Frictionally excited thermoelastic instability in the presence of contact resistance

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Abstract: In sliding systems, frictional heating generates a well-known instability above a certain critical speed V_{cr}, which is a function of geometrical and material properties only. Similar instabilities are known to occur in the static problem, driven by temperature differences, in the presence of thermal contact resistance. Thermal contact resistance at the interface has seldom been considered and gives rise to full coupling of the problem. Generally, the resistance decreases non-linearly when pressure is increased. Here, the critical condition (in terms of heat flux and sliding speed) for the stability of the uniform pressure solution for a half-plane in frictional contact with a rigid wall at fixed temperature is studied for a general resistance function $R(p)$. It is found that the heat flux direction increases the instability as in the case of zero speed, i.e. when directed into the half-plane (which is the only distortive material), whereas frictional heating can have also a stabilizing effect, for a given heat flux, specifically when $R(p) + pR'(p) < 0$. Also, an isothermal critical speed has been defined, and the actual critical speed is found to be smaller or larger depending on the temperature difference sign. Longer wavelengths are always more unstable so that the critical wavelength is still dictated by the real size of the system.

Keywords: thermal contact resistance, thermoelastic instability, hot spotting

NOTATION

- b growth rate (s^{-1})
- c_p specific heat of half-plane (J/kg °C)
- E Young's modulus of half-plane (N/m^2)
- f frictional coefficient
- k diffusivity of half-plane (m^2/s)
- K thermal conductivity of half-plane (W/m °C)
- m wave number (m^{-1})
- p contact pressure (N/m^2)
- p_0 contact pressure in the steady state (N/m^2)
- q heat flux (positive if entering the half-plane) (W/m^2)
- q_0 heat flux in the steady state (W/m^2)
- $\hat{q}_0$ dimensionless heat flux in the steady state
- R thermal contact resistance (m^2 °C/W)
- R_0 thermal contact resistance in the steady state (m^2 °C/W)
- t time (s)
- T_H temperature field in the half-plane (°C)
- T_W wall temperature (°C)
- T_0 temperature field in the half-plane at steady state (°C)
- T_1 temperature perturbation in the half-plane (°C)
- V sliding speed (m/s)
- V_{cr} critical sliding speed (m/s)
- $\hat{V}$ dimensionless sliding speed

- α thermal expansion coefficient of the half-plane (°C^{-1})
- δ distortivity of the half-plane (m/W)
- μ shear modulus of the half-plane (N/m^2)
- ν Poisson's ratio of the half-plane
- ρ density of the half-plane (kg/m^3)

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1 INTRODUCTION

In sliding systems such as brakes, clutches and seals, frictional heat generation depends on the local pressure, and it is well known that, for a given friction coefficient f, there will be a certain sliding speed V_{cr} above which the system will be unstable; this is believed to lead eventually to localization of the contact load in a small region of the nominal contact area and to high local temperatures, known as hot spots [**1–4**]. This phenomenon is known as frictionally excited thermoelastic instability (TEI) [1, 5] and is of critical importance in the design of brakes and clutches. Several methodologies have been proposed over the years to study the problem: firstly, Burton *et al.* [6] used a perturbation method to investigate analytically (as an eigenvalue problem) the stability of the uniform pressure solution for the contact between two sliding half-planes, defining the critical speed V_{cr} as the speed for which the growth rate is zero. This technique has been used by Lee and Barber [7] to analyse the stability of a layer sliding between two half-planes. For more realistic geometries, numerical methods need to be used, and the Burton *et al.* method was extended by Du *et al.* [8] using the finite element method to develop the matrix defining the eigenvalue problem. A significant further improvement of the Burton *et al.* eigenvalue technique was devised by using Fourier reduction in axisymmetric problems by Yi *et al.* [9], and the resulting code is available as HOTSPOTTER [10]. These techniques have a significant advantage over the alternative full transient simulation with the finite element method, since coupled transient thermoelastic contact problems in time are still very computationally demanding [11–13].

Generally, the influence of thermal contact resistance R is neglected in these studies (except for a few, such as that by Johannson [12]), and either zero resistance (perfect thermal contact) or infinite resistance (open gap) is assumed to occur at the interface. This simplifies the problem considerably; as the system is linear as long as there is full contact, it is possible to define the critical condition clearly as a function only of geometrical and material characteristics. However, even if there is full contact between the two bodies, there will generally be a thermal contact resistance at the interface because nominally flat surfaces are always rough at the microscopic scale. Thermal contact resistance has been a subject of extensive experimental [14–16] and theoretical [17, 18] investigations. There is no general agreement on quantitative models for the resistance and particularly for the interaction with friction and the partition of frictional heating (see references [12] and [19] and references therein). However, it is generally agreed that it is a monotonically decreasing function of contact pressure, principally because increasing the contact pressure increases the number of actual contact areas and hence reduces the constriction effect.

