Proceedings of The Fifteenth (2005) International Offshore and Polar Engineering Conference
Seoul, Korea, June 19–24, 2005

ISBN 1-880653-64-8 (Set); ISSN 1098-6189 (Set)

A Novel Method for Power Flow Design and Control Based on Power Flow Mode Theory

Y. P. Xiong[1, 2], *J. T. Xing*[1] *and W. G. Price*[1]
[1]School of Engineering Sciences, Ship Science, University of Southampton, Southampton, UK
[2]Institute of Engineering Mechanics, Shandong University, Jinan, People's Republic of China

ABSTRACT

In a previous study, a generalized power flow mode theory was proposed to describe the power flow behaviour of a dynamical system based on the inherent characteristics of the system's damping distribution. By extending this theory, a power flow design and control mathematical model is developed which allows control of energy flow patterns, thus reducing or retaining vibratory energy flow in a particular vibration mode of the system. This is achieved through analyzing energy flow characteristics and designing an appropriate damping distribution in the system to adjust its characteristic damping factors and power flow mode vectors. To meet different vibration control requirements, new design criteria are proposed so as to dissipate maximum vibration energy and/or to control power flow in a specific vibration mode of the system. This mathematical model is demonstrated through an example of a suspension system with two degrees of freedom for which the power flow dissipation corresponding to selected control cases are presented. This study provides a novel approach to design a dynamical system from the perspective of energy flow patterns.

KEY WORDS: Power flow mode theory; Characteristic damping factor; Damping distribution; Power flow design; Power flow control; Mode control factor.

INTRODUCTION

Many types of engineering structures, such as ships and offshore structures, are subjected to high-frequency excitation, in the sense that high modal densities are experienced in the high-frequency range. Typical examples include dynamic responses of various ship subject to explosion waves in sea, ship machinery foundations excited by engines and auxiliaries, etc. A power flow analysis provides a technique to describe the dynamical behaviour of various structures and coupled systems at medium to high values of frequency. It focuses on the flow of vibrational energy rather than the detailed spatial pattern of the structural response. The fundamental concept of power flow was discussed by Goyder and White (1980) and Pinnington and White (1981), with significant developments and advances reported by Fahy and Price (1999).

In recent years, this approach has been applied to model complex structures and various methods are proposed. For example, Langley (1990) described a direct-dynamic stiffness method to analyse the power flow in beams and frameworks. Cuschieri (1990) used a mobility approach to analyse the power flow in periodically connected beams. Wohlever and Bernhard (1992) applied a power flow analysis to rods and beams. Bouthie and Bernhard (1992) developed power flow models for the propagation of flexural waves in isotropic membranes and thin plates. From a more generic viewpoint, Xiong *et al.* (2001) proposed a generalized mobility / impedance power flow mathematical model to analyse the dynamical behaviour of a complex coupled system consisting of any number of substructures with various configurations and multiple interaction interfaces. Park *et al.* (2001) derived the energy governing equations for in-plane waves in finite coupled thin plates. Based on continuum dynamics, Xing and Price (1999) presented a generic power flow theory in which the concept of energy flow density vector and energy flow line are defined. This was further investigated by Wang *et al*. (2002a, 2002b, 2004) for indeterminate rod / beam system, L-shaped plates and a complex coupled plate-cylindrical shell system.

Enhancements to the traditional linear passive, active and hybrid vibration controls incorporated into a power flow analysis have been proposed and investigated by Miller *et al.*(1990); Pan and Hansen, (1993); Royston and Singh (1996); Xiong *et al.*(1996); Gardonio *et al.* (1997); Xiong *et al.*(2000, 2003) to name but a few selected studies.

Investigations into nonlinear behaviour of power flow are limited. Royston and Singh (1996, 1997) studied vibratory power flow through a nonlinear isolator path to a compliant, linear receiver associated with an automotive hydraulic engine mount. In a recent paper, Xiong *et al.*(2005) studied an interactive system comprising of equipment, nonlinear isolator and travelling flexible ship excited by waves to examine its nonlinear power flow characteristics and vibration isolation effectiveness.

A mobility-based power flow mode approach, as proposed by Ji *et al*. (2003), uses the eigen properties of the real part of the mobility matrix and power mode force vector to describe the time-averaged power input to a system. However, to predict input power and power transmission requires full knowledge of the system's physical properties such as inertia, elastic and damping parameters as well as the frequency of the external forces. To overcome this complexity, Xiong *et al.* (2004) developed a generalized power flow mode theory based on the inherent characteristics of the system's damping distribution to describe the power flow behaviour of a dynamical system. In this theory, the characteristic damping matrix is constructed and its eigenvalues and eigenvectors are defined as the characteristic damping factor and the power flow mode vectors of the system, respectively. The system's velocity vector is decomposed in the power flow space spanned by eigenvectors of the characteristic damping matrix from which it is determined. From this information, the total time averaged power dissipated in the system is represented by a summation of the N independent energy dissipations of which each corresponds to a respective power flow mode. This damping-based power flow mode approach provides insights into the energy flow dissipation mechanisms of dynamic systems.

Based on these developed mathematical models, this study aims to answer the following questions.

i) For a system with prescribed vibrations excited by external forces, how is appropriate damping distribution designed and arranged to achieve the most effective vibration control?

ii) In practical applications of vibration control, it is often required to suppress a particular vibration mode of a system. How can damping distributions be designed to achieve prescribed suppressions?

iii) For the objective of ii), how to retain a particular vibration mode for which the designed damping distribution dissipates minimum energy of this mode?

