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Brief note: Some observations on oscillating tangential forces and wear in general plane contacts

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Abstract – For general plane contact of elastically similar materials, including cases where there are multiple regions of contact, general properties of the partial slip solution for conditions of constant normal force and monotonically increasing shearing force have been found recently by the first author. An extension is given here to cover the unloading and cyclic loading cases. Further, it is shown that, if the tangential load varies between two fixed limits, the region of stick does not change, *even if relative microslip causes wear*, changing continuously the profile of the indenter. The contact area will change, but wear will not enter the original region of adhesion. The theoretical limit to which wear will eventually, asymptotically proceed is established, viz. almost complete contact over what is the initial stick zone, although it may, in practice, take a long time to reach this state. © Elsevier, Paris

1. Introduction

In many engineering components, the normal load for contacting bodies often does not vary significantly, whereas the tangential load varies cyclically with time. For the case of Hertzian contacts, the problem of partial slip of the contact when first a constant normal load is applied, and then a monotonically increasing tangential force is added, was first solved by Cattaneo (1938). More general loading paths applied to the Hertzian geometry have been considered by Mindlin and coworkers in a series of papers (Mindlin, 1949; Mindlin and Deresiewicz, 1953). The effect of dissimilar material constants, or bulk loads in the contacting bodies, have been considered numerically (see (Hills et al., 1993, §4.6)). Further solutions for problems of this type, but with alternative contact geometries, are much more recent, e.g. the wedge and the cone indenter (Truman et al., 1995). In all cases, regions of slip develop at the contact area boundary, whilst adhesion prevents all relative micro-displacement within the interior. Experimental evidence of these characteristics was found by Johnson (1995), for the shape and dimension of the slip zone, tangential compliance and energy dissipation. The presence of wear was also evident, although it is not clear if there was any significant effect on the measured quantities.

Recently the Cattaneo partial slip contact problem has been revisited by the first author for a general plane contact (Ciavarella, 1998a,b). It has been shown that the contact problem can be reduced to a difference between two normal indentation cases, specifically the actual normal contact problem, and a corrective normal contact problem for a reduced load; this provides the shear correction in the stick zone. In particular, apart from the Hertzian parabolic profile and the wedge geometry, a wedge with rounded apex (Ciavarella et al., 1998a), a flat punch with rounded corners (Ciavarella et al., 1998b), or the general case of a spline profile have recently

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Slip invariably starts at the edge of each contact area, and, following Cattaneo (1938), we consider the shearing tractions as the superposition of a full sliding component and a corrective part $q^*(x)$, non-zero only in the stick zone, whose location and extent are presently unknown

$$q(x) = \begin{cases} fp(x) + q^*(x), & x \in S_{\text{stick}}, \\ fp(x), & x \in S_{\text{slip}}. \end{cases} \tag{12}$$

The integral equation for relative displacements in the tangential direction, Eq. (4) then states, using (12) and (11), and finally substituting Eq. (3) for the full sliding component,

$$\frac{1}{\pi}\int_{S_{\text{stick}}} \frac{-q^*(\xi)/f}{x-\xi}\,\mathrm{d}\xi = \frac{1}{A}h'(x), \quad x \in S_{\text{stick}} \tag{13}$$

which can be recognized as being of the same *form* as the original equation for normal contact Eq. (3), with $p(x)$ replaced by $-q^*(x)/f$, and the function $h'(x)$ is still given by the *actual* normal contact problem, so that the corrective problem corresponds to the same rotation, but with a lower normal load than that of the actual contact problem. A rigorous proof that the analogy holds in the face of the inequalities is given by Ciavarella (1998a).

This analogy allows us to solve a sequential tangential contact problem, as simply as the corresponding normal one. For example, in the case of a single, simply connected contact region, involving *incomplete* symmetrical contact, i.e. with bounded contact pressure at the edges $x = \pm a$, the normal load can be found from (Ciavarella, 1998a)

$$P = -\frac{1}{A}\int_{-a}^{a} \frac{h'(t)t\,\mathrm{d}t}{\sqrt{a^2-t^2}} \tag{14}$$

so that it is clear that there is no need to calculate the normal pressure explicitly. In the same way, as a consequence of the analogy, the tangential load vs. size of stick zone may be calculated from (Ciavarella, 1998a)

$$Q = fP + \frac{f}{A}\int_{-c}^{c} \frac{h'(t)t\,\mathrm{d}t}{\sqrt{c^2-t^2}}, \tag{15}$$

where c is the semi-dimension of the stick zone. Moreover, the magnitude of relative shear displacement in the slip zones may be found directly from (Ciavarella, 1998a)

$$g'(x) = \frac{1}{\pi}\int_{S} \frac{q(\xi)\,\mathrm{d}\xi}{x-\xi} = \frac{f}{A}h'(x) + \frac{1}{\pi}\int_{S_{\text{stick}}} \frac{q^*(\xi)\,\mathrm{d}\xi}{x-\xi}. \tag{16}$$

We now return to a consideration of whether the analogous properties hold also in the unloading phase, when Q reaches a maximum, is decreased back to zero, and indeed a cyclic loading is applied.

