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Numerical methods for the optimisation of specific sliding, stress concentration and fatigue life of gears

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Abstract

Presented in this paper is a set of modern tools for the design of gearing: kinematic optimisation (minimisation and balancing of specific sliding), static stress analysis (to minimise stress concentrations) and crack propagation studies (to estimate fatigue life under a pre-existing defect). All three aspects are integrated in a software package developed by the authors. In particular, Boundary Element (BE) and Finite Element (FE) grids are automatically generated corresponding to gears manufactured by means of user defined tools with known shape and cutting parameters. BE models are used for a complete and automatic subcritical propagation analysis of cracks. FE models are used mostly for cases without crack propagation but requiring a greater versatility.

Tests conducted on cases found in the literature demonstrate the accuracy of the methods used and the effects of rack shift factors and of rim thickness are studied in example cases.

It is found that the fatigue life depends significantly on the cracking path mode, which in turn is particularly sensitive to the rim thickness in gears manufactured on thin hollow shafts as are typical in aeronautical applications. Further, the rack shift factors significantly change the stress concentrations (and therefore the maximum torque transmittable, in general in a beneficial manner). However, for designs with same concentration factor the fatigue life is considerably different, and in particular is lower on gears with a low number of teeth. This clearly indicates that the use of a complete crack propagation analysis from the early stages of the design process is highly recommended.

1. Introduction

The design of gearing is one of the classical topics of machine design, for obvious reasons of technological importance. Providing a complete bibliography on the subject of gearing and gear boxes here is impossible if not inappropriate. For this we recommend standard textbooks on machine design [1,2], or consideration of the monographs [3–5].

The classical route followed for the design of gears is to appeal to standards, like AGMA, DIN or ISO (see References [18–28]). These standards are based on extremely large collections of results and empirical rules from practical experience in a vast range of engineering applications. They provide a set of formulae, rules and charts to design the gearing taking into account various working conditions and several aspects of their performance, such as the power level, degree of service intermittence, noise, lubrication conditions, wear rate, likelihood of impact, pitting, and corrosion. A great part of the design process is concerned with the choice of materials and their treatments. The standards are so detailed and complex that various associations of gear manufacturers offer them in software form (see the reference list for standards) and other packages of various kind are widely available to the engineering community to help the gear design process (a quick search into the WEB provides information about this).

The degree of complexity and refinement of the design process varies considerably from one application to the next: in some cases, a quick calculation for static strength as a function of the torque transmitted is sufficient (from the celebrated Lewis' formula). The standards, for strength calculation, are based on a more refined version of Lewis' formula, taking into account fatigue and other phenomena with appropriate coefficients; for example AGMA2101 gives the following formula:

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$$s_t = W_t \frac{K_a}{K_v} \frac{P_d}{f} \frac{K_s K_m K_B}{J} < s_{at} \frac{K_L}{K_T K_R} \quad (1)$$

where s_t is the bending stress, W_t the tangential load, K_a, K_v, K_s, K_m, K_B are respectively application, speed, size, load distribution and rim factors. P_d is the diametral pitch, f the face width, J the geometry factor. The allowable stress index depends on the material (s_{at}), and K_L, K_T, K_R are the life, temperature and reliability factors.

Nowadays, a number of other factors are almost invariably also considered. For example, surface fatigue. Typically, a surface stress index is defined, depending on the Hertzian contact stress and a formula analogous to Eq. (1) is used to evaluate the allowable maximum contact stress. More specialised applications require new and more precise design procedures, and in the literature various studies are appearing at an increasing frequency which consider various aspects of the optimisation of gear shape and the evaluation of their strength using modern numerical methods. For example, we can classify kinematic, stress analysis and crack propagation studies.

In the first category, we cite References [6] and [7]; they attempt to study the balancing of specific sliding, according to what is recommended in principle already in the classical handbook [3], the localised bearing contact or the misalignment error, which affects mainly the noise and vibrations produced.

In the second category, we cite for example Reference [8], for the use of an approximate solution with the complex variable formulation of elasticity, in obtaining a more accurate estimate of stress concentration factors at the fillet root, which allows a better fatigue analysis according to classical fatigue design handbooks (e.g. References [1 and 2]).

Finally, in the third category References [9–13] all consider sub-critical propagation of fatigue cracks from gear tooth roots, using different methods. Some experimental results are also usually provided. In particular, References [9] and [11] use a special FE code (FRANC) to automatically propagate the crack without manual update of the grid, and concentrate their attention on gears of reduced hubs, and successfully compare experimental results of crack path test with the numerical predictions. Blarasin et al. [10] concentrate on the residual stress treatments (carburized or carburized and shot peened gears) and employ 2D and 3D FE techniques, or the Weight Function technique, which also gives good agreement with experimental results of fatigue life. Finally, Pehan et al. [13] moves towards a more realistic treatment of the load applied to the gear, considering 3D effects.

