

Use of Response Surface Methods to Aid Understanding and Visualization in Aircraft Design

Carren M.E. Holden, BAE SYSTEMS, ATC Sowerby Building, FPC 267, PO Box 5, Filton, Bristol BS34 7QW, United Kingdom. E-mail: Carren.Holden@baesystems.com.

Andy J. Keane, Professor of Computational Engineering, Room 2009, Building 25, University of Southampton, Main Campus, Highfield, Southampton, Hants, SO17 1BJ, United Kingdom. E-mail: Andy.Keane@soton.ac.uk.

1. Abstract

A two dimensional visualization of a representative military aircraft problem is described and then extended to five dimensions. The paper includes the optimization problem specification and introduces a *modus operandi* for a design process. The visualization developed is compared to alternative techniques, such as scatter plots and Kohonen map visualisations. Possible applications of the different visualization methods are also described.

Keywords: Visualization, Response surface methods, Optimization

2. Introduction

There are considerable cost benefits to be derived from improving designs both by better searches in current design spaces and by satisfying more complex design criteria, for instance by striking the right balance between multiple objectives from the same discipline or from different disciplines. At present designers spend too long preparing individual analysis cases and, therefore, are constrained to sampling only small areas of the design space close to previously explored regions. In a design process, complex design tasks need to be broken down into manageable portions so that specialists in individual disciplines can resolve different parts of the design. Currently these portions are too small and there is too much iteration between disciplines which prohibits the challenging of the constraints of one discipline by another, where the most progress is likely to be made. However, this situation can be ameliorated if small numbers of individuals can assimilate, appreciate and understand large amounts of data rapidly and efficiently. With this in mind, this work is aimed directly at the designers in the design process of the future, who communicate with and steer the design search and optimization process and will extract previous work from databases transparently and flexibly. They will use software and data perhaps as web services, so that the source of this data and software will also be unknown and unimportant. Such designers make use of high cost analysis codes within automated search and response surface models to map and optimize domains using computationally expensive evaluations. They will use low fidelity (or computationally inexpensive) physical models to navigate high fidelity design spaces. The placement of analyses in the design space are defined by design of experiments (DoE) methods both as schemes defined *a priori* and as a result of optimization or error-defining processes. These engineers, armed with more information and understanding, can be expected regularly to challenge cross-discipline constraints. They are more concerned with problem definition, design space shape and optimization than how any one analysis is obtained and use advanced visualisation techniques to provide enhanced comprehension and effective navigation of hyper-dimensional design spaces.