2.1. *Unloading and cyclic loading*

So far, we have considered the case of applying a normal load P, giving rise to stick everywhere in the contact area; then P is held constant, while Q is increased monotonically from zero. When Q is first applied, slip zones develop, which start from the last point which comes into contact during the normal loading phase (this is apparent from the analogy). The stick zone therefore recedes as Q is increased, and the slip zones advance. When the tangential load is initially reduced, stick immediately takes place *everywhere*, as a consequence of $\partial g/\partial t$ changing sign, but the shearing tractions remain unaltered. A further small reduction in Q causes backslip within a region close to the edge of the contact (precisely where forward slip started when Q was increased), as the contact pressure is unable to sustain stick in that region. This can also be appreciated from a different point of view: consider the new load Q^u as a perturbation from the maximum load $Q^{\max}$, i.e. as an application of a $\Delta Q = Q^{\max} - Q^u$. Then, if slip were prevented, the tangential tractions would, as in the case of the increasing tangential load phase, become singular at the edge of the contact area, with the important difference that the sign is reversed. Thus a region of counter-slip *must* from, and grow inwards. Consider, now, a new shear stress distribution $q^u(x)$ in the partial slip regime during unloading, so that again the resultant shear is a superposition, this time of the partial slip solution previously obtained, together with a reverse full sliding component (to model correctly the reverse slip in the new slip zones) and a new corrective part $q^{**}(x)$ in the new stick zone S^u_{stick}—location and dimension presently unknown—as

$$q^u(x) = fp(x) + q^*(x) - 2fp(x) + q^{**}(x), \tag{17}$$

where $q^*(x) = 0$, $x \in S_{\text{stick}}$, $q^{**}(x) = 0$, $x \in S^u_{\text{stick}}$. In the new stick zone S^u_{stick}, $g'(x) = 0$ only in the part where there had not been slip in the previous increasing shear load phase, i.e. $S^u_{\text{stick}} \cap S_{\text{stick}}$, whilst in the remaining zone the amount of slip displacement is fixed at the value it reached at the end of the loading phase, given by Eq. (16). The integral equation for relative displacements in the tangential direction (4) states that, using (17), in the new stick zone upon unloading, $x \in S^u_{\text{stick}}$

$$\begin{aligned}\frac{1}{A}g'(x) &= \frac{1}{\pi}\int_S \frac{q^u(\xi)\,d\xi}{x-\xi} \\ &= \frac{f}{A}h'(x) + \frac{1}{\pi}\int_{S_{\text{stick}}} \frac{q^*(\xi)\,d\xi}{x-\xi} - 2f\frac{1}{A}h'(x) + \frac{1}{\pi}\int_{S^u_{\text{stick}}} \frac{q^{**}(\xi)\,d\xi}{x-\xi}.\end{aligned} \tag{18}$$

Then, on substituting Eq. (16) into (18) we find that $q^{**}(x)$ is the solution of the following integral equation. Note that $\frac{1}{A}g'(x)$ on the left hand side cancels out with the first two terms on the right hand side, so that

$$\frac{1}{\pi}\int_{S^u_{\text{stick}}} \frac{q^{**}(\xi)\,d\xi}{x-\xi} = 2\frac{f}{A}h'(x), \quad x \in S^u_{\text{stick}} \tag{19}$$

which is again the normal indentation problem, *rescaled* into the new unloading stick zone S^u_{stick} (note that this time the sign is the same as that of the pressure, and the additional factor 2). All the inequalities for the problem of finding $q^{**}(x)$ in S^u_{stick} can again easily be shown to correspond to the inequalities for the normal pressure $p(x)$, for the contact area S^u_{stick}. Thus, the analogy continues to apply in the unloading, and even reloading, regimes.

