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EFFICIENT GENETIC ALGORITHM BASED ROBUST DESIGN METHOD FOR COMPRESSOR FAN BLADES

Apurva Kumar*, A. J. Keane, P. B. Nair
Computational Engineering and Design Group
School of Engineering Sciences
University of Southampton
Southampton, SO171BJ, UK

S. Shahpar
Aerothermal Methods
Rolls-Royce plc
PO Box 31
Derby, DE248BJ, UK.

ABSTRACT

This paper presents an efficient genetic algorithm based methodology for robust design that produces compressor fan blades tolerant against erosion. A novel geometry modeling method is employed to create eroded compressor fan blade sections. A multigrid Reynolds-Averaged Navier Stokes (RANS) solver HYDRA with Spalart Allmaras turbulence model is used for CFD simulations to calculate the pressure losses. This is used in conjunction with Design of Experiment techniques to create Gaussian stochastic process surrogate models to predict the mean and variance of the performance. The Non-dominated Sorting Genetic Algorithm (NSGA-II) is employed for the multi-objective optimization to find the global Pareto-optimal front. This enables the designer to trade off between mean and variance of performance to propose robust designs.

INTRODUCTION

During operation, compressor fan blades are exposed to a number of erosion processes [1]. This can lead to reduction of the blade chord, alteration in the shape and increase in the surface roughness [2]. This is critical to the blade performance and can lead to degraded overall engine efficiency. Roberts [3] has shown that geometric variability in the form of leading edge erosion in compressor airfoils may account for an increase of 3 % or more on the thrust specific fuel consumption. Erosion can cause upto 5 % deterioration in total pressure loss of compressor fan blades [4]. Replacing the eroded compressor fan blades can prove to be expensive. Hence, it is desirable to design compressor fan blades that are robust to erosion processes ie, blades whose performance does not degrade significantly in the presence of erosion.

Traditional deterministic optimization methods seek to optimize the mean performance of the system. These methods when used for product design tend to produce solutions that perform well at the design point but have poor off-design characteristics. In recent years, there has been a resurgent interest in computational analysis and design methods that rationally accommodate uncertainty arising from sources such as varying operating conditions, manufacturing errors or inaccurate system parameters [5]. In most cases, removing the causes of uncertainty can be prohibitively expensive. Robust Design is concerned with minimizing the effect of uncertainty in design parameters on a design without eliminating the source of uncertainty [6].

In the 1970's Taguchi emphasized the need to reduce variation in product and processes to improve their quality [7, 8]. An overview of Taguchi's experimentation strategy and parameter design method can be found in [9,10]. The system design method and the selection of Signal-to-Noise (SN) ratio as a measure of robustness proposed by Taguchi had several limitations [11, 12]. Welch et al [13, 14] proposed a system for quality improvement via computer experiments as an alternative to Taguchi's methods.

*ak3@soton.ac.uk

Statistical decision theory has also been used to formulate robust design as an optimization problem. Minimax strategy [15] can be used to find a design with optimal worst case performance. This method is conservative as it seeks to protect the decision maker against the worst case scenario [12]. Huyse et al [16, 17] use the Bayes principal to achieve consistent improvement of the performance over a given range of uncertainty parameters. The problem in using their formulation is that the evaluation of the objective function is very expensive.

Many researchers have treated robust design as a multi-objective problem [18], where the goal is to (1) optimize the mean of the performance, and (2) minimize the variance of the performance of the system. Parkinson [19] has discussed conventional Weighted Sum (WS) methods to develop a single objective function to be utilized for robust design. The WS methods can only be used if the Pareto front is convex and fails to produce an even distribution of points from all parts of the Pareto set [20]. Chen et all [21] solved the bi-objective robust design problem from a utility perspective. They employ a Compromise Programming (CP) approach based on the Tchebycheff method. Genetic Algorithms (GA) are inherently well suited for Multi-objective problems like robust design, as they have the ability to find multiple Pareto-optimal solutions in one single simulation run. In our study we employ the Non-dominated Sorting GA (NSGA II) proposed by Deb et al [22]. The NSGA-II method uses an elitism based non-dominated GA to ensure much better spread of solutions and better convergence near the true Pareto-optimal front compared to conventional GA methods.