Although kinematic studies do consider the actual shape of the gears, this is an aspect that is not generally taken into account in the other two categories. However, this aspect of the analysis is important if a refined method is used for the stress analysis and the residual life calculation: in other words, it may well be that the error introduced by using an approximate geometry is larger than the error removed with a more refined stress analysis with respect to the hand calculation methods. Two examples are worth elucidating: the use of rack shift and the need for light gearing (for which the rim thickness of the gear is reduced, and in the limit the inner diameter of the gear is coincident with the outer diameter of the shaft). It is well known that the tooth profile would be completely external to the base circle only for gears of more than 35 teeth. However, the non-interference conditions are satisfied if the pinion has more than 17 teeth, but many applications need an even lower number of teeth, so that it is common practise to use rack shift, altering the geometry of the tooth considerably. Indeed it is not unusual to find as few as 8–11 teeth instead of 17 teeth.[1] On the other hand, in many cases, for instance in aeronautical applications, it is necessary to have extremely light gearing, and this need is usually associated with the use of a very high number of teeth, avoiding the need for rack shift.

In several applications, e.g., speed shift or gearboxes for automotive applications, earth-mechanics, and more general industrial applications, it often happens that gearing ends up having, at the end of its (usually the vehicles) service life, a considerable residual life. This is unlikely to be true for other parts like internal combustion engines, clutches, brake systems, actuators. It would then be worthwhile to optimise the design in order to have the same service life for every part of the machine.

Currently, 'optimal' solutions are *not* achieved in the design process because it is considered impractical to study a great number of configurations obtained by varying the various relevant parameters (number of teeth, modulus, rack shift factors, material properties). The design process would take too long, even assuming it was possible to discern the best way to proceed. Some fundamental problems, i.e., the amount of force shared between teeth when more than one tooth is meshing, the effect of the backlash on the shared force, are not accurately assessed with the empirical formulae. However, it would be possible and desirable to have a more efficient and accurate understanding if ad hoc tools were available.

This paper presents a set of routines designed to consider these aspects, and some examples are presented to show the kind of analyses that are then possible.

[1] These considerations apply strictly only when the ratio between addendum and modulus is 1.

2. Kinematic optimisation

2.1. Gear cutting and mesh simulation

The geometries of the gears are obtained by simulating the cutting process. The data needed are the numbers of teeth of both the gears, the modulus and the shape of the tool to be used. The modulus (defined as the ratio of the diameter to the tooth number) is the basic dimension for gear sizing.

The shape of the cutting tool is given as a set of piecewise linear parametric curves (in Cartesian or cylindrical coordinate systems), i.e., straight segments, arcs of circle, arcs of spirals. A library of standard tool shapes (according to AGMA, ISO) has been implemented. The choice of a further fundamental geometric parameter, the pressure angle, i.e. the angle formed by the force direction with respect to the ideal tangential direction, is automatically included in the choice of the tool.

In the simulation process, the coordinates of the points of the profile are calculated by finding, in the relative motion of the tools with respect to the gear, the point of the tool whose trajectory is tangent to the tool profile. In this way it is possible to obtain, in the envelope cutting process of involute gears, the accurate shape of the parts of the profile having a non-elementary analytical representation. This is generally needed for the fillet at the base of the tooth, where stress concentration arises. Basic geometry, terminology and examples of the manufacturing and meshing simulation, are shown in Fig. 1(a)–(c), respectively.

2.2. Specific sliding for wear rating of gears

In the simulation of the gear meshing the code performs calculation in order to evaluate the specific sliding, defined for each gear as

$$K_{s1}=\frac{V_{s1}-V_{s2}}{V_{s1}};\ K_{s2}=\frac{V_{s2}-V_{s1}}{V_{s2}} \tag{2}$$

where V_{s1} and V_{s2} are the tangential components of the absolute velocity of the points in contact under consideration (Fig. 1(d)). In gears, specific sliding changes during meshing. Therefore, in order to achieve a reliable and regular working life of the gear set, it is desirable to limit the maximum value and modify the gear design such as to make it of the same order for both gears (balanced design). Note that $K_{s1}=-$Infinity when the contact point is at the end of the pressure line. In fact, one or the other of the profiles has in that point its involute starting point, having locally an infinite curvature.

