ELLIPSE DETECTION VIA GRADIENT DIRECTION IN THE HOUGH TRANSFORM

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ABSTRACT

Detecting ellipses is an important computer vision task. In order to overcome the excessive time and storage requirements associated with the Hough transform, some techniques decompose the required parameter space into several sub-spaces. The decomposition is achieved by using geometric constraints which define relative positions between a set of points. This paper shows how positional constraints can be avoided by using the local properties of the position function to decompose the parameter space. This decomposition requires the combination of a pair of edge points and their gradient direction. The decomposition is used to extract ellipses in synthetic and real images.

INTRODUCTION

The detection of geometric primitives from an image is one of the basic tasks of computer vision. The Hough transform (HT) and its extensions (Illingworth and Kittler (1), Leavers (2)) constitute a popular method for extracting geometric shapes. Primitives on the HT are represented by parametric curves with a number of free parameters. The principal concept of the HT is to define a mapping between an image space and a parameter space. Each edge point in an image is transformed by the mapping to determine cells in the parameter space whose associated parameters are such that the defined primitive passes through the data point. The chosen cells are accumulated and after all the points in an image have been considered, local maxima in the accumulator correspond to the parameters of the specified shape.

Because a curve with n parameters requires an n-dimensional parameter space, many applications of the HT concern line and circle detection. In order to overcome the excessive time and space requirements for ellipse extraction, proposed techniques (Yip et al. (3), Pao et al. (4), Yoo and Sethi (5), Wu and Wang (6), Ho and Chen (7)) decompose the five dimensional parameter space into several sub spaces of less dimensions. The decomposition is achieved by using geometric features which define constraints in the organization of edge data. These constraints include distance and angular relationships which define relative positions between a set of edge points. Hence, the parameters are computed after labelling the points which satisfy the constraints in a computational intensive approach.

In this paper we show how it is possible to decompose the parameter space, based on the polar definition of an ellipse. Angular information, defined from the variation of a position function, represents local change in the curvature of border points. This information is used to formulate expressions which define an ellipse by including local shape properties.

Ballard (8) originally suggested that the number of free parameters can be reduced by including derivatives. This is used to improve speed, but still requires a five dimensional parameter space. We show that in order to achieve a parameter decomposition (due to the ellipse anisotropy) it is necessary to consider the angular change of the second derivative (tangent angle of the second directional derivative).

In order to accumulate evidence from image data, we compute angular information by taking a pair of points and their gradient direction. The use of the local ellipse properties avoids the constraints which define relative position, as required by other approaches.

INCLUDING GRADIENT DIRECTION

Gradient direction information has been used to reduce the computational requirements of the HT (Ballard (8), Davis (9), Conker (10)). Here we use gradient information for ellipse extraction by relating local geometric features of a parametric representation to the local features on an image.

We represent an ellipse by the locus defined by the differentiable vector-valued function $z(\theta)=xU_x + yU_y$ for $U_x=[1,0]$, $U_y=[0,1]$ and

$$\begin{aligned} x &= a_0 + a_x \cos(\theta) + b_x \sin(\theta), \\ y &= b_0 + a_y \sin(\theta) + b_y \cos(\theta). \end{aligned} \qquad (1)$$

In this expression there are five free parameters defined by the two centre parameters (a_0,b_0) and only three of the axis parameters (a_x,a_y,b_x,b_y), θ is a position index.

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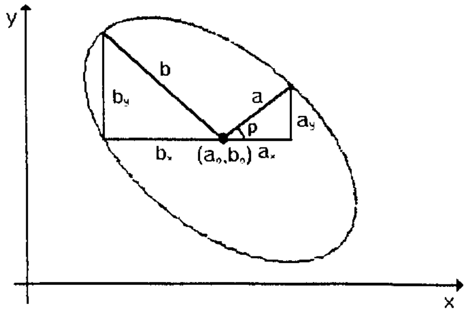

Figure 1: Definition of ellipse parameters.

Based on this representation, the remaining axis parameter can be computed by the orthonormality relationship defined by: $a_x\ b_x + a_y\ b_y = 0$. Figure 1 shows the geometry of the definition of the ellipse parameters for the polar form.

In order to define a relationship between adjacent points in an ellipse we consider the local angular change on the position function. The tangent of the angle of the first and second order derivatives of the position vector function $z(\theta)$ are,

$$\frac{y'}{x'} = \frac{-a_y \sin(\theta) + b_y \cos(\theta)}{-a_x \sin(\theta) + b_x \cos(\theta)}, \qquad (2)$$

$$\frac{y''}{x''} = \frac{-a_y \cos(\theta) - b_y \sin(\theta)}{-a_x \sin(\theta) - b_x \sin(\theta)}. \qquad (3)$$

where x', y', x'' and y'' are the first and second derivatives with respect to θ.