The robust design approach proposed here combines conventional robust design methods with Design of Experiments (DOE) techniques, sophisticated surrogate modeling and GA to suggest an efficient hybrid method. In this method DOE techniques are used to create an initial system design of inner control and outer noise array. A sophisticated grid generation routine is employed to generate meshes which are then used for computational fluid dynamics (CFD) simulations. NSGA-II is used in conjunction with Gaussian stochastic process model (Krig) to search the design space for Pareto-optimal solutions. This method is used to seek compressor fan blade sections that are robust to erosion processes. The remainder of this paper is organized as follows. In the next section we present the methodology used for robust design. This is followed by a description of the geometry parameterization, grid generation and CFD simulation. The following section will describe the surrogate modeling approach employed in the study. The latter sections will talk about the GA search method, numerical analysis and results.

ROBUST DESIGN METHODOLOGY

This sections discusses the methodology proposed for robust design of compressor blade sections against erosion.In the first step we select the design space and use DOE techniques to rationally choose a set of compressor fan blade sections as initial m candidate points. Subsequently a second level of DOE is run to suggest n different erosion geometries on each candidate compressor fan blade section. This is very similar to the inner control factor array of m points and noise factor array of n points used in Taguchi's system design. The eroded compressor blades are modeled using a novel parameterization technique based on Hicks-Henne functions, discussed in the next section. A Parametric Design and Rapid Meshing (PADRAM) tool is used to produce high quality hybrid meshes. A multigrid Reynolds-Averaged Navier Stokes (RANS) solver HYDRA with Spalart Allmaras turbulence model is used for Computational Fluid Dynamics (CFD) simulations to calculate the total pressure loss over the compressor blade section at each of the $m \times n$ points. The mean and standard deviation of the total pressure loss (over n erosion types settings) is calculated for all the m blade sections.

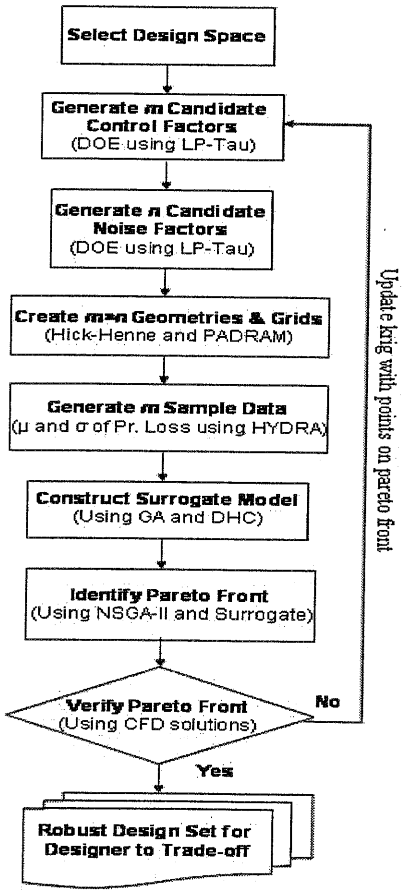

Figure 1. FLOWCHART OF ROBUST DESIGN METHODOLOGY

Unlike conventional robust design procedures we do not limit ourselves to just searching the initial design space. A Gaussian stochastic process model (Krig) is employed to generate a computationally less expensive surrogate to predict the mean and standard deviation of the total pressure loss. The hyperparameters of the krig are calculated using a combination of GA and Dynamic Hill Climbing (DHC) search methods. The elitist Non-dominated Sorting Genetic Algorithm (NSGA-II) is then used to search the entire design space to obtain Pareto-optimal solutions. The prediction, using the surrogate model, at the points on Pareto-front are then verified by running full scale CFD simulations. If the Krig is not accurate enough, a low-crowding algorithm is used to select points for update near the Pareto front. The surrogate model (Krig) is then updated at the suggested points and this process is performed iteratively until predictions are satisfactory. After the optimizer converges to the true Pareto-front the designer can use the optimal design set to trade off between mean performance and variance to obtain robust designs. This methodology is presented in the flowchart in figure 1.

