Letter to the Editor

On the Ekberg, Kabo and Andersson calculation of the Dang Van high cycle fatigue limit for rolling contact fatigue

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ABSTRACT Recently, various methods have been proposed to assess the risk of rolling contact fatigue failure by Ekberg, Kabo and Andersson, and in particular, the Dang Van multiaxial fatigue criterion has been suggested in a simple approximate formulation. In this note, it is found that the approximation implied can be very significant; the calculation is improved and corrected, and focused on the study of plane problems but for a complete range of possible friction coefficients. It is found that predicted fatigue limit could be much higher than that under standard uniaxial tension/compression for 'hard materials' than for 'ductile materials.' This is in qualitative agreement, for example, with gears' design standards, but in quantitative terms, particularly for frictionless condition, the predicted limit seems possibly too high, indicating the need for careful comparison with experimental results. Some comments are devoted to the interplay of shakedown and fatigue.

Keywords HCF fatigue; rolling contact fatigue; safe-life design.

Rolling contact fatigue (RCF) occurs in railways, but also in gears and rolling bearings, and many other mechanical applications. It has various forms (plastic deformations, macro- and micro-pitting, spalling, crack initiation from inclusions, etc.) and is generally interacting with other forms of surface damage, and in particular wear. Both phenomena (RCF and wear) strongly depend on surface roughness and lubrication conditions, and therefore it is not surprising that a general understanding or design methodology has so far been lacking. Recently, Ekberg, Kabo and Andersson (EKA),[1] have suggested an approach to the problem with independent consideration of various mechanisms at play, corresponding to various indexes. The first (surface-initiated fatigue) derives from the classical approach based on plasticity theory, shakedown and ratcheting, as described, for example, in Chapter 9 of Johnson's book.[2] Ratcheting in particular would seem to dominate the process at sufficiently large pressures, and its modelling was initially apparently successful with simple analyses using elastic–perfectly plastic constitutive equations. More recent FEM plasticity models quite efficiently take account of more complex constitutive laws and also can compute the steady-state response (if there is one) directly,[3–5] but the ratcheting response modelling remains today one of the most complex in plasticity theory. For example, in the celebrated RCF experiments by Merwin[6] also reported in Merwin and Johnson,[7] the choice of yield limit was quite arbitrary (1% of permanent strain in the monotonic curves for dural and mild steel, but 25% for copper) and probably motivated by the need to justify the experimentally observed beginning of ratcheting under RCF conditions, rather than the measured yield limit in standard monotonic tests of the material.[8,9] The use of shakedown maps and the EKA Eq. (4) seems therefore a little questionable without a clearer idea of what the yield limit should be. Further, as recently remarked in Afferrante *et al.*,[10] another celebrated set of RCF experimental results, those by Clayton and Su[11] and Su and Clayton,[12] show a fatigue limit around $p_0/k_y = 2.5$ (p_0 being the Hertzian peak pressure and k_y the nominal and initial yield limit in shear) rather than the $p_0/k_y = 4$ expected from shakedown theory. After all, this is not really surprising, as it is well known that the shakedown and fatigue limit are not directly connected. Indeed, according to Dang Van *et al.*,[13] they involve at most the same process but occurring at different scales (fatigue limit being the shakedown at grain, so-called 'mesoscopic' level). This

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is why fatigue limit properties of materials are generally directly measured from experiments, rather than extrapolated from plasticity constants, which perhaps would be a lot faster and cheaper to obtain! It is true that Dang Van's criterion[13] permits the use of shakedown theories, but this is limited to the examination of the effect of multiaxiality of the stress cycle by making assumptions on the relationship between macroscopic and microscopic stresses, but ultimately makes use of fatigue limit properties (generally, the fatigue amplitude limit under bending σ_e and under torsion τ_e, where we use the subscript 'e' standing for 'endurance'). As a general rule (see [14]) very soft metals and alloys show cyclic hardening and a large fatigue ratio ($\sigma_e > \sigma_y$) whereas heavily work-hardened metals tend to cyclically soften and have low fatigue ratios ($\sigma_e < \sigma_y$). There is perhaps a better correlation of fatigue limit with cyclic yield strength, as fatigue limit is often close to correspond to cyclic strain amplitudes of 0.2–0.35%.