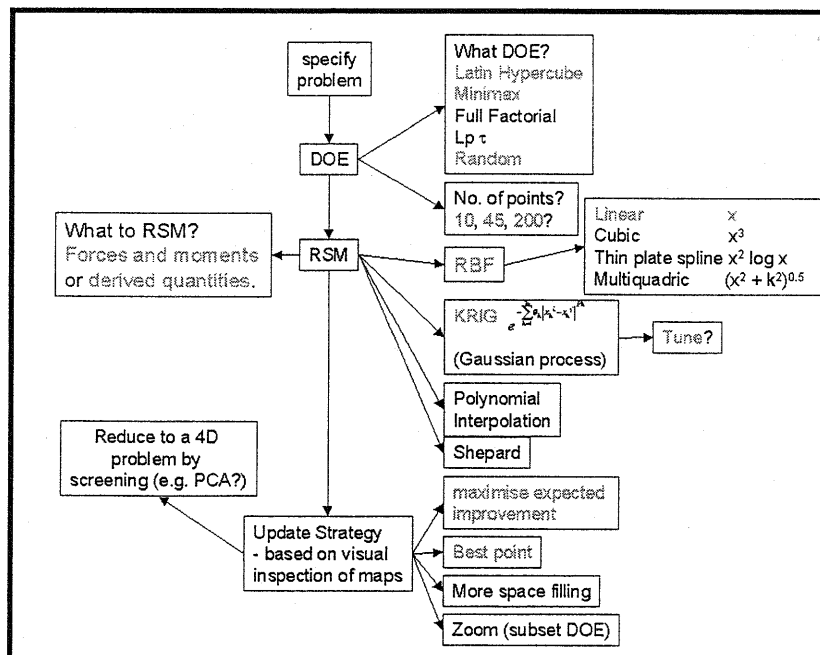


Figure 1: RSM based Design Process

3. Modus Operandi

The large amounts of data required for design space appreciation are here provided by response surface method technology. Visualization is part of a *modus operandi*, shown in Figure 1. This comprises first, a problem specification, in terms of objectives, constraints and design variables. Then a DoE (e.g. Latin Hypercube [1], LP_r [2]) is performed and a response surface methodology applied (for instance: kriging, radial basis functions, etc. [3]). Finally, an update strategy is determined by statistical methods, such as expected improvement or the optimum in the domain, based on the visual inspection of maps. As part of this process an assessment takes place as to the importance of the design variables in the problem.

4. Problem Specification

Minimise the drag coefficient (C_D) of a representative Military Aircraft trapezoidal body/wing tail-less aircraft in cruise at $M=0.85$, using five design variables, subject to:

$$\begin{aligned} -3.0 &\leq \text{wing linear twist (washout, } \theta) \leq 7.0 \\ -3.0 &\leq \text{wing angle of attack } (\alpha) \leq 7.0 \\ 0.0 &\leq \text{mid-camber} \leq 0.07 \\ 0.0 &\leq \text{flap deflection angle } (\delta) \leq 10.0 \\ 9.557 &\leq \text{leading edge (l.e.) tip location} \leq 14.227 \\ C_L &\geq 0.1994 \end{aligned}$$

$$C_{m_{total}} = C_{m_{le}} + hC_L \cos \alpha + hC_D \sin \alpha$$

$$\Rightarrow c_1 = \frac{C_L + a_3}{-\frac{C_{m_{le}}}{h \cos \alpha} - C_D \tan \alpha + a_3} = 1.0$$

or alternatively $C_m < |0.02|$ in the 2-D problem, where C_L is the lift coefficient and C_m the pitching moment coefficient.

CFD evaluations are performed throughout using the BAE SYSTEMS in-house code FLITE3D [4] in inviscid, Euler mode.

5. Visualization

Figure 2 shows a visualization of a related two-dimensional design space, with computational noise and failed evaluations. (Here, a simpler solver is used, with two trailing edge parameters.) Computational noise often causes ridges in the design space, see also, for instance [5]. It is clear that gradient searches in this domain will not find the global optimum, but a local one, stopping at the bottom of a ridge. These problems are likely to be amplified in higher dimensional space. Inability to visualize high dimensional spaces can lead to an inability to be able to fully exploit additional flexibility. Computational noise and expensive evaluation leads naturally to the use of response surface methodologies in design space optimization.

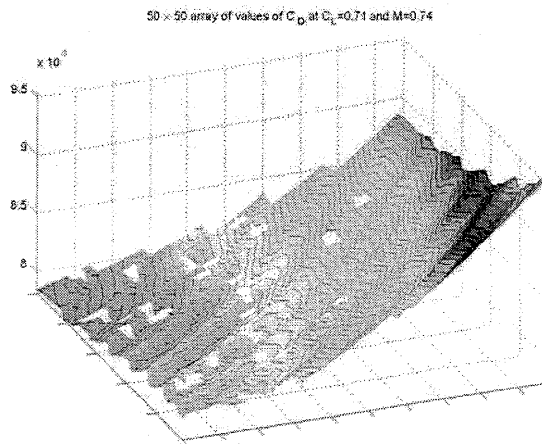


Figure 2: 50*50 evaluations of a two-dimensional design space (C_D at $C_L=0.71$, $M=0.74$) using a panel method. The two-dimensional design space describes the vertical movement of two B-spline poles adjacent to the trailing edge of an aerofoil.