Therefore, R varies with local contact pressure and a perturbation in pressure causes a corresponding perturbation in the heat flux; this can be *per se* a source of thermoelastic contact instability [20]. The inclusion of the contact resistance is crucial to explain instabilities in heat conduction across an interface without frictional heating, as in duplex tube exchangers [21] or in solidification of a metal against a plane mould where thermoelastic contact between the partially solidified casting and the mould can become unstable, leading to significantly non-uniform pressure distribution and alloy composition [22, 23]. Barber [24] in particular studied the contact of two half-planes and found, in the limit case of one rigid perfect conductor material, that the necessary condition for instability of a perturbation of given wavelength was that the heat flow had to be directed into the half-plane (more distortive material) and had to be of a certain critical amount (wavelength dependent) as a sufficient condition; the steady state is believed to involve stable separate solutions. In the more general case of two elastic and conducting materials, the condition is more complicated (eigenvalues can be complex), and it was shown that the steady state could be unique but unstable, involving oscillatory solutions.

Additional interest to introduce the thermal contact resistance from the theoretical point of view comes from the fact that it makes the resistance function continuous and therefore easier to treat some effects that otherwise would only be present with partially separate conditions. Recently [25], an attempt has been made to study the connection between 'frictionally excited' TEI and 'static' TEI, in the context of the simple Barber *et al.* rod model [20]. Existence could only be guaranteed below a certain critical speed $\mathbf{V}_\infty$ as above $\mathbf{V}_\infty$, for some initial conditions, the pressure grows without limit, causing seizure, similar to what had been suggested in a shaft rotating in a bearing [26, 27]. In the more general three-dimensional contact problem against a rigid non-conducting wall, extension of the Duvaut theorem [28] was proved by Andersson *et al.* [29], only for sufficiently low speeds (this suggests that, for high speeds, non-existence associated with seizure is possible). In the case of two conducting materials in contact, particularly in the case of frictional heating, non-existence of a steady state may be associated also with oscillatory solutions, or with moving contacts, so that it is not surprising that the existence of a solution has not been proved.

An extension of the rod model is where the total force is prescribed in a system of various rods in parallel, the so-called 'Aldo model' originally devised by Comninou and Dundurs [30] and recently extended to the effect of frictional sliding by Afferrante *et al.* [31]. For any given imposed temperature difference, various critical speeds can be defined: the first, V_u, for which the uniform pressure solution becomes non-unique and another, V_s, for which it becomes unstable. One of the interesting results is that, when the solution is unique (uniform pressure), it is always stable whereas, when there are multiple solutions, the uniform-pressure solution is not necessarily stable. Also, if V_s is defined as the critical condition (as is more sensible in view of the typical analysis of nominal steady states in sliding systems), it is found that separate solutions may exist below V_s and are locally stable and also (non-uniform) full contact solutions may exist above V_s. In a continuous problem, non-uniform

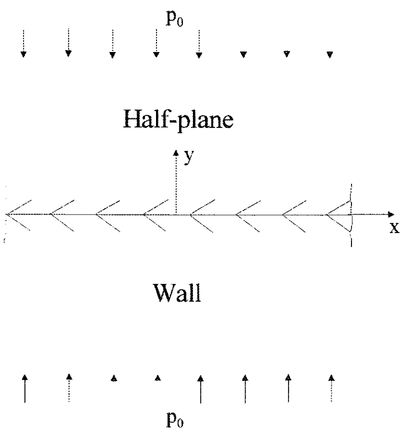

Fig. 1 Elastic half-plane in contact with a rigid perfect conductor wall

pressure is routine for a separated contact, and there is no obvious equivalent to this condition.