For a given system, several guidelines are presented to design appropriate damping distributions to meet different control requirements. Theoretical analysis and numerical simulations provide a means of demonstrating the applicability of the developed mathematical model using a suspension system.

BRIEF REVIEW OF POWER FLOW MODE THEORY

Generalized Formulation of a Vibration System

For generality, the dynamic equation governing a generalized linear system with N degree-of-freedom is represented in the matrix form

$$\mathbf{M}\,\ddot{\tilde{\mathbf{X}}} + \mathbf{C}\dot{\tilde{\mathbf{X}}} + (\mathbf{K} + \mathrm{i}\eta\overline{\mathbf{K}})\tilde{\mathbf{X}} = \tilde{\mathbf{f}}e^{\mathrm{i}\omega t} = \tilde{\mathbf{F}}\,, \tag{1}$$

where $\tilde{\mathbf{F}}$ denotes an excitation force vector, $\mathbf{M}$ and $\mathbf{K}$ represent real symmetric and semi-positive definite mass and stiffness matrices, respectively, $\mathbf{C}$ is a damping matrix which may be non-symmetric in form and $\overline{\mathbf{K}}$ is a real symmetric stiffness matrix relating to hysteretic damping involving a loss factor η .

For a solution $\dot{\tilde{\mathbf{X}}} = \tilde{\mathbf{V}}e^{\mathrm{i}\omega t}$, Eq. 1 reduces to

$$\tilde{\mathbf{f}} = \tilde{\mathbf{Z}}\tilde{\mathbf{V}}\,, \tag{2a}$$

$$\tilde{\mathbf{Z}} = \mathrm{i}\omega\mathbf{M} + \mathbf{C} + (\mathbf{K} + \mathrm{i}\eta\overline{\mathbf{K}})/\mathrm{i}\omega\,. \tag{2b}$$

In general, the impedance $\tilde{\mathbf{Z}}$ is described by a non-symmetric matrix due to the non-symmetric nature of the damping matrix $\mathbf{C}$.

Power Flow Mode Vector and its Characteristic Factor

Through a detailed analysis, Xiong *et al*. (2004) developed a mathematical model in which the following results are derived and validated. Namely,

$$\tilde{\mathbf{V}}^{\mathrm{H}}\overline{\mathbf{C}}\tilde{\mathbf{V}} = \tilde{\mathbf{V}}^{\mathrm{H}}\tilde{\mathbf{f}} + \tilde{\mathbf{f}}^{\mathrm{H}}\tilde{\mathbf{V}}\,, \tag{3a}$$

where

$$\overline{\mathbf{C}} = (\mathbf{C} + \mathbf{C}^{\mathrm{T}})/2 + \eta\overline{\mathbf{K}}/\omega \tag{3b}$$

is a real and symmetric matrix called the characteristic damping matrix of the system and H denotes the Hermitian transpose of a matrix. The time-averaged input power to the system is expressed as

$$P = \frac{1}{2}\mathrm{Re}\{\tilde{\mathbf{V}}^{\mathrm{H}}\overline{\mathbf{C}}\tilde{\mathbf{V}}\}\,. \tag{4}$$

The characteristic matrix $\overline{\mathbf{C}}$ is decomposed into the form

$$\overline{\mathbf{C}} = \mathbf{\Phi}\mathbf{\Lambda}\mathbf{\Phi}^{\mathrm{T}}\,, \tag{5}$$

where $\mathbf{\Lambda} = \mathrm{diag}(\lambda_j)$ is a real diagonal matrix of the eigenvalues λ_j of the matrix $\overline{\mathbf{C}}$ and $\mathbf{\Phi} = [\varphi_1 \quad \varphi_2 \quad \cdots \quad \varphi_N]$ is its corresponding matrix of eigenvectors satisfying the orthogonal relation $\mathbf{\Phi}^{\mathrm{T}}\mathbf{\Phi} = \mathbf{\Phi}\mathbf{\Phi}^{\mathrm{T}} = \mathbf{I}$, where $\mathbf{I}$ is a unit matrix.

We define λ_j as the j-th characteristic damping factor and eigenvector φ_j as the j-th power flow mode vector of the system. The power flow mode vectors are linearly independent of each other and are chosen as a set of base vectors spanning the power flow space. This set, therefore, allows a complete description of the power flow of the system.

The velocity vector $\tilde{\mathbf{V}}$ is represented in the power flow space and decomposed into the form

$$\tilde{\mathbf{V}} = \mathbf{\Phi}\tilde{\mathbf{q}}\,, \tag{6a}$$

$$\tilde{\mathbf{q}} = \mathbf{\Phi}^{\mathrm{T}}\tilde{\mathbf{V}}\,, \tag{6b}$$

$$\tilde{q}_j = \varphi_j^{\mathrm{T}}\tilde{\mathbf{V}}\,, \tag{6c}$$

where $\tilde{\mathbf{q}}$ is defined as a complex *characteristic velocity* vector. The time-averaged power flow expressed in Eq.4 is represented as

$$P = \frac{1}{2}\mathrm{Re}\{\tilde{\mathbf{q}}^{\mathrm{H}}\mathbf{\Lambda}\tilde{\mathbf{q}}\} = \sum_{j=1}^{N} P_j\,, \tag{7a}$$

$$P_j = \frac{1}{2}\lambda_j\left|\tilde{q}_j\right|^2. \tag{7b}$$

This result provides a *damping-based power mode* expression of the power flow in which P_j represents the energy dissipated by the *j*-th power flow mode. It relates to only one component of the characteristic velocity vector and its corresponding characteristic damping factor. Eq.7 demonstrates that the total power dissipated in the system is a summation of the power dissipated by all the *N* independent power flow modes.

