10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference
30 August - 1 September 2004, Albany, New York

AIAA 2004-4449

Visualization Methodologies in Aircraft Design

Carren M.E. Holden*
BAE SYSTEMS, SEIC, Holywell Park, Loughborough University, Leicestershire, LE11 3TU.
UNITED KINGDOM

and

Andy J. Keane†
University of Southampton, Highfield, Southampton, SO17 1BJ, UNITED KINGDOM.

This paper reviews aspects of low dimensional visualization methods, which are used currently in aerodynamic design and how these can be extended to higher dimensions. The shortcomings in current visualization methods are described and methods introduced which extend other visualization methods to this application. These simplify complex ideas into a small number of plots that the designer can understand and so use to gain insight into design. In general these take the form of maps which reduce the problem dimensionality from 5 and 8D to just 2 dimensions. This enables screening and optimization to be performed visually. Much of this is made possible only because the large amounts of data required for high dimensional design space appreciation are provided by response surface method technology. A modus operandi is proposed and the possibilities for visualization as an aid to understanding design are illustrated via aircraft aerodynamic design. Application to a sample problem that deals with a military aircraft optimization problem in 2, 5, 8 and 14 dimensions is discussed.

Nomenclature and Abbreviations

α = angle of attack
θ = wing twist angle
δ = flap deflection angle
C_D = drag coefficient
C_L = lift coefficient
C_m = pitching moment coefficient
CFD = computational fluid dynamics
DoE = design of experiments
GTM = generative topographic map
HAT = hierarchical axes technique
SOM = self organizing map
M = Mach Number
Tip l.e. loc = tip leading edge location

I. Introduction

THERE are considerable cost benefits to be derived from improving designs both by better searches in current design spaces and by satisfying more complex design criteria, for instance by striking the right balance between multiple objectives from the same discipline or from different disciplines. At present designers spend too long preparing individual analysis cases and, therefore, are constrained to sampling only small areas of the design space close to previously explored regions. In the design process, complex design tasks need to be broken down into manageable portions so that specialists in individual disciplines can resolve different parts of the design. Currently these portions are too small and there is too much iteration between disciplines which prohibits the challenging of

* Systems Engineer.
† Professor of Computational Engineering.

the constraints of one discipline by another, where progress is likely to be made. However, this situation can only be ameliorated if small numbers of individuals can assimilate, appreciate and understand large amounts of data rapidly and efficiently.

With this in mind, this work is aimed directly at the designer in the design process of the future, who communicates with and steers the design, search and optimization process and will extract previous work from databases transparently and flexibly. They will use software and data perhaps as web services, so that the source of this data and software will also be relatively unknown and unimportant. This designer makes use of high cost analysis codes within automated search and response surface models to map and optimise domains using computationally expensive evaluations. He or she then uses low fidelity (or computationally inexpensive) meta models to navigate high fidelity design spaces. The placement of analyses in the design space are defined by design of experiments (DoE) both as a scheme defined a priori and as a result of an optimization or error-defining processes. This engineer armed with more information and understanding, regularly challenges cross-discipline constraints. They are more concerned with problem definition, design space shape and optimization than how any one analysis is obtained and use advanced visualization techniques to provide enhanced comprehension and effective navigation of hyper-dimensional design spaces.

This paper reviews some currently used visualization methods: schematics, scatter plots, optimization tracing and parallel axes methods. It proposes a design process in which visualization is integral, introduces a scalable aircraft design problem and applies some currently used and more novel visualizations to this problem, including self-organizing maps (SOM) for visual screening and Mihalalisin's hierarchical axis technique (HAT) plots for optimization tracing in high dimensions. Finally, an 8 dimensional design space is reduced to a 2 dimensional design space and its potential role in optimization is discussed. This is achieved using a combination of these approaches, performing the transformation using the generative topographic mapping (GTM), to give a method of optimization tracing in high dimensions. Here, no new visualizations are developed, but the SOM, HAT plot and GTM are applied in novel ways.

II. Currently used visualization

Current multi-dimensional design space visualizations include, for example the general arrangement of an aircraft, a drawing in which the plan, front view and side elevation of an aircraft are shown. This potentially illustrates all of the geometrical variables of the aircraft simultaneously and is used extensively in commercial aircraft conceptual design to show starting, final and intermediate aircraft designs. The experienced design engineer can at a glance discern all the key features and assess whether or not he or she likes the design, based on similarity to previously obtained designs with good performance.