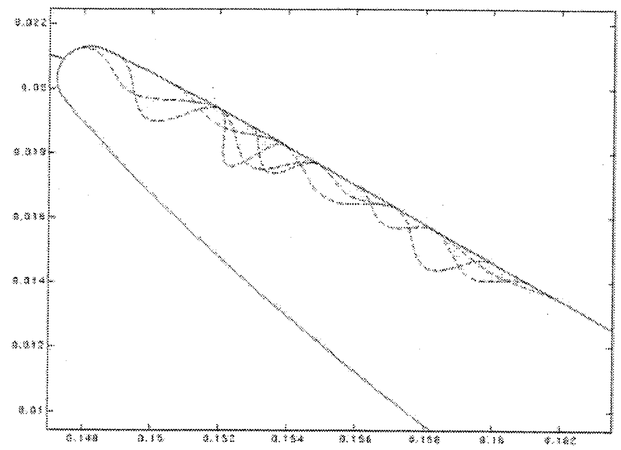

Figure 2. NOISE FACTORS: ERODED BLADE MODELS

MODELING AND PARAMETRIZATION

Robust design methods require definition of noise factors and control (design) factors in the system design. For the noise factors we need to develop a parametric model of the eroded compressor blade section. Erosion leads to blade surface deterioration and causes a depression in the original airfoil. Hence for modeling eroded geometries, a tool that can model local dents in the original airfoil shape is required. Hicks Henne functions [23] provide a flexible tool to model local variation in the form of bumps. Erosion patterns observed in compressor fan blades can be very complex. A combination of piece-wise cubic polynomial and Hicks-Henne function is used here to create a simple but realistic model of the erosion patterns. The eroded compressor fan blade section is parametrized in terms of the location, depth and the width of the eroded section. The Hicks-Henne functions can be expressed as:

$$b(x) = A\left[sin\left(\pi x^{\frac{log5}{log t_1}}\right)\right]^{t_2}, 0 \leq x \leq 1. \quad (1)$$

Here, A is the maximum bump magnitude, t_1 locates the position of the maximum of the bump at $x = t_1$, and t_2 controls the width of the bump. This provides us with three Noise factors - location ($t1$), width ($t2$) and depth (A), for the robust design system. Figure 2 show some typical eroded blade models used to represent noise factors this study.

The compressor blade geometry itself also needs to be parametrized to create a set of control factors. To parametrize the blade section geometry we use the Hicks-Henne method proposed in [23]. Wu et al [24] discuss and compare the efficacy of Hicks-Henne shape functions to other methods for modeling compressor blade sections. In this specific case we have used 10 Hicks-Henne functions to define the compressor fan blade section. The shape functions are added to a typical Rolls-Royce compressor fan blade section to form new shapes. The weights of these shapes are used as design parameters. Hence, for our robust design study we have 10 design parameters which can be treated as the control factors. Figure 3 shows the variations in geometry caused by changing the control factors.

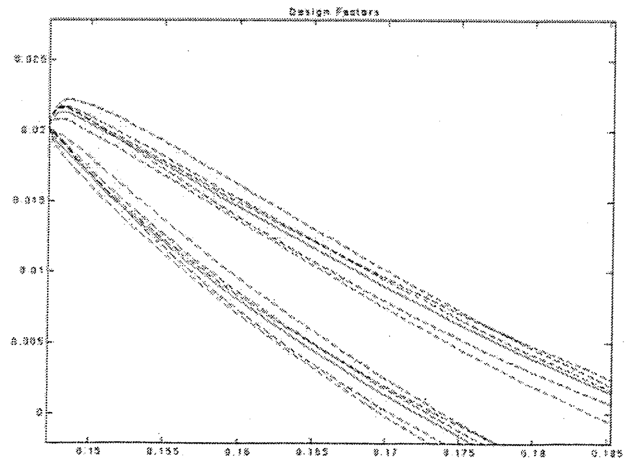

Figure 3. VARIATIONS IN CONTROL FACTORS

The model discussed above is combined with the Rolls-Royce propriety code PADRAM, a parametric design and meshing routine employed for automating the geometry creation and grid generation process. PADRAM makes use of both transfinite interpolation and elliptic grid generation to generate hybrid C-O-H meshes. An orthogonal body fitted O mesh is used to capture the viscous region of the airfoil whilst an H mesh is used near the boundary where stretched cells are required, for example in the wake region. After Grid refinement studies we select a mesh of the order of 28,000 cells in two dimensions. Figure 4 shows a typical compressor fan blade section geometry with the CFD mesh.