By combining expressions (1) and (3) we obtain a mapping which relates the ellipse centre parameters and angular information,

$$\frac{y''}{x''} = \frac{(y - b_0)}{(x - a_0)}. \qquad (4)$$

A map which relates the axis parameters and angular information is formulated by considering the property of orthonormality of the ellipse axes,

$$\tan(\phi_1 - \rho)\tan(\phi_2 - \rho) = N^2, \qquad (5)$$

where,

$$\phi_1 = atan\left(\frac{y'}{x'}\right),$$

$$\phi_2 = atan\left(\frac{y''}{x''}\right),$$

$$\rho = atan(K), \quad K = \frac{a_y}{a_x}, \quad N = \frac{b_y}{a_x}.$$

Therefore, by defining equations (4) and (5) as a mapping between an image and a parameter space it is possible to compute four ellipse parameters represented by (a_0,b_0) and the ratios of axes (K,N).

In order to obtain the axes after computing the ratio values, it is necessary to carry out a histogram accumulation process which obtains a_x. An expression for the axis component a_x given an edge point and the values of a_0, b_0, K and N is

$$a_x = \sqrt{\frac{y_0^2 + x_0^2 N^2}{N^2 (1 + K^2)}}, \qquad (6)$$

for x_0, y_0 defined by,

$$x_0 = \frac{(x - a_0)}{\sqrt{K^2 + 1}} + \frac{(y - b_0) K}{\sqrt{K^2 + 1}}, \qquad (7)$$

$$y_0 = \frac{(x - a_0) K}{\sqrt{K^2 + 1}} + \frac{(y - b_0)}{\sqrt{K^2 + 1}}, \qquad (8)$$

Consequently, by using equations (4), (5) and (6) it is possible to decompose the parameter space for ellipse extraction into two independent accumulators and a histogram accumulator. This decomposition is based on the local change of a curve defined by the angle of the tangent vector functions $z'(\theta)$ and $z''(\theta)$.

Note that the angular information of the parametric form does not define the local behaviour of an edge point. Therefore, in order to extract an ellipse, it is necessary to formulate an expression which makes explicit the relationships between positional angular features and data regularities on an image. Here we consider the use of edge direction information to match image data to the local particulars of an ellipse description.

Although the tangent angle of the first directional derivative can be related to gradient direction of an edge point by using the chain rule of differentiation, the angle of the tangent vector $z''(\theta)$ is not related to the change of gradient on an image. Therefore, the positional angular information does not depend only on the local information.

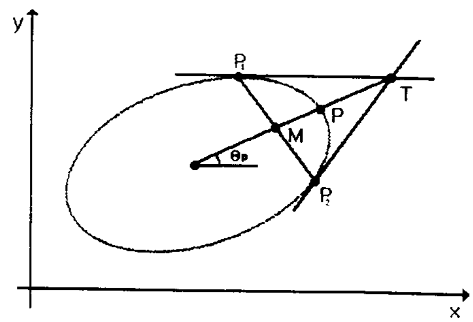

Figure 2: Geometry of two points on an ellipse.

In order to obtain the angle of the first and second order directional derivatives on an edge point P, we consider the positional and directional information of two arbitrary points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$, by following the ellipse geometry presented in Yuen et al (11).

Figure 2 shows the geometry of two ellipse points and the relationship defined by their derivatives. The point P can be obtained by computing the intersection of the line MT with the ellipse. If θ_1, θ_2 define the positions of the points P_1 and P_2, then the value of θ which specifies the point P is,

$$\theta_p = \frac{1}{2}(\theta_1 + \theta_2). \tag{9}$$

By substitution of this expression in equation (1) when $a_y=b_x=0$, we obtain the angle of the first and second directional derivatives for the point P,

$$\frac{y'}{x'} = \frac{Y}{X}, \tag{10}$$

$$\frac{y''}{x''} = \frac{2\,M_1\,X - Y\,M_2}{X\,M_2 - 2\,Y}, \tag{11}$$

for

$$Y = y_2 - y_1, \qquad X = x_2 - x_1,$$
$$M_1 = m_1\,m_2, \qquad M_2 = m_1 + m_2,$$

and where m_1 and m_2 are the tangents of the angle of inclination of a tangent line to the points P_1 and P_2 respectively. This information can be obtained by the change of gradient on an image by using a local edge operator.

EXPERIMENTAL RESULTS

By combining equations (4) and (5) with equations (10) and (11) we derive an algorithm which accumulates evidence by taking pairs of points from an image.

Although equations (10) and (11) define the angular relationship for arbitrary points, to compute accurate values it is necessary to provide pairs of points for which equations (10) and (11) do not suffer bias due to discretisation.

The error due to discretisation increases when M and T are close. Consequently, we avoid pairing points spaced by less than 25 pixels as well as points whose tangent is similar (i.e. $|P_1P_2|\geq 25$ and $|m_1-m_2|>10°$).

If the centre of the ellipse is within the image, the accumulator (a_0,b_0) and the histogram a_x are congruent to the image space. In order to accumulate evidence of the axis relationship (K,N) in a finite range of values, atan(K) and atan(N) are stored in the accumulator.