Let us assume that λ_1 and λ_N denote the smallest and largest eigenvalues of the characteristic damping matrix $\overline{\mathbf{C}}$. The time-averaged power *P* can be estimated to lie in the range defined by the lower bound P_{low} and the upper bound P_{up} as given by

$$P_{\text{low}} = \frac{1}{2}(\sum_{j=1}^{N}\left|\tilde{q}_j\right|^2)\ \lambda_1 < P < \frac{1}{2}(\sum_{j=1}^{N}\left|\tilde{q}_j\right|^2)\ \lambda_N = P_{\text{up}}. \tag{8}$$

Noting that $\tilde{\mathbf{q}}^{\text{H}}\tilde{\mathbf{q}} = \tilde{\mathbf{V}}^{\text{H}}\mathbf{\Phi}\mathbf{\Phi}^{\text{T}}\tilde{\mathbf{V}} = \tilde{\mathbf{V}}^{\text{H}}\tilde{\mathbf{V}} = \left|\tilde{\mathbf{V}}\right|^2$, it follows that

$$P_{\text{low}} = \frac{1}{2}(\sum_{j=1}^{N}\left|\tilde{v}_j\right|^2)\ \lambda_1 < P < \frac{1}{2}(\sum_{j=1}^{N}\left|\tilde{v}_j\right|^2)\ \lambda_N = P_{\text{up}}, \tag{9}$$

$$\frac{\lambda_1}{2} < \frac{P}{|\tilde{\mathbf{V}}|^2} < \frac{\lambda_N}{2}. \tag{10}$$

Physically, $P/|\tilde{\mathbf{V}}|^2$ represents the power dissipated per unit velocity square norm. Therefore Eq. 10 provides an estimation of time-averaged power flow in terms of the maximum and minimum characteristic damping factors relating only to the damping of the system and does not require any information of the vibration sources and responses of the system.

POWER FLOW DESIGN APPROACHES

To dissipate maximum vibration energy

From Eq.7b, the time-averaged power dissipated in the *j*-th mode depends on λ_j and $|\tilde{q}_j|^2$. For a unit power flow response $\tilde{q}_j$, the time-averaged power flow given in Eq. 7 depends only on the system's natural characteristic damping factor λ_j, which is independent of any external force. A large value of λ_j implies significant energy dissipation in the *j*-th power flow mode φ_j. Hence, to obtain the maximum energy dissipation for various power flow responses $\tilde{q}_j$ involving external forces, it is necessary to design a damping distribution with a maximum summation of the system's natural characteristic damping factor λ_j. That is

$$\lambda_1 + \lambda_2 + \cdots + \lambda_N = \text{Max}. \tag{11}$$

Based on matrix theory (Nering, 1963), the orthogonal transformation represented by Eq. 5 does not change the trace of the real symmetric matrix $\overline{\mathbf{C}}$ and therefore

$$\lambda_1 + \lambda_2 + \cdots + \lambda_N = \text{tr}\,\overline{\mathbf{C}}. \tag{12}$$

To suppress or retain a specific vibration mode

Le us assume that $\mathbf{\Psi} = [\boldsymbol{\psi}_1,\ \boldsymbol{\psi}_2,\ ...,\ \boldsymbol{\psi}_N]$ represents the matrix of normalized natural vibration modes of a system. These vibration modes satisfy the following orthogonal relations (Bishop and Price, 1979)

$$\mathbf{\Psi}^{\text{T}}\mathbf{\Psi} = \mathbf{I}, \tag{13a}$$

$$\mathbf{\Psi}^{\text{T}}\mathbf{M}\mathbf{\Psi} = \text{diag}(M_j), \tag{13b}$$

$$\mathbf{\Psi}^{\text{T}}\mathbf{K}\mathbf{\Psi} = \text{diag}(K_j), \tag{14a}$$

$$\omega_j^2 = K_j / M_j, \tag{14b}$$

where ω_j^2 denotes the natural frequency of the system. The velocity vector $\tilde{\mathbf{V}}$ can be decomposed in the vibration mode space into the form

$$\tilde{\mathbf{V}} = \mathbf{\Psi}\tilde{\mathbf{Q}}, \tag{15}$$

where $\tilde{\mathbf{Q}}$ is a general velocity vector. Substituting Eqs. 5 and 15 into Eq. 4 we find that

$$\begin{aligned} P &= \frac{1}{2}\text{Re}\{\tilde{\mathbf{Q}}^{\text{H}}\mathbf{\Psi}^{\text{T}}\overline{\mathbf{C}}\mathbf{\Psi}\tilde{\mathbf{Q}}\} \\ &= \frac{1}{2}\text{Re}\{\tilde{\mathbf{Q}}^{\text{H}}\mathbf{\Psi}^{\text{T}}\mathbf{\Phi}\mathbf{\Lambda}\mathbf{\Phi}^{\text{T}}\mathbf{\Psi}\tilde{\mathbf{Q}}\} \end{aligned} \tag{16}$$

and Eq.6b takes the form

$$\tilde{\mathbf{q}} = \mathbf{\Phi}^{\text{T}}\mathbf{\Psi}\tilde{\mathbf{Q}}, \tag{17}$$

in which the *j*-th power flow response $\tilde{q}_j$ is

$$\tilde{q}_j = \sum_{i=1}^{N}\varphi_j^{\text{T}}\boldsymbol{\psi}_i\tilde{Q}_i = \sum_{i=1}^{N}\gamma_{ji}\tilde{Q}_i. \tag{18}$$

Here,

$$\gamma_{ji} = \varphi_j^{\text{T}}\boldsymbol{\psi}_i \qquad (i, j = 1, 2, 3 \ldots, N) \tag{19}$$

is defined as the *i*-th component of the characteristic velocity $\tilde{q}_j$. This is a dot product of the *i*-th natural vibration mode and *j*-th power flow mode and is referred to the *mode control factor*.