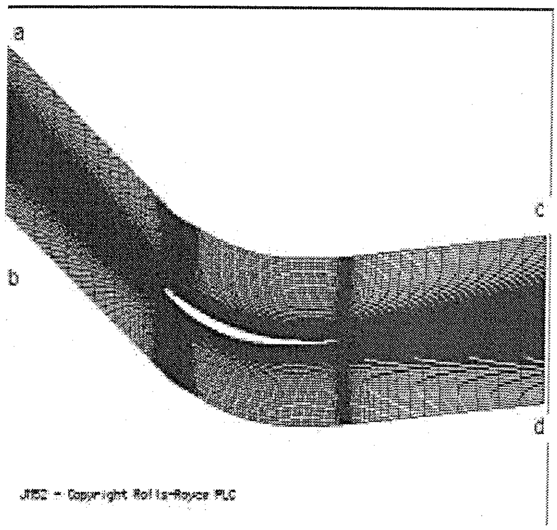

Figure 4. A typical C-O-H mesh used for CFD analysis

In Figure 4 *abdc* is the CFD domain where boundary *ab* is the inlet and boundary *cd* is the exit. A non-linear, unstructured viscous flow solver HYDRA is used for the CFD simulation. It solves the Reynolds Averaged steady Navier-Stokes equations with the Spalart-Allmaras turbulence model. To accelerate the convergence to steady-state it uses preconditioning and multi-grids [25]. A four level multigrid is used for the present simulations. The inlet boundary conditions for the CFD analysis are Total temperature = 290 Kelvin, Total Pressure = 63400 Pascal, Whirl Angle = -37.28 Degrees and the outlet boundary condition is Static Pressure = 52000 Pascal. An initial uniform flow condition with Density = 0.7675 kg/m^3, Velocity = 0 and Pressure = 66932 Pascal is considered. The converged CFD solution is used to calculate the pressure loss at the nominal geometry. The equation for the pressure loss is:

$$Loss = \frac{P_{inlet} - P_{exit}}{P_{inlet}} \times 100. \tag{2}$$

where P_{inlet} is the total pressure at the inlet and P_{exit} is the total pressure at the exit.

SURROGATE MODELING

In probabilistic analysis the computational cost involved in solving high-fidelity simulation models many times over is very high. Surrogate modeling uses the basic idea of analyzing an initial set of design points to generate data which can be used to construct approximations of the original high fidelity model. The high-fidelity model CFD simulation in this study can be represented by a functional relationship $y = f(\mathbf{x})$, where $\mathbf{x}$ is the vector of inputs to the simulation code and y is the output. The objective is to construct an approximate model $\hat{y} = \hat{f}(\mathbf{x}, \alpha) \approx f(\mathbf{x})$, that is computationally cheaper to evaluate. α is a vector of undetermined hyperparameters which is estimated by employing a black-box approach [26] to the input-output data. In general, black-box surrogate modeling involves the following steps: (1) data generation, (2) model structure selection, (3) parameter estimation and (4) model validation.

Data Generation

A surrogate modeling approach needs a set of training data and the quality of the approximate model crucially depends on the location of these training points. Design of experiments (DOE) techniques offer a way to choose the training points so that the maximum quantity of information can be extracted about the underlying input-output relationship. As computer experiments are deterministic, it is important to choose the training points which fill the design space in an optimal sense [27]. The Monte Carlo Simulation (MCS) technique is among the most robust and simple techniques, wherein the basic idea is to employ a random number generator to sample the design space. However MCS is used as the method of last resort as the points are not essentially space filling and the computational cost involved with high fidelity models can be prohibitively high. Other commonly used methods are stratified MCS, Orthogonal arrays, Latin Hypercube Sampling (LHS) and minimum discrepancy sequences. We use one of the minimum discrepancy methods based on Sobol sequences [28] also known as the LP_τ method. LP_τ sampling is based on uniformly distributed sequences in space and gives a mechanism for generating points in n-dimensional space which are uniformly distributed.

Model Structure Selection

In our study we use a Gaussian stochastic process model to build the surrogate model. The foundations of this method were developed in the field of geostatistics, where this model is referred to as Kriging and has been in use since the early 1960's [29]. It is also widely used in the neural network community where it is referred to as Gaussian process regression [30]. The model structure typically used in stochastic process approximation of the relationship $y = f(\mathbf{x})$ can be compactly written as

$$Y(\mathbf{x}) = \beta + Z(\mathbf{x}), \tag{3}$$

where β is an unknown hyperparameter to be estimated from the data and $Z(\mathbf{x})$ is a Gaussian stochastic process with zero mean and covariance

$$\mathrm{Cov}(Z(\mathbf{x},\mathbf{x}')) = \Gamma(\mathbf{x},\mathbf{x}') = \sigma^2{}_z R(\mathbf{x},\mathbf{x}'). \tag{4}$$

In other words, the observed outputs of the simulation code $\mathbf{y} = \{y^1, y^2, ..., y^l\}$ are assumed to be realizations of a Gaussian random field with mean β and covariance Γ. Here $R(.,.)$ is a parametrized correlation function that can be tuned to the training dataset and $\sigma^2{}_z$ is the so called process variance. A commonly used choice of covariance function is the stationary family which obeys the product correlation rule [27].