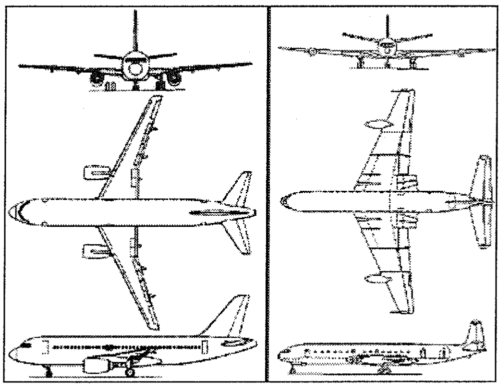

Figure 1: Schematic of Airbus A320 and DeHavilland Comet Aircraft.

An example of two such schematics is shown in Figure 1, in which the geometrical properties of the 1958 de Havilland *Comet* aircraft can be compared and contrasted to its successor, the Airbus *A320* aircraft of 1988. These schematics also illustrate the vast numbers of design variables involved in a real aircraft design. For aircraft with a similar range and thrust, the fuel consumption is much lower for the A320 aircraft and it carries half as many passengers again as the *Comet*. An indication as to why this is the case is given by the relatively large wing plan form of the *Comet* and its much narrower fuselage. The performance of the aircraft has, however, to be inferred from the geometry.

In this paper we aim to bring together a variety of data sampling and hyper-surface fitting technologies to draw multi-dimensional maps of key performance quantities. The visual inspection of these will aid the designer in the design process. However, with higher dimensional data the trends, clusters, hyper surfaces and anomalies, that may be of interest to a designer, are often difficult to identify and interpret because higher dimensional data is often sparse (described by Huber[4] as *the curse of dimensionality*). Structure in the data may also be difficult to locate or comprehend.

The work described in this paper builds on earlier work see, for example, Tweedie[1], in which an electronics design application is discussed, and extends this to an aircraft design application. This work additionally expands the visualization methods discussed in for instance, Chatterjee[2] into four dimensions and for more complicated functionality, without necessarily resorting to the use of parallel co-ordinates, although the use of this technique is also illustrated. An aircraft application is presented in Grubel[3], which also uses parallel co-ordinates, but the design space visualization does not include four variable visualization with constraint application as presented here. A nice review of other visualization methods is presented in Messac[17].

III. Taxonomy of a Design Process

The modules available in the design process discussed here are illustrated in **Figure 2**. The design problem is first specified in terms of an objective function with equality and inequality constraints, which vary with respect to design variables. It is assumed here that calculation of these quantities is expensive and that any such calculations must be sparingly applied.

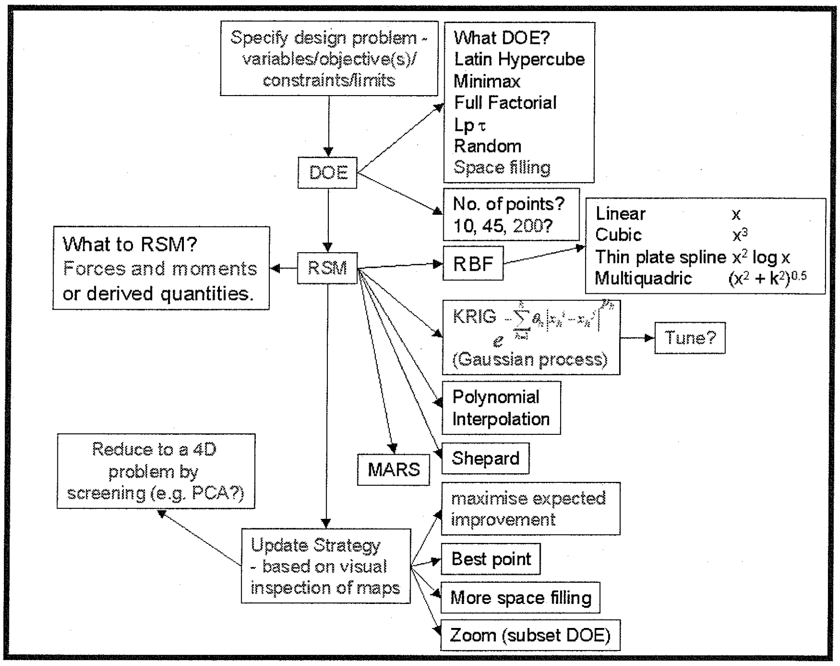

Figure 2: Taxonomy of a Design Process. The items highlighted in red indicate a possible set of selections.

Next a Design of Experiments (or DoE) is implemented to populate the sample space. A key question to be answered is exactly how many samples are required in a design space. Often the answer is those that can be afforded. The next question is how best to place these. Full factorial and random experiments are common in complex design situations, but techniques such as Latin hypercubes described by McKay[10], minimax space filling as discussed in Audze[1] or LP_τ arrays described by Statnikov[14] sample the design space more effectively. It is usually best to use relatively few initial samples and then insert additional points based on goodness of fit metrics such as root mean square error and expected improvement.