More in general, rod models have only a limited relationship with the behaviour of more complex continuous structures (including two-dimensional and three-dimensional solids). Hence, to make a further step towards more realistic cases, the problem of an elastic half-plane in contact with a rigid perfect conductor wall at a temperature T_W is considered (Fig. 1). The half-plane has a relative speed V with respect to the rigid wall; a uniform pressure p_0 is remotely applied to ensure complete contact along the interface and is assumed to be constant at the interface. Further, a uniform heat flux q_0 is exchanged between the bodies because of initial temperature drop across the interface and frictional heat flux is produced during the sliding contact, fVp. Note that, when a thermal contact resistance is postulated at the interface, it is generally necessary to define the partitioning of this heat flux into each of the bodies. However, the temperature of the wall does not change ex-hypothesis, and hence attention can be focused on the heat flux portion which enters the half-plane, which without loss of generality will still be denoted fVp.

With these preliminaries, the steady-state heat flux q, which is positive if entering the half-plane $y > 0$, must satisfy the equations

$$q = \frac{T_W - T_H}{R} + fVp \tag{1}$$

$$q = -K \frac{\partial T}{\partial y}\bigg|_{y=0} \tag{2}$$

where $T_W - T_H$ is the temperature drop across the sliding interface. The stability of the system to small perturbation (the system being non-linear) can be investigated by a linear perturbation analysis about the steady state.

2 STABILITY

The general transient solution for the temperature field in the elastic half-plane can be written

$$T_H(x, y, t) = T_0(x, y) + T_1(x, y, t) \tag{3}$$

where $T_0(x, y)$ is the temperature field in the half-plane in the steady state and $T_1(x, y, t)$ is a temperature perturbation. Differentiating equation (1), and if Δq, ΔT_H and Δp are small perturbations in q, T_H and p respectively such that

$$\Delta q\, R_0 + q_0\, \Delta R = -\Delta T_H + fVR_0\, \Delta p + fVp_0\, \Delta R \tag{4}$$

and noting that

$$\Delta R = \frac{\partial R}{\partial p} \Delta p = R' \Delta p \tag{5}$$

it can be mentioned that the contributions due to frictional heating have the opposite signs, since R' is generally negative. This suggests already that, if the effect of the derivative of resistance is large enough, it would make the effect of frictional heating opposite to the usual case, as will be made clear at a later stage. Returning to equation (4), using equation (5), it is found that

$$\Delta q\, R_0 = -\Delta T_H - [q_0 R' - fV(R_0 + p_0 R')]\, \Delta p \tag{6}$$

where q_0, p_0 and R_0 are the values of q, p and R in the steady state.

Since the steady state certainly satisfies the governing equations and the boundary conditions, the system has a trivial solution $T_1(x, y, t) = 0$ in which the contact pressure is uniform and equal to p_0. In such conditions, the temperature field T_H is linear in y and independent of x as is the heat flux which is equal to q_0. The linearity of the governing equations (for small perturbations about the steady state) permits the temperature perturbation to be written

$$T_1(x, y, t) = \theta(y) \exp(bt) \cos(mx) \tag{7}$$

where m is the wave number and defines the spatial frequency of the sinusoidal perturbation in the direction x, and b is the growth rate of the perturbation. The function $\theta(y)$ must be obtained by substituting equation (7) in the heat conduction equation

$$\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2} = \frac{1}{k} \frac{\partial T_1}{\partial t} \tag{8}$$

in which $k = K/\rho c_p$ is the half-plane thermal diffusivity (K, ρ and c_p are the thermal conductivity, the density and the specific heat respectively of the half-plane). As $\theta(\infty) = 0$, the function $\theta(y)$ assumes the form

$$\theta(y) = A \exp(-\lambda y) \tag{9}$$

where A is an arbitrary constant and

$$\lambda = \sqrt{m^2 + \frac{b}{k}} \tag{10}$$

with $\mathrm{Re}(\lambda) > 0$. Therefore, the temperature and heat flux perturbations become

$$\Delta T_{\mathrm{H}} = T_1(x, 0, t) = A\exp(bt)\cos(mx) \tag{11}$$

$$\Delta q = KA\lambda\exp(bt)\cos(mx) \tag{12}$$

For the pressure perturbation, using the work of Barber and Hector [**32**],

$$\Delta p = \frac{m\alpha EA}{(1-\nu)(\lambda + m)}\exp(bt)\cos(mx) \tag{13}$$

If equations (11), (12) and (13) are substituted into equation (6), the characteristic equation is found to be

$$\left(\frac{\lambda}{m}\right)^2 + \left(1 + \frac{1}{mKR_0}\right)\frac{\lambda}{m} + \left\{\frac{1}{mKR_0} + \frac{4M\delta}{mR_0}[q_0R' - fV(R_0 + p_0R')]\right\} = 0 \tag{14}$$