As a second criterion, EKA suggest considering for sub-surface initiated fatigue a very simple approximate calculation of the Dang Van criterion (see their Eqs (6)–(14)),

$$\sigma_{eq} = \max_t(\tau_a(t) + a_{DV}\sigma_h(t)) < \tau_e, \tag{1}$$

where $\tau_a(t)$ is a time-dependent shear-stress amplitude, defined as

$$\tau_a(t) = |\tau(t) - \tau_m|. \tag{2}$$

Here, $\tau(t)$ is the shear-stress vector, and τ_m is its mid-value during the stress cycle. Also (in Eq. (1)), a_{DV} is a material parameter, which can be computed from two known values of the equivalent stress. In particular, from alternating bending and alternating torsion limits, it is simply defined as

$$a_{DV} = 3\frac{\tau_e}{\sigma_e} - \frac{3}{2}. \tag{3}$$

Finally, $\sigma_h(t)$ is the hydrostatic pressure, and includes both the elastic component and the residual stresses. EKA show that the Dang Van criterion in pure rolling, in the absence of residual stresses, does not depend greatly on the hydrostatic pressure term, and obtained their Eq. (11)

$$\sigma_{eq,PR} = \frac{\tau_{max}}{2}. \tag{4}$$

Applying this result for the 2D pure rolling problem for which $\tau_{max} = 0.3p_0$ (Section 4.47 of Ref. [2]), this would give $(p_0/\tau_e)_{lim} \approx 2/0.3 = 6.67$. If we consider the in-plane shear stress, then $\tau_{max} = 0.25p_0$ and $(p_0/\tau_e)_{lim} \approx 2/0.25 = 8$. Finally, they also suggest Eq. (13) in terms of normal load, F,

$$\sigma_{eq,PR} = \frac{F}{4\pi ab}, \tag{5}$$

where a, b are ellipse semiaxes for Hertzian 3D contact. In this case, taking $F/\pi ab$ as the average pressure, and specializing for 2D pure rolling $p_{med} = \pi p_0/4$ (Section 4.45 of Ref. [2]), we would obtain $(p_0/\tau_1)_{lim} \approx 16/\pi = 5.09$. These various options are already a little confusing.

A detailed calculation considering the full 3D state of stress in the plane strain problem and the full Dang Van criterion, using Poisson's ratio 0.3, has been attempted. Figure 1 shows the results for the line contact problem using the frictionless stress distribution of the Hertz solution (see Ref. [2]), as a function of the material constant a_{DV}, varying between 0 (when $\sigma_e/\tau_e = 2$) and 1.5 (when

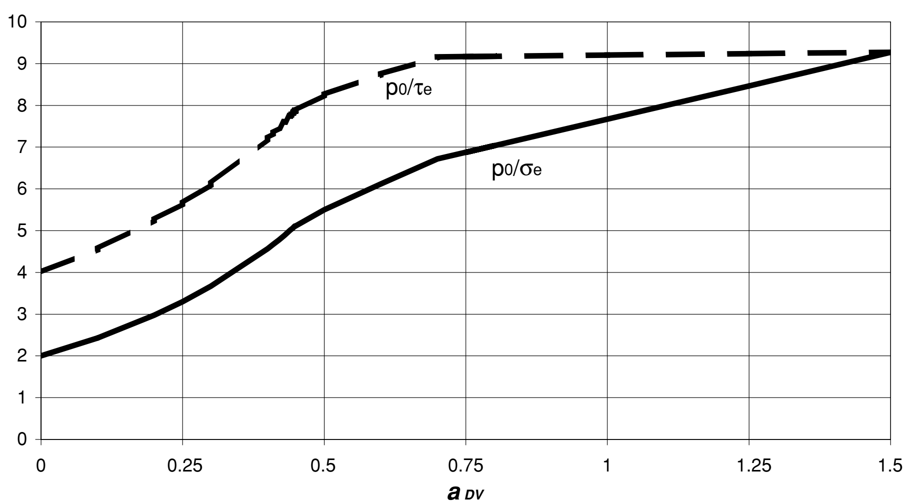

Fig. 1 The RCF limits $(p_0/\tau_e)_{lim}$, $(p_0/\sigma_e)_{lim}$ according to the Dang Van criterion for line rolling contact and no friction (in terms of ratio between limit pressure and fully reversed shear stress or tension/compression fatigue endurance limits), as a function of the material constant $a_{DV} = 0.5$.