The most used technique for the purpose of balancing and minimising specific sliding is the use of rack shift or addendum modification. It consists in the radial shift of the tool (the hob or the rack), that changes the portion

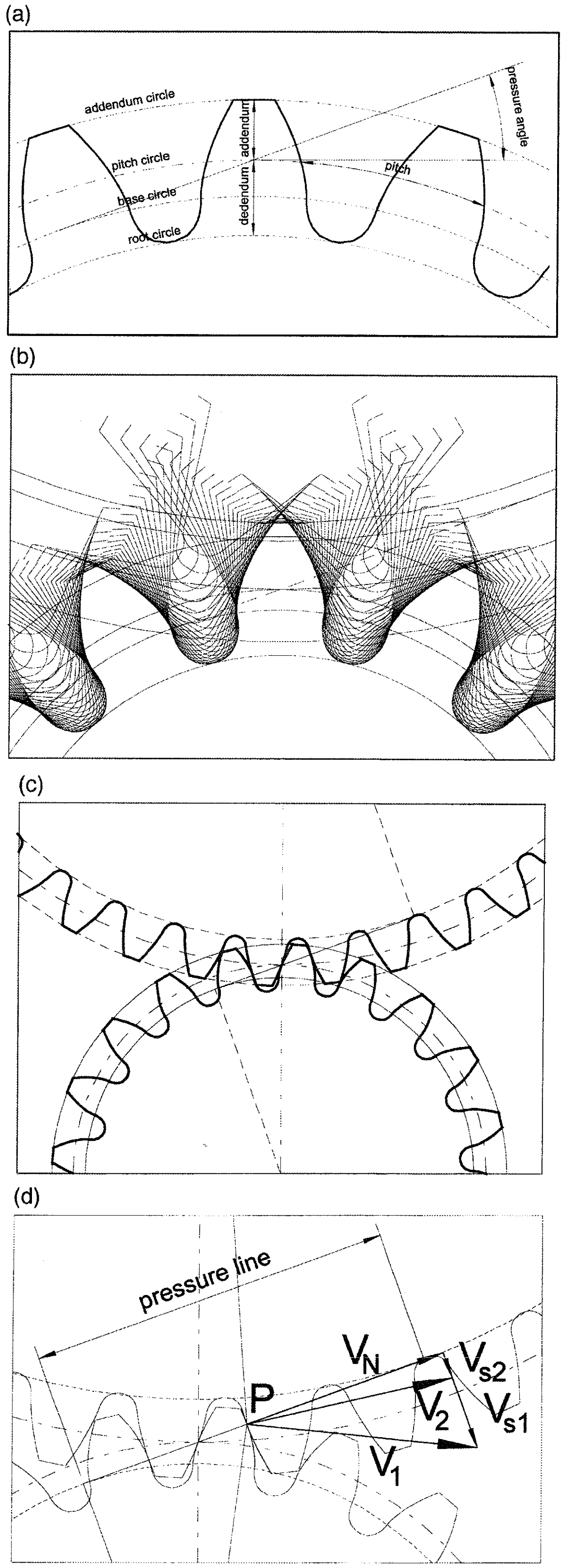


Fig. 1. (a) Basic geometry and symbols; (b) Simulation of the manufacturing process; (c) Simulation of the meshing; (d) Velocity vectors at the contact point.

of involute to be included in the tooth shape. Different rack shifts can be given for each gear, so that contact starts (and finishes) where the maximum of the absolute value of specific sliding is less than design specification for both gears.

2.3. Rack shifting (addendum modification)

Rack shifting (or addendum modification) is a manufacturing technique, which can be adopted to change the shape and the addendum diameter of the tooth. It changes the extension of the contact arc and, as a consequence, the values of the specific sliding. In standard sizing, the addendum of gears is set equal to the modulus, whereas rack shift changes the external radius and so addendum is changed.

3. BE (and FE) model generation

Using the shape of the tooth obtained numerically, a BE grid is generated for use with various standard computer programs. Currently, 2 BE programs are supported [14,15], with the typical options for the user to introduce the range of the element dimensions and grid refinement. For FE use, paving techniques [16,17] are used, an internal grid is generated using triangular elements (currently only the FE program ANSYS is supported, see Fig. 2 for an example). Fig. 3 shows typical results from an FE analysis (here displacement vectors are represented in a 3 tooth model, to examine the appropriate conditions for the successive use of a simplified model with only 1 tooth in the detailed crack propagation analysis using BE). A typical BE mesh is presented in Fig. 4.