$$R(\mathbf{x}^1,\mathbf{x}^2) = \prod_{j=1}^{p} exp(-\theta_j |\mathbf{x}_j^1 - \mathbf{x}_j^2|^{p_j}), \tag{5}$$

where $\theta_j \geq 0$ and $1 < p_j \leq 2$ are the hyperparameters. The values of the hyperparameter θ can be used to understand the relative importance of each parameter on the performance of the airfoil. Hence θ_j, which is the value corresponding to the jth parameter, is an indicator of the its influence on the airfoil performance. If a Gaussian process prior over functions is used, the posterior process is also Gaussian. Hence using standard statistical results from Bayesian inferencing, the posterior mean and covariance can be stated as

$$\hat{y}(\mathbf{x}) = \beta + \tau(\mathbf{x})^T \mathbf{R}^{-1}(\mathbf{y} - \mathbf{1}\beta), \tag{6}$$

and

$$C(\mathbf{x},\mathbf{x}') = \sigma_z{}^2 \left(R(\mathbf{x},\mathbf{x}') - \tau(\mathbf{x})^T \mathbf{R}^{-1} \tau(\mathbf{x}') \right). \tag{7}$$

Here $\mathbf{R}$ is the correlation matrix whose ijth element is calculated as $R(\mathbf{x}^{(i)},\mathbf{x}^{(j)})$ and $\tau = \{R(\mathbf{x},\mathbf{x}^{(1)}), R(\mathbf{x},\mathbf{x}^{(1)}), ..., R(\mathbf{x},\mathbf{x}^{(1)})\}^T \in \mathbb{R}^n$ and $\mathbf{1} = \{1,1,...,1\} \in \mathbb{R}^n$. This approach finally leads to an approximation of the computational model as a multidimensional Gaussian random field. The randomness in equation (7), given by posterior variance $\sigma^2(\mathbf{x}) = C(\mathbf{x},\mathbf{x}')$, can be interpreted as an estimate of the uncertainty involved in predicting the output at any new points using the given finite dataset.

Parameter Estimation

After choosing an appropriate covariance function for the surrogate model the next task at hand is to estimate the set of unknown model parameters. The covariance function Γ can be parametrized in the term of vectors $\theta = \{\theta_1, \theta_2, ..., \theta_p\}$. Given the training dataset, we need to estimate θ and the other hyperparameters β and $\sigma_z{}^2$. Martin et al [31] have compared Maximum Likelihood Estimation (MLE) and Cross-Validation (CV) technique for estimating the parameters. Here we use the MLE technique as proposed in the Design and Analysis of Computer Experiments (DACE) [27]. The MLE approach leads to those values of the undetermined parameters that are most likely to have generated the training dataset. For the Gaussian process interpolation, since we assume that the observed outputs have a Gaussian distribution, the negative log-likelihood function to be minimized becomes

$$L(\theta,\beta,\sigma_z{}^2) = -\frac{1}{2}\left[n ln \sigma_z{}^2 + ln|\mathbf{R}| + \frac{1}{2\sigma_z{}^2}(\mathbf{y} - \mathbf{1}\beta)^T \mathbf{R}^{-1}(\mathbf{y} - \mathbf{1}\beta) \right]. \tag{8}$$

By taking the derivative of the log-likelihood function with respect to β and $\sigma_z{}^2$ and equating them to zero, we get a closed form solution for the optimal values of β and $\sigma_z{}^2$ as

$$\hat{\beta} = (\mathbf{1}^T \mathbf{R}^{-1} \mathbf{1})^{-1} \mathbf{1}^T \mathbf{R}^{-1} \mathbf{y}, \tag{9}$$

and

$$\hat{\sigma_z}{}^2 = \frac{1}{n}(\mathbf{y} - \mathbf{1}\beta)^T \mathbf{R}^{-1}(\mathbf{y} - \mathbf{1}\beta). \tag{10}$$

A closed-form solution does not exist for θ, requiring an iterative optimization procedure to minimize L as a function of θ. For a given value of θ, estimates of β and $\sigma_z{}^2$ can obtained using equation (10) and (11), respectively. These computed values of β and $\sigma_z{}^2$ can be substituted into equation (9) to calculate the log-likelihood function. In principle, any standard optimization routine can be employed to compute the maximum likelihood estimate of θ.

