stress distribution

Three different tooth profiles have been optimised using the two-stage optimisation strategy described in the previous section. Comparisons are made here between the following three pairs: optimised single-arc fillet and conic fillet; optimised conic fillet and double-arc fillet and optimised double-arc fillet and cubic fillet.

6.1. Comparison between conic fillet and single-arc fillet

Replacing the single-arc fillet with a conic fillet brings explicit control over the bedding length and therefore the crushing stress. Then it becomes possible to decrease the crushing stress below the material criteria while at least maintaining the notch stress level at those seen in single-arc fillet designs, as shown in Figure 7. However, it is difficult to reduce the notch stress as the conic fillet offers little more flexibility than the single arc fillet.

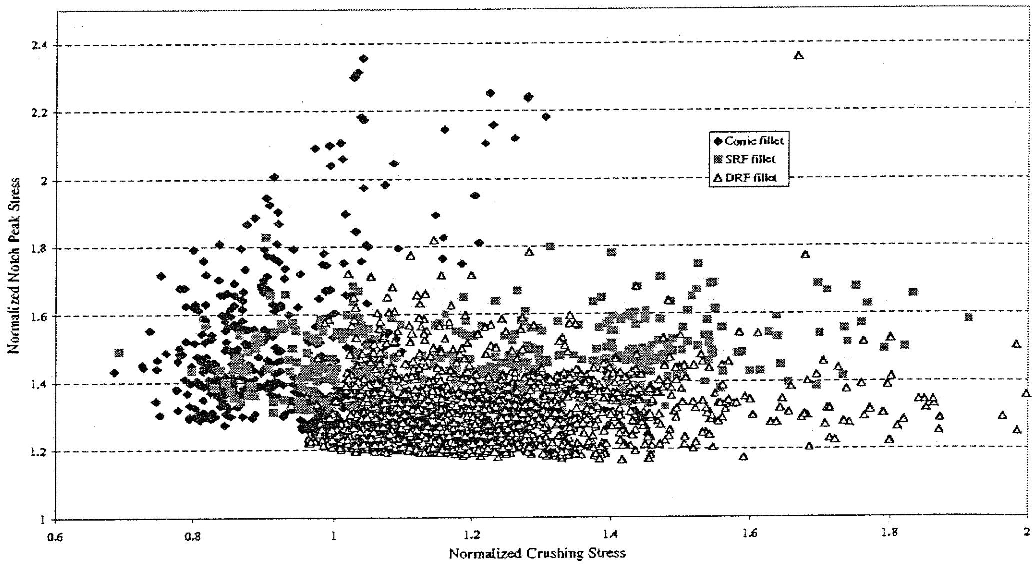

Figure 7: Normalized maximum notch stress against normalized maximum crushing stress. (normalization is defined as stress/material stress and all results considered during searches)

6.2. Comparison between double-arc fillet and conic fillet

Before moving on to the more flexible cubic fillet, a double arc fillet is next used to provide a basis for a general notch profile. The use of a double-arc fillet can further reduce the peak notch stress, and this

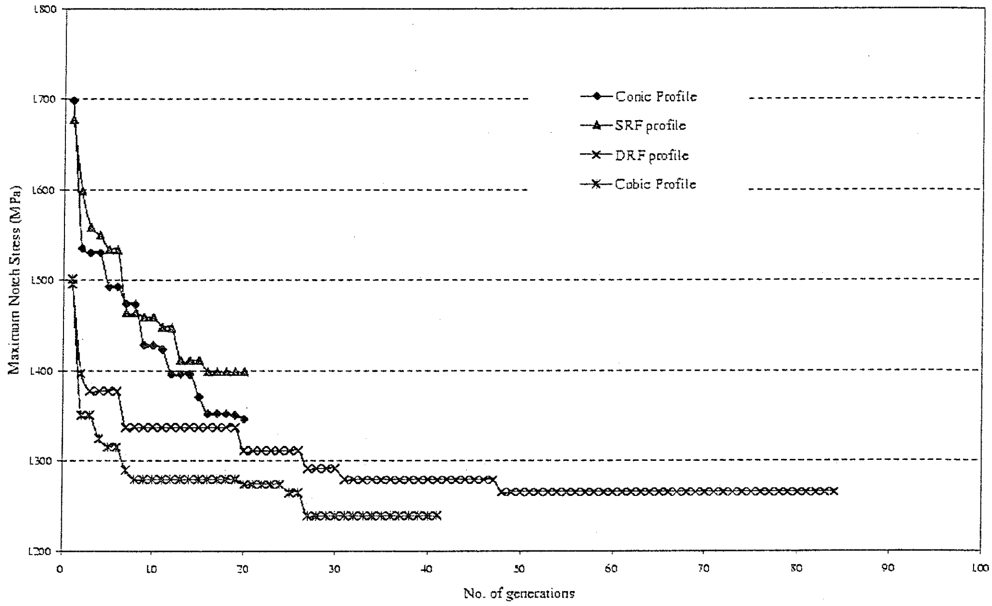

Figure 8: Convergence curves for different types of notch fillet using the GA.