$\sigma_e/\tau_e = 1$). It is found that, contrary to what was suggested by EKA, there is a significant effect of the constant a_{DV} (i.e., of the hydrostatic component), because the obtained range of limit is $(p_0/\tau_e)_{lim} \approx 4-9.2$ and $(p_0/\sigma_e)_{lim} \approx 2-9.2$. A detailed analysis shows that the critical cycle is always found subsurface for pure rolling, the depth being $z/a = 0.5$ for $a_{DV} = 0$; and slowly moving further subsurface for increasing a_{DV}, moving slowly to $z/a = 0.6$ for $a_{DV} = 0.45$; and then suddenly jumping to $z/a = 0.833$ and staying constant, for larger a_{DV}.

Moreover, in terms of the ratio $(p_0/\sigma_e)_{lim}$ the variation is close to linear, whereas in terms of $(p_0/\tau_e)_{lim}$ the variation is rapid up to $a_{DV} = 0.7$ and then stops, indicating that the following variation in terms of $(p_0/\sigma_e)_{lim}$ is only due to the variation of the ratio $\sigma_e/\tau_e = 2$ with a_{DV}. Finally, notice that the EKA approximate calculation in its various possible tentative forms, $(p_0/\tau_e)_{lim} \approx 5.09, 6.67, 8$, indicates possible results but only for special values of a_{DV}, and in particular for low a_{DV} in the range 0.2–0.5.

Next, the effect of frictional tractions is considered, in full sliding. EKA suggest adding a quadratic decay of the limit with the friction coefficient, but again this is found to be a strong and inaccurate approximation. In Figs 2 and 3, results are presented in terms of $(p_0/\tau_e)_{lim}$ and $(p_0/\sigma_e)_{lim}$, respectively, as a function of friction coefficient and for various values of $a_{DV} = 0, 0.5, 1, 1.5$. It is evident that the effect of friction is much weaker for ductile materials (those with high a_{DV}), and indeed for $a_{DV} = 0$, the limit pressure stays constant. Also, notice that for high friction coefficients, the criterion becomes essentially uniaxial in the sense that the limit cycle moves to the surface and is dominated by the tensile stresses that are nearly fully reversed. In these conditions, since the maximum tensile

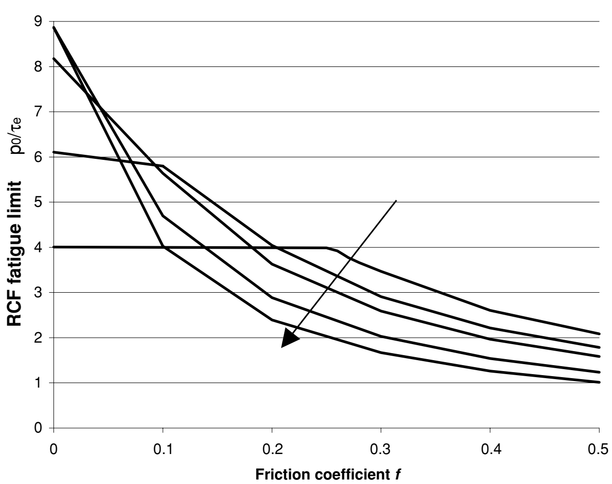

Fig. 2 The RCF limit according to the Dang Van criterion (in terms of ratio $(p_0/\tau_e)_{lim}$ between limit pressure and fully reversed shear stress fatigue endurance limit) for line rolling contact and full slip conditions, for various material constants a_{DV}.

