

**STABILITY ISSUES OF FINITE PRECISION CONTROLLER STRUCTURES
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Abstract: This paper present a new effective algorithm for the optimal realization of sampled-data SISO two order controller structures using the delta operator subject to Finite-Word-Length (FWL) constraints. The problem is formulated as a non-linear programming problem with a maximum of four independent variables to provide an easy and efficient optimization tool to solve such complex problems. Simulation results of the optimum FWL realizations of digital SISO two order controller structure applied to an electrohydraulic actuator system are presented to illustrate the effectiveness of the proposed strategy. *Copyright © 1999 IFAC*

Keywords: control applications of computers, stability robustness, sampled data systems, optimization problems, linear systems.

1. INTRODUCTION

The recent advances in fixed-point implementation of digital controllers such as the design of dedicated fixed-point Digital Signal Processors (DSP) and new Digital Control Processors (DCP) architectures made Finite Word Length (FWL) implementation an important issue in modern digital control engineering design applications. Improved control performance

and increased levels of integration are especially important in many areas such as consumer electronic products, automotive and electro-mechanical control systems. This is because hardware controller implementation with fixed-point arithmetic offer the advantages of speed, memory space, cost and simplicity over floating-point arithmetic.

The FWL effects have been well studied in digital

signal processing, especially in digital filter implementation since the 1970's (Roberts and Mullis, 1987). The results have recently been extended to the study of FWL effects of digital controller on control systems. Moroney *et al.* (1980) studied the effects of FWL implemented digital controller on the degradation of an LQG cost function from a statistical point of view. Fialho and Georgiou (1994) analyzed the effects of FWL implemented digital controller on the stability and performance of sampled data systems. A FWL stability measure μ_0 was presented by Moroney *et al.* (1980) and Fialho and Georgiou (1994), but computing explicitly this measure seems very hard and is still an open problem. Based on the first order approximation, Li and Gevers (1996) and Istepanian *et al.* (1998b) developed two tractable FWL stability measures which are the lower bounds of μ_0 respectively. More recently, Istepanian *et al.* (1996, 1998a, 1998c), Madieviski *et al.* (1995), Qiu *et al.* (1993), and Whidborne *et al.* (1998) dealt with FWL implementation issues using different approaches.

In all these studies of FWL effects of digital controllers were described and realized using the usual shift operator z structure. It is well known that a discrete time system can also be described and realized with a different discrete operator called delta operator (Middleton and Goodwin, 1990). The delta operator parametrization provides a unified formulation between the continuous time and discrete time systems. This paper presents the FWL stability issues of sampling data control systems using the delta operator domain. The results of Li and Gevers (1996) and Istepanian *et al.* (1998b) are extended using the delta operator formulation and a delta-based FWL stability measure is derived. The relevant procedures for computing this measure and the algorithmic approach for the optimal FWL realizations providing the maximum stability measures are also presented. The theoretical approach is illustrated by a numerical example.

2. THE DELTA OPERATOR FORMULATIONS

From a continuous time transfer function $G(s)$, as the results of a discretization procedure with the shift operator z and a sampling period h , a discrete time transfer function $G_z(z)$ can be obtained. Define

$$\delta = \frac{z-1}{h} \tag{1}$$

with this definition, the transfer function $G_z(z)$ can be re-expressed in δ form:

$$G_z(z) = G_\delta(\delta) \tag{2}$$

which means $G_z(z)$ and $G_\delta(\delta)$ are two different but equivalent parametrizations representing the same object. These two input-output relationships

can be represented by a shift operator (resp. δ -operator) state space model as follows:

$$\begin{cases} zx_z(k) = A_z x_z(k) + B_z u(k) \\ y(k) = C_z x_z(k) + D_z u(k) \end{cases} \tag{3}$$

$$\begin{cases} \delta x_\delta(k) = A_\delta x_\delta(k) + B_\delta u(k) \\ y(k) = C_\delta x_\delta(k) + D_\delta u(k) \end{cases}$$

The following relationships relate the internal and external representations:

$$\begin{aligned} G_z(z) &= C_z (zI - A_z)^{-1} B_z + D_z \\ &= G_\delta(\delta) = C_\delta (\delta I - A_\delta)^{-1} B_\delta + D_\delta \\ A_z &= hA_\delta + I \\ B_z &= hB_\delta \\ C_z &= C_\delta \\ D_z &= D_\delta \end{aligned} \tag{4}$$

Define

$$S_\rho = \{A_\rho, B_\rho, C_\rho, D_\rho\} : G_\rho(\rho) = C_\rho(\rho I - A_\rho)^{-1} B_\rho + D_\rho$$

where $\rho = z$ or δ is called generalized operator.