The next question is which response surface algorithm to use to interpolate or regress the available data. Possibilities include radial basis functions without basis function tuning, such as linear, cubic and thin plate spline or with a single tuning parameter, for instance multi-quadric. An alternative is kriging, a form of Gaussian processing, with statistically motivated basis function tuning as described in Jones *et al*[6], Jones[7] and Sacks *et al*[13]. In addition, simple multi dimensional polynomial interpolation and localised Shepard's methods may also be applicable in some cases.

At this point we may decide we want to reduce the size of the problem to more manageable dimensions. This area of research can be termed *data screening*. Techniques such as principal component analysis can be utilized to orthogonalize and then prioritize the design variables. The tuning hyper parameters in kriging can also be used to provide the order of importance of the design variables.

We will normally next want to put more points into the sample, in promising regions, where for instance the expected improvement is a maximum (see Jones *et al*[6]) or we may be interested in identifying the best point or a variety of local optima. Then a range of stochastic and gradient based optimization methodologies may be brought to bear in the response surface design space to identify these points. New sample points are then obtained at these optima. Alternatively, we may wish to zoom in, in terms of variable range or numbers of variables, or both, on a smaller subset of the original design space.

A variety of maps can be made available to the designer to aid this process. It is the construction and use of such maps that is the main subject of this paper.

IV. Airplane Design Problem

The problem chosen here to illustrate design space visualization is to minimize the drag coefficient C_D of a representative Military Aircraft. This is a trapezoidal tail-less aircraft in cruise at M=0.85, subject to constraints on lift coefficient (C_L) and pitching moment coefficient (C_m). Different problems are considered, representative of different phases of the design process.

Figure 3 shows the forces and moments on the aircraft in straight and level flight, from which the trim condition is derived.

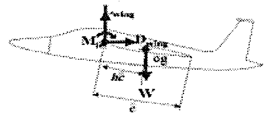

Figure 3: Major forces and moments on a military aircraft in straight and level flight, showing sign conventions utilized.

A BAE SYSTEMS in-house 3-dimensional inviscid, unstructured CFD method is used to perform the evaluations, which is computationally relatively expensive. A single evaluation takes approximately 10 c.p.u. minutes on a single processor computer. Morgan *et al*[12]describes the methodology used in the CFD code. To begin with, 200 calculation results were used in the DoE.

V. 2-dimensional search space

In order to understand the requirements of visualization to support optimization, let us first examine a two-dimensional design space that we can readily understand and use it to examine the additional appreciation that visualization provides. Discussion of this type of visualization for constrained optimization is provided for example in Jones[5] and in the determination of feasible solutions section of Statnikov and Matusov[14]. In order to perform an optimization successfully, it is helpful to the designer to be able to view the design space, to see the path of the optimization algorithm and the constraint boundaries restricting access to certain sub domains and highlighting the feasible region. This is relatively easy to do in two dimensions, where the variation of the objective against two design variables can be drawn as a contour plot or three-dimensional surface (provided that enough data points are available to construct the plot).

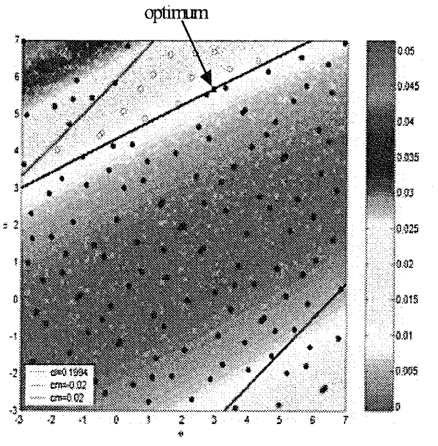

Figure 4: Visualization of 2 dimensional design space for Military Aircraft Wing Alone Problem. C_D is shown shaded, with the constraints C_L >= 0.1994 and $|C_m|$<= 0.02. The infeasible region is indicated by use of black circles to represent DoE points and white circles are used in the feasible region. Also shown are white crosses to represent the evaluations of simulated annealing on the Kriged design space. The square represents the domain optimum, where the constraints are satisfied and the C_D is a minimum.