On using Eqs 16 and 18, the energy dissipated in the *i*-th vibration mode $\boldsymbol{\psi}_i$ with unit-generalized velocity $\tilde{Q}_i$ is expressed as

$$P_i = \frac{1}{2}\sum_{j=1}^{N}\lambda_j\,|\gamma_{ji}|^2. \tag{20}$$

This expression relates the energy dissipation of the i-th vibration mode to the power flow modes in the power flow mode space. A large value of $|\gamma_{ji}|$ represents a large component of $\tilde{q}_j$ produced by the i-th natural vibration mode and therefore a large energy dissipation $\frac{1}{2}\lambda_j |\gamma_{ji}|^2$ in the j-th power flow mode.

Suppression of the i-th vibration mode can be realized by designing a power flow mode j that provides a large absolute value of mode control factor γ_{ji}. For the reverse case, to retain the i-th vibration mode, we need to design a power flow mode φ_j that provides a zero mode control factor γ_{ji}.

To suppress or retain a prescribed motion

From Eq. 6c, it is concluded that for a prescribed velocity response $\tilde{\mathbf{V}}$, a designed power flow mode φ_j^{T} orthogonal to $\tilde{\mathbf{V}}$ produces a zero characteristic velocity $\tilde{q}_j$ and therefore there is no energy dissipated by this energy flow mode to suppress the motion. This design principle is applied in the following section to retain the specific vibration mode of interest.

On the other hand, it follows from Eq. 6c that a designed power flow mode producing a large characteristic velocity $\tilde{q}_j$ dissipates a large amount of energy and therefore it can be used to suppress the prescribed motion.

APPLICATION OF POWER FLOW DESIGN METHOD

Model of a Suspension System

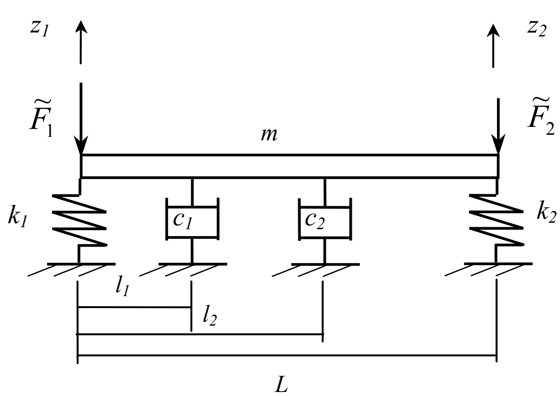


Fig.1. A suspension system with two degrees of freedom.

Fig.1 illustrates a suspension system with two-degrees-of-freedom used to demonstrate application of the proposed energy flow design and control method. Here a rigid uniform rod of length L and mass density m per unit length is supported at its two ends by two springs of stiffness k_1 and k_2. Two dampers of damping coefficients c_1 and c_2 are arranged as shown at different distances l_1 and l_2 from the left end of the rod. Two forces $\tilde{F}_1$ and $\tilde{F}_2$ are applied to excite the vibration of the rod. We choose the displacements z_1 and z_2 at the ends as two independent variables describing the rod motion. The two non-dimensional parameters $L_1 = l_1 / L$ and $L_2 = l_2 / L$ are adopted in the following equations.

Dynamic Equation. The dynamic equation of this system is derived as

$$\begin{bmatrix} ml/3 & ml/6 \\ ml/6 & ml/3 \end{bmatrix} \begin{Bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{Bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{Bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{Bmatrix} \tag{21}$$

$$+ \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} = \begin{Bmatrix} \tilde{F}_1 \\ \tilde{F}_2 \end{Bmatrix} = \begin{Bmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{Bmatrix} e^{i\omega t}.$$

The characteristic-damping matrix $\overline{\mathbf{C}}$, defined in Eq. 3, is derived as

$$\overline{\mathbf{C}} = \mathbf{C} = \mathbf{a}^{\mathrm{T}} \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \mathbf{a} = \left[c_{ij} \right], \tag{22}$$

where

$$\mathbf{a} = \begin{bmatrix} 1 - L_1 & L_1 \\ 1 - L_2 & L_2 \end{bmatrix}. \tag{23a}$$

A representative element c_{ij} of this characteristic damping matrix is calculated by the following expression

$$c_{ij} = \sum_{s=1}^{2} a_{si} c_s a_{sj}, \qquad (i, j = 1, 2). \tag{23b}$$

Eq. 22 is rewritten as

$$\overline{\mathbf{C}} = c_1 \begin{bmatrix} (1 - L_1)^2 \pm (1 - L_2)^2 & L_1(1 - L_1) \pm L_2(1 - L_2) \\ L_1(1 - L_1) \pm L_2(1 - L_2) & L_1^{\,2} \pm L_2^{\,2} \end{bmatrix}. \tag{24}$$

for the two special cases $c_2 = \pm c_1$.