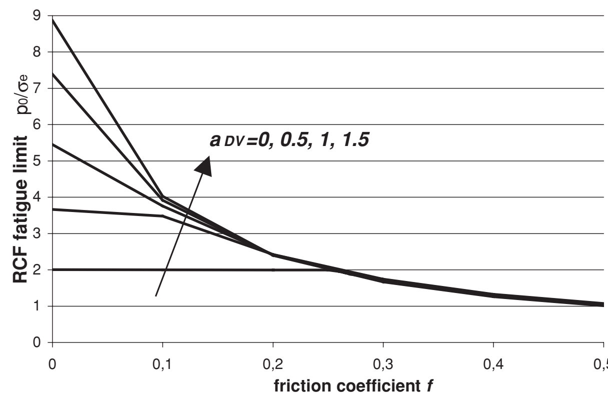

Fig. 3 The RCF limit according to the Dang Van criterion (but in terms of ratio $(p_0/\sigma_e)_{lim}$ between limit pressure and fully reversed tensile stress fatigue limit) for line rolling contact and full slip conditions, for various material constants a_{DV}.

stress is $2fp_0$, the limit varies in an inversely proportional manner. Notice, moreover, that in this case it is the curve $(p_0/\sigma_e)_{lim}$ which shows the simplest behaviour as for a low friction coefficient, the limit is a clearly monotonic increasing function of a_{DV}, at all friction coefficients, but it becomes independent of a_{DV} for a friction coefficient larger than about 0.27. Vice versa, for the curve in terms of $(p_0/\tau_e)_{lim}$, we obtain for low friction, again that the limit is a monotonic increasing function of a_{DV}, but for a friction coefficient greater than about 0.27, the situation is reversed, and the limit is decreasing with a_{DV}, and not independent of it.

It appears that very hard materials resists a lot better under pure compressive (frictionless) rolling, than under frictional rolling (where there are tensile stresses). This is not particularly surprising, and qualitatively in agreement with gears design standard such as BS ISO 6336-2[15] or ANSI/AGMA 2001-B88,[16] for pitting resistance of gears, for which materials such as grey cast iron have a lot higher value of recommended pressure than bending stress. The typical ratio between pitting fatigue limit pressure and uniaxial bending fatigue limit varies between 1.5 and 5–6, and therefore although the design practice evidently takes account of many factors and is essentially empirical, the ratio seems to be of the same order of magnitude as the Dang Van criterion applied here. Notice that the gears standards give both bending strengths and pitting strength as various linear functions of hardness of the material based on equations originally recommended by Buckingham, such as for steel with a fatigue limit at 10^7 cycles of[1]

$$p_0^{lim} = (2.8HB - 69) \quad \text{(MPa)}, \tag{6}$$

[1]Since Brinell hardness has [Kg f mm^{-2}] units, the notation $3HB$ really indicates roughly 0.3 the pressure in the hardness test, and the notation $10HB$ roughly the hardness pressure itself.

and for the entire life curve, a factor C_{Li} is used (i.e., $C_{\mathrm{Li}} = 1$ at 10^7 cycles). For example, $C_{\mathrm{Li}} = 1.7$ at 10^4 cycles, and $C_{\mathrm{Li}} = 0.65$ at 10^{11} cycles. A number of other linear relationships are found for particular materials, and for the bending fatigue strength. An equivalent of (6) for a very approximate estimate is perhaps

$$\sigma_{\mathrm{e}} = 1.7HB \quad (\mathrm{MPa}), \tag{7}$$

which would give a ratio $p_0^{\mathrm{lim}}/\sigma_{\mathrm{e}}$ around 1.6, which is lower than any of the results from the Dang Van criterion, and particularly for the large a_{DV}. The reasons for these discrepancies may be many, for example, the multiaxial Dang Van criterion is not accurate in the region of compressive hydrostatic stresses where very few experimental data are available (in particular, it is possible that the increase of resistance with hydrostatic compression is not as beneficial as indicated by Eq. (1) in the entire range), or that a special modification is needed for the non-proportional conditions induced by RCF. More comparison with experimental values would certainly be beneficial. The overall philosophy in gears standard seem to be appropriate also for rail materials,[10] although they become particularly conservative at low number of cycles.