A two-dimensional visualization of a kriged design space of C_D of the Military Aircraft trapezoidal wing alone case, with the design variables angle of attack (α , y-axis) and wing twist angle (θ , x-axis) is shown in Figure 3. Two additional Krigs of C_L and C_m are also made. The *infeasible* region, where the inequality constraint on C_L is *not* satisfied (i.e. where $C_L < 0.1994$ and $|C_m| < 0.02$) is indicated by use of black circles to represent DoE points in this region. The quality of the C_D fit is shown by the fact that the design space is relatively monotonic. The faint, white crosses represent the evaluations of the model using simulated annealing to find the domain

optimum. A blue square represents the location of the optimum in the domain i.e. the location where both the constraints are met and C_D has the lowest (reddest) value. Large numbers of evaluations of the simulated annealing search centre on the optimum. The optimum is given by CFD: $C_m = -0.007081$, $C_D = 9.999e-03$, $C_L = 0.1994$ and the Krig model: $C_m = -0.007238$, $C_D = 9.918E-03$ and $C_L = 0.1994$. Alternative plots for visualizations of optimizations can be used to show the trace of the optimization through the design space and for genetic algorithm populations and cluster or island centres.

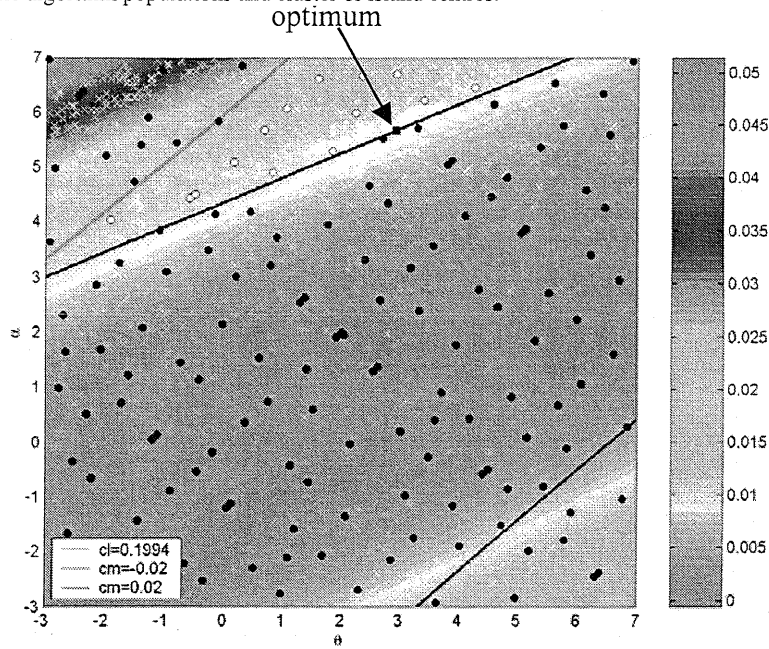


Figure 3: Visualization of a Krig of 150 CFD evaluations in a two-dimensional design space. The data points are shown as white circles in the feasible region and black circles in the infeasible region. The faint white crosses represent evaluations by a simulated annealing optimization on the response surface. The optimum is shown as a square.