Experimental results have been assessed both on synthetic and real images. Figure 3(a) shows a 256x256x8-bit real image. Figure 3(b) shows the results of applying the Canny edge detector operator and Figure 3(c) shows the detected ellipse. Figure 4 shows a comparison of the resulting accumulator arrays. For a synthetic image, exact edge data provides accumulators with very clear maxima (Figure 4(a)). Noise increases when the edge direction is computed from the image (Figure 4(b)). However, the final result is still clear for ellipse extraction as shown in the accumulators present in Figure 4(c) which correspond to the image in Figure 3(b).

DISCUSSION AND CONCLUSIONS

A mapping for ellipse extraction has been developed which includes edge tangent information. This mapping combines local information computed from pairs of edge points in order to decompose the space required for the HT.

Although the analysis on synthetic images shows that there is a reduction in resolution on the parameter space due to the use of gradient directional information, the results show that this decomposition can be used when ellipse data is incomplete or occluded by another shape. The new mapping can then retain the original advantages of the HT.

An important point in the implementation consists of defining pairs of points which produce accurate information, as well as points which belong to the same geometric feature. This is generally not considered in methods which include several points to decompose the parameter space. Since we include edge direction information, the proposed method only involves pairs of points without any geometric constraints.

Current work focuses on incorporating more sophisticated information of the object in order to avoid noise in the accumulators caused by pairing points which do not belong to the same geometric primitive.

REFERENCES

1. Illingworth J, Kittler J, 1988, "A survey of the Hough transform", Computer Vision, Graphics and Image Processing, 44, 87-116.
2. Leavers V F, 1993, "Which Hough transform", CVGIP:Image Understanding, 58, 250-264.
3. Yip R K K, Tam P K S, Leung D N K, 1992, "Modification of the Hough transform for circles an ellipses detection using a 2-dimensional Array", Pattern Recognition, 25, 1007-1022.
4. Pao D, Li H F, Jayakumar R, 1993, " A decomposable parameter space for the detection of ellipses", Pattern Recognition Letters, 14, 951-958.
5. Yoo J H, Sethi I K, 1993, "An ellipse detection method from the polar and pole definition of conics", Pattern Recognition, 26, 307-315.
6. Wu W, Wang M J, 1993, "Elliptical object detection by using its geometrical properties", Pattern Recognition, 26, 1499-1509.
7. Ho C, Chan L, 1995, "A fast ellipse/circle detector using geometric symmetry", Pattern Recognition, 28, 117-124.
8. Ballard D H, 1981, "Generalizing the Hough transform to detect arbitrary shapes", Pattern Recognition, 13, 111-122.
9. Davies E R, 1986, "Image space transform for detecting straight edges in industrial images", Pattern Recognition Letters, 4, 185-192.
10. Conker R S, 1988, "A dual-plane variation of the Hough transform for detecting non-concentric circles of different radii", Computer Graphics and Image Processing, 43, 115-132.
11. Yuen H K, Illingworth J, Kittler J, 1989, "Detecting partially occluded ellipses using the Hough transform", Image and Vision Computing, 7, 31-37.

(a) (b) (c)

Figure 3. Ellipse detection example

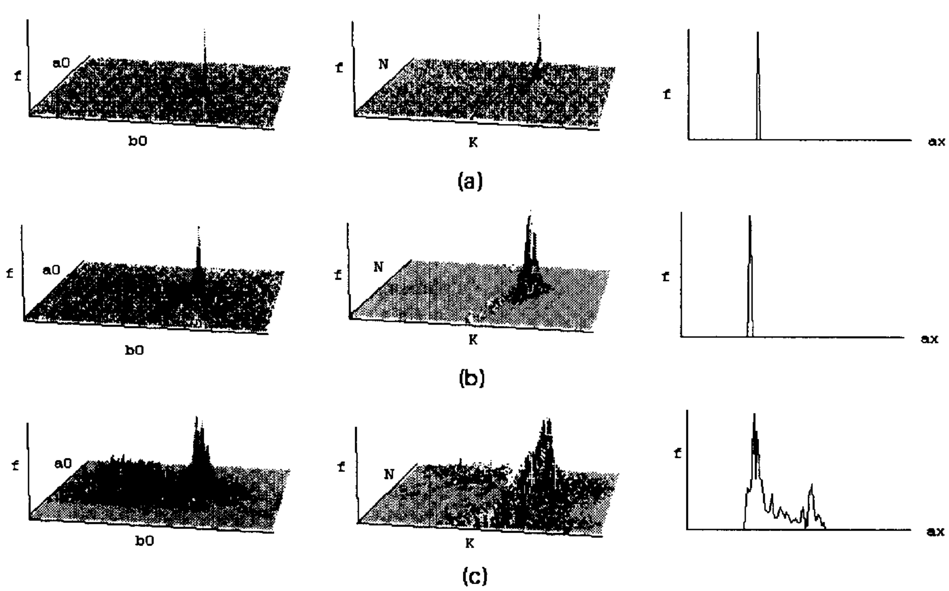

Figure 4. Comparison of results for synthetic and real images