To illustrate this, a 2-dimensional visualization of a Kriged search space of $C_D(\theta,\alpha)$, where θ is the linear twist down the wing or wash out and α is its angle-of-attack is shown in Figure 4. The design variables act within user specified bounds: $-3^{o} <= \theta <= 7^{o}$ and $-3^{o} <= \alpha <= 7$. Two additional Krigs of C_L and C_m are also made. The infeasible region, where the inequality constraint on C_L is not satisfied (i.e. where $C_L < 0.1994$) or on C_m (i.e. where $|C_m| > 0.02$) is indicated by use of black circles to represent DoE points in this region. Notice that the C_D design space is relatively smooth. The faint, white crosses represent the evaluations of the Krig model using simulated annealing to find the domain optimum. A white square represents the location of the optimum in the Krig domain, *i.e.* the location where both the constraints are met and C_D has the lowest (whitest) value. Large numbers of evaluations of the simulated annealing search center on the optimum. The optimum is given by CFD as $C_m = -0.007081$, $C_D = 9.999e\text{-}03$, $C_L = 0.1994$ and from the Krig model *as* $C_m = -0.007238$, $C_D = 9.918E\text{-}03$ and $C_L = 0.1994$, the differences illustrating the slight lack of precision in the Krigs.

VI. 5-dimensional search space

We next consider a 5-dimensional search space with the additional design variables: mid-camber (all sections assumed to be the same and scaled according to local chord), flap deflection angle (δ) and tip leading edge location (***tip l.e. loc***), which impacts on both leading and trailing edge sweep angles. In this case ***200*** CFD calculations were made for a body/wing aircraft.

In the first instance, scatter plots of the data, as shown in Figure 5, can provide some appreciation of the design space and establish that known trends are being followed, for instance, the quadratic nature of the relationship between C_L and C_D can easily be identified. Any off trend points, which can represent failed CFD evaluations can also easily be identified and either repeated or eliminated from the data set.

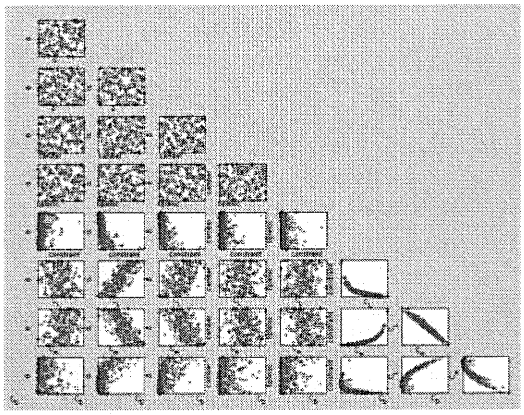

Figure 5: Scatter plot.

It is sometimes useful to know the order of importance of the design variables in the problem, particularly when eliminating variables to reduce problem size. The results from screening this data using conventional PCA and kriging (as described in Jones *et al*[6]) give α as the most important design variable in the problem, then ***camber*** and θ and then ***tip l.e. loc.*** and δ.

A parallel co-ordinates plot for the independent variables, constraints and objective is shown in Figure 6. Here, the variables are sorted so that variables which are related are adjacent, for instance α and C_D. As we are attempting to minimize C_D, the lines for the lowest *2* values of C_D, which satisfy the equality constraint on ***trim*** and the inequality on C_L are shown as bolder lines. The independent variables, give in general uncorrelated responses as defined by the DoE. The negative correlation between C_L and C_m and the positive correlation between C_L and C_D and C_m and ***trim*** are also clear.

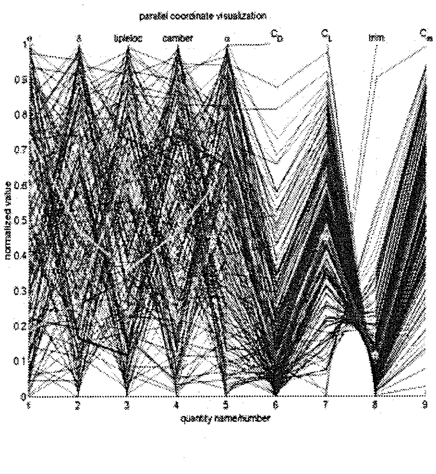

Figure 6: Parallel Co-ordinates technique for Body/Wing Problem design space visualization.

The self-organizing map (SOM, after Kohonen[8]) is a diagrammatic representation rather than a physical map of the design space. The SOM is a projection and visualization of a high-dimensional signal space on a two-dimensional display. Coarsely defined SOM's are presented here in Figure 7. In the SOM's the trends of the maps of C_L, C_D, C_m and ***trim*** are generally from red to pink in diagonal bands, showing that they are all related (C_D, C_L, α are positively correlated to each other and negatively correlated to C_m and ***trim***). The orientation of the SOM's is arbitrary, but the relative orientation is significant. The SOM's also show that the independent variables in the problem are unrelated, as they should be. In higher dimensional problems, the SOM has been shown to indicate the first two principal components, as their SOM's can be related although often in opposite directions (*e.g.* in opposite diagonal directions or vertically and horizontally). This can be used as a visual representation of *non-linear* principal component analysis. Bland regions in the SOM can give an indication of rogue data *e.g.* from unconverged CFD evaluations) and completely bland SOM's can help to identify unimportant variables. Further discussion of the establishment of variable relationships using SOM's is given for different design spaces in Matthews[9].

