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An efficient experiment methodology to investigate product design: an acoustic sounder case study.

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Abstract

An innovative experiment methodology is described which enables the investigation, through planned experiment, of component aspects of a manufactured product whose values cannot be economically set to specified values in an experiment. These aspects are called semi-controlled factors and their presence in an investigation means that conventional experiment plans cannot be employed. The proposed method of experimentation integrates the study of the manufacturing system and the product design. It involves obtaining samples of components from production and careful measurement of their relevant features. The new approach is demonstrated through a case study on the design and analysis of an experiment to investigate how seven factors influence the performance of an electro-mechanical product called an acoustic sounder. A computer search is used to select from a batch of available components those sets which, when assembled into products, will enable maximum information to be gained from product testing. To facilitate this extensive search, an exchange algorithm and a genetic algorithm have been employed. These algorithms are described and their performances in searching for a plan are compared for the case study.

Keywords:

assembly; component; design of experiments; D-optimality; electro-mechanical; factorial experiment; genetic algorithm; search algorithm; semi-controlled

1. Introduction.

The need for manufacturing industry to produce high quality goods at low cost has encouraged the use of designed experiments to investigate factors affecting product performance. These investigations often employ statistical experiment plans such as orthogonal arrays (Myers and Montgomery, 2002) which specify combinations of factor values to be used in prototype products for the experiment. A planned experiment on a manufactured product typically requires special components to be obtained which have the particular dimensions or features specified by the experiment plan. These components may be specially fabricated, or carefully selected from a large sample of measured components taken from production. For some products both these methods of obtaining components for the experiment may be prohibitively expensive or impractical. In this paper a methodology is presented which enables efficient experimentation on manufactured products for which it is not economically feasible to set factors to pre-specified values.

The approach is illustrated through a case study of an acoustic sounder manufactured by Hosiden Besson Ltd and used in products such as fire alarms and telephones. The company wished to investigate the effects of seven factors on sound output (in dB SPL) in a planned experiment. The factors included both manufacturing variables, such as heat treatment conditions for batches of components, and product design variables such as armature pip height. A conventional experiment plan could not be used for this product as two of the factors to be investigated could not be consistently produced at pre-specified values. Although the values of such factors cannot be fully controlled in the experiment, the choice of which factor values are combined together is under the experimenter's control. These factors will be called semi-controlled. The method presented involves obtaining samples of components and carefully measuring the features relevant to these factors. A tailored search algorithm is then needed to select sets of components from the samples which, when assembled into products, enable maximum information to be gained. The algorithm simultaneously seeks the best combinations of values of the semi-controlled factors and the pre-specified settings for the remaining factors. Other applications where these semi-controlled experiments have been exploited include hydraulic gear pumps for the automotive industry and precision fuel pumps for the aeronautical industry (see, for example, Sexton *et al*, 2000).

In section 2 the acoustic sounder is described and the factors included in the experiment are discussed. The plan used in the case study is introduced in section 3 and section 4 outlines two competing algorithms for finding plans for this kind of study, a genetic algorithm and an exchange algorithm. The performances of these algorithms in finding a plan for the case study are compared. In section 5 the analysis of the data from the experiment is presented and conclusions are given in section 6.

2. The problem

The investigation concerns an electro-acoustic transducer of the type shown in Figure 1 whose operation and manufacture are briefly described. An oscillating input electrical signal fed to the bobbin coils is converted into changes in the magnetic circuit that cause the armature to rock on its pivot using a "push-pull" action. By careful adjustment, the torsionally restrained armature is held in a balanced condition by virtue of the permanent magnet, allowing the transducer to be adjusted to achieve a high sensitivity. One end of the armature is connected to a lightweight diaphragm that, due to the back and forth motion, radiates sound. The air gaps between the rocking armature and the yoke need to be carefully adjusted so that the balanced state of the armature is maintained during operation. This is done using an electrical measurement (the electrical gap adjustment), and this sensitive process is undertaken manually during manufacture. The magnetic charge of the permanent magnet is then adjusted to achieve the required capsule sensitivity (a process known as "demagging"). Other factors believed to influence sound output include the pip height of the pivot, unevenness in the distribution of the windings between the bobbin coils, and the force used to crimp the front cover to the capsule frame.