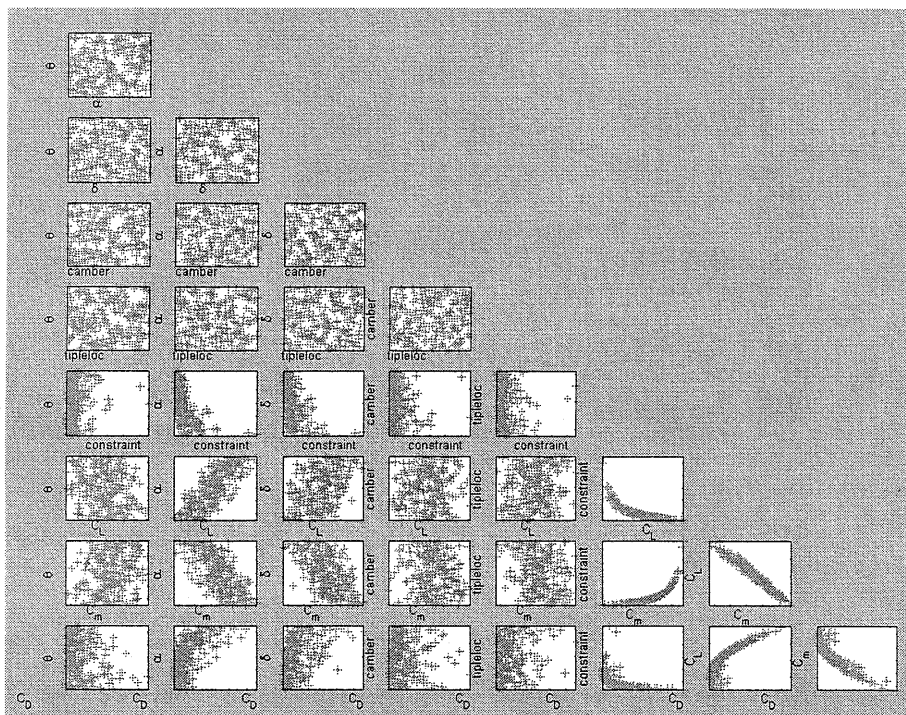


Figure 4: Scatter plot of every dependent and independent variable in the 5D problem against every other variable.

The usual approach for visualizing such data in higher dimensions is to use scatter plots, as illustrated in Figure 4, in which a graph is plotted of every design variable against every other variable (scatter plots are widely used by the statistics community, see for

example [6]). These plots enable a good appreciation of the design space and to establish whether or not known trends are being followed, for instance, the quadratic nature of the relationship between C_L and C_D can easily be identified. Additionally the independent variables show no relation with each other, as should be expected, given the DoE used to collect the data. Alternatives to this include parallel co-ordinates [7,8] or Kohonen's self-organising maps (SOMs, [9]). These latter methods also enable relationships between variables to be obtained [10] and, possibly, to establish the most important variables in the problem. However, they do not show graphically where the optimum lies in the domain. To orient ourselves with the SOMs we first look at the picture of the glyphs shown top right. In this image, each of the glyphs is placed according to the value of α , θ and δ . These are then colored according to the value of C_L . The SOM is a diagrammatic representation rather than a physical map of the design space as illustrated by comparison between the SOM of C_L and the glyph image: the trends of the data are the same and have similar features. In the SOMs the diagonal trends of the maps of C_D , C_L , C_m , c_l and α show that they are all related (C_D , C_L , α are positively correlated to each other and negatively correlated to C_m and c_l). The SOMs also show that the independent variables in the problem are unrelated, as they should be. In higher dimensional problems, the SOM has been shown to indicate the first two principal components, as their SOMs can be related although in opposite directions (e.g. in opposite diagonal directions or vertically and horizontally). Bland regions in the SOM give an indication of rogue data (e.g. from unconverged CFD evaluations) and completely bland SOMs can help to identify unimportant variables.

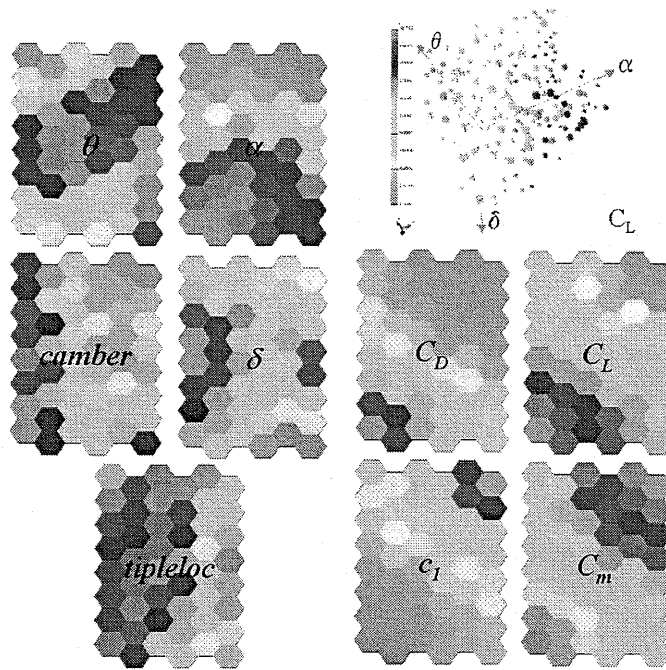


Figure 5: Self-organizing maps for 5D problem.

