A FAST MODEL FOR SEQUENTIAL COMBINATORIAL OPTIMISATION PROBLEMS IN FINITE ELEMENT STUDIES: APPLICATIONS IN AEROSPACE WELDING

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ABSTRACT
Combinatorial optimisation usually involves many iterations, which makes such approaches computationally expensive, especially when dealing with complex finite element models. This paper proposes to use a surrogate model that can bring down the computational expense of sequential combinatorial finite element problems. The idea of using such model is illustrated on a welding example.

KEYWORDS: combinatorial optimisation, surrogate models

1. INTRODUCTION

The welding of a vane to the inner ring of a gas turbine tail bearing housing (TBH) is chosen to illustrate the method. A typical TBH is shown in Figure 1. Its major structural details are the outer ring, the inner ring and the vanes. Vanes are usually welded to the rings using Gas Tungsten Arc Welding, frequently known as TIG welding.

The inner ring and the vane are both clamped during the welding. The welding causes a contraction which occurs when the melted material in the weld pool is cooled. This causes stresses of large magnitude to be distributed throughout the component and also imparts deformations to the workpiece. In our example the vane deforms and the deformation is characterized by the displacement of the two nodes opposite to the welding event, identified as Node 10 and Node 96 (the front tip at the leading edge and the back tip at the trailing edge as shown in Figure 2).

The welding parameters such as weld speed and power are defined by the welding process itself and can only to a little extent be altered to reduce the deformation. Another possible method can be to divide the welding path into smaller intervals and try to find the sequence of welds that causes minimal or no deformation; the latter method will be used here

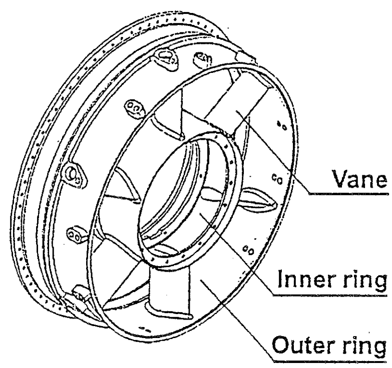


Figure 1 – Tail bearing housing

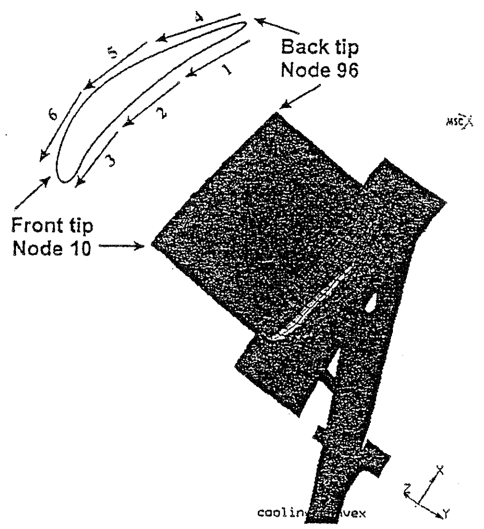


Figure 2 – Welding paths and FE simulations

2. PROBLEM NOTATION AND ANALYSIS

Figure 2 (top) shows a cross-section of the base of the vane that has to be welded to the inner ring. The welding takes place on the contours of this diagram. Here, the contour has been divided into six welding paths, as shown on the diagram. The welding is performed on one section at a time and there is a 5 second gap between the end of one weld and the beginning of the next.

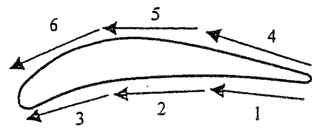
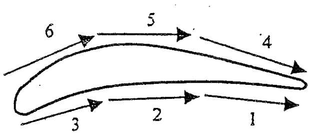
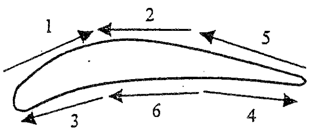
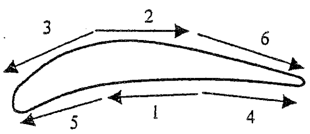
Sequence Number	Welding position						Diagram
	1	2	3	4	5	6	
1	1	2	3	4	5	6	
2	-1	-2	-3	-4	-5	-6	
6	-6	5	3	-1	4	2	
20	2	-5	6	-1	3	-4	

Table 1 – Problem notation

The notation adopted to describe some possible welding sequences is presented in Table 1. Four sequences (labelled 1, 2, 6 and 20) are shown for illustration. In the header row, labels 1 to 6 designate the 6 places in a welding sequence. Each welding sequence contains the names of welding events as given in Table 1, Sequence 1. The *minus sign corresponds to the opposite direction* as shown in Table 1, Sequence 2. The dashed line running vertically between two columns represents the 5 second gap, used to change and reposition the welding tool. The numbers in the heading columns correspond to the order in the sequence in which the welds have been carried out. Arrows show the direction and position.