Characteristic Damping Factors and Power Flow Modes. The eigenvalue problem of the characteristic-damping matrix $\overline{\mathbf{C}}$ of the system is defined as

$$\overline{\mathbf{C}} \varphi = \lambda \varphi, \tag{25}$$

which has its characteristic equation

$$\begin{vmatrix} c_{11} - \lambda & c_{12} \\ c_{21} & c_{22} - \lambda \end{vmatrix} = 0. \tag{26}$$

The solutions of Eq. 26 give the characteristic damping factors

$$\lambda_{1,2} = \frac{1}{2} [(c_{11} + c_{22}) \pm \sqrt{(c_{11} - c_{22})^2 + 4c_{12}^2}\,]. \tag{27}$$

The corresponding power flow modes are denoted by

$$\varphi_i = \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix}, \quad (i = 1, 2), \tag{28}$$

which satisfy the normalized relation used in Eq. 5 and is obtained by solving Eq. 25. That is

$$\begin{bmatrix} c_{11} - \lambda_i & c_{12} \\ c_{21} & c_{22} - \lambda_i \end{bmatrix} \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} = 0 \cdot \tag{29}$$

Trace of $\overline{\mathbf{C}}$**.** The trace of the characteristic-damping matrix $\overline{\mathbf{C}}$ is obtained as

$$\mathrm{tr}\overline{\mathbf{C}} = c_1(2L_1^2 - 2L_1 + 1) + c_2(2L_2^2 - 2L_2 + 1). \tag{30}$$

Natural Frequencies and Modes. The physical velocity response of the system is expressed as

$$\begin{Bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{Bmatrix} = \tilde{\mathbf{V}} e^{\mathrm{i}\omega t}. \tag{31}$$

For simplicity, let us examine the case $k_1 = k_2$. The natural frequencies and the corresponding natural modes of the system are calculated as follows.

Mode 1

$$\omega_1^2 = 2k_1 / mL, \tag{32a}$$

$$\psi_1 = \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}. \tag{32b}$$

Mode 2

$$\omega_2^2 = 12k_1 / mL, \tag{33a}$$

$$\psi_2 = \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}. \tag{33b}$$

Mode Control Factors. On using Eqs. 19, 28, 32 and 33, we express the mode control factors of the system as

$$\gamma_{ji} = \frac{\sqrt{2}}{2}[\beta_j + (-1)^{i+1}\alpha_j], \qquad (i, j = 1,2). \tag{34}$$

Damping Distribution Design

Maximum $\mathrm{tr}\overline{\mathbf{C}}$. For the given dampers, we now examine how to arrange the positions of the two dampers to obtain the maximum $\mathrm{tr}\overline{\mathbf{C}}$ to give the maximum energy dissipation for unit power flow response $\tilde{q}_j$, i.e. $|\tilde{q}_j|^2 = 1$.

Fig. 2 shows the $\mathrm{tr}\overline{\mathbf{C}}$ values as a function of the position parameters L_1 and L_2. From this figure it is observed that the position of the minimum value of $\mathrm{tr}\overline{\mathbf{C}}$ occurs at the point $(L_1, L_2) \equiv$ (0.5,0.5) or its maximum at points (0,0), (0,1), (1,0) and (1,1).

From these results, we conclude that to obtain maximum characteristic damping factors the two dampers are located at the ends of the rod, which includes the four cases for maximum $\mathrm{tr}\overline{\mathbf{C}}$ value. For the obverse, the two dampers are located at the middle point of the rod.

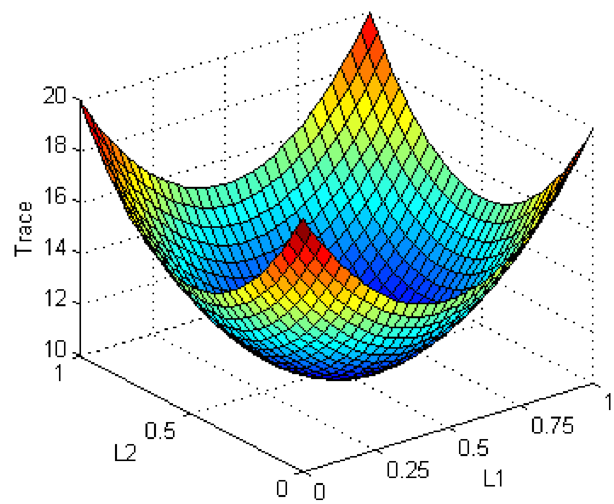


Fig.2. The value of $\mathrm{tr}\overline{\mathbf{C}}$ varies as a function of the position parameters L_1 and L_2.

Mode or Motion Control. Now we investigate how to suppress or retain a heave or pitch motion of the system using the power flow design method.

Heave Motion

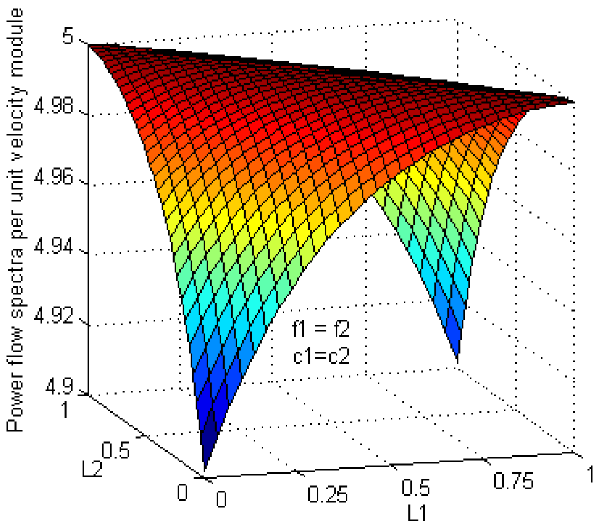
On using Eq. 4 or 20, the energy dissipation of the system in heave motion given by Eq. 32 is obtained as