Model Validation

In this study we employ an efficient method to verify and update the surrogate model. After each iteration the NSGA-II predicts a set of Pareto optimal points. A low crowding algorithm is used to select points from the Pareto optimal set for verification and update. This ensures that the user is concentrating his computational effort in the area of interest. Also the points with the lowest rankings, predicted by the GA, are removed from the design space before the hyperparameters of the metamodel are trained. This ensures high accuracy of the metamodel in the region of interest.

NSGA-II FOR ROBUST DESIGN

Robust design can be formulated as a multi-objective problem with the goal of minimizing (1) the mean (μ) performance of the system, and (2) the standard deviation (σ) of the performance. This can be expressed as

$$\begin{aligned} &\text{Minimize}: \mu = \tfrac{1}{k}\textstyle\sum_{i=1}^{k} Pl_i \quad \text{and} \\ &\text{Minimize}: \sigma = \sqrt{\tfrac{1}{k-1}\textstyle\sum_{i=1}^{k}(Pl_i-\mu)^2}, \quad i=1,2,....k. \end{aligned} \tag{11}$$

where Pl is the pressure loss and k is the number of eroded compressor blade types used for representing the sample space. The presence of multiple objectives in a problem leads to a set of optimal solutions, rather than a single solution. Such a solution set is referred to as Pareto-optimal set in the optimization literature. Each point in the set is optimal in the sense that no improvement can be achieved in one objective without worsening the other objective. In the absence of further information about the relative importance of the objectives, it is not possible to decide which design is better than the rest. Hence, it is important to find as many Pareto-optimal solutions as possible for the benefit of the designer.

Classical optimization methods suggest converting the multiple objective optimization problem to a single objective optimization problem emphasizing one particular Pareto-optimal solution at a time. Such methods prove to be computationally expensive and do not ensure convergence to true optimal Pareto sets in non-convex problems [33,34]. In contrast, Genetic Algorithms are inherently suited for multi-objective problems as they have the ability to find multiple Pareto-optimal solutions in one simulation run. Since GAs work with a population of solutions, it is easier to extend them to maintain high diversity in finding multiple Pareto-optimal solutions at each stage, while moving toward the true Pareto-optimal region [35].

In recent years several approaches have been proposed to solve Multi-objective problems using GAs [36,37]. The elitism based Non-dominated Sorting Genetic Algorithm (NSGA-II) proposed by Deb et al [22] is employed here to seek the true Pareto-optimal front. The NSGA-II method is fast as it has a computational complexity of $O(MN^2)$ (where M is the number of objectives and N is the population size) when compared to other non-dominated GA with computational complexity $O(MN^3)$. NSGA-II method also uses elitism to enhance the performance of the GA and prevent the loss of good solutions once they are found. Traditional GA methods ensure diversity in a population by relying on the concept of sharing. In such methods it is necessary to specify the sharing parameter (σ_{share}) beforehand by the user. The performance of sharing functions in ensuring diversity, is dependent upon the choice of σ_{share}. In practice it is not very obvious how to select the best σ_{share}. In NSGA-II the sharing function approach is replaced by a crowded comparison approach. The crowded comparison approach has a better computational complexity and eliminates any user defined parameter for maintaining diversity among population members. In this study NSGA-II is employed in conjunction with surrogate models to identify the true Pareto-optimal front to seek robust designs.

NUMERICAL STUDIES AND RESULTS

The robust design method discussed above is applied to typical Rolls-Royce compressor fan blade section. LP_τ based DOE is employed to create an initial control factor set of m (m = 50) compressor fan blade shapes. This is followed by another set of LP_τ based DOE to create n (n=15) types of erosion on each of the m blade shapes. The noise factors - location ($t1$), width ($t2$) and depth (A) are represented by a uniform distribution. Figure 2 shows the shapes generated. PADRAM, a parametric and automatic grid generator is employed for generating high quality CFD mesh. A multigrid RANS based CFD simulation is performed at the $m \times n$ candidates points to evaluate m mean and variance of the pressure losses. The hyperparameters $\theta, \beta, {\sigma_z}^2$ of the Krig are evaluated using a hybrid Genetic Algorithm (GA) and Dynamic Hill Climbing (DHC) search method. The multi-objective algorithm NSGA-II is used in conjunction with the krig to seek the pareto optimal solutions. After each search iteration 10 points are selected on the Pareto front using a low crowding algorithm. The quality of the Krig is evaluated using the selected points after each iteration. Figure 5 and 6 show the regression coefficient for the values of standard deviation and mean of the pressure losses, predicted by the Krig, versus the values of standard deviation and mean of the pressure losses predicted by the CFD simulations.