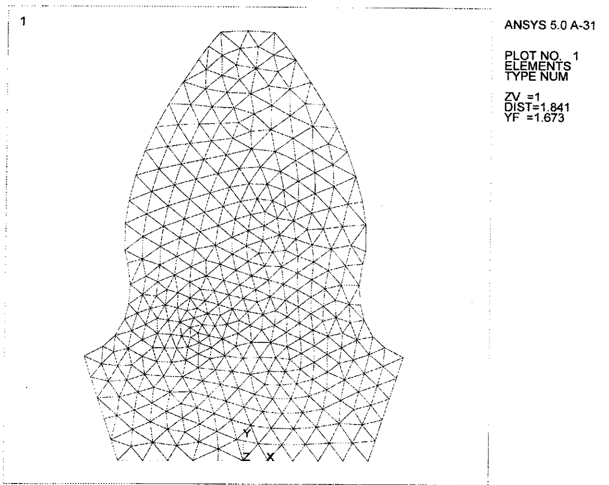


Fig. 2. Basic sector of gear tooth for the FE mesh automatically generated for the program ANSYS.

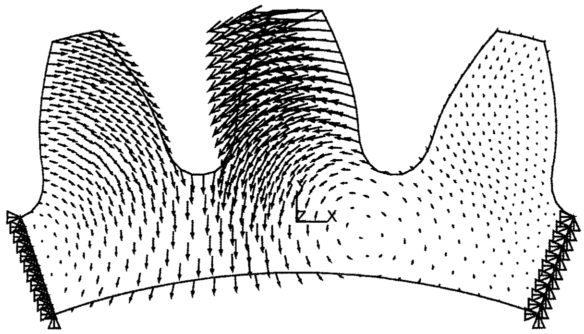


Fig. 3. Vectorial representation of displacements in a sector of a 3 tooth gear crown used to obtain most appropriate boundary conditions in the single tooth model.

Fig. 4. Shape of the tooth when rack shift is introduced and path of the cracks in the cases analysed.

3.1. Stress analysis and crack propagation

Some basic considerations that must be made before proceeding further into the analysis are the following, based on geometrical and working conditions:

1. at the start and end of the meshing there are two or more teeth in contact (in some applications many more);
2. the load moves along the flank of the tooth (in intermediate gears or satellites of epicycloidal gearing the load is alternating), and there are 3D effects;
3. the tooth thickness at the tip is slightly reduced due to tip relief introduced to avoid impact in the first meshing contact;
4. the material has a thermal treatment, and therefore its hardness varies with depth, affecting Paris' law parameters;

5. the material has often been treated (commonly by shot peening), so there are significant residual stresses in a layer of a certain thickness, which in general would require a detailed non-linear material analysis to be determined with accuracy.

For the solution of crack propagation problems, we make some simplifying *assumptions*. We consider only one tooth is in contact, with the contact force being applied at a point fixed in space but varying in intensity from zero to a maximum value. Moreover, we neglect the tip relief, as consistent with the latter choice, and we ignore 3D effects. Finally, we consider the material to be perfectly elastic, without any thermal treatment, and with a crack propagation law of classical Paris' form, although the BE computer code used [15] allows the adoption of different user defined propagation laws.

3.2. Comparison with a test case (Ballarini et al. [11])

The reference case studied in order to evaluate the present numerical method is the one discussed in (Ballarini et al. [11]). The gear ratio adopted in the cited reference is τ=1.0 whilst here a τ=0.5 ratio has been adopted to perform different rack shift designs, in order to evaluate the effect of the rack shift on this particular gearing. The change of τ does not significantly affect the comparison purposes in the case without rack shift.

A pair of gears in mesh has been generated, having 28 and 56 teeth and a modulus of 3.2 mm. The tool used to generate the teeth is a 20° pressure angle standard AGMA rack, having the ratio between fillet radius and the modulus equal to 0.35. The BE model has been generated for the 28 teeth gear only, because the other gear is more robust even when a negative rack shift is introduced.

The mesh has been optimised by finding the minimum specific sliding condition and by adopting two different pairs of rack shift. The first preserves the value of the centre distance with opposite rack shifts having the same absolute value. The second is more efficient in the reduction of specific sliding but the rack shifts are different for the gears. The data for the two cases are summarised in Table 1, where specific sliding Ks_1 and Ks_2 are defined as in Section 2.

Using the code for the generation of the gear profiles, either for the case of normal cut or for that with rack shift, we have determined the phase of meshing which corresponds to a single tooth in contact and to the maximum distance of the point of application of the force exchanged from the axes of rotation. In this configuration, a BE grid has been produced in the absence of the crack and the most stressed point at the notch is determined.