REFERENCES

1 Ekberg, A., Kabo, E. and Andersson, H. (2002) An engineering model for prediction of rolling contact fatigue of railway wheels. *Fatigue Fract. Engng Mater. Struct*. **25**, 899–909.

2 Johnson, K. L. (1985) *Contact Mechanics*. Cambridge University Press, Cambridge.

3 Dang Van, K. and Maitournam, M. H. (2002) On some recent trends in modelling of contact fatigue and wear in rail. *Wear* **253**, 219–227.

4 Dang Van, K., Maitournam, M. H. and Prasil, B. (1996) Elastoplastic analysis of repeated moving contact: Application to railways damage phenomena. *Wear* **196**, 77–81.

5 Maouche, N., Maitournam, M. H. and Dang Van, K. (1997) On a new method of evaluation of the inelastic state due to moving contacts. *Wear* **203**, 139–147.

6 Merwin, J. E. (1962) Plastic deformation of surfaces in rolling, PhD dissertation, Cambridge University, UK.

7 Merwin, J. E. and Johnson, K. L. (1963) An analysis of plastic deformation in rolling contact. *Proc. Inst. Mech. Engng* **177**, 676.

8 Ponter, A. R. S., Afferrante, L. and Ciavarella, M. (2002) Shakedown in rolling contact. Fatigue 2002. In: *Proceedings of the Eighth International Fatigue Congress*, Stockholm, Vol. **2.** EMAS Cradley Heath (UK), pp. 1355–1363 (ISBN 1901537307).

9 Ponter, A. R. S., Afferrante, L. and Ciavarella, M. (2004) A note on Merwin's analysis of forward flow in rolling contact. *Wear* **256**, 321–328.

10 Afferrante, L., Ciavarella, M. and Demelio, G. (2004) A re-examination of rolling contact fatigue experiments by Clayton & Su with suggestions for surface durability calculations. *Wear* **256**, 329–334.

11 Clayton, P. and Su, X. (1996) Surface initiated fatigue of pearlitic and bainitic steels under water lubricated rolling/sliding contact. *Wear* **200**, 63–73.

12 Su, X. and Clayton, P. (1997) Ratcheting strain experiments with a pearlitic steel under rolling/sliding contact. *Wear* **205**, 137–143.

13 Dang Van, K., Cailletaud, G., Flavenot, J. F., Le Douaron, A. and Lieurade, H. P. (1989) Criterion for high cycle fatigue failure under multiaxial loading. In: *Biaxial and Multiaxial Fatigue* (Edited by M. W. Brown & K. J. Miller). Mechanical Engineering Publications, London, UK, pp. 459–478.

14 Fleck, N. A., Kang, K. J. and Asbhy, M. F. (1993) Overview 112: The cyclic properties of engineering materials. *Acta Metall. Mater*. **42**, 365–381.

15 BS ISO 6336-2. (1996) Calculation of load capacity of spur and helical gears (Part 2: Calculation of surface durability (pitting); Part 5: Strength and quality of materials). British Standard, ISBN 0580 26750 4, ISBN 0580 27098 X.

16 ANSI/AGMA 2001-B88. (1988) Fundamental rating factors and calculation methods for involute spur and helical gears. ISBN: 1 55589 647 2. American Gear Manufacturers Association, Alexandria, VA, Canada.

Response

Answer to the Letter to the Editor from M. Ciavarella and H. Maitournam

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It should first be noted that the model, referred to by Ciavarella and Maitournam was developed for the prediction of rolling contact fatigue (RCF) of railway wheels. In the letter to the editor, Ciavarella and Maitournam apply the model to two-dimensional line contact for which it was not intended. However, from this extrapolation, some interesting findings arise as will be discussed below.

Regarding surface-initiated fatigue, the choice of a shakedown map approach is obviously a sacrifice in accuracy for computational speed. The entire material characteristics then need to be incorporated in the yield limit in shear, which has to be chosen to account for plastic hardening. More sophisticated models of surface-initiated fatigue may be found in the literature (see, e.g., Ref. [1]). However, incorporating plastic simulations calls for extensive simulations and is inappropriate if the model is to be integrated into multi-body simulations of train–track interaction, which was a primary goal.