where

$$\delta = \frac{\alpha(1+\nu)}{K} \tag{15}$$

is the half-plane distortivity and M depends on the shear modulus and Poisson's ratio of the half-plane according to

$$2M = \frac{\mu}{1-\nu} \tag{16}$$

Equation (14) permits determination of the parameter λ and hence the growth rate of the temperature perturbation b [see equation (10)]. Since the perturbation must decay away from the sliding interface, $\mathrm{Re}(\lambda) > 0$, and hence the only admissible solution of equation (14) is

$$\frac{\lambda}{m} = -\frac{1}{2}\left(1 + \frac{1}{mKR_0}\right) + \Gamma^{1/2} \tag{17}$$

if and only if

$$\Gamma = \frac{1}{4}\left(1 - \frac{1}{mKR_0}\right)^2 - \frac{4M\delta}{mR_0}[q_0R' - fV(R_0 + p_0R')] > 0 \tag{18}$$

If $fVp = 0$, equation (17) returns to the characteristic equation given by Barber [**24**].

Setting $\lambda = m$ (i.e. $b = 0$), from equation (17) it is possible to establish the critical condition, at which the system becomes unstable, which together with condition

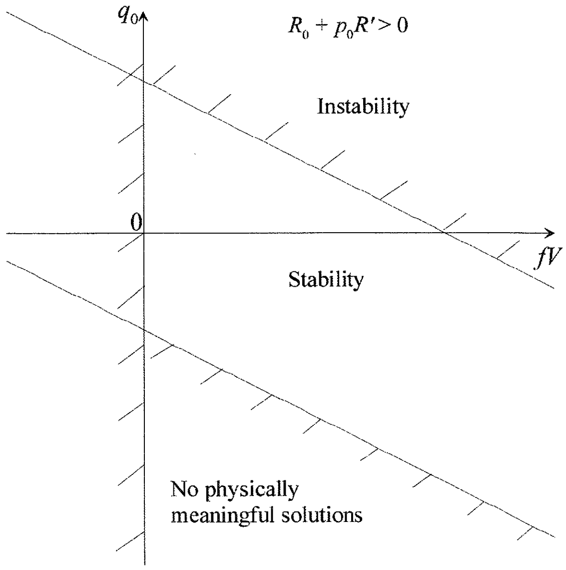

Fig. 2 Stability zone for the system for $R + pR' > 0$; frictional heating is a destabilizing factor

(18) gives

$$q_0R' < fV(R_0 + p_0R') + \frac{(mKR_0 - 1)^2}{16M\delta mR_0K^2} \tag{19}$$

$$q_0R' < fV(R_0 + p_0R') - \frac{mKR_0 + 1}{2M\delta K} \tag{20}$$

It is clear that condition (20) is more restrictive than condition (19). Figures 2 and 3 show the possible zones of instability of the system in example cases, for $R_0 + p_0R' > 0$ and $R_0 + p_0R' < 0$ respectively. Note that for $q_0 > 0$, i.e. when heat flow is directed into the more distortive material (the direction giving possible

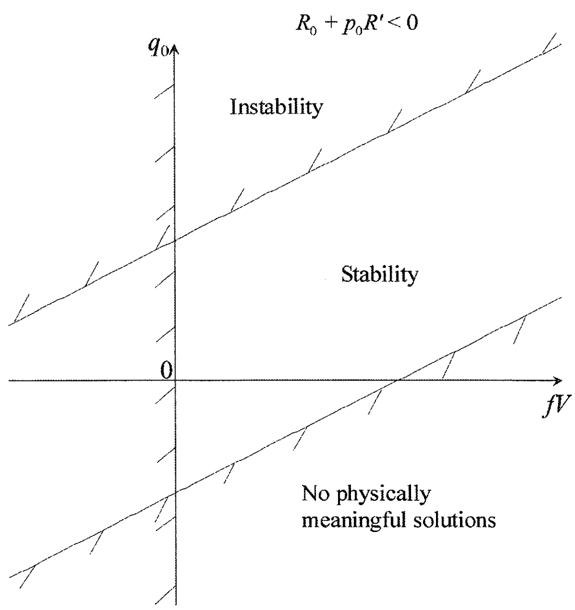

Fig. 3 Stability zone for the system for $R + pR' < 0$; frictional heating is a stabilizing factor

instability in the non-sliding situation), an increase in q_0 is always a destabilizing factor.