$$P_1 = \psi_1^{\mathrm{T}} \mathbf{C} \psi_1 = c_1 + c_2. \tag{35}$$

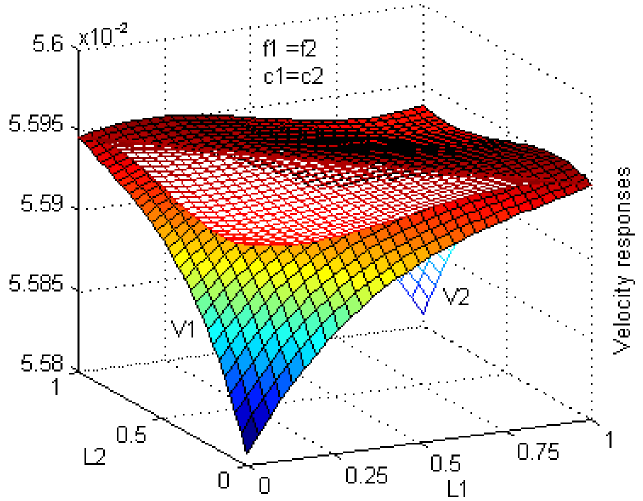
This result demonstrates that the heave motion is independent of the damper positions. However, if the two dampers are arranged in a non-symmetrical manner about the centre of the rod, the motion may not be pure heave.

To retain validity of Eq. 35, it is required that the two dampers are symmetrically arranged. That is

$$L_1 + L_2 = 1. \tag{36}$$



a)



b)

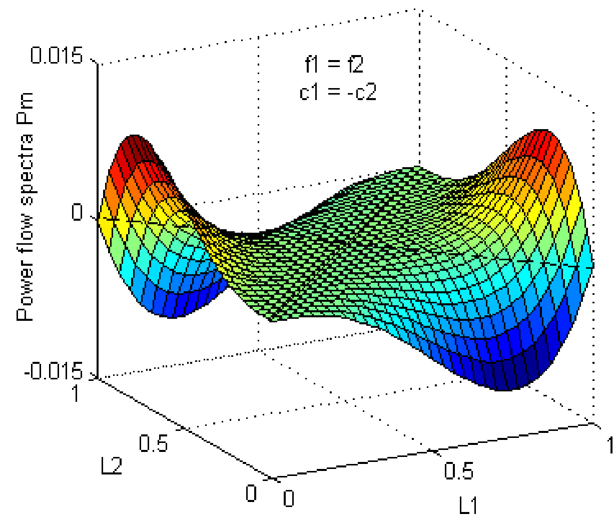
Fig.3. a) Power flow dissipation and b) velocities at the two ends of the rod affected by the mounting position of the two passive dampers

Fig.3a) shows the time averaged energy dissipation per unit velocity square norm of the system, i.e. $P_m = P/|\tilde{\mathbf{V}}|^2$, varying with the location parameters L_1 and L_2 of the two dampers ($c_1 = c_2$) under the same excitations $f_1 = f_2 = 10\,\text{N}$. It is demonstrated from this figure that maximum vibration power dissipation is achieved on the line $L_1 + L_2 = 1$ along which the velocities at both ends of the rod have the same value as shown in Fig 3b) and therefore the motion of the rod is heave as defined in Eq. 32.

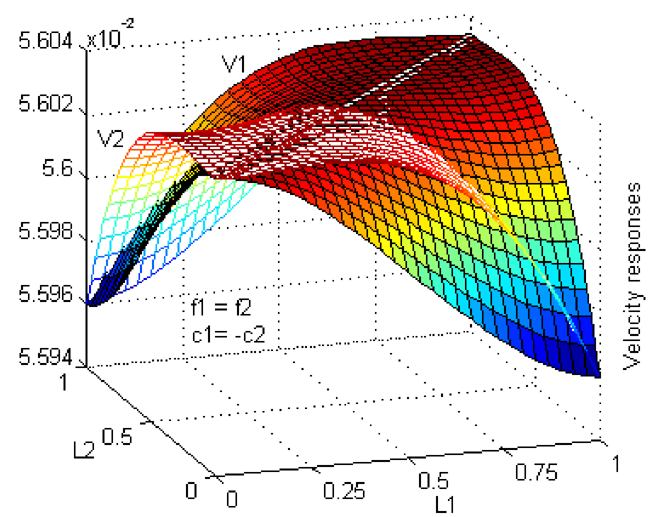
To obtain minimum energy dissipation in the heave motion, an active damper with negative damping coefficient (Xing *et al.* 2005) is required to make

$$c_1 + c_2 = 0\,. \tag{37}$$

Fig. 4 shows the energy dissipation of the system when an active damper satisfying Eq. 37 is applied. It is found that along the line described by Eq. 36 as shown by the dotted line in Fig. 4 a), there is no energy dissipation as demonstrated by the equivalent line in Fig. 4 a). Also along the line $L_1 = L_2$, the energy dispassion vanishes. This is because the two dampers are arranged at the same point, which produces zero resultant damping.