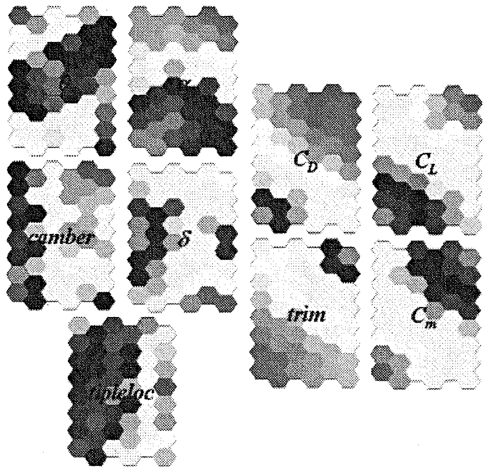

Figure 7: Use of Kohonen's Self--Organizing Maps for design space visualization.

Figure 6 and Figure 7 are clearly not easy to interpret and so the next question is whether or not the type of visualization presented in Figure 4 can be extended to higher dimensions to provide equally useful information. A Hierarchical Axes Technique (HAT) plot, after Mihalasin[11], is shown in Figure 8. Here, within an individual tile ***camber*** and δ vary. As we go from tile to tile in the horizontal direction of the main plot θ varies and in the vertical direction α varies. Then the top *81* tiles of this image represent the highest value of the fifth design variable, leading edge tip location. Each block of *81* tiles going down then represent lowering values of leading edge tip location. Only the top of this image is presented so that the smaller tiles are seen clearly. Two optima are shown: the first the result of a simulated annealing optimization (filled square) and the second the result of an exhaustive search in the representation data. This picture is only possible because response surface methodologies enable the large amount of data required for plotting to be provided. The image gives confidence that the same optimum is being located in both cases and that the curve fit is well behaved. To prevent confusion with the trace of the optimization, the initial DoE has not been presented here, although this is clearly possible and useful, in the same way.

To validate our procedure some results from this process are given in Table 1. (For clarity of presentation the GTM results for the following section are also presented in Table 1). The variation in the results given in Table 1 could be cause for concern, except that reference to the visualization shows that these results are close to each other and are therefore close to the same optimum. The CFD result is significantly different from the RSM, which means that there is mileage in building a new model, including this optimum and iterating until the optimum and CFD result obtained converge. Such differences are common in higher dimensional RSM work. Often multiple updates are required.

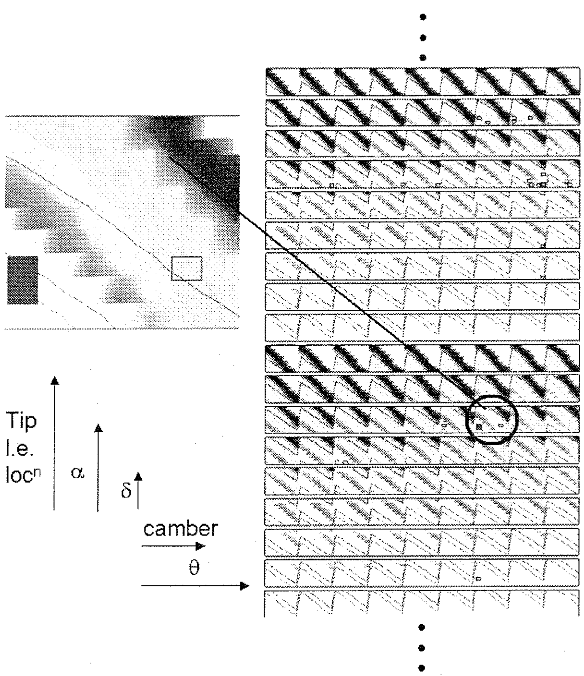

Figure 8: HAT Visualization of the 5 dimensional Airplane Problem Design Space. Also shown is a trace of a simulated annealing optimization, in which all the evaluations made during the optimization are superimposed on the plot as unfilled squares.