The $R^2 = 0.789$ for the values of σ and $R^2 = 0.9527$ for the values of μ for the last update. The quality of the Krig can be improved by further updating the Krig. The NSGA-II algorithm with the update method is employed for 12 updates to find the optimal robust design set. Figure 7 shows the initial dataset with the initial Pareto front. The solid line represents the final Pareto

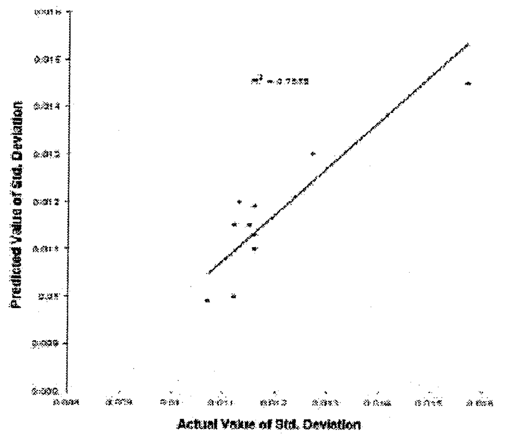

Figure 5. R^2 VALUES FOR σ FOR PREDICTED VS ACTUAL VALUES

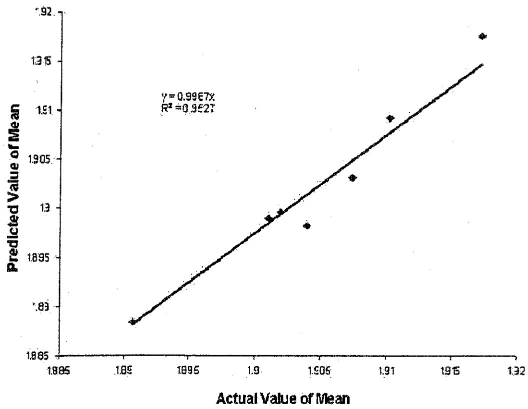

Figure 6. R^2 VALUES FOR μ FOR PREDICTED VS ACTUAL VALUES

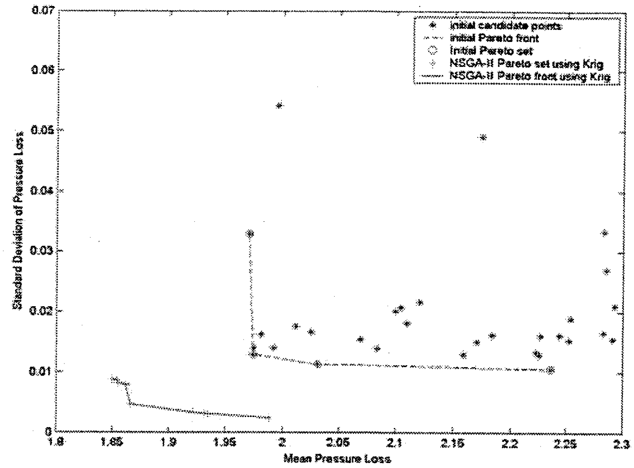

Figure 7. INITIAL AND NSGA-II PARETO SET

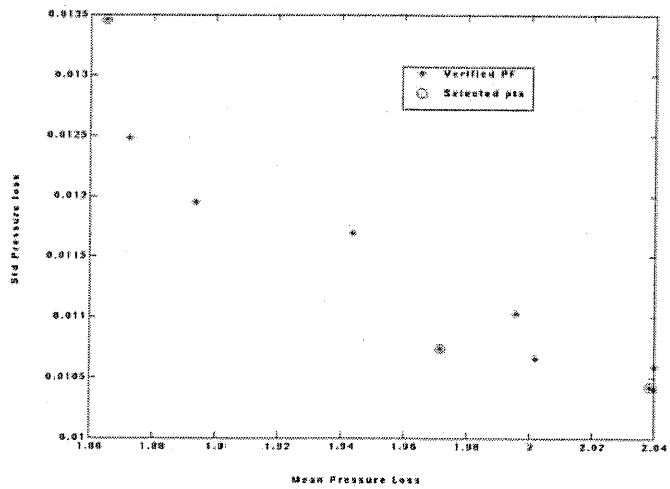

Figure 8. VERIFIED PARETO FRONT

front obtained by using NSGA-II in conjunction with Krig. The improvement in the Pareto set is obvious from the figure 7. The points on the Pareto front obtained by NSGA-II using the Krig are verified by CFD simulations. Figure 8 shows the verified Pareto points. The encircled points on the verified Pareto front are selected for further analysis. A 50 point LP_τ based DOE with noise factors - location ($t1$),width ($t2$) and depth (A) is executed for each of the selected geometry on the verified Pareto front. The data is used to train a surrogate model (Krig), which is further used for a Monte Carlo Simulation (MCS). A MCS of 10000 runs is executed for each selected blade geometry and the histograms of the pressure loss are generated. Figure 9 shows the histograms of pressure loss for the 3 selected blade geometries.