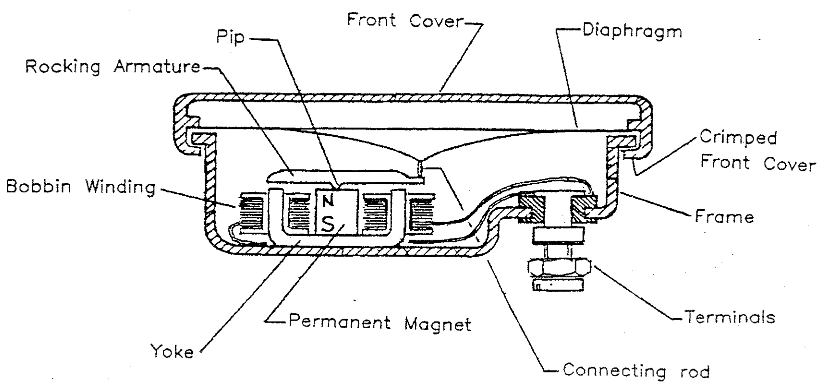

Figure 1. Schematic of electro-acoustic transducer type which forms the capsule investigated

From discussions with engineers at Hosiden Besson Ltd., the seven factors shown in Table 1 were identified as the most likely to affect the sound output and hence as the factors that should be included in an experiment. The objective of the experiment was to find out which, if any, of these factors had a large effect on product performance and whether or not there was scope for improving the product or manufacturing process through changing the settings of some of the factors. The first five factors in the table can be set to specific values in the experiment. The inclusion of the bobbin windings factor aims to explore changes from normal manufacture in which there is an equal or "even" number of windings on each bobbin (with a tolerance of three windings). Thus "even" windings was one value used for this factor in the experiment. The two remaining values represented uneven windings outside tolerance: 16 more windings on the bobbin nearer to the centre of the capsule than on the other bobbin (denoted by +8/-8), and a reversal in windings on the two bobbins (-8/+8). The armature pip (or pivot) height had two values in the experiment: the current nominal height and an increased height of three times the current tolerance. Values used for the electrical gap adjustment related to the gap size currently used which is within the central one third of the sector on the oscilloscope used for the adjustment. This sector was divided into three equal sectors. Three adjustments were made in the experiment: within the central sector (much tighter adjustment than usual) and adjustment to one of the two outer sectors. The crimping force was investigated with the crimping machine air pressure at its current setting of 6 bar, and a lower setting of 5 bar. The impact of the demagging operator on sound output was investigated through the use of a skilled operator (A) and a less experienced operator (B).

The final two factors shown in Table 1, armature permeability and yoke permeability, are semi-controlled factors. They cannot easily be set to pre-specified values for the experiment because magnetic permeability varies due to the source metal, the cutting and stamping procedures and the heat treatment process used during manufacture. It was not practical to make yokes and armatures with specified permeability values and, although these properties could be measured, there were insufficient resources to measure the large quantities necessary to find yokes and armatures with the required permeabilities to fit the requirements of a conventional plan. Therefore samples of yokes and armatures were obtained from normal production and these components were arranged with combinations of the conventional

Factor	Factor Values		
Bobbin winding distribution	+8/−8	even windings	−8/+8
Armature pip height	nominal		augmented
Electrical gap adjustment	left-hand third of existing scale	centre third of existing scale	right-hand third of existing scale
Crimping force	5 bar		6 bar
Demagging operator	operator A (more skilled)		operator B (less skilled)
Armature permeability	semi-controlled (sampled from production)		
Yoke permeability	semi-controlled (sampled from production)		

Table 1. Factors investigated in the experiment

factors to form the assembled products for the experiment. A total of 139 yokes and 113 armatures were made available for the experiment.