If the effect of frictional heating due to sliding (for a given heat flow q_0) is considered, it is found that the critical speed decreases with increasing heat flux if $R_0 + p_0 R' > 0$. However, the critical speed increases with heat flux if $R_0 + p_0 R' < 0$. This may appear surprising but in fact is consistent because, in both cases, heat flux into the more distortive material makes the system more unstable, as the instability condition occurs for speeds lower than the critical speed if $R_0 + p_0 R' < 0$. Note also that $m = 0$ always gives the most unstable perturbation, as usual,

$$fV_0 = \frac{1}{R_0 + p_0 R'}\left(q_0 R' + \frac{1}{2M\delta K}\right) \tag{21}$$

but it is a minimum critical speed for the system if $R_0 + p_0 R' > 0$ and a maximum critical speed if $R_0 + p_0 R' < 0$ respectively.

Also, the heat flow q_0 depends also on frictional heating, from equation (1). Hence, for a fixed temperature difference, the critical condition (20) becomes

$$\left(\frac{T_W - T_H}{R_0} + fVp_0\right)R' < fV(R_0 + p_0 R') - \frac{mKR_0 + 1}{2M\delta K} \tag{22}$$

and, rearranging for fV, the two effects cancel each other to give

$$fV > \frac{m}{2M\delta} + \frac{1}{2M\delta KR_0} + \frac{T_W - T_H}{R_0^2}R' \tag{23}$$

i.e. the critical speed is a function of the temperature difference $fV(T_W - T_H)$. If the 'isothermal' critical speed* for zero temperature difference is defined as

$$fV_{iso} = \frac{m}{2M\delta} + \frac{1}{2M\delta KR_0} \tag{24}$$

then, since $R' < 0$, the actual critical speed is higher than the isothermal speed if $T_W - T_H < 0$, and lower if $T_W - T_H > 0$. Also, this form of the stability condition makes clear that the system always has a critical speed, and the highest stability is obtained when the wall is at the lowest possible temperature, although in a real system even the temperature difference is likely to change with speed, adding another non-linearity to the system.

Note that these results are obviously consistent with the case $V = 0$, which can be treated directly from equations (20) and (19), coincidentally with equations (33) and (34) from the work by Barber **[24]**.

*Note that this critical speed coincides with the Burton *et al.* critical speed for out-of-plane sliding of a half-plane against a rigid non-conductor when $R_0 = \infty$.

3 SPECIAL CASES

3.1 $R = R_0 =$ constant

With a constant thermal contact resistance the instability condition assumes the form

$$f\hat{V} > f\hat{V}_{cr} = \frac{1}{2}\left(1 + \frac{1}{mKR_0}\right) \tag{25}$$

where

$$\hat{V} = \frac{M\delta V}{m} \tag{26}$$

is the dimensionless sliding speed. Note that V_{cr} quickly decreases with increasing dimensionless thermal contact resistance mKR_0 and does not depend on q_0 and p_0 but only on R_0.

3.2 $R = A/p$

If a hyperbolic dependence of the resistance R on the contact pressure p is assumed, equation (20) becomes

$$\hat{q}_0 > \hat{q}_{cr} = \frac{1}{2}\left(1 + \frac{1}{mKA/p_0}\right) \tag{27}$$

where

$$\hat{q}_0 = \frac{M\delta q_0}{mp_0} \tag{28}$$

is the dimensionless heat flux.

Therefore, an interesting result has been obtained; the stability condition for $V = 0$ (no sliding) coincides with the stability condition for $V \neq 0$ when an inverse dependence of the thermal contact resistance on the pressure is assumed, with the only difference that the heat flow now depends also on frictional heating, from equation (1). In particular, the dependences of the dimensionless critical heat flux $\hat{q}_{cr}$ (for $R = A/p$) and the dimensionless critical speed $f\hat{V}_{cr}$ (for $R = R_0$) on the dimensionless thermal contact resistance mKR are identical, as shown in Fig. 4.

