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A Study of Shape Parameterisation Methods for Airfoil Optimisation

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This paper presents a study on parameterisation methods for airfoil shape optimisation within a CAD-based design optimisation framework. The objective of the paper is to study the effect of different methods on airfoil shape optimisation when using computational fluid dynamics (CFD). Parameterisation of geometry is one of the essential requirements in shape optimisation, and it presents further challenges when carrying out multidisciplinary design optimisation, as it is critically important to maintain shape consistency between analysis domains, while providing different analysis models from the same CAD definition. It is usually the case that there are numerous possibilities in defining the parametric model, and it will prescribe to a large extent the scope of the search space and landscape of the objective function. This paper adopts design of experiments and optimisation approaches to study several representative parameterisation methods in terms of flexibility and accuracy of the methods for aerodynamic shape optimisation.

I. Introduction

AIRCRAFT design is a complex decision making process and according to Raymer,[1] can usually be broken down into three phases, conceptual design, preliminary design, and detailed design. Aerodynamic design occurs throughout these steps. Two different approaches are often employed in the aerodynamic design: 1) inverse design and 2) direct numerical optimisation. The first method tries to solve for a geometry that produces a prescribed pressure distribution. On the other hand, direct numerical optimisation methods couple a geometry definition and aerodynamic analysis code in an iterative process to produce optimum designs subject to various constraints. Depending on whether the goal is to improve on an existing design or to create a completely new design, different parameterisation methods are often required. If the new design only requires small changes to the initial geometry, a localized parameterisation approach is often used. But when conducting a study of a radically new concept, the parameterisation method needs to accommodate a wider range of new shapes.

Airfoils have been represented in a number of different ways in the past. For example, coordinates have been directly used to fit airfoil shapes using B-splines and Bzier curves[2] via interpolation methods. Analytical functions have also been derived to represent families of airfoils, for example, in the work reported by Hicks and Henne.[3] In a more recent work,[4] Non-uniform rational B-splines (NURBS) were used first to approximate existing airfoils, then adopted as a general parameterisation method to be used in optimisation. The concept of using relatively few orthogonal functions to represent a large number of functions has also been exploited, for example in a work reported by Robinson and Keane,[5] where a set of orthogonal functions was developed using numerical methods. These functions were then used to represent a family of airfoils in a wing design study. However, the basis functions derived by Robinson and Keane[5] were believed to be dependent on the particular familiar of airfoils. Although these numerically derived basis functions can be used in the design of a particular set of airfoils, other airfoils may not be adequately represented using them.

The choice of parameterisation method, when coupled with optimisation techniques to find desirable shapes in terms of user-defined objective functions and constraints, has a major effect on the final results, efficiency and effectiveness of particular search strategies. Giving the same CFD models, the parameterisation

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effectively defines the optimisation problem formulation, the topology of the design space, and the landscape of the objective functions. Although it is vitally important, it is also very difficult to come up with a set of effective criteria that can be readily used to evaluate the pros and cons of different parameterisation methods. Wu[6] compared three geometric representations in three case studies of cascade blade design using adjoint methods. After carrying out the optimisation, it has concluded that one of the methods using geometry parameters is not suitable for two occasions.

There are a number of key issues that need to be addressed in the choise of parameterisation methods. The first issue is the flexibility of any parameterisation method. Flexibility is interpreted here as the ability to represent a wide range of different shapes. Some parameterisation methods, for example, coordinate-based methods, can accurately represent a variety of dramatically different shapes and can also reflect subtle changes in local areas, however it would be very difficult to use such an approach for optimisation problems using high fidelity codes due to the large number of design variables and complexity of the design space. On the other hand, methods using fewer variables may not be capable of generating shapes with high accuracy, especially when used in inverse design problems where a target pressure distribution is sought. The second issue when considering parameterisation methods is the accuracy or the optimum objective functions that the final shape can achieve either in an inverse design study or direct optimisation work, respectively. The accuracy should be measured in both geometric and aerodynamic senses. However, the optimal objective function cannot be obtained without actually carrying out the optimisation, therefore, here an inverse design approach is adopted to compare different parameterisations.

In this work, three datum airfoils, two from the NACA supercritical airfoils family (NACA0406 and NACA0610) and the third being the RAE2822, are used as reference shapes to compare different parameterisation methods for airfoil design. The paper is organised as follows. Section two describes different parameterisation methods for airfoil design. The geometry modelling and flow analysis of the airfoil problems are described in section three. Results and discussions are presented in section four, with conclusions given in section five.

II. Parameterisation of Airfoil Geometry

Geometry parameterisation methods have attracted renewed interests in recent years, especially in the context of multidisciplinary design optimisation (MDO). Samareh[7] identified three categories of parameterisation methods in the context of MDO. These include the discrete approach, CAD-based approaches, and free-form deformation methods. Indeed, all these different approaches could be implemented in most modern CAD systems. Several parameterisation methods have been proposed in previous papers for airfoil geometry, for example, the NACA supercritical airfoils are defined as a series of y-coordinates at prescribed chord wise locations.[8] The second approach models the geometry as the linear combination of a basis airfoil and a set of perturbation functions, defined either analytically[4] or numerically,[5] as shown in Eq. (1). The coefficients of the perturbation functions involved are then considered as the design variables. A set of such orthogonal basis functions derived from a group of base airfoils was developed by Robinson and Keane[5] to provide an efficient means to define the airfoil for optimisation study in preliminary design, for example.

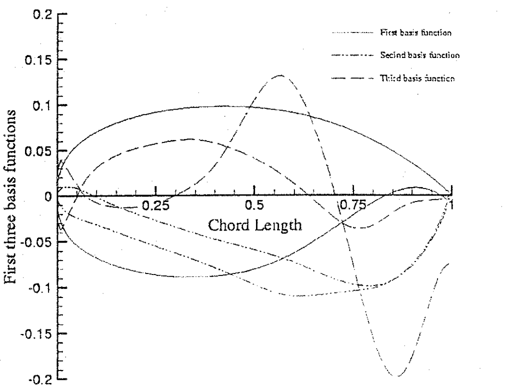

Figure 1. The first three numerically derived orthogonal basis functions.

$$y(x) = \alpha_0 y_0(x) + \sum w_i f_i(x) \tag{1}$$

A third, and more geometrically intuitive method, is to use geometric parameters such as leading edge radius, thickness-to-chord ratio or maximum thickness to define the airfoil shape. An airfoil parameterisation using 11 geometry parameters was presented by Sobieczky[9] and used by Oyama etc.[10] A fourth method uses the control points of Non-uniform Rational B-splines (NURBS) curves to define the airfoils.[4] This method is also used by Li[11,12] in which a B-pline interpolation through 35 points is used to define the airfoil geometry. The advantage of this approach is that free-form geometrical shapes can be accommodated with fewer design variables compared to the direct use of coordinates. However, due to difficulties in controlling the relative positions of the control points, free-form parameterisations are usually used in the inverse design approach, where only a subset of control points are allowed to change in a relatively small range to meet a target pressure distribution.

One of the key issues in deciding the parameterisation method is the balance the requirements of robustness and flexibility, and these decisions are also strongly dependent on the goal of the design activity. Although free-form parameterisations may well be able to generate radical new shapes, this is not suitable for designs where the aim is to meet a specific pressure distribution, due to the poor efficiency caused by the large search space that arises in the optimisation process. Another disadvantage of free form parameterisation is the inherent difficulties encountered when trying to generate airfoil-like shapes: Usually additional geometrical constraints need to be imposed. Two different parameterisation methods are implemented in the current work to compare their effectiveness. The first approach is to use a set of numerically derived basis function to define the airfoil,[5] in this case, only a small number of design variables are involved. The basis functions used are illustrated in Figure 1. The second approach uses a B-spline interpolation based on 34 points as shown in Figure 2. The y coordinates of the points are used as design variables while the chord wise coordinates of the points are fixed. Both methods are implemented in the CAD system ProEngineer.[13] Airfoil shapes from the family of supercritical airfoils[3] and RAE2822 are chosen in the current work as reference airfoils in the comparison. The number of parameters involved in the two parameterisation methods are summarized in Table 1.

Table 1. Summary of different parameter methods for airfoils

Method	Number of parameters	Description of the parameters
Numerical Basis Functions	5	Weights for the basis airfoil functions
B-spline interpolation	34	Point coordinates

III. Geometry Modelling and Flow Analysis

To compare the flexibilities of different parameterisation methods in representing different airfoil shapes, three existing airfoils NACA0406, NACA0610, and RAE4822 are chosen as the modelling targets for these parameterisations. The difference between the target airfoil and approximated airfoil is defined as

$$diff = \sum abs(f_{target}(x_i) - f(x_i)) \qquad (2)$$

where $x_i (i = 1, ..., n)$ are the chordwise coordinates used in the definition of the target airfoils. This quantity is minimised using a global optimisation algorithm for different parameterisation methods and then used as an indication of the flexibility of the methods. To remove the effect of the optimisation techniques that may not produce the optimum results, a large number of iterations has been carried out for the minimisation problem.

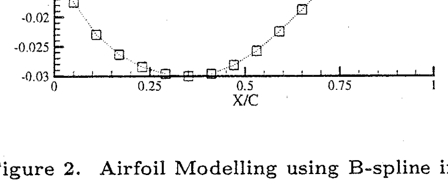

Figure 2. Airfoil Modelling using B-spline interpolation.

The airfoil models are all implemented using ProEngineer and exported in the form of a STEP file, which is then imported into Gambit. A boundary layer is attached to the lower and upper surface of the airfoils, and size functions are also used to give better control of the mesh and to reduce the computational time for the problem. An unstructured mesh generated using Gambit[14] for solving the N-S equations is shown in Figure 3. The mesh contains 11356 cells (compared with 87305 cells without using the size function). In both cases, the node spacing on the airfoil surfaces and farfield circle are the same, with size functions giving better control of the transition of size of the cells in between. The computation time is reduced from around 40 minutes to less than 20 minutes for most geometries on a Xeon 2.4Ghz compute node with 1Gb memory.

The flow model used in the current work is based on the Navier-Stoke model from Fluent.[14] The pressure distribution of the upper and lower surfaces are used in the comparison. Here, the cruise condition ($M_\infty = 0.73$) is used when calculating the lift and drag values. The Spalart-Allmaras viscosity model is used.

IV. Results and Analysis

It is not straightforward to compare alternative parameterisation methods. There are two important considerations when a parameterisation model is built around an existing geometry: the first is the flexibility of the model, i.e., how many different shapes can this model represent. The second is be the robustness of the model, i.e., can the model generates the desired shape for large number of different designs. In general models with more design variables will be able to represent more complex shapes, and will be more likely to produce novel designs using optimisation. However, that will be more expensive in the search as the design space will have higher dimensions, and chances of failure or not generating desired shapes will be higher.

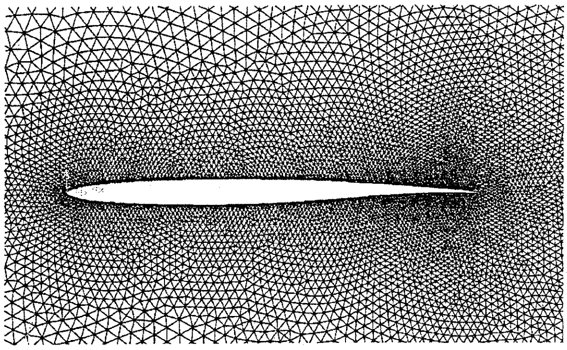

Figure 3. Unstructured mesh used for solving the N-S equations by Fluent

The best results for approximations of the target airfoils are shown in Table 2. The results are produced by minimising the objective function computed using (2). A genetic algorithm (GA) from OPTIONS[15] is used in the current work, however, the first population is not generated randomly, rather, it is generated using a Design of Experiment (DoE) method plus one point describing a user specified base design to obtain more uniform coverage of the design space as well as to provide the best possible guess. The DoE method used here is a Latin Hyper Cube method. The base design specified by the user can play an important role in accelerating the search process as the GA used in this work always maintains the best solution in the population. A single base design is used in the orthogonal basis function approach for all three target airfoils and in the B-spline approach for RAE2822 approximation. The two base designs used in B-spline interpolation for NACA0406 and NACA0610 are NACA0403 and NACA0606, respectively.

It can be seen that the approximations using orthogonal basis function can produce better results for the NACA supercritical airfoils NACA0406 and NACA0610 than for the RAE2822, this is not surprising, as the set of basis functions used were derived from a family of airfoils containing these two. The errors are believed to be caused by the smoothing process in the derivation of these basis functions.[5] For NACA0406 and NACA0610, the orthogonal basis functions also produce better results than the B-spline interpolation approach, this is because more variables are used in the B-spline interpolation and so it would be much more expensive to obtain the optimum if a comparable number of iterations were used for both cases. However, a different story arises for RAE2822. Since it was not included in the process of deriving the basis functions, this leads to greater error when compared with the B-spline interpolation approach. This indicates the wider applicability of the B-spline approach, at the higher cost of reaching the optimum.

Table 2. Best Approximation of target airfoils for different parameterisations

Method	RAE2822	NACA0406	NACA0610
Numerical Basis Functions	0.2217	0.0582	0.1595
B-spline interpolation	0.1552	0.0758	0.1993

However, similarity in geometry does not always guarantee similar pressure distribution, especially for transonic flows where small perturbations in shape will lead to large variations in pressure. Therefore, the pressure distributions of the approximated shape are also compared to that of the original airfoils, as shown in Figure 4. It can be seen from Figure 4 that the orthogonal basis function approach always produces smooth pressure distributions; also errors are generally bigger in the leading and trailing edge areas than in the middle section of the airfoils.

Figure 5 shows the results of approximation using the B-spline interpolation approach. It can be seen that close agreement can be achieved using B-spline interpolation through 34 points apart from the leading edge area, which indicates that more points need to be placed within this area to achieve better results. Moreover the chordwise coordinates can also be varied, but this would involve higher computational cost while not necessarily increasing the accuracy of the approximation. Another advantage of the B-spline approach is its ability to carry out local shape tunning by varying a subset of the coordinates, while it would be difficult to perform this with the basis function approach, in which, any changes in the coefficients will change the shape globally.

The approach adopted in the current work is essentially an inverse design method. However, it is not used to seek a prescribed pressure distribution, as the definition of the pressure distribution itself is a design problem and accurately re-generating the prescribed pressure distribution often leads to degradation of performance in other conditions. This method can be used to evaluate different parameterisations before carrying out optimisations using the high fidelity codes.

V. Conclusions

Two airfoil parameterisation approaches are studied in this paper to analyse their flexibility and robustness in producing optimal shapes when used in optimisation studies. Global optimisation methods are used to analysis the accuracy these two parameterisations can achieve when used to model three target airfoils. The B-spline approach produces better results in terms of accuracy at a higher computational cost while the basis function approach is more efficient while producing less accurate results. Further work will involve the combination of the basis function approach in the initial stages of design combined with B-spline interpolation for the final tuning of the shapes.

Acknowledgement

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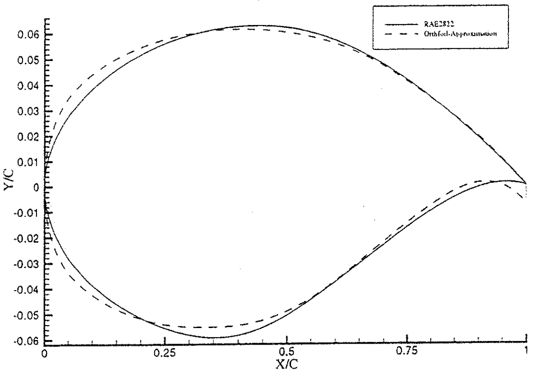
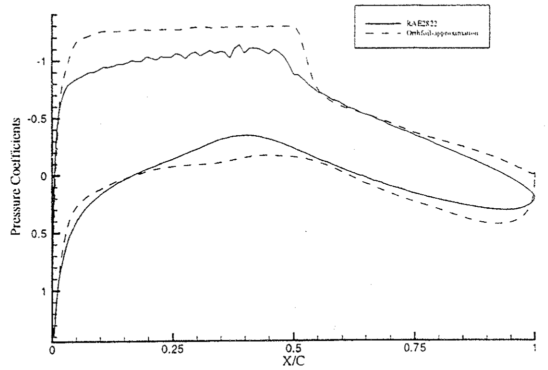
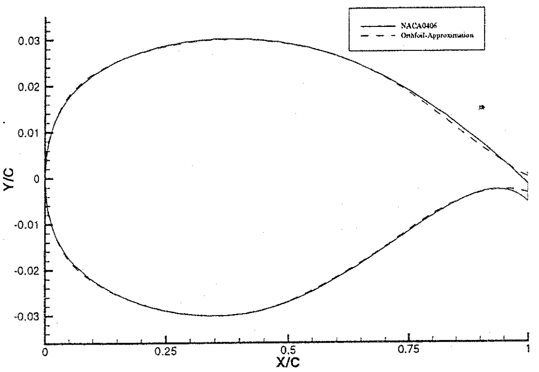


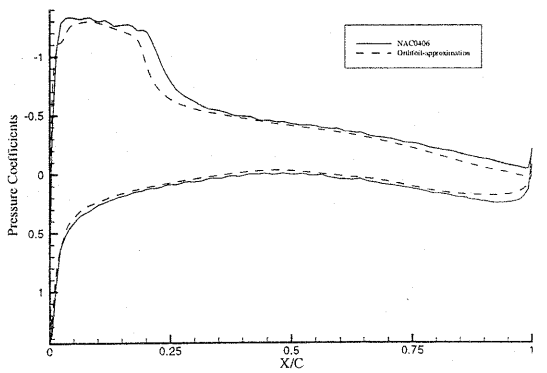
(a) Approximation of geometry to RAE2822 by basis functions approach



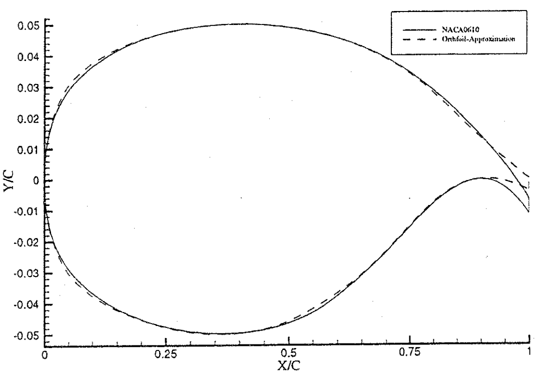
(b) Comparison of pressure distributions for RAE2822 using basis functions



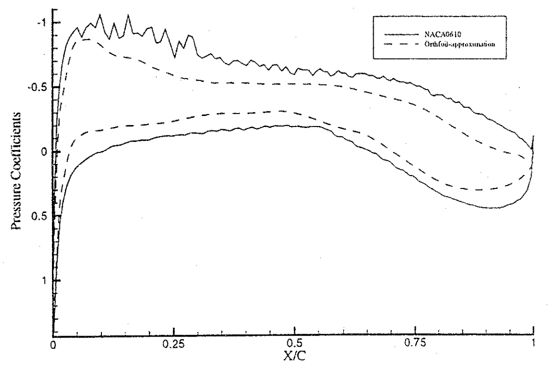
(c) Approximation of geometry to NACA0406 by basis functions approach



(d) Comparison of pressure distributions for NACA0406 using basis functions

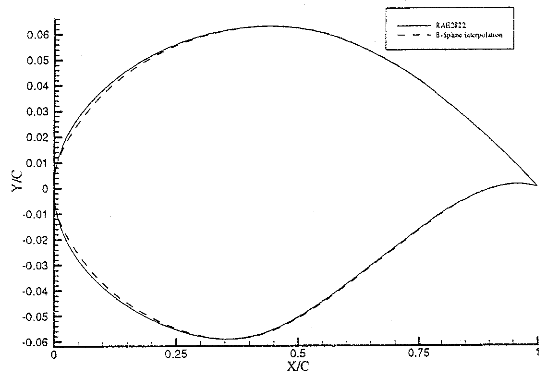


(e) Approximation of geometry to NACA0610 by basis functions approach

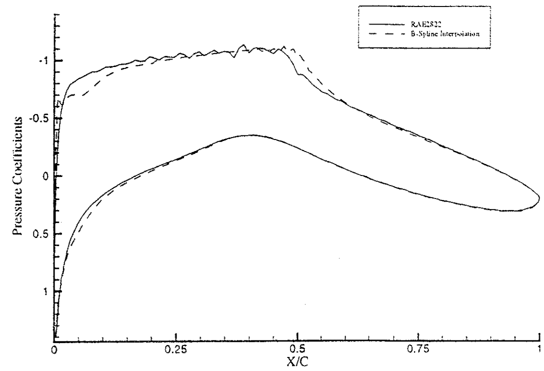


(f) Comparison of pressure distributions for NACA0610 using basis functions

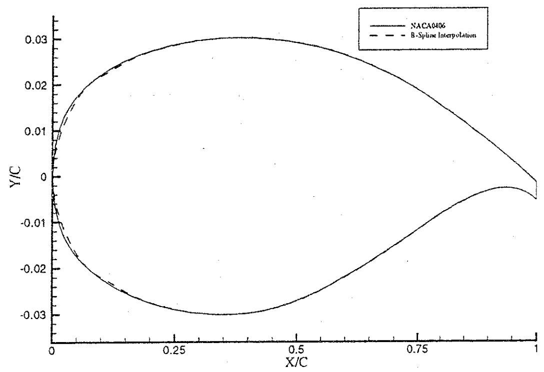
Figure 4. Comparisons of geometry and pressure distributions for approximations using orthogonal basis functions



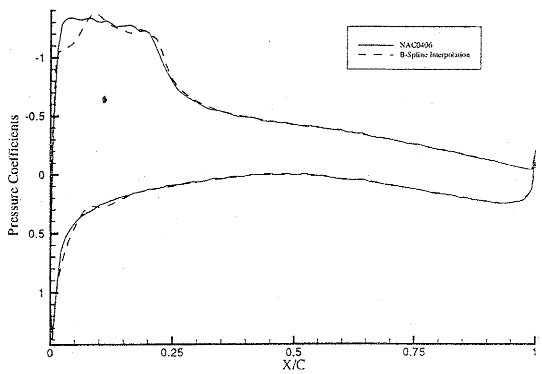
(a) Approximation of geometry to RAE2822 by B-spline interpolation



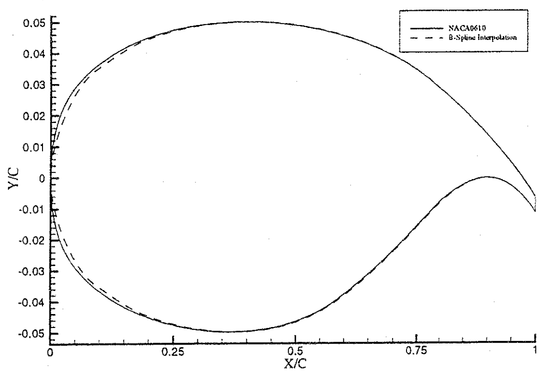
(b) Comparison of pressure distributions for RAE2822 using B-spline interpolation



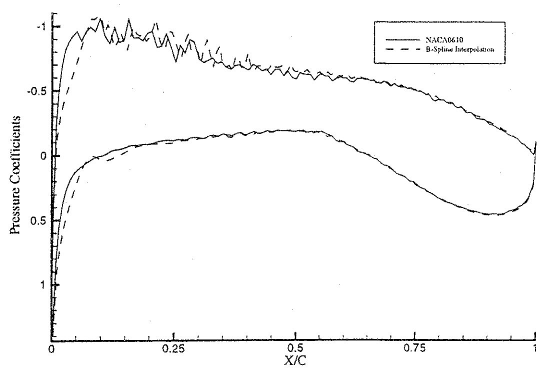
(c) Approximation of geometry to NACA0406 by B-spline interpolation



(d) Comparison of pressure distributions for NACA0406 using B-spline interpolation



(e) Approximation of geometry to NACA0610 by B-spline interpolation



(f) Comparison of pressure distributions for NACA0610 using B-spline interpolation

Figure 5. Comparisons of geometry and pressure distributions for approximations using B-spline interpolations

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