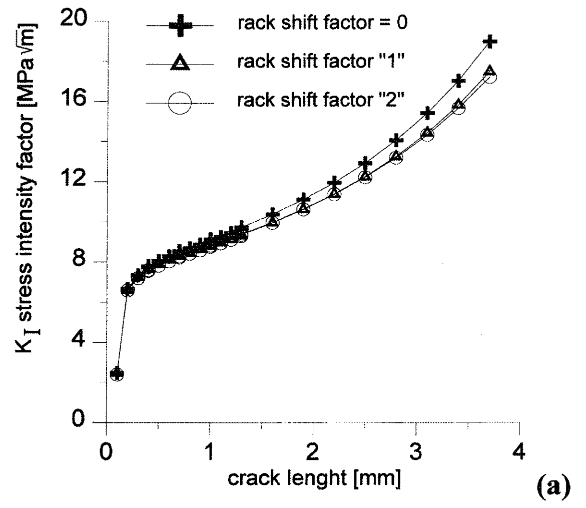
Correspondingly, the value of the force is computed which will produce a maximum principal tension in this point at the notch of 300 N/mm^2. The value of 300 N/mm^2 is used since it is typically in the range of the fatigue limits of the materials used in gears.

In the following generation of the grid, a crack has been introduced: the crack has dimension 0.1 mm, and is situated normally to the surface of the fillet, in correspondence of the most severely stressed point. The parameters used in the Paris law of propagation are m=3.3, C=2.8$e-$14, with propagation speed expressed in mm/cycle and the variation of the stress intensity factor in N/mm$^{3/2}$. Notice that as the load is moving along the face of the tooth, the variation of stress intensity factor would have a quite complex cyclic history, but in the use of Paris' law it is, practically, important to consider only the maximum and minimum values, and therefore the calculation of the maximum is correctly given here (the minimum being zero).

The results obtained are shown in Figs. 4 and 5. For every case studied, two analyses have been completed, each one composed by twelve steps of propagation, the first one with increases of the crack of 0.1 mm, the second with increases of 0.3 mm. In Fig. 5(a) the variation of the stress intensity factor K_I as a function of the dimension of the crack is shown. The results obtained for the case without rack shift are practically coincident with those obtained in Reference [11], showing just a slightly improved accuracy in the interval of the dimensions of the crack up to 1 mm. For the two types of rack shift studied, the values of the stress intensity factors in mode I are slightly inferior to those corresponding to the case without rack shift and practically coincident with these. Notice that for very long cracks, the assumptions of dominant elastic behaviour in the fatigue propagation process may be inadequate, but the life of the gear is at

Table 1
Rack shift, centre distance, specific sliding and pressure angle for the case studied

Case	x_1/m	x_2/m	centre distance [mm]	Ks_1	Ks_2	pressure angle
No rack shift	0	0	134.4	−1.81	−1.01	20°
Rack shift '1'	0.17	−0.17	134.4	−1.28	−1.22	20°
Rack shift '2'	0.34	0.17	135.966	−0.96	−0.99	21.74°



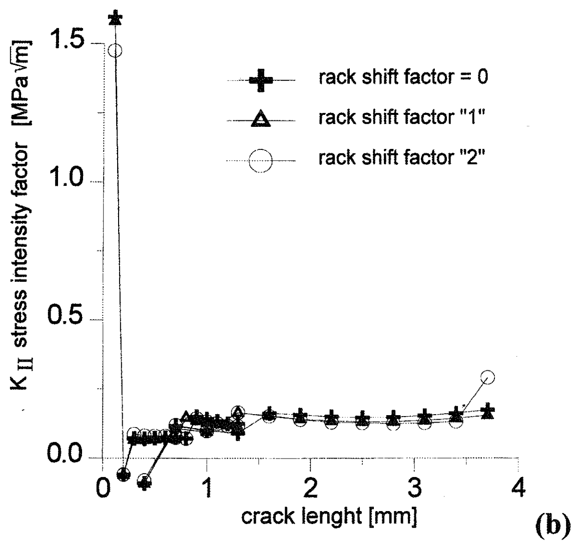


Fig. 5. Effect of the rack shift factor on the stress intensity factors for the two cases analysed, (a) mode I, (b) mode II.

that point very short and therefore such inaccuracies can be neglected.

In order to really estimate the effect of the rack shift, it is necessary to consider that rack shift makes the tooth stronger, and so the comparison is made keeping constant the maximum stress in correspondence with the fillet. This means that the point of application of the forces has been fixed nearer to the top of the tooth, and the resulting bending moment is bigger at the root of wheels with rack shift. In this sense, the transmitted couple increases using rack shift, in comparison to the case of a wheel without any rack shift, by 7.1% for the first type of rack shift and by 7.5% for the second type.

With an equal transmitted couple, rack shift therefore provides a considerable increase in the residual life of the gear. As regards the variation of the mode II stress intensity factor, shown in Fig. 5(b), the values calculated for very small cracks are coincident with that reported in Reference [11], and so are the values for crack longer than 1.3 mm. In the intermediate range, relatively significant numerical errors are obtained, which are sensitive to the fatigue propagation crack step dimension (0.1 mm in comparison to 0.3 mm step). Such errors are, however, negligible for the modest absolute value assumed by the more important opening stress intensity factor K_I.