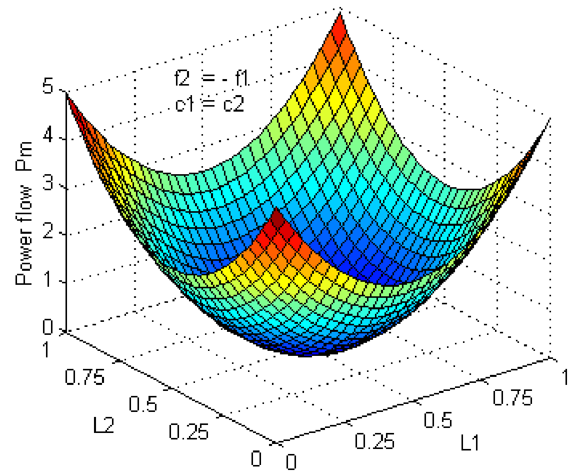


a)

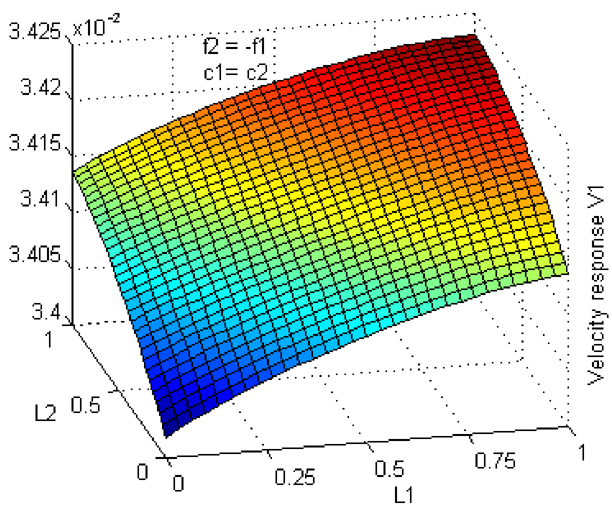


b)

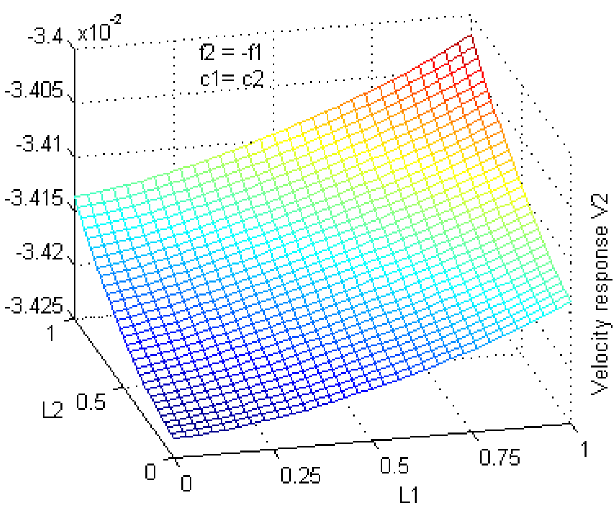
Fig. 4. a) Power flow dissipation and b) velocities at the two ends of the rod affected by the mounting position of a passive damper and an active damper satisfying Eq. 37.



a)



b)



c)

Fig.5 The energy dissipation a) and velocities b, c) affected by the locations of the two passive dampers under excitations of $\tilde{f}_1 = -\tilde{f}_2$.

Pitch Motion

In a similar way, the energy dissipation of the system in pitch motion described by Eq. 33 is obtained as

$$P_2 = \psi_2^{\mathrm{T}} \mathbf{C} \psi_2 = c_1(1-2L_1)^2 + c_2(1-2L_2)^2 . \tag{38}$$

From Eq. 38, it is concluded that

$$P_2 = \begin{cases} 0, & L_1 = L_2 = 0.5, \\ c_1 + c_2, & L_1 = 0,1 \quad L_2 = 0,1. \end{cases} \tag{39}$$

To excite a pitch motion of the rod, anti-symmetric forces $\tilde{f}_1 = -\tilde{f}_2$ are applied to the system characterized by a symmetric arrangement of two springs and dampers.

The energy dissipation in this case is shown in Fig. 5 a) and demonstrates the result given in Eq 39. For an arrangement of two dampers located at the middle point of the rod, $(L_1, L_2) \equiv (0.5, 0.5)$, there is no power dissipated. Therefore, the pitch vibration mode is retained. However, arranging the two dampers located at the ends of the rod, both dampers at one end ($L_1 = L_2 = 0$, or $L_1 = L_2 = 1$) and either damper at each end ($L_1 = 0, L_2 = 1$, or $L_1 = 1, L_2 = 0$), provides maximum energy dissipation. To keep the pitch motion of the rod, the symmetric arrangement of two dampers ($L_1 = 0, L_2 = 1$, or $L_1 = 1, L_2 = 0$) is appropriate. Figs. 5b and 5c show that for this case the velocities at the two ends of the rod satisfy the pitch motion $V_1 = -V_2$.

CONCLUSIONS

A power flow design and control approach based on power flow mode theory is developed to control energy flow transmission patterns, to reduce vibratory energy flow in a particular vibration mode or to retain a specific vibration mode of interest. This is achieved through analyzing energy flow characteristics and designing appropriate damping distributions in the system by adjusting its characteristic damping factor, the power flow mode vectors of the system.

It is demonstrated that the trace of the characteristic-damping matrix of the system represents its power flow dissipation with unit characteristic velocity. This provides the necessary criteria to design a damping distribution with large characteristic damping factor.

Moreover, *mode control factor* is introduced to reveal the coherence of a natural vibration mode or a motion form with a power flow mode. A particular vibration mode or motion form can be suppressed or retained through modifying the mode control factor by adjusting the damping distribution.

An example of a suspension system with two degrees of freedom is used to explain and demonstrate the theory and the power flow design approach. The positions of two dampers are investigated to obtain maximum characteristic damping factors. The power flow dissipation corresponding to heave and pitch modes of the system are analyzed to illustrate how to suppress or retain a particular vibration mode through a suitable damper arrangement.

The proposed power flow design approach offers insights into the behaviour of the energy dissipated in the entire system or in a particular mode. It provides a way of designing appropriate energy flow mode and corresponding characteristic damping factors to meet control requirements for a wide range of different dynamical systems. This study provides a new approach to control vibrations or to design a dynamical system from a power flow mode perspective.