Method	x_1	x_2	x_3	x_4	x_5	trim	C_L	C_D	C_m
Simulated annealing on response surface	3.583	5.194	1.184	0.0156	10.24	1.000	0.1993	68.07e-04	-0.2153
Additional CFD evaluation	3.583	5.1936	1.1835	0.0156	10.2366	1.0004	0.2035	86.31e-04	-0.2153
9^6 point search on response surface	5.75	5.75	1.125	0.035	9.5573	0.9999	0.1994	74.03e-04	-0.2153
Simplex method on response surface	6.6985	4.8302	5.7286	0.0004	9.5808	0.9973	0.1974	66.19E-04	-0.2188
GTM	2.7877	4.0083	0.5342	0.0506	10.1614	0.9803	0.1966	64.0 E-04	-0.2284
Additional CFD evaluation	2.7877	4.0083	0.5342	0.0506	10.1614	0.9920	0.1934	54.0E-04	-0.2221

Table 1: Results for Design Space Optimization using different methods, but note that results were achieved only using the initial DoE

VII. Transformation of high dimensional spaces to low dimensional spaces using the GTM

Although the coarse SOM's already discussed enable the detection of relationships between functions and identification of their relative importance, they do not facilitate the scrutiny of trades between domain constraints. In addition, the location of function minima is also desirable, along with tracking the paths of optimizations and neither are possible in the SOM.

One approach to this problem is the triangle plot presented in Grossman[18], which is a mapping of 28 dimensional space into 2 dimensions. To do this 3 points are chosen to form the corners of the triangle. These may be local optima or perhaps two local optima and one intermediate point. All the intermediate points are then obtained as being a certain distance away from these reference points. The actual objective and constraints are then plotted as usual in a 2 dimensional plot. This technique necessarily incurs a loss in data structure, and a triangle is a relatively complicated mapping, particularly from higher dimensions. Additionally, optimization is difficult in this transformed space, as the corners of the triangle are defined by known points (the optima are used) and therefore exactly how far away other optima are and how far to extend the space is not known *a priori*.

To obtain a plot that gives the designer better insight a transform is required which has a more intuitive 2 dimensional manifold. The SOM is a projection and visualization of a high-dimensional signal space on a two-dimensional display. However, it is based on a heuristic algorithm rather than sound statistical techniques and therefore errors are difficult to control when transforming between design spaces.

Kohonen[8] introduces the Generative Topographic Mapping (GTM)[19] as an alternative computation of the SOM in which the metric relations between the models on the map grid and the data space are defined with improved fidelity. The trade-off is a higher computing load.

Therefore, in a new development, we transform data from a high-dimensional to a low-dimensional design space and, by inspecting the transformed data in the low-dimensional design space, enable an optimum value for a functional representation of the transformed data to be established. This permits effective exploration of the high-dimensional design space. A patent has been filed and is pending regarding this process in Holden and Keane[20] and an example for an 8 dimensional aircraft design problem is shown in Figure 9. This Figure was obtained using 516 points in the initial DoE.

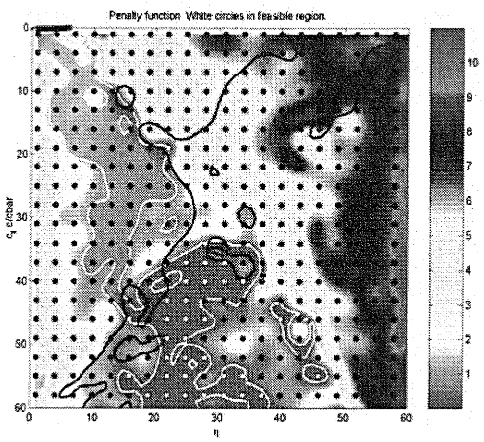

Figure 9: GTM for 8D aircraft design space.

The end solution to the 5 dimensional problem is given in Table 1. The sweep is reduced compared to the initial configuration as large amounts of sweep give a large amount of downward pitching moment, which has been reduced.

The solution to the 8 dimensional problem obtained using the GTM after a number of iterations is at: x_1*=0.05058*, x_2*=0.01256*, x_3*=0.005531*, x_4*=0.00668*, x_5*=0.00953*, x_6*=-0.00360*, x_7*=4.8462*, x_8*= 1.7266*, and the equivalent CFD solution is at C_L=*0.2022*, C_D=*58.83* counts, C_m=*-0.00359661*. This is probably reasonable compared to other optimizations. For instance, a gradient search in a similar 3 dimensional version of this problem also gave C_D= *58.8* counts, with C_L=*0.1994* and C_m=*-0.02* (higher C_L and lower C_m in this case), although the GTM solution is not fully optimized as it is not on the boundary. This optimum could possibly be found using a gradient search within the GTM space.

The pressure distribution, loading distribution and geometry of the starting and final 8D configurations are shown in Figure 10 to Figure 13. In this case the loading distribution is nearly elliptic, as expected, except for the removal of twist to reduce wave drag in the outboard wing and similar increase inboard to maintain lift.