A finite element (FE) model is used to calculate the displacement at Node 10 and Node 96 (here only Node 10 is considered, as Node 96 shows similar behaviour, as shown in Figure 3). The analysis shows that only the X component of the displacement needs to be considered for optimisation, as it is the most significant and changes in both directions (plus and minus). Component Z is an order of magnitude lower than X and therefore does not have significant influence on the resultant displacement. Component Y only changes in the positive direction, and cannot be compensated for by changing the welding sequence. The model thus takes as input the welding sequence and produces a displacement diagram over the welding process.

The three components of the displacement at node 10 for welding sequence number 6 are shown in Figure 3. The curve labelled 'speed' shows the speed of the welding tool, and may be used to visualise and distinguish between the six welding processes and the 5 second cooling gap. The process completes after 193 seconds and then until second 300 the weld is cooled with the clamps on. At second 300, the clamps are released and the cooling continues to room temperature. The work described here attempts to minimise the displacements while the component is clamped.

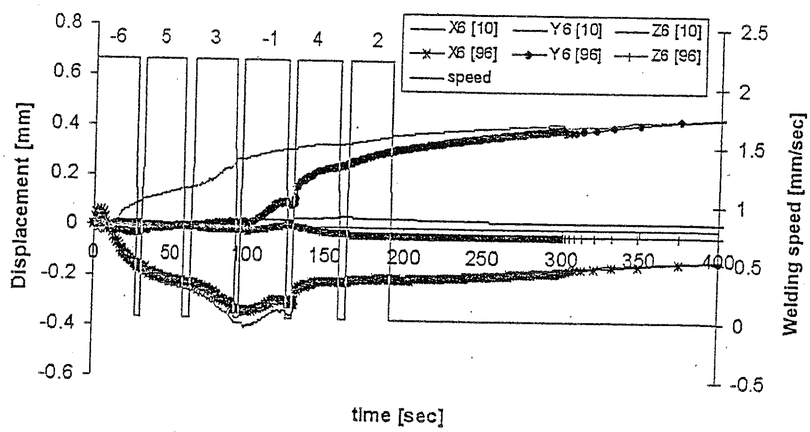


Figure 3 – X, Y and Z components of the displacement at node 10, for welding sequence number 6

Displacements X for runs 1, 2, 6 and 20 from Table 1 are shown in Figure 4. Note that variation of the welding sequence can significantly change the displacement, and therefore can be used for optimisation purposes.

In summary, the aim is to reduce the final displacement, by changing the welding sequence. We have six variables (one for each welding position), each of which can take 12 non-numerical values (6 for one direction and 6 for the opposite). An obvious solution is to run all possible 46080 ($2^6 \times 6!$) combinations and select the best amongst them. However, this is not feasible when taking into account that one combination takes 32 hours to compute, using the FE approach.

Considering this high computational expense, the aim here is to optimise the process with a maximum of 50 FE runs. Most of the available combinatorial optimisation methods in the literature would far exceed this number, i.e., integer programming, graphs, branch and bound, binary trees [1], etc. Therefore, the aim is to reduce the computational expense, by constructing a surrogate model that requires less effort to compute. This model is discussed next.