Figure 4 and Figure 5 are clearly not so easy to interpret and so the next question is whether or not the type of visualization presented in Figure 3 can be extended to higher dimensions to provide equally useful information. In this case we increase the dimensionality of the problem by also including the design variables mid-camber, δ and wing tip leading edge location (which influences both leading and trailing edge sweep angles) in addition to α and θ considered in Figure 3. A body/wing configuration was considered in this case and 200 CFD evaluations were used to obtain the Krig. A hierarchical axes technique (HAT) plot after [11] is shown in Figure 6. Here, within an individual tile camber and δ vary. As we go from tile to tile in the horizontal direction of the main plot θ varies and in the vertical direction α varies. Then the top 81 tiles of this image represent the highest value of the fifth design variable, leading edge tip location. Each block of 81 tiles going down then represent lowering values of leading edge tip location. Only the top of this image is presented so that the smaller tiles are seen clearly. Two optima are shown: the first the result of a simulated annealing optimization (red square) and the second the result of an exhaustive search in the representation data. This visualization is similar in spirit to the triangles used in [12], but with a less complex design space transformation. The HAT plot can, however, only be used for data appreciation in up to 6 dimensions, as the tiles become too small thereafter. This picture is only possible because response surface methodologies enable the large amount of data required for plotting to be provided. The image gives confidence that the same optimum is being located in both cases and that the curve fit is well behaved.

6. Results

To validate our procedure some results from this process are given in Table 1. The variation in the results given in Table 1 could be cause for concern, except that reference to the visualization shows that these results are close to each other and are therefore close to the same optimum. The CFD result is significantly different from the RSM, which means that there is mileage in building a new model, including this optimum and iterating until the optimum and CFD result obtained converge. The loading distribution and geometry of the final configuration are shown in Figure 7. In this case the loading distribution is nearly elliptic, as required, except

for the compensation for wave drag in the outboard wing. The sweep is reduced compared to the initial configuration as the large amounts of sweep give a large amount of downward pitching moment, which has been reduced.