REFERENCES

Bishop, RED, and Price WG (1979). "*Hydroelasticity of Ships*," Cambridge: Cambridge University Press.

Bouthie, OM, and Bernhard, RJ (1992). "Models of Space-averaged Energetics of Plates," *American Institute of Aeronautics and Astronautics Journal*, Vol 30, pp 616-623.

Cuschieri, JM (1990). "Vibration Transmission through Periodic Structures Using a Mobility Power Flow Approach," *J Sound and Vibration* Vol 143, pp 65-74.

Fahy, FJ, and Price, WG (1999). "*IUTAM Symposium on Statistical Energy Analysis*," Kluwer Academic.

Gardonio P, Elliott SJ, and Pinnington, RJ (1997). "Active Isolation of Structural Vibration on a Multiple-Degree-of-Freedom System, Part II: Effectiveness of Active Control Strategies," *J Sound and Vibration* Vol 207, pp 95-121.

Goyder, HGD, and White, RG (1980). "Vibrational Power Flow From Machines into Buildup Structures, Part III: Power Flow Through Isolation Systems," *J Sound and Vibration*, Vol 68, pp 97-117.

Ji L, Mace BR, and Pinnington, RJ (2003). "A Power Mode Approach to Estimating Vibrational Power Transmitted by Multiple Sources," *J Sound and Vibration*, Vol 265, pp 387-399.

Langley, RS (1990). "Analysis of Power Flow in Beams And Frameworks Using The Direct-Dynamic Stiffness Method," *J Sound and Vibration* Vol 136, pp 439-452.

Miller, DW, Hall, SR, and von Flotow, AH (1990). "Optimal Control of Power Flow at Structural Junctions," *J Sound and Vibration*, Vol 140, pp 475-497.

Nering, ED (1963). *Linear Algebra and Matrix Theory*. John Wiley

Pan, JQ, and Hansen, CH (1993). "Active Control of Power Flow from a Vibrating Rigid Body to a Flexible Panel through Two Active Isolators," *J Acoustical Society of America*, Vol 93, pp 1947-1953.

Park, DH, Hong, SY, Kil, HG, and Jeon JJ (2001). "Power Flow Models and Analysis of In-plane Waves in Finite Coupled Thin Plates," *J Sound and Vibration*, Vol 244, pp 651-668.

Pinnington, RJ, and White, RG (1981). "Power Flow through Machine Isolators to Resonant and non-Resonant Beam," *J Sound and Vibration*, Vol 75, pp 179-197.

Royston, TJ and Singh, R (1996). "Optimization of Passive and Active Nonlinear Vibration Mounting Systems Based on Vibratory Transmission," *J Sound and Vibration*, Vol 194, pp 295-316.

Royston, TJ and Singh, R (1997). "Vibratory Power Flow through a Non-Linear Path into a Resonant Receiver," *J. Acoust. Soc. Am*. Vol 101 pp 2059-2069.

Wang, ZH, Xing, JT and Price, WG (2002a). "Power Flow Analysis of Indeterminate Rod / Beam System Using a Substructure Method," *J Sound and Vibration,* Vol 249, pp 3-22.

Wang, ZH, Xing, JT and Price, WG (2002b). "An Investigation of Power Flow Characteristics of L-Shaped Plates Adopting a Substructure Approach " *J Sound and Vibration,* Vol 250, pp 627-648.

Wang, ZH, Xing, JT and Price, WG (2004). "A Study of Power Flow in a Coupled Plate–Cylindrical Shell System," *J Sound and Vibration,* Vol 271, pp 863-882.

Wohlever, JC, and Bernhard, RJ (1992). "Mechanical Energy Flow Models of Rods and Beams," *J Sound and Vibration*, Vol 153, pp 1-19.

Xing, JT, and Price, WG (1999). A Power-Flow Analysis Based On Continuum Dynamics, *Proceedings of the Roy. Soc. Lond. A*, Vol 455, pp 401-436.

Xing, JT, Xiong, YP, and Price, WG (2005) "Passive-Active Vibration Isolation Systems to Produce Zero or Infinite Dynamic Modulus: Theoretical and Conceptual Design Strategies," *J Sound and Vibration*. (in press and appeared on web).

Xiong Y.P., Song K. J., Wang C (1996). "Optimum Control of Vibration and Structure-Borne Noise for Machinery on Flexible Foundation," *Chinese J Mechanical Eng*, Vol 9, pp 13-20.

Xiong, YP, Xing JT, and Price, WG (2000). "Hybrid Active And Passive Control of Vibratory Power Flow in Flexible Isolation System," *Shock and Vibration*, Vol 7, pp 139-148.

Xiong, YP, Xing JT, and Price, WG (2001). "Power Flow Analysis of Complex Coupled Systems By Progressive Approaches," *J Sound and Vibration*, Vol 239,pp275-295.

Xiong, YP, Xing JT, and Price, WG (2003). "A General Linear Mathematical Model of Power Flow Analysis and Control of Integrated Structure-Control Systems," *J Sound and Vibration*, Vol 267, pp 301-334.

Xiong, YP, Xing JT, and Price, WG (2004). "A Power Flow Mode Theory Based on Inherent Characteristics of Damping Distributions in Systems and its Applications," CD Proc 21th International Congress of Theoretical and Applied Mechanics, IUTAM, Warszawa, 15-21 August, 2004.

Xiong, YP, Xing JT, and Price, WG (2005). "Interactive Power Flow Characteristics of an Integrated Equipment ~ Nonlinear Isolator ~ Travelling Flexible Ship Excited by Sea Waves" *J Sound and Vibration*, (in press and appeared on web).