The same MCS analysis was conducted for the baseline geometry. Figure 10 shows the histograms of the nominal geometry and one of the selected robust geometries. It can be noted that the robust design has a better mean performance as well as lower variation as compared to the baseline geometry. This is not usually the case in real life problems where more often than not there is a trade-off between mean performance and variation. The above observation can be explained by the fact that the baseline geometry used here was not optimized for pressure loss alone. Other factors such as strength, cost and manufacturability would have effected the decision, which were not accounted for in this

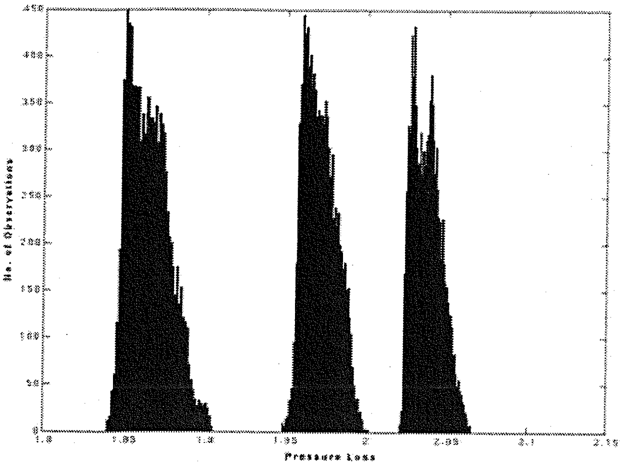

Figure 9. HISTOGRAMS OF SELECTED ROBUST GEOMETRIES

study. Hence, it was not surprising to find designs with better nominal performance as well as lower variability.

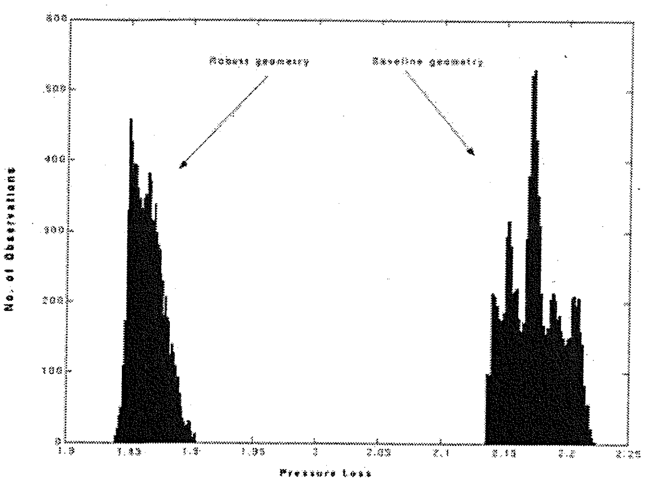

Figure 10. HISTOGRAM: ROBUST AND NOMINAL GEOMETRIES

CONCLUSIONS

In this paper we have presented an efficient and fast genetic algorithm based robust design methodology. A novel parametrization method was developed to model eroded compressor fan blade sections. A LP_τ based DOE technique was used to construct an initial inner control array and outer noise array. A parametric grid generation routine was used to automate geometry creation and grid generation to construct high quality CFD meshes. Gaussian stochastic process models (Krigs) were used as computational surrogates to the high fidelity CFD simulations.

The Robust design problem was formulated as a multi-objective problem. An elitism based non-dominated sorting genetic algorithm was employed in conjunction with the surrogate model to search the design space. A Pareto-optimal set was identified for trade off between the mean and standard deviation of the pressure loss. The efficiency of the proposed solutions would depend on the quality of the Krig used. The Pareto-optimal set suggested by the NSGA-II, using the Krig, was verified using CFD simulations and few points were selected from the verified Pareto front for further analysis. A Monte Carlo simulation based on surrogate models was executed for the selected blades and the results were found to be considerably better and robust than the baseline design. The method presented can be employed to seek robust optimal sets which can be presented to the designers to find compressor blade designs that are robust to erosion processes.

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