Random samples of yokes and of armatures were taken from production for use in the experiment. These components were heat treated in two batches of approximately equal size. Pre-experiment analysis of the measured yoke and armature permeabilities revealed a marked difference in the permeabilities of the yokes and armatures from different batches. This can be seen in Figure 2 where the permeabilities for the armatures are smaller for the second heat treatment batch than for the first. A similar pattern was seen in the permeabilities of the yokes. In order to assess the importance of the batch-to-batch variation in the heat treatment, the experiment plan allowed for estimation of the batch effect. Approximately half the armatures were manufactured with the increased pip height prior to heat treatment. Thus each pip height was fixed at one of the two levels before the armature permeabilities were measured. Hence if an armature is selected for its permeability value, the associated pip height is selected too. Thus there is not a free choice of the combination of pip height and permeability. Similarly, the selection of a particular component gives automatic selection of a heat treatment batch.

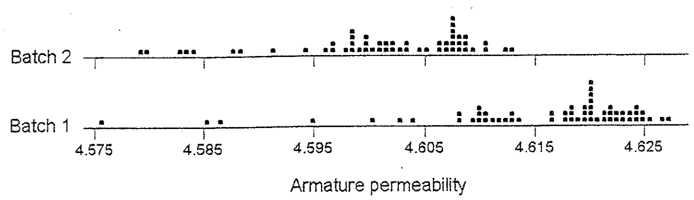

Figure 2. Distribution of armature permeability for each heat treatment batch.

3. The plan for the experiment

In this section we describe the plan used for the case study. The method of choosing the plan is discussed in the following section. In deciding on the size of the experiment it is necessary to consider the form of statistical analysis that is expected to be applied to the data. In this case study, a linear regression model with the usual error assumptions was thought likely to provide a reasonable approximation to the relationship between the factors and the sound output. The model proposed includes linear terms for each of the seven factors, interaction terms between all pairs of factors and quadratic terms for each of the bobbin winding, electrical gap adjustment, and the permeabilities of the yoke and the armature. In addition, the model includes dummy variables that indicate the heat treatment batches of the yoke, armature combinations, giving in total 36 model coefficients. In order to obtain sufficient data to estimate these model coefficients, allowing for the possibility of results being lost, the assembly and testing of 60 products (experiment runs) was considered adequate. Hence 60 yokes and 60 armatures were to be selected from the pool of 139 yokes and 113 armatures, each of which was labelled and had carefully measured permeabilities. The purpose of the algorithm, described in the following section, is to select the yokes and armatures in an optimal manner based on the values of their permeabilities, their heat treatment batches, and the armature pip height, and to combine these with the values of the four remaining factors in Table 1. The algorithms ensure that the full range of factor values are explored which is desirable for accurate estimation of the coefficients. A critical examination of such plans often indicates that the coverage of the central or nominal values of the factors is relatively thin leading to less accurate predictions in this region. A standard approach to improve the coverage of such a plan is to augment with additional experimental runs. Therefore, in addition to the 60 sounders chosen by the algorithm, 18 additional sounders were built in which the values of the factors were roughly in the middle of their ranges.

Arm. ID	Yoke ID	Arm. perm	Yoke perm.	Pip ht.	Batch	Bob. wind.	Elec. Gap	Crimp force	Op. skill
E26	F23	−0.53	−0.87	−1	4	1	−1	−1	1
A09	B10	0.93	0.73	1	1	−1	0	−1	0
F21	A30	−0.50	0.81	−1	4	1	1	1	1
D03	E18	0.70	−0.63	−1	2	−1	1	−1	0
D04	F25	−0.63	−0.77	−1	2	−1	1	1	1
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Table 2. Part of the experiment plan

The first five rows of the plan for the experiment, as determined by the algorithmic searching described in section 4, are shown in Table 2. (Factor values are given in coded form). This plan tells the experimenter which of the sampled armatures and yokes to use for each assembled product (experiment run) and how the remaining manufacturing factors under investigation should also be set.