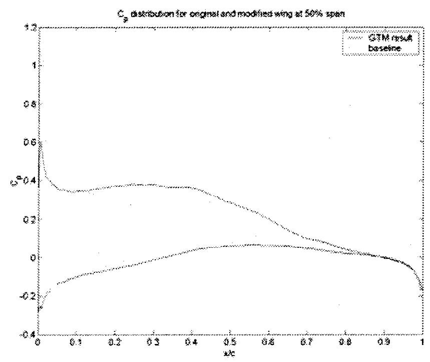

Figure 10: Pressure distribution from 8D optimization using the GTM compared to baseline.

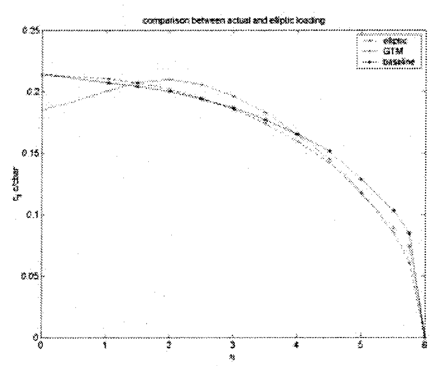

Figure 11: Spanwise loading distribution for 8D optimization using the GTM compared to baseline and elliptic.

A visualization is also presented for the corresponding 14D design space in Figure 14. The similarity with the 8D space should be noted. However, good results were more difficult to obtain in 14D. This is probably because the transformation is doing more work, *i.e.* going from 14 to 2 dimensions. To ameliorate this problem we transform to 4 dimensions (instead of 2) as presented in Figure 15. (This image is easier to understand if the similarity with the HAT plot in Figure 8 is appreciated.)

The quantitative results obtained in 14 dimensions were not as good as might be possible (although this is quite a high dimensional space which is difficult to navigate) and therefore further work is required to either use larger numbers of Gaussian centers, to use curvatures when obtaining physical distances or perhaps to use the Neuroscale transformation instead of the GTM. (See for instance Nabney[21] for further information.) However, the GTM is still useful qualitatively, for example it could be used to pre-process a problem so that a subsequent method could look at a reduced problem area or start from a better location.

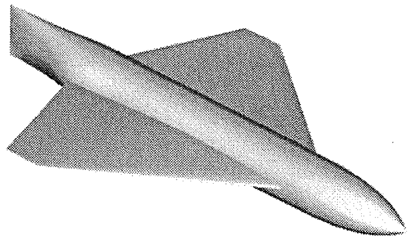

Figure 12: Baseline geometry, 8D problem.

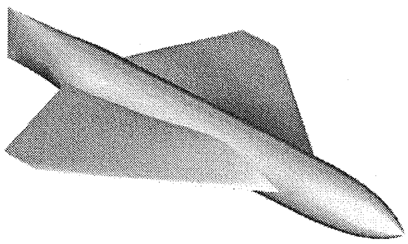

Figure 13: Geometry after optimization, 8D problem.

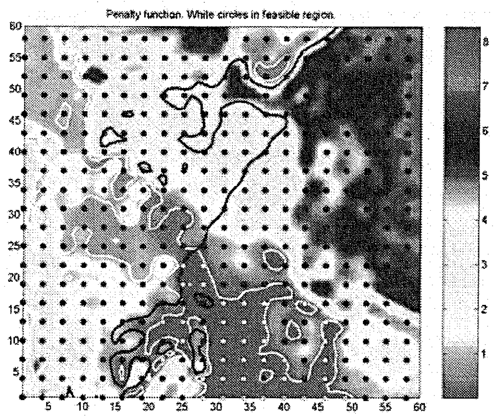

Figure 14: 2D GTM for 14D aircraft design space.

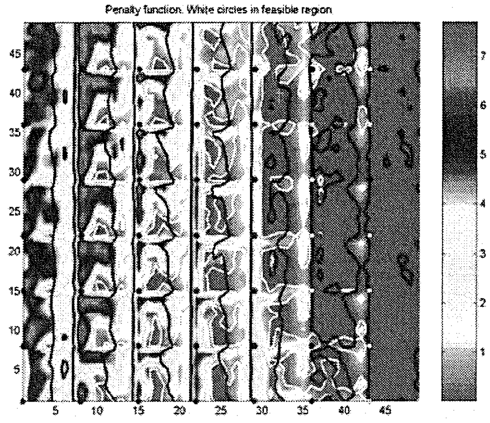

Figure 15: 4D GTM for 14D aircraft design space

VIII. Conclusion

In summary the work done in this paper achieves:

- The review, testing and ranking of a small number of existing high dimensional visualization methodologies,
- the use of the coarse SOM for pictorial screening in design,
- the establishment of relationships between variables,
- the use of Kriging to produce HAT plots in four to six dimensions in sparsely sampled domains,
- the use of the GTM to perform graphical optimization in, so far, up to 14 dimensions,
- the play off between the objective function and constraints,
- Optimization and optimization tracing.