In Fig. 4 the trajectory of propagation of the cracks for the three cases examined are shown. The patterns are homogeneous and tend to fracture the teeth according to a direction normal to their mean line.

When the gear is not a solid block, i.e. is supported by a rim of reduced thickness connected to the shaft, or it is directly manufactured on a hollow shaft, it is extremely important to find the minimum dimension of the rim that guarantees a suitable safety factor. Note that if the crack propagates through the rim, an unexpected catastrophic failure may follow; on the other hand, a crack crossing the tooth can be detected in the early stages, since it produces an unusual periodic noise or vibrations before the defect becomes critical. The crack propagation is usually very fast in cases when the crack crosses the rim almost perpendicularly. The analysis requires the modelling of a congruous number of teeth in order to simulate the compliance of the whole gear. In a simplified model with only one tooth used, it is necessary to set up a proper constraint scheme.

For this purpose a grid has been generated for a sector of a gear having three teeth. Various ratios between the rim and heights of the tooth in the range 0.3–1 have been considered. A qualitative example of the results obtained is shown in Fig. 3. As in the BE grid adopted for the crack propagation study only one tooth was typically modelled,[2] the qualitative FE results for displacement fields have been used to understand the best boundary conditions to apply in the simplified model. From Fig. 3 it is clear that a justifiable assumption is to consider the radial boundary lying in the same half plane of the applied force as being completely constrained. On the opposite radial boundary, only the normal displacements of the elements are constrained. Indeed, in the tangential direction a uniform traction (directed radially) has been introduced. The value of the resulting force is equal to the 70% of the applied force.

It has been verified that the patterns of the cracks and the values of the stress intensity factors are not influ-

[2] There is no real need for this simplification. Even for the full crack propagation analysis, the computer requirement is very small. However, in our version of the BE code, the program was running on a PC machine and was limited to about 140 elements, of which only 20 have boundary constraint. Still there was no real need for a larger analysis.

enced by the value of this force. The analysis of the propagation of fatigue cracks has been performed on the tooth without any rack shift assuming different values of the ratio between least radial dimension of the rim t and height of the tooth h (Fig. 6). The patterns of the cracks, are shown in Fig. 6. Note that in the case 'a' (t/h=1) the crack crossed the tooth, while in the case 'b' (t/h=0.75) the crack grows through the rim (although it eventually deviates in bending mod). For a suitable value of the ratio t/h the crack will propagate with a pattern having the maximum extension. This would be the 'optimal' choice for service life duration.

However, a better choice for the rim thickness would be the one a little higher than this, just ensuring that the crack propagates through the tooth. In the range considered for the ratio t/h the curve of the K_I versus the crack length results close to the curve for the solid gear (Fig. 7). In further analysis performed for t/h=0.5, the increase of the stress intensity factor turns out to be more sensitive than the increase found in Reference [11], a fact that may be due to the slightly different boundary conditions of the analysis.

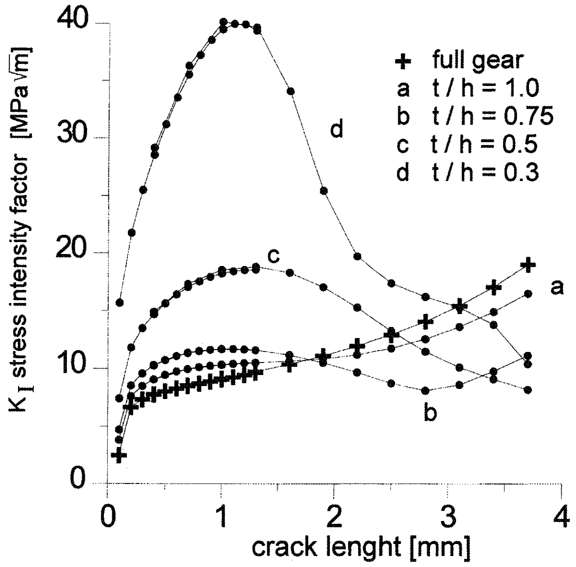


Fig. 7. Mode I stress intensity factors for the cases of rim thickness examined.

4. An example of integrated design

Having validated the codes in the previous section, we move on with an example of more realistic design. The complete design possible with our set of codes is shown for a spur gear pair with a pinion and a gear having, respectively, 10 and 20 teeth. The pinion and the gear have been generated with a 3.2 mm modulus and 20° pressure angle standard AGMA rack.