Hence if $(A_\rho, B_\rho, C_\rho, D_\rho) \in S_\rho$, $(T^{-1}A_\rho T, T^{-1}B_\rho, C_\rho T, D_\rho) \in S_\rho$ if and only if T is nonsingular. The following lemmas result directly.

Lemma 1: $\lambda_i(A_z) = 1 + h\lambda_i(A_\delta)$, where $\lambda_i(\cdot)$ is the eigenvalue.

It is well known that the discrete time system (A_z, B_z, C_z, D_z) is stable if and only if $|\lambda_i(A_z)| < 1$. From lemma 1, we can get the condition of the stability of the discrete time system described with the delta operator:

Lemma 2: The discrete time system $(A_\delta, B_\delta, C_\delta, D_\delta)$ is stable if and only if $|\lambda_i(A_\delta) + \frac{1}{h}| < \frac{1}{h}$.

3. DELTA-BASED STABILITY ROBUSTNESS MEASURES WITH FWL CONSIDERATION

Considering the sampled data system Σ_1 shown as Figure 1.

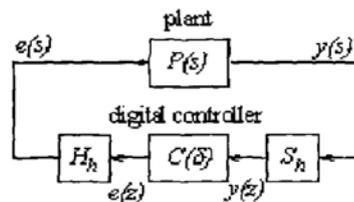


Fig. 1. Sampled data system

where $P(s)$ is the continuous time plant, $C(\delta)$ is the discrete time controller, S_h is the sampler with sampling period h ,

$$y(z) = S_h y(s) : y(k) = y(kh)$$

H_h is the hold device with sampling period h .

$$e(s) = H_h e(z) : e(t) = e(k), (kh < t \leq (k+1)h)$$

Suppose $P(s)$ is strictly proper. Let $(A_p, B_p, C_p, 0)$ be a state description of $P(s)$ i.e.

$$P(s) = C_p (sI - A_p)^{-1} B_p \quad \text{where} \\ A_p \in R^{m \times m}, B_p \in R^{m \times l}, C_p \in R^{q \times m}.$$

Let (A_c, B_c, C_c, D_c) be a state description of $C(\delta)$, i.e. $C(\delta) = C_c (\delta I - A_c)^{-1} B_c + D_c$ where

$$A_c \in R^{n \times n}, B_c \in R^{n \times q}, C_c \in R^{l \times n}, D_c \in R^{l \times q}.$$

In this paper, (A_c, B_c, C_c, D_c) is also called a realization of $C(\delta)$. The realizations of $C(\delta)$ are not unique.

In fact, if $(A_c^0, B_c^0, C_c^0, D_c^0)$ is a realization of $C(\delta)$. Then so is $(T^{-1}A_c^0T, T^{-1}B_c^0, C_c^0T, D_c^0)$ for any similarity transformation $T \in R^{n \times n}$. Considering the behaviour of the sampled data system Σ_1 only at its sampling instants, we arrive at a discrete time feedback system Σ_2 :

$$\begin{cases} y(z) = S_h P(s) H_h e(z) \\ e(z) = C(\delta) y(z) \end{cases} \quad (5)$$

The plant $P(\delta) = S_h P(s) H_h = C_\delta (\delta I - A_\delta)^{-1} B_\delta$ is the discretized $P(s)$ whose state description is $(A_\delta, B_\delta, C_\delta, 0)$, where

$$A_\delta = \frac{1}{h} (e^{A_p h} - I) \in R^{m \times m},$$

$$B_\delta = \frac{1}{h} \int_0^h e^{A_p \tau} B_p d\tau \in R^{m \times l}, C_\delta = C_p \in R^{q \times m} \quad (6)$$