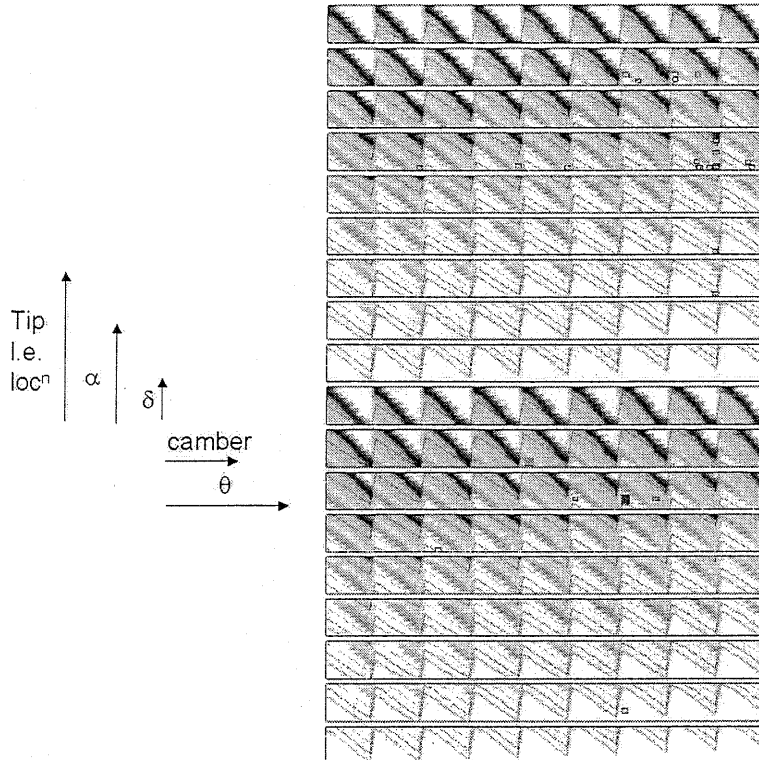


Figure 6: Zoom into upper part of 5-D HAT plot. Filled squares denote optima located in the model using different optimization methods. The other squares show the locations of evaluations in the final stages of simulated annealing of the response surface. The value of C_D is colored within the tile. Coloring does not take place if the inequality constraint on C_L is violated. The contour lines are contours of trim constraint.

Method	x_1	x_2	x_3	x_4	x_5	c_1	C_L	C_D
Simulated annealing on response surface	3.583	5.194	1.184	0.0156	10.24	1.000	0.1993	68.07e-04
Additional CFD evaluation	3.583	5.1936	1.1835	0.0156	10.2366	1.0004	0.2035	86.31e-04

Table 1: Results for Design Space Optimization. The additional CFD evaluation result is illustrated in Figure 7. This is the result of a sparsely sampled DoE, repeat iterations where an improved model is obtained using additional CFD evaluations at optima would improve the agreement between the model and CFD.

7. Conclusions

Visualization technology helps the designer to spend time inspecting results and ensuring correctness rather than dealing with large volumes of flow data. After a large number of numerical evaluations have been performed there is a need to identify any off trend results. The scatter plot method presented here is ideal for this purpose. The SOM also helps with this to give an overview. Response surface method technology eliminates the inefficient design space sampling in the final stages of a gradient search optimization. A balance is required between the number evaluations in the DoE, the number of cycles in the CFD, the number of points in the initial DoE and the number of further evaluations used. The use of one data set for training and a second for error analysis proved effective. Work is required to establish the relationship between the statistical measures of error with and without a second data set. The trade-off between low-dimensional comprehension and high-dimensional accuracy impacts optimization.

These type of visualization methodologies are utilised to support design decisions such as the relative scaling of the design variables and constraints and choice of penalty function, which are important in optimization. Also possible is the recognition of problems such as numerical noise and boundaries of evaluation failure. Initial design of experiments can be improved in a systematic way by

calculation of additional points dictated by maps of statistical and other error criteria. Although this technology has been developed with reference to aircraft aerodynamic design in particular, the design space visualisation and curve-fitting technology developed is general. It should therefore be equally applicable to other disciplines such as cost analysis, structures and computational electromagnetics, in which expensive analysis tools are used to find optima for complicated design problems. It is expected to be particularly useful in multi-disciplinary design and situations where there are multiple optima in the design space.