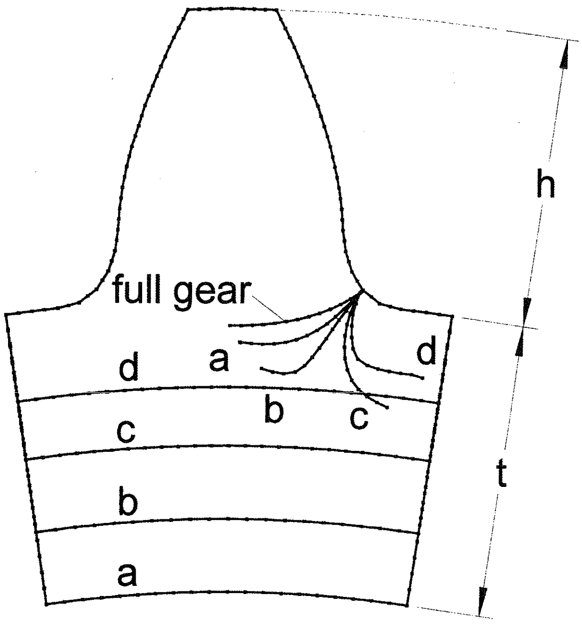


Fig. 6. Effect of rim thickness on the crack paths. ((a) corresponds to t/h=1, (b) to t/h=0.75, (c) to t/h=0.5, and finally (d) to t/h=0.3).

Fig. 8(a) shows the gear pair without addendum modification. In this case undercutting is necessary to prevent interference and the pinion is considerably weakened. The first set of rack shift introduced (x/m=0.2, x'/m=−0.2) holds fixed the centre distance (see Fig. 8(b)). The section modulus is increased at the base of the root but the specific sliding of the pinion is still infinite. The second set of rack shift (x/m=0.4, x'/m=0.2) introduces a remarkable increment of the section modulus at the base of the root and gives reduced and well balanced specific sliding (Fig. 8(c)). The changes in the shape of the gears can be appreciated in Fig. 9. As for the previous cases the effect of the modulus modification on the transmitted load has been evaluated generating a BE model. For each case the position of the highest point of single tooth contact has been determined. For a fixed value of the maximum allowable stress in the fillet root, the calculated ratio between the load transmitted in the case 'b' and the load transmitted in the case 'a' is 1.19. In case 'c' a major increment of the load transmitted is achieved and the ratio respect to case a is 1.58.

The crack propagation problem has been solved by introducing a 0.2 mm crack in the root fillet of the pinion. In this case a solid blank has been considered. The crack patterns for the three cases analysed are shown in Fig. 10.

Although the transmitted loads are different (the maximum local stress is the same) the rack shift pair that produces a balanced reduction of the specific sliding gives also the best performances in the case of a crack emanating from the root fillet as shown in Fig. 11.

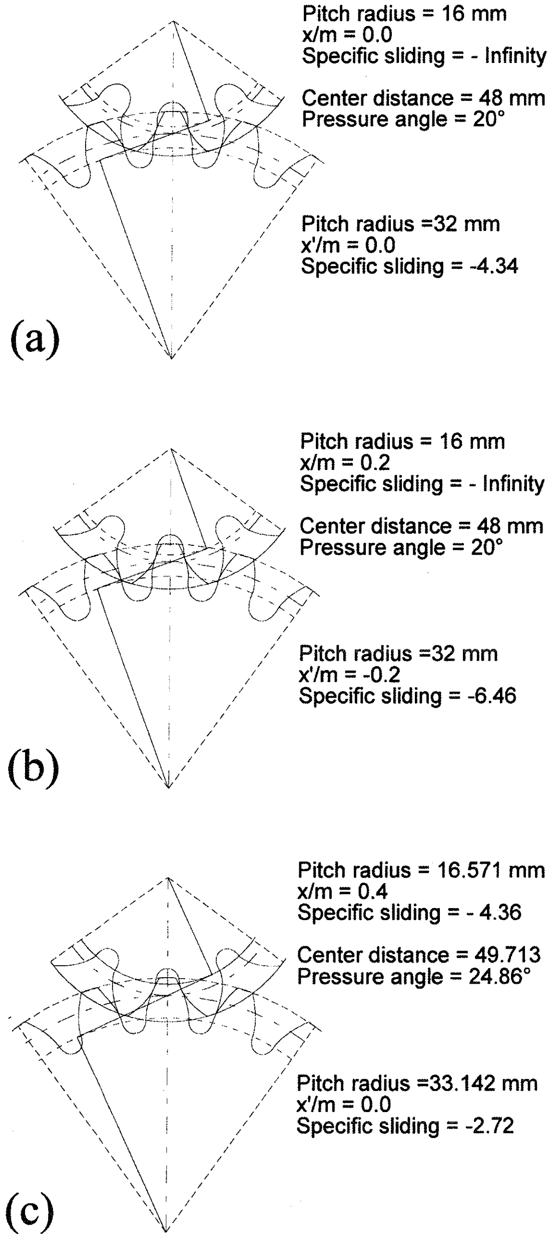


Fig. 8. (a–c) Example of specific sliding optimization.