It can be easily seen that the corresponding state description $(\bar{A}, \bar{B}, \bar{C}, \bar{D})$ of the discrete time closed loop system Σ_2 without FWL effect is given by:

$$\bar{A} = \begin{bmatrix} A_\delta + B_\delta D_c C_\delta & B_\delta C_c \\ B_c C_\delta & A_c \end{bmatrix}$$

$$= \begin{bmatrix} A_\delta & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} B_\delta & 0 \\ 0 & I_n \end{bmatrix} \begin{bmatrix} D_c & C_c \\ B_c & A_c \end{bmatrix} \begin{bmatrix} C_\delta & 0 \\ 0 & I_n \end{bmatrix} \quad (7)$$

$$= M_0 + M_1 X M_2 = \bar{A}(X)$$

$$\bar{B} = \begin{bmatrix} B_\delta \\ 0 \end{bmatrix}, \bar{C} = [C_\delta \quad 0], \bar{D} = 0 \quad (8)$$

where $M_0 \in R^{(m+n) \times (m+n)}, M_1 \in R^{(m+n) \times (l+n)}, M_2 \in R^{(q+n) \times (m+n)}$ are fixed matrices that depend on $P(s)$ and h ; $X \in R^{(l+n) \times (q+n)}$ is called the controller matrix; I_n denotes $n \times n$ identity matrix. Suppose $C(\delta)$ has been given to make the sampled data system Σ_1 stable, and the realization of $C(\delta)$ is

$$X_0 = \begin{bmatrix} D_c^0 & C_c^0 \\ B_c^0 & A_c^0 \end{bmatrix}.$$

Since the sampled data system Σ_1 is stable if and only if system Σ_2 is stable (Chen and Francis, 1991), it follows from lemma 2 that

$$\forall i \in \{1, \dots, m+n\}, \quad \left| \lambda_i(\bar{A}(X_0)) + \frac{1}{h} \right| < \frac{1}{h}.$$

When realization $(A_c^0, B_c^0, C_c^0, D_c^0)$ of $C(\delta)$ is implemented with a DCP, the controller matrix

$$X = \begin{bmatrix} p_1 & p_2 & \dots & p_{q+n} \\ p_{q+n+1} & p_{q+n+2} & \dots & p_{2(q+n)} \\ \vdots & \vdots & \dots & \vdots \\ p_{(l+n-1)(q+n)+1} & p_{(l+n-1)(q+n)+2} & \dots & p_{(l+n)(q+n)} \end{bmatrix} \quad (9)$$

is perturbed into:

$$X_0 + \Delta X = \begin{bmatrix} D_c^0 & C_c^0 \\ B_c^0 & A_c^0 \end{bmatrix} + \begin{bmatrix} \Delta p_1 & \Delta p_2 & \dots & \Delta p_{q+n} \\ \Delta p_{q+n+1} & \Delta p_{q+n+2} & \dots & \Delta p_{2(q+n)} \\ \vdots & \vdots & \dots & \vdots \\ \Delta p_{(l+n-1)(q+n)+1} & \Delta p_{(l+n-1)(q+n)+2} & \dots & \Delta p_{(l+n)(q+n)} \end{bmatrix} \quad (10)$$

due to the FWL effect, where each element of ΔX is bounded by $\frac{\varepsilon}{2}$, i.e.

$$\mu(\Delta X) = \max_{i \in \{1, \dots, (l+n)(q+n)\}} |\Delta p_i| \leq \frac{\varepsilon}{2} \quad (11)$$

For a fixed point processor of B_s bits,

$$\varepsilon = 2^{-(B_s - B_x)} \quad (12)$$

where 2^{B_x} is a normalization factor such that each parameter of $2^{-B_x} X_0$ is absolutely not bigger than 1. With the perturbation ΔX , $\lambda_i(\bar{A}(X_0))$ may be moved to $\lambda_i(\bar{A}(X_0 + \Delta X))$. The sampled data system Σ_1 is unstable if and only if there is $i \in \{1, \dots, m+n\}$

such that $\left| \lambda_i(\bar{A}(X_0 + \Delta X)) + \frac{1}{h} \right| \geq \frac{1}{h}$.

When ΔX is small, $\forall i \in \{1, \dots, m+n\}$, we have (Li and Gevers, 1996 and Istepanian et al., 1998a)

$$\Delta \lambda_i = \lambda_i(\bar{A}(X_0 + \Delta X)) - \lambda_i(\bar{A}(X_0)) = \sum_{j=1}^N \frac{\partial \lambda_i}{\partial p_j} \Big|_{X=X_0} \Delta p_j \quad (13)$$

where $N = (l+n) \times (q+n)$ is the number of elements of X . It follows that:

$$|\Delta \lambda_i| \leq \sum_{j=1}^N \left| \