can also be seen from Figure 7, where the objective function is set to minimise the notch stress. It may be seen that both the crushing stress and notch stress are reduced compared to single-arc fillet, which is very difficult to achieve in this example if the notch shape is restricted to a traditional single-arc design. The comparison between conic and single-arc fillet shows that the use of NURBS can bring benefits and that a double-arc fillet provides a good basis for a NURBS model of the notch profile.

6.3. Comparison between cubic fillet and double-arc fillet

The introduction of the cubic fillet provides further reductions to the disc notch peak stress, while all the other stresses can be maintained below the material allowable levels. Therefore an increase of fatigue life can be expected using a cubic fillet. Convergence curves are shown in Figure 8 for the different types of notch fillet, from which the benefits of introducing NURBS fillets can be clearly seen. A stress contour map for the notch region is shown in Figure 9, in which peak stresses before and after optimisation are noted. However, whether or not the introduction of more flexible notch fillet will cause difficulties in manufacture or incur significant cost increases remains to be further investigated.

7. Conclusion

A combined geometric parameterisation method using both basic geometric features and free-form shapes is presented in this paper and applied to the local shape optimisation of a turbine blade firtree root problem. This is used to further improve the firtree

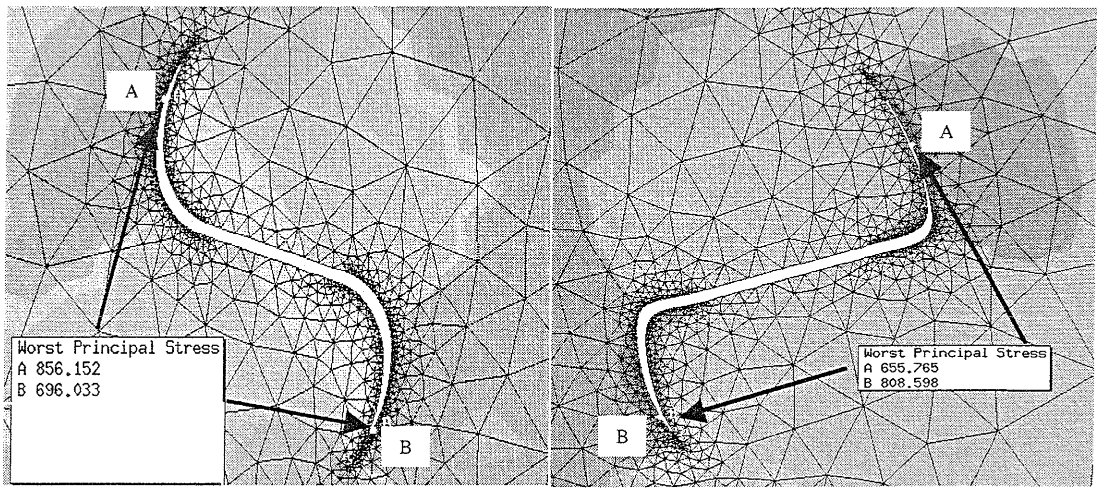

Figure 9: Stress contour maps for the notch region before and after optimisation.

design compared with traditional tooth profiles consisting of straight lines and circular arcs only. Non-uniform rational B-splines (NURBS) are used to model the tooth profile. The optimisation problem is defined in terms of various geometric properties and stresses. The geometry is constructed from a number of geometric features and localized free form shape control points. The control points and related weights of the NURBS curves along with several geometric feature dimensions are then chosen as design variables. The geometry is modelled using the intelligent CAD system ICAD. Stresses are obtained from finite element analysis. A fully integrated system of ICAD and FE is incorporated into an optimisation loop. A two-stage hybrid search strategy is then used to find better designs. Improvements in the objective function are achieved compared to designs obtained using traditional manual experience–based methods.

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