4. Algorithms for selection of a plan

Two types of algorithm were written to search for experiment plans that involve both semi-controlled factors and factors whose values can be pre-set. The first is a genetic algorithm and the second is a development from exchange algorithm methodology. There are a number of different criteria that can be used for selection of a good plan. Both the algorithms described here use the D-criterion (Myers and Montgomery, 2002, p. 393). This criterion seeks plans with minimum value of the standardised determinant $D = n(\det(\mathbf{X}^T\mathbf{X}))^{-1/p}$ where the matrix $\mathbf{X}$ is obtained from the statistical model which approximates the response (sound output) in terms of the factors to be investigated and the combinations of factor values in the plan under consideration, n is the number of products in the experiment and p is the number of coefficients in the statistical model. The D-criterion is appropriate in this case study because it maximizes the accuracy of the estimated model coefficients, and these coefficients measure the influence of the factors on the response.

4.1 The genetic algorithm

Genetic algorithms (see Goldberg, 1989, or Mitchell, 1996) are based on an abstraction of biological evolution. The optimisation process evolves from one generation of design solutions to the next by a process of natural selection. Each generation is formed from a population of a set of chromosomes that are themselves strings of numbers (normally binary) representing all of the variables whose values are set to optimise the fitness function. The fitness of each particular chromosome is evaluated by the fitness function, D, where the smaller the value of D for each evaluated chromosome, the better the experiment plan that it represents.

The genetic algorithm is initialised with a random pool of chromosomes. Each subsequent generation of this pool is then formed via three key operations. A number of the chromosomes from the current pool (generation) are selected in such a way that those with greater fitness have a higher probability of selection. Some of these chromosomes are then mated in pairs; two mating chromosomes swap information beyond either one or two crossover points that are randomly selected, and two offspring thus result. The last operation, mutation, involves randomly changing, with a small probability, each chromosome. This provides random diversity in the evolution and helps to prevent premature convergence before too little evolutionary experience has been gained. Only the basic operations necessary to define a genetic algorithm have been described above. There are many additional operations which may be applied to improve performance, but are not discussed here.

Normally the best solution encountered through the entire optimisation is taken as the result. This is achieved using an elitist strategy so that the best-so-far solution is guaranteed to survive into the next generation. The mechanism by which genetic algorithms achieve better solutions is by discovering promising solutions, emphasising their importance in each population and recombining them to (possibly) produce even better solutions (Mitchell, 1996).

Genetic algorithms have been used in the selection of near-optimal plans for experiments where all the factors can be pre-set. Montepiedra *et al* (1998), use binary chromosomes to find designs for example experiments with one, two and four factors. Heredia-Langer *et al* (2003) investigate plans for multi-factor experiments with linear constraints on the values of the factors that can be combined, where the factor values are independent. In the case considered here the factor values cannot be set independently: the problem is the optimization of combinations of factor values (or products) that define the experiment plan. The components are selected from a pool of available components and, once one component is selected, it is not available for subsequent selection. This property, that components cannot be re-used, is accommodated in the genetic algorithm by defining the chromosome as a string of concatenated permutations that assigns the components within the pool to the products forming the experimental plan. This is the essential difference between the coding used in this algorithm and those available in the previously mentioned literature. In addition, appropriate crossover and mutation operations must be used to ensure that valid permutations always result from these actions.

Existing permutation-based genetic algorithm operators have been reported (see, for example, Goldberg, 1989), but here a new coding was used which allows the normal GA crossover operator to be utilized, and only a slightly modified mutation operator to be used. This was found to provide better results for the problem reported here. For further details, see Anthony (2003).

4.2 Exchange algorithm

Standard exchange algorithms are in regular use for finding plans for conventional experiments where all the factors can be set at specified values in combinations determined by the plan. The algorithm of Fedorov (1972) is commonly used to find a plan for an experiment with n runs; it is described below in the context of the D criterion.

1. A candidate list of all feasible combinations of the factor values is formed.
2. From this list, n combinations are selected to form a starting plan, for instance from a random selection of runs, and the value of D for this plan is calculated.
3. Each combination of factor values in the plan is then exchanged with each combination of factor values in the candidate list of feasible combinations and the value of D for the resulting plan calculated. This process is repeated and the exchange that leads to the largest reduction in D is accepted.
4. Step 3 is iterated until the improvement in D is smaller than some specified tolerance.
5. Steps 2 to 4 are repeated several times to try to avoid local optima, and the best plan found is used.