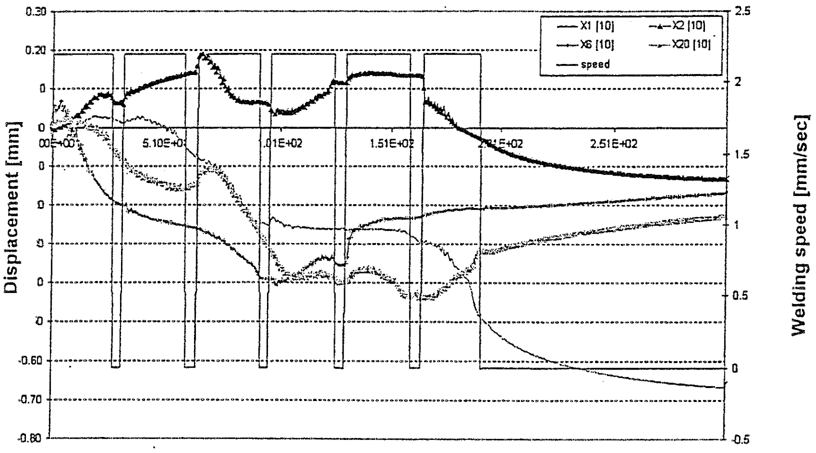


Figure 4 – Displacements for 4 welding sequences

3. SURROGATE MODELS

The idea of using surrogate models in optimisation is widely explored for problems related to expensive computations. A well-known surrogate method is kriging, but could be any other approximation method that can be suitable for the specific problem.

Figure 5 (left) shows a conventional optimisation cycle for a finite element (FE) model. Optimisation may require thousands of iterations to converge which due to the computationally expensive FE model could be a very lengthy procedure. Figure 5 (right) shows an optimisation cycle built on around a surrogate model. This model is an approximation of the FE model, and such that is very fast to compute. For instance a single FE run could take about 48 hours, while a 50000 surrogate evaluations could take less than 10 minutes.

The surrogate cycle begins with several initial runs, that are needed to 'train' the surrogate model. The number of these initial runs depends on the complexity of the studied behaviour. Usually around 20-30 initial runs are sufficient to obtain reasonable initial approximation. Once the model is trained, it is included in an optimisation cycle. For more complex and multimodal functions, it is feasible to allow more iterations if using simulated annealing or genetic algorithms. Once an optimum result is obtained, one needs to bear in mind that this is a result based on approximation and might not be accurate. That is why it is feasible to verify it with the value produced by an FE run at the same design point. If there is significant difference between the two values, one trains the surrogate model again, including the new FE run. The optimisation cycle is then applied again. The procedure continues until the values produced by the surrogate model and the FE model agree. In this way, the surrogate model becomes more accurate around the area of optimum, i.e., accuracy is added only where it is needed, which therefore reduces the computational time.

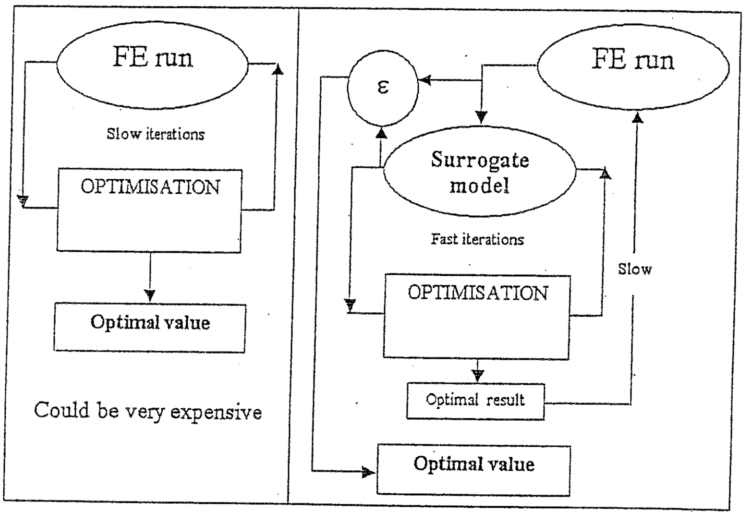


Figure 5 – The usage of surrogate models

As mentioned earlier, many methods for function approximation are available through out the literature when the function depends on variables that have continuous or discrete nature and can be physically or quantitatively expressed. However none exist in the combinatorial domain. This is a domain, where the variables have no physical meaning, for instance they are just an order in a sequence.

Such models however, have been developed and currently tested. It will be a subject of further publication. Within the scope of this one, some results related to such models can be shown.