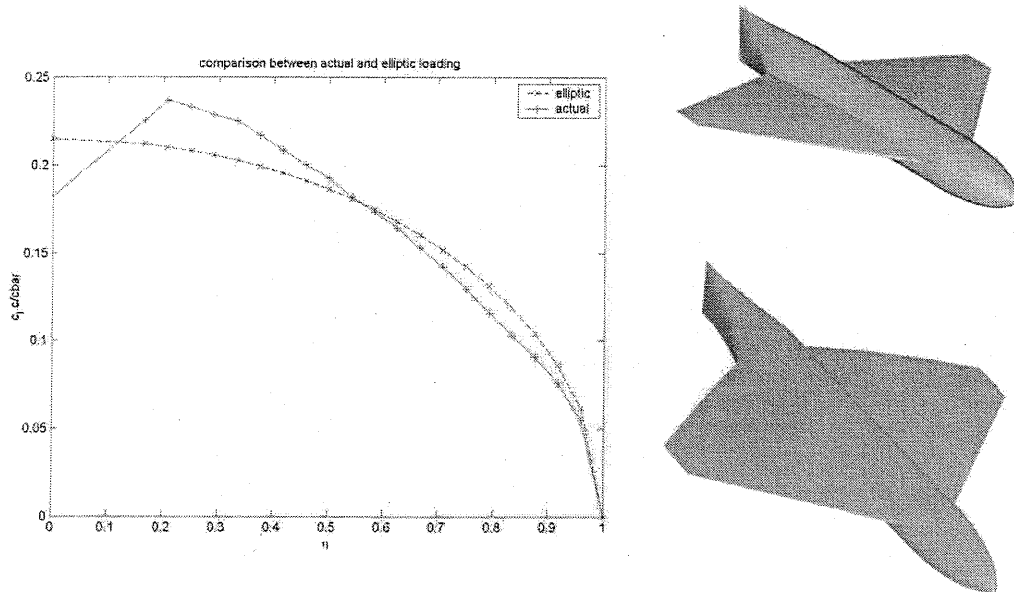


Figure 7: Loading distribution of optimized wing design. Also shown are the starting and final CFD models.

8. References

1. M.D. McKay, R. J. Beckham and W. J. Conover, "A comparison of three methods for selecting values of input variables in the analysis of output from a computer code.," *Techometrics*, vol. 21, no. 2, pp. 239-245, 1979.
2. R.B. Statnikov and J.B. Matusov. *MultiCriteria Optimization and Engineering*. Chapman & Hall Publishers, ISBN 0-412-99231-0.
3. D. R. Jones, "A taxonomy of global optimization methods based on response surfaces," *Journal of Global Optimization*, vol. 21, pp. 345-383, 2001.
4. K. Morgan, J. Peraire and J. Peiro, "Unstructured grid methods for compressible flows", Special Course on Unstructured Grid Methods for Advection Dominated Flows, AGARD Report 787, March 1992.
5. V. Torczon, C. M. Siefert, and M. W. "Model-assisted pattern search." In *Proceedings: First International Workshop on Surrogate Modelling And Space Mapping for Engineering Optimization*, held at the Technical University of Denmark, November 16-18, 2000.
6. R. A. Bates, R. Fontana, C. Randazzo, E. Vaccarinao and H.P. Wynn, "Empirical Modelling Of Diesel Engine Performance for Robust Engineering Design" In: *Statistics for Engine Optimisation* pp.163-173, 1998.
7. Inselberg and B. Dimsdale, "Parallel coordinates: A tool for visualizing multi-dimensional geometry," in *Proceedings, Visualization 1990 IEEE CS Press Los Alamitos Calif.*, 1990, pp. 361-370.
8. Goel, A., Baker, C., Shaffer, C., Grossman, B., Haftka, R., Mason, W., and Watson, L. (2000). *VizCraft: A Problem Solving Environment for Configuration Design of a High Speed Civil Transport*. Computing in Science and Engineering. to appear. <http://citeseer.nj.nec.com/goel00/vizcraft.html>.
9. T.Kohonen. *Self-Organizing Maps*. Number {ISBN}: 3-540-67921-9. Springer-Verlag, 2000.
10. P.C. Matthews, *The Application of Self-Organising Maps in Conceptual Design*, Ph.D. thesis, Fitzwilliam College, University of Cambridge, June 2001.
11. T. Mihalalisin, J. Timlin, and J. Schwegler, "Visualizing multivariate functions, data and distributions," in *Readings in Information Visualization: Using Vision to Think*, S. K. Card, J. Mackinlay, and B. Shneiderman, Editors., ISBN 1-55860-533-9, pp. 115-125. Morgan Kaufmann, 1999.
12. B. Grossman, R. T. Haftka, W. H. Mason, L. T. Watson, C. Baker, S. Balabanov, S. Cox, A. Giunta, H. Kim, D. Knill, and D. Krasteva, "Effective use of surrogate models in aircraft design," Presentation at First International Workshop on Surrogate Modelling and Space Mapping for Engineering Optimization - held in Lyngby, Denmark, November 16-18th 2000.