Several variants of this algorithm exist, for example the Modified Fedorov Algorithm of Cook and Nachtsheim (1980), in which, at step 3, any exchange that reduces the value of D is made as soon as it is found, thus speeding up the algorithm without sacrificing the quality of the final plan. Algorithms of the above type cannot be used in the situation presented by the case study because each component can only be used once to form a single run (product) in the experiment. Thus the value of any factor associated with that component, for example the permeability of the armature, can only be used once, unless there happens to be another armature in the sample with exactly the same permeability value.

The following algorithm was developed from that of Sexton, Lewis and Please (2001), using exchange algorithm ideas, to find a plan for the acoustic sounder experiment:

1. A list is formed of all feasible combinations of the values of those factors that can be pre-set. This is known as the candidate list.
2. A starting set of 60 combinations of the pre-set factor values are randomly selected from the candidate list, where combinations may be selected more than once. These are combined at random with 60 yokes and 60 armatures chosen at random from the samples available, to form 60 runs for the experiment. Each armature and each yoke may be selected only once.

3. The value of D is calculated from the pre-set factor values and the values of the semi-controlled factors.
4. Keeping the components in the plan fixed, a modified Fedorov algorithm is applied to the pre-set factor value combinations.
5. Keeping the values of the pre-set factors fixed, an attempt is then made to improve the plan by swapping the components as follows:
   5.1. Starting with the first combination of components in the plan (the first row) each armature in the plan is exchanged in turn with each armature in the pool of remaining armatures and the values of the armature permeability, pip height and heat treatment batch are updated. The value of D for each new arrangement is calculated and if an exchange leads to an improvement in the determinant value, it is retained.
   5.2. Step 5.1 is applied to the yoke components.
   5.3. Each pair of armatures in the plan are then interchanged and the interchange is retained only if D decreases.
   5.4. Step 5.3 is applied to the yoke components.
   5.5. Steps 5.1 to 5.4 are repeated until the reduction in D is less than some specified tolerance.
6. Steps 4 and 5 are repeated until the improvement in D is less than a tolerance.

4.3 Comparison of the algorithms on the case study

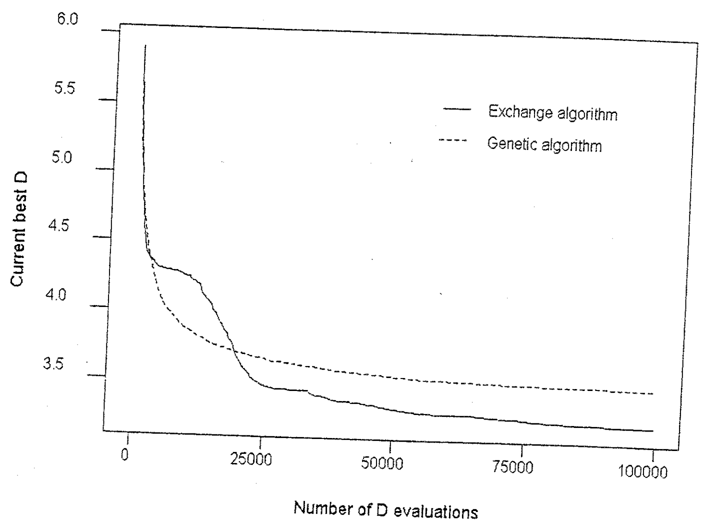

Figure 3. The standardized determinant (D) against the number of evaluations of D for the exchange and the genetic algorithm.

As the main computational effort in each algorithm is the calculation of D, the efficiency of each algorithm can be assessed by considering the current best value of D against the number of evaluations of D that have been performed. Figure 3 shows this relationship for each of the exchange algorithm and the genetic algorithm. As both algorithms have random components, the results displayed are the average values of the current best D calculated over 10 independent runs of each algorithm. The curves are plotted at intervals of 50 evaluations of D; for the genetic algorithm 50 is the number of chromosomes (and evaluations of D required) for each of the 2000 generations used. The figure shows that the genetic algorithm produces plans with smaller D than the exchange algorithm for up to around the 20,000th evaluation; thereafter the exchange algorithm has the better performance. However, the difference between the best determinants at the 100,000th evaluation is not substantive and a suitable design could be found using either algorithm.