- It maps the non-continuous space to a continuous one, allowing the conversion of a sequential combinatorial problem to a conventional representation.
- The model is applicable to a variety of sequential combinatorial problems.
- The model is built sequentially – every new combination presented to it makes it more accurate.
- The model is efficient - it extracts all the useful information out of the presented FE runs, and arranges this in a priority list, based on its usefulness.
- The model is fast - All possible 46080 combinations may be calculated in less than 5 minutes on 800 MHz Pentium III machine, using a MATLAB code. A code written in C or FORTRAN would produce results much faster.
- 27 initial FE calculations were sufficient to obtain a high accuracy, fast prediction, which produced the following value for the optimal displacement –

0.00023*mm*, which is very close to the FE predicted value of 0.000231*mm*. The final optimisation results are shown in Figure 6, where it is seen that run 28, crosses the horizontal axis for the displacement at the moment of clamps removal, as desired. Figure 6, shows the displacement values for all 28 runs, showing the last as being minimal.

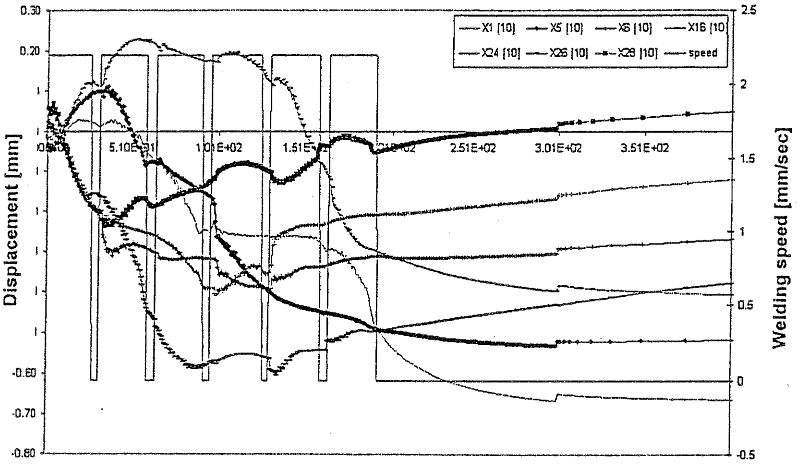


Figure 6 – Run 28 is the optimised displacement with a clamped structure

Figure 7 visualises the displacements obtained for all 28 runs, showing that its absolute value comes to 0 for the final 28th run.

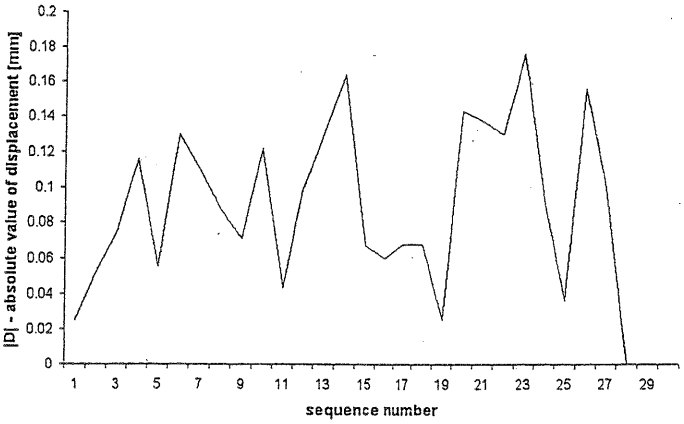


Figure 7 – Absolute value of the displacement for the all 28 runs, showing the last as being optimal

4. CONCLUSIONS

The paper has introduced the idea of using a surrogate model that can be applied to sequential combinatorial finite element (FE) problems. The model proves to be fast and accurate and can be used for optimisation purposes. The method has been applied to the optimisation of a welding sequence, and has demonstrated that optimum post-weld distortions can be achieved in 28 FE runs, out of possible 46080 combinations. This greatly reduces the computational expense needed in weld planning, without significant loss of accuracy. The model has been adapted so that it can be used with any volume of information, allowing for dynamic expansion as more data is made available. This poses the interesting problem of developing a dynamic optimal design of experiments for sequential combinatorial problems. Being a link between combinatorial and continuous domain, this model might be used to explore widely this branch of space mapping and optimisation techniques.

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REFERENCES

[1] Winston., W, 1995, Introduction to mathematical programming. Second Ed., Duxbury Press, California, USA.