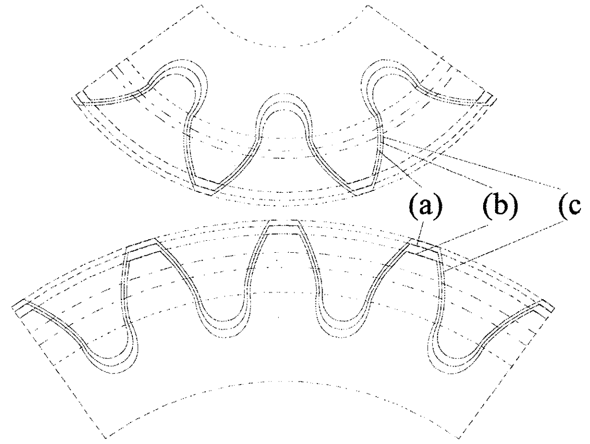


Fig. 9. Change of shape of the teeth during the process of specific sliding optimisation.

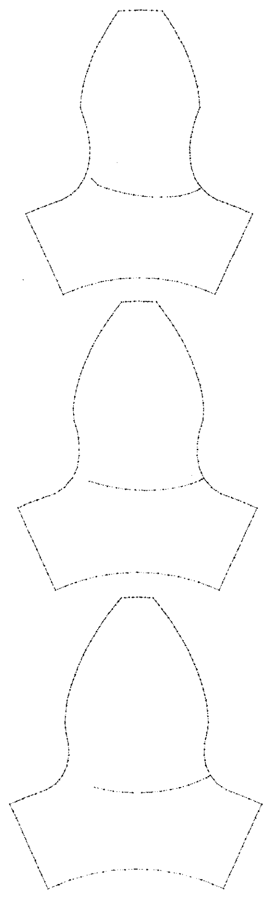


Fig. 10. Cracking path for the integrated design example.

5. Conclusions

A software for the solid modelling of gears and integration with numerical methods such as stress analysis using FE and BE, and crack propagation analysis have been produced by the authors and their use and possible results shown. The adoption of these methods of analysis is extremely rapid and efficient in each phase (meshing simulation, kinematic optimisation, stress and crack propagation analysis) thanks to the use of state of the art methods for each kind of analysis. The comparison with the available numerical and experimental results in the literature has shown, at the same time, the accuracy of the procedures.

In the case of gear designed with a rim of reduced radial thickness, the results confirm the existence of a limit of the thickness of the rim under which the crack

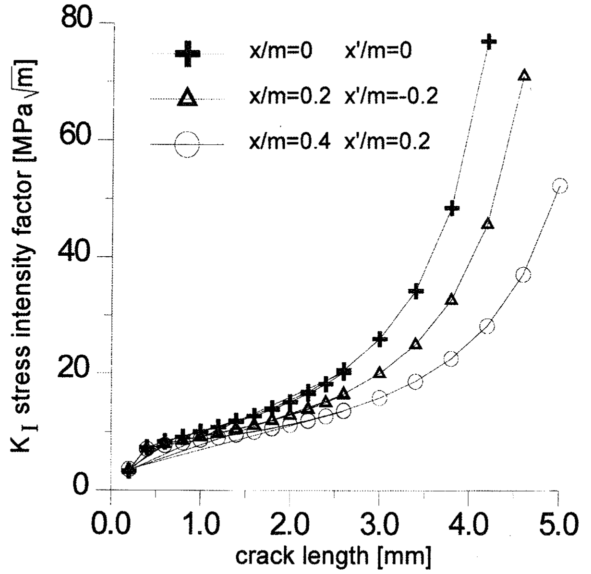


Fig. 11. Mode I and II stress intensity factors for the for the integrated design example.

has the tendency to propagate radially through the rim in a small number of cycles.

As an example of design, a typical gearing has been considered. The effects of addendum modifications (rack shift) have been examined, showing quantitatively the advantages in fatigue life in the case of the propagation of cracks initiated from the root fillet. The effects of rack shift tends to be very significant from stress analysis and crack propagation points of view in cases of low numbers of teeth, which are quite common in engineering practise. For the example shown, choosing the same maximum local stress (the transmitted loads are consequently different) the rack shift pair that produces a balanced reduction of the specific sliding gives the best performances for crack propagation.

However, it is not possible to draw general conclusions, and indeed the use of all three tools of analysis (kinematic, stress analysis, crack propagation) is always recommended. The speed of the routines developed renders this a realistic interactive possibility for design optimisation, and indeed future work will be directed towards the integrating of automatic optimisation procedures, after an adequate set of performance indexes has been defined.

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