5. Analysis of the experiment results

The analysis of the experiment data involves fitting the model described in section 3 by least squares regression. The data used to fit this model consisted of the sound output measurements (in decibels) from the 60 sounders specified by the search algorithm, plus the 18 additional sounders. Two extreme results were omitted where the model provided a poor fit, so that a better fitting model could be produced to explain the bulk of the responses. The full model, which explained 81% of the variation in the data, was simplified by discarding terms, one at a time, that made the least contribution to the model, until the only terms remaining are those that are either significant at the 5% level or are of borderline significance. Where an interaction between two factors was found to be significant, the main effects of the factors were included in the model for ease of interpretation. The reduced model is

$$Y = 124.54 - 0.008\,YP - 0.323\,PH + 0.033\,BW + 0.002\,EG + 0.053\,CF + 0.122\,DO$$
$$-0.100\,YP \times BW + 0.097\,YP \times DO + 0.073\,PH \times CF + 0.073\,EG \times DO$$

where Y is the output in dB, YP is the coded yoke permeability, PH is the coded pip height, BW is the coded bobbin winding, EG is the coded electrical gap adjustment, CF is the coded crimping force and DO is the coded demagging operator. (The coding is on the scale [-1, +1].) The reduced model explains 70% of the variation in the data.

Factor	Coefficient estimate	Standard error	t statistic	P-value
Intercept	124.54	0.030	4115	<0.0001
Yoke permeabiliy	−0.008	0.048	−0.17	0.8655
Pip height	−0.323	0.030	−10.74	<0.0001
Bobbin winding	0.033	0.038	0.88	0.3815
Electrical gap adj	0.002	0.037	0.05	0.9615
Crimping force	0.053	0.030	1.77	0.0813
Demag. Op.	0.122	0.030	4.02	0.0002
Yoke × Bob	−0.100	0.051	−1.95	0.0549
Yoke × Op	0.097	0.048	2.02	0.0477
Pip × Crimp	0.073	0.030	2.43	0.0179
Gap × Op	0.073	0.037	1.98	0.0515

Table 3. Details of the fitted model

Table 3 shows the details of the fitted model. The p-value in the table gives the statistical significance of the contribution of the coefficient to the model, the smaller the p-value, the more evidence that the coefficient is non-zero. The conventional cut-off point is 0.05, that is a p-value of 0.05 or less indicates the effect is of statistical significance. The table shows that pip height ($p < 0.0001$) and demagging operator ($p = 0.0002$) are highly significant. Also very significant is the interaction between pip height and crimping force ($p = 0.0179$). Of less significance are the interactions between yoke permeability and bobbin winding ($p = 0.0549$), yoke permeability and demagging operator ($p = 0.0477$), and electrical gap adjustment and demagging operator ($p = 0.0515$). The armature permeability does not feature either as a main effect or as an interaction with another variable. No evidence has been found from these data that the armature permeability has any effect on the performance of the capsule.