Acknowledgments

This work was wholly funded by BAE SYSTEMS and the contribution of our colleagues: Peter Young from BAE SYSTEMS Air Systems, Warton and Alan Gould of the BAE SYSTEMS Advanced Technology Centre is acknowledged.

References

[1] P. Audze and V. Eglais, "New approach for planning experiments." *Problems of Dynamics and Strengths*, 35:104-107, 1977. Riga Zinatne Publishing House.

[2] A.Chatterjee, P.P. Das, and S. Bhattacharya. "Visualization in linear programming using parallel coordinates." *Pattern Recognition*, 26(11):1725-1736, 1993.

[3] G. Grubel, H. D. Joos, M.Otter, and R. Finsterwalder. "The {ANDECS} design environment for control engineering." *In IFAC 12th Triennial World Congress, Sydney, Australia*, pages 789-796, 1993.

[4] P. J. Huber. "Projection pursuit." *The Annals of Statistics*, 13:435-475, 1985.

[5] C.V. Jones. "Visualization and Optimization." *Operations Research/Computer Science Interfaces.* Kluwer Academic Publishers, 1996.

[6] D. Jones, M. Schonlau, and W. Welch. "Efficient global optimization of expensive black-box functions." *Journal of Global Optimization*, 13:455-492, 1998.

[7] D. R. Jones, ``A taxonomy of global optimization methods based on response surfaces," *Journal of Global Optimization*, vol. 21, pp. 345-383, 2001.

[8] T. Kohonen. *Self-Organizing Maps.* Number ISBN: 3-540-67921-9. Springer-Verlag, 2000.

[9] P.C. Matthews, *The Application of Self-Organising Maps in Conceptual Design*, Ph.D. thesis, Fitzwilliam College, University of Cambridge, June 2001.

[10] M.D. McKay, R. J. Beckham and W. J. Conover, ``A comparison of three methods for selecting values of input variables in the analysis of output from a computer code.," *Techometrics*, vol. 21, no. 2, pp. 239-245, 1979.

[11] T. Mihalalisin, J. Timlin, and J. Schwegler, ``Visualizing multivariate functions, data and distributions,'' in *Readings in Information Visualization: Using Vision to Think*, S. K. Card, J. Mackinlay, and B. Shneiderman, Editors., ISBN 1-55860-533-9, pp. 115-125. Morgan Kaufmann, 1999.

[12] K. Morgan, J. Peraire and J. Peiro, "Unstructured grid methods for compressible flows", *Special Course on Unstructured Grid Methods for Advection Dominated Flows*, AGARD Report 787, March 1992.

[13] J. Sacks, W.J. Welch, T.J. Mitchell, and H.P. Wynn. "*Design and analysis of computer experiments.*" Statistical Science, 4(4):409-435, 1989.

[14] R.B. Statnikov and J.B. Matusov. *MultiCriteria Optimization and Engineering.* Chapman & Hall Publishers, ISBN 0-412-99231-0.

[15] L. Tweedie, R. Spence, H. Dawkes, and Hua Su. "Externalising abstract mathematical models." In S.K. Card, J. Mackinlay, and B. Shneiderman, editors, *Readings in Information Visualization: Using Vision to Think*, number 1-55860-533-9, pages 276-284. Morgan Kaufmann, 1999.

[16] W.J. Welch, R.J. Buck, J.Sacks, H.P. Wynn, T.J. Mitchell, and M.D. Morris. "Screening, predicting, and computer experiments." *Techometrics*, 34:15-25, 1992.

[17] Messac, A., and Chen, X., "Visualizing the Optimization Process in Real-Time Using Physical Programming," *Engineering Optimization*, Vol 32, No. 5, May 2000.

[18] B. Grossman, R. T. Haftka, W. H. Mason, L. T. Watson, C. Baker, S. Balabanov, S. Cox, A. Giunta, H. Kim, D. Knill, and D. Krasteva, ``Effective use of surrogate models in aircraft design," *Presentation at First International Workshop on Surrogate Modelling and Space Mapping for Engineering Optimization* - held in Lyngby, Denmark, November 16-18th 2000.

[19] Markus Svensen. "*GTM: The Generative Topographic Mapping.*" PhD thesis, Aston University, Birmingham, UK, 1998.

[20] C.M.E. Holden and A.J. Keane, "*Design by Space Transformation from high to low dimensions*", PATENT filed, October 2003, UK GB, Application GB0323848.2.

[21] I.T. Nabney. "*Algorithms for Pattern Recognition*", ISBN 1-85233-440-1. Springer, 2001.