Turning to subsurface initiated RCF, the influence of the material parameter a_{DV} is interesting. It should first be noted that a_{DV} might be evaluated from the fatigue limit in alternating ($\pm\sigma_e$) and pulsating ($\sigma_{ep} \pm \sigma_{ep}$) bending as

$$a_{DV} = \frac{3}{2}\frac{\sigma_e - \sigma_{ep}}{2\sigma_{ep} - \sigma_e}. \tag{1}$$

The choice of $a_{DV} = 0.32$ as used in Ref. [2] is motivated by the engineering rule $\sigma_{ep}/\sigma_e = 0.85$. According to (1), the choice of $a_{DV} = 0$ predicts $\sigma_{ep} = \sigma_e$, which is non-physical.

Study the case of two-dimensional line contact as discussed by Ciavarella and Maitournam and in particular stresses occurring in two shear planes: a plane parallel to the surface and a plane inclined 45° to the surface (below denoted as the 45° plane). The evolution of the shear stress and the 'Dang Van stress' at a normalized depth of $z/b = 0.6$ at these two shear planes are plotted in Fig. 1. The 'Dang Van stress' is here defined as $\sigma_{EQ}(\eta) = |\tau(\eta) - \tau_{mid}| + a_{DV}\sigma_h(\eta)$ with $\eta = y/b$ being the normalized lateral position (for a moving load, each position is equivalent to a certain instant in time). It is seen that for $a_{DV} = 0$, the plane parallel to the surface will experience the highest σ_{EQ}. This will occur at positions marked 1 and 2 in Fig. 1a. The non-influence of an applied frictional load in this case (as noted by Ciavarella and Maitournam) is probably due to an equal increase of the shear stress at positions 1 and 2 causing $|\tau(\eta) - \tau_{mid}|$ to be unaltered. As a_{DV} increases, the magnitude of τ_{DV} at the plane parallel to the surface will decrease. As seen from Fig. 1a, an approximation of σ_{EQ} at this shear plane is $\sigma_{EQ} = (1 - a_{DV})p_0/4$. Eventually, as a_{DV} increases, the 45° plane will become critical. The highest σ_{EQ} will now occur close to position 2 in Fig. 1b. As a_{DV} increases further, the position of maximum σ_{EQ} will shift towards 3 in Fig. 1b. Here the hydrostatic stress is zero and we will have no further influence of the hydrostatic stress.

Naturally, the Dang Van stress has to be evaluated at all possible shear planes (which is made automatically by the original procedure outlined by Dang Van and coworkers) and at all material points to identify the critical combination. However, we believe that Fig. 1 shows the basic principles behind the influence of a_{DV} in line contact.

In rolling contact of railway wheels, the issue is different. Here the contact patch is elongated in the rolling direction (line-like contacts may appear, but typically correspond to benign contact conditions with very large contact patches). The evaluation of the Dang Van stress is more complex due to the three-dimensionality of the problem. The tip of the shear stress vector (at a chosen shear plane) will form a closed path during a cycle. At the critical shear plane, the path will resemble a pulsating evolution for most magnitudes of a_{DV} (similar to the case for the 45° plane in the line contact case); see Ref. [4]. This 'pulsating' evolution is the assumption underlying Eq. (1) in Ref. [2].

Results for some simulations with realistic geometries and loads are compiled in Table 1. The case of $a_{DV} = 0$ here corresponds to a critical shear plane parallel to the surface, but the effect on σ_{EQ} is much less dramatic than in the line contact case.

From the fourth row of Table 1 it can be noted that an applied friction (F_x, F_y) will have a pronounced effect at $a_{DV} = 0$. However, as mentioned above, this is a fairly academic case.