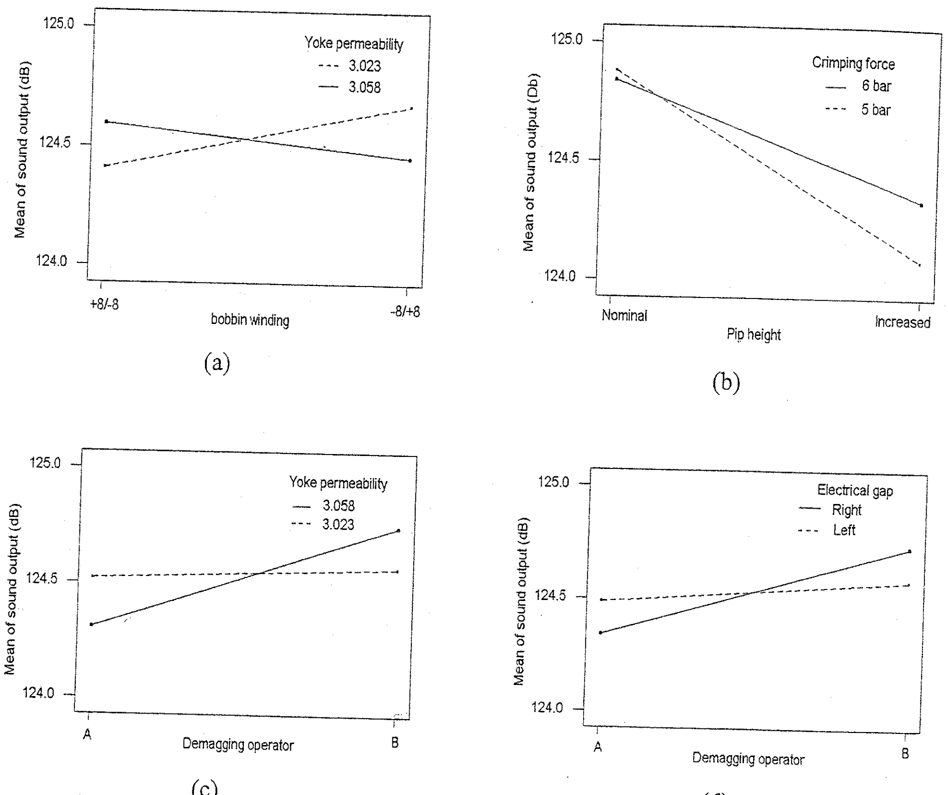

Figure 4. Predicted interactions between (a) yoke permeability and bobbin winding, (b) pip height and crimping force, (c) demagging operator and yoke permeability and (d) demagging operator and electrical gap adjustment.

The model was used to simulate a full factorial experiment by predicting the response for all possible combinations of the factor values taken at the end points of their experimental ranges. From these predictions, plots have been produced to illustrate the four interactions found to be of statistical significance, so that their engineering importance can be assessed. These are shown in Figures 4(a), (b), (c) and (d).

While these interactions are of statistical significance, the figures show that the changes in sound output that they produce are not of engineering importance. Consider, for example, Figure 4(b), which illustrates the interaction between the armature pip height and crimping force. At the nominal value of pip height there is little impact on sound output from changing the machine air pressure for the crimping. At the increased pip height, the effect of increasing the air pressure from 5 bar to 6 bar is to increase the predicted mean sound output by just 0.25 dB.

The data indicate that the sound output due to the less skilled operator B is, somewhat surprisingly, greater than that of the more experienced operator A, see Figures 4 (c) and (d). The explanation for this finding is that the more experienced operator was able to apply just sufficient demagging to allow the capsule to pass the final production test (fall within an acceptable band of sound output). The inexperienced operator appears to apply more demagging in order to ensure that the sound output of the capsule is above the acceptance lower level.

The lack of substantial impact on the product performance arising from the interaction effects is beneficial new knowledge to the company as it indicates that it is unnecessary to use resources and incur costs by tightening up tolerances on the factors concerned.

The most important factor identified from this study is the pip height which, when changed from nominal to the increased height, gives a decrease in response of approximately 0.65 dB. This finding suggests that in future experiments an exploration should be made of the effect of reducing the pip height away from nominal. The indications from this experiment are that a decrease in pip height might be a design improvement. However, an important issue is whether a smaller pip height may produce a greater chance of "poll over'' during demagging, when the armature touches the yoke and consequently remains in permanent contact, thus producing a non-operational capsule.

6. Conclusions

The methodology presented in this paper proved beneficial to Hosiden Besson in allowing them to efficiently investigate whether certain factors influenced the performance of the sound transducer. In particular it facilitated the investigation of the yoke and armature permeabilities. It would be difficult to investigated these two factors by conventional methods as pre-determined values cannot be easily achieved. The results of the experiment provided useful information to the company through indicating that variations in the factors investigated had little impact on the produce performance. In particular, setting the pip height and bobbin wind factor values beyond the normal tolerance limits did not cause failures or worsen the current performance. This indicates that the product is already robust to these variations and that tolerances on these factors can be relaxed, allowing any further investigations to focus on other factors.

Currently, work is in progress on software to produce a user-interface that will facilitate use of the exchange algorithm presented in this paper to find plans for experiments for products assembled from components for which one or more factors are semi-controlled.

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