This is, however, only a consequence of a stronger result, which is obtained if it is noted that, when $R = A/p$, equation (1) becomes

$$q = \frac{T_W - T_H}{A}p + fVp = \frac{(T_W + AfV) - T_H}{A}p \tag{29}$$

i.e. the entire solution of the problem can be obtained for an equivalent static problem for an increased wall temperature. Since stability is unaffected by the wall temperature in the unperturbed solution, it follows that sliding has no effect on the stability boundary and this applies to any geometry, not just the half-plane. Since $R = A/p$ is in the class $R + pR' = 0$, this makes it

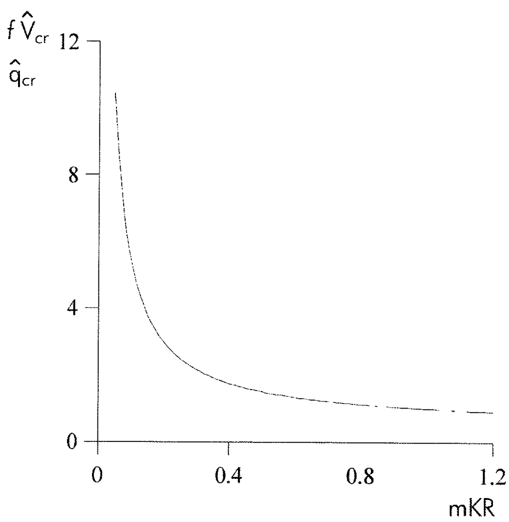

Fig. 4 Dependence of the dimensionless critical heat flux $\hat{q}_{cr}$ (for $R = A/p$) and the dimensionless critical speed $f\hat{V}_{cr}$ (for $R = R_0$) on the dimensionless thermal contact resistance mKR

possible to argue that, for $R + pR' > 0$, destabilization by sliding to more general geometries is expected.

3.3 $R = R_\infty + A/p$

In this case $R' = -A/p^2$; hence $R_0 + p_0 R' = R_0 > 0$ and the general conclusions of the previous section hold. In particular, the stability condition (20) becomes

$$f\hat{V} > f\hat{V}_{cr} = -\frac{A}{R_\infty}\frac{M\delta q_0}{mp_0^2} + \frac{1 + mK(R_\infty + A/p_0)}{2R_\infty mK} \tag{30}$$

The critical speed decreases with increasing heat flux if $q_0 > 0$. Also, let us analyse two limit cases:

1. For $p_0 \to \infty$ the critical condition (25) for $R = \text{constant} = R_\infty$ is obtained. The critical condition, therefore, decreases with increasing dimensionless thermal contact resistance mKR_0 and does not depend on q_0 and p_0, but only on R_0. Note that such a limit condition is equally obtained if A tends to zero.
2. For $p_0 \to 0$, equation (30) becomes

$$f\hat{V} > -\frac{A}{R_\infty}\frac{M\delta q_0}{mp_0^2} \to \begin{cases} -\infty & \text{for } q_0 > 0 \\ +\infty & \text{for } q_0 < 0 \end{cases} \tag{31}$$

Therefore, in the limit of $R_\infty \to 0$ or when the pressure tends to zero, the instability condition coincides with the usual condition [24] that for instability the heat flow has to be directed into the more distortive material (the half-plane), and greater than a certain threshold. Note also that, in the limit of large frictional heating (or at least much greater than heat flow due to interface temperature drop), then $q = fVp$, giving

$$p_0 > -\frac{A}{R_\infty} \to \text{always} \tag{32}$$

meaning that, since the frictional heating is always directed in the half-plane, it is always a destabilizing effect.

4 CONCLUSIONS

The critical condition for instability of the uniform-pressure solution for a half-plane sliding against a rigid perfect conductor wall, which depends on both the heat flow across the interface and frictional heating, has been studied. In other words, a critical speed can be defined for a given heat flux, or vice versa a critical heat flow can be defined for a given speed.

However, the sign of the factor $R + pR'$ is extremely important, as for a given heat flux, the following hold:

1. For $R + pR' > 0$, frictional heating tends to enlarge the region of possible instabilities, and there is a finite critical speed for any given heat flux.
2. For $R + pR' < 0$, frictional heating tends to make the system more stable, as the system is unstable only *below* a given critical speed, and there is such a critical speed only for a large positive heat flux. This speed becomes larger, the larger the heat flux.

In order to study the case the effect of frictional heating, for a given temperature difference, is considered; an 'isothermal' critical speed has been defined and it was found that the actual critical speed is

$$fV_{cr} = fV_{iso} + \frac{T_W - T_H}{R_0^2}R' \tag{33}$$

i.e. is larger than the isothermal value if $T_W - T_H < 0$, but smaller if $T_W - T_H > 0$. This may suggest that in practical systems it may be useful, in principle, to keep the temperature of the better conductor higher than that of the counterpart (e.g. in a typical brake system, of the metal discs and the frictional part respectively), although this of course may not be practical from other practical points of view.

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