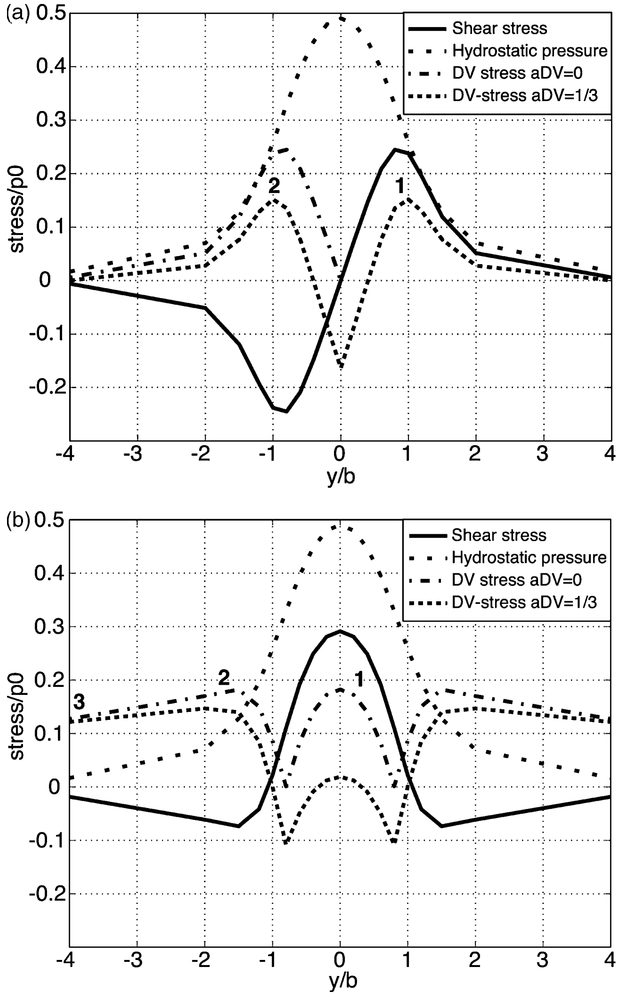

Fig. 1 Evolution of shear stress and Dang Van stress (normalized with peak contact pressure, p_0) in line contact at a shear plane: (a) parallel to the surface and (b) inclined 45° to the surface. Stresses adopted from Ref. [3].

The approximate friction correction employed in Ref. [2] is, as seen from Fig. 5 and Table 1 in Ref. [2], targeted at low to moderate friction for which it works well. In cases of high friction the material damage will occur at the surface (as mentioned in Ref. [2] and also noted by Ciavarella and Maitournam). This will result in surface-initiated fatigue for which FI_{sub} is irrelevant.

In conclusion, the results of Ciavarella and Maitournam stem from their application of the criteria to a case of two-dimensional line contact. This is of practical interest since today's RCF testing is often carried out as twin disc tests, resulting in line contact. In cases of wheel–rail contact for which the criterion was developed, the accuracy is good as evidenced by results presented here and in Ref. [2].

Table 1 The influence of the material parameter a_{DV} in wheel–rail rolling contact conditions.

a_{DV}	F_z (kN)	F_y (kN)	F_x (kN)	Rail radius (m)	σ_{EQ} (MPa)	FI_{sub} (MPa)	Error (%)
0.00	185	0	0	0.15	340	331.8	2.5
0.32	185	0	0	0.15	319	331.8	4.0
1.00	185	0	0	0.15	315	331.8	5.3
0.00	185	18.5	18.5	0.3	305	254.8	19.7
0.32	185	18.5	18.5	0.3	246	254.8	3.6

Wheel diameter 0.88 m, no residual stresses. FI_{sub} is the approximation of σ_{EQ} proposed in Ref. [2].

REFERENCES

1 Ringsberg, J. W. (2001) Life prediction of rolling contact fatigue initiation. *Int. J. Fatigue* **23**, 575–586.
2 Ekberg, A., Kabo, E. and Andersson, H. (2002) An engineering model for prediction of rolling contact fatigue of railway wheels. *Fatigue Fract. Engng Mater. Struct.* **25**, 899–909.
3 Lundberg, G. and Sjövall, H. (1958) *Stress and Deformation in Elastic Contacts*, The Institution of Theory of Elasticity and Strength of Materials, Chalmers University of Technology, Gothenburg, Publication no. 4.
4 Ekberg, A. (1997) Rolling contact fatigue of railway wheels—A parametric study. *Wear* **211**, 280–288.