It is clear that this superposition procedure can be repeated *ad libitum*, and that the only care required is in the imposition of the correct direction of slip in the slip zones. However, in plane problems only two conditions are possible (slip and reverse-slip), depending of the sign of the tangential load. We repeat that the results are completely general, providing only that a half-plane formulation is appropriate to the contact problem itself; multiply connected contacts, associated with surface roughness and individual asperify effects are examples of cases where these results may be used, providing that the normal contact problem itself may be solved. It would even be possible to use the approach to solve problems involving a very complex profile, perhaps solved, for the normal contact case, by finite elements, providing the half-plane requirement is met. This is particularly useful when the partial slip solution is difficult to obtain, as is often the case, by purely numerical procedures.

3. Implications for the wear process

Fretting involves partial slip under normal contact; it is therefore inevitable that some wear takes place, as these are the two ingredients needed. Results of fretting fatigue tests are sometimes regarded skeptically because the question is asked '*As wear has taken place, has the contact geometry changed substantially?*', and hence, by implication, modified the conditions assumed for fretting itself. This question is not easy to answer, although various deductions could be made, if a wear law were available. As these are notoriously difficult to formulate and render 'geometry-independent', it is worth investigating the results deduced above to see if any light can be shed on this problem. First, we regard it as essential to have both contact pressure and relative movement in order to have wear; it follows that within the stick zone wear cannot take place, and clearly nothing can happen exterior to the contact. Wear must therefore be confined to zones of slip, where the profile of the indenter will change. Now, as long as the relative rotation of the bodies is fixed, the minimum size of the stick zone during a load cycle, i.e. when the full tangential load is applied, is determined by the corrective normal contact problem, which is always the same, and independent of what is happening in the slip zones. In other words, although the pressure distribution will change (over the entire contact area, and *not* only in the slip zones), and hence so will the tangential relative slip displacement in the slip zones, the position and dimension of the stick zone will remain *fixed*, so that wear will never enter these zones.

There are therefore two issues here, one is the timescale of the evolution of wear, which is very difficult to attack without a great deal of experimental data, and the second is the steady state limit towards which the system will ultimately move.

3.1. *Evolution and Archard's law*

If we assume that wear proceeds according to Archard's law, which is probably the most universally accepted, the rate of material removal, w, is proportional to relative micro-slip velocity and local pressure

$$w \approx A \cdot \frac{\delta u_x(x)}{\delta t} \cdot f p(x), \tag{20}$$

where A has dimensions of $[L^2/F]$. Further, on noting that $u_x(x)$ is given by Eq. (16), and considering that the integral on the right hand side of (16) is constant during the wear process (as neither $q^*(x)$ nor S_{stick} changes) we deduce that the wear rate will have a component directly dependent on the pressure plus a second component dependent on the value assumed by $h(x)$. Because of wear, the contact pressure will decrease in the slip zones, and hence the wear rate itself will also eventually decrease with time. We could argue that wear at the highest contact points occurs first, so as to reduce the contact pressure to a spatially constant value in the slip areas. This simple assertion is physically reasonable for *sliding* contacts. However, it is difficult to sustain under partial slip conditions, for some geometries. For example, for a Hertzian contact, it would imply that wear would occur adjacent to the stick-slip interface, just within the slip zones. But wear could not occur there without a loss of contact, as clearly adjacent material just within the stick zones will not erode. Hence, the only way in which a constant pressure state could be approached would be the *addition* of material, which is extremely unlikely to occur unless wear debris volume exceeds that of the wearing material.

3.2. *Steady state limit*

In most fretting problems the applied normal load is controlled, so that the pressure in the stick area will *increase* to support the load (i.e. a complete contact will evolve, where the pressure is singular at the stick area edges), and ultimately wear will cease. For this to arise, because the relative slip displacements are never zero in the slip zones, the *only* possibility is that the pressure goes to zero there, i.e. there is incipient loss of contact. An asymptotic contact problem can therefore be postulated for which the load is supplied entirely over the minimum stick zone, and the contact profile just touches the regions of slip. It is possible to calculate the amount of material to be worn away, simply from the boundary value problem for the complete contact in the stick area, knowing the overlap of material that would arise for the original profiles. This result should be of some use in determing the length of time for which wear may be expected to occur, and hence give an indication of the rate of change of the contact profile.

4. Conclusions

The case of oscillating tangential forces has been treated for a general configuration of the contact for elastically similar materials. The implications for the wear process in fretting of plane contacts has been considered, and some general properties have been given, as related to the changes in the geometry of the contacting materials. It has been shown that the wear operates only in the region of initial micro-slip, and that wear must always proceed so as to give a steady state, of full adhesion. This property is independent on the assumed number or shape of the contact areas, and also serves to give a clear idea of the amount of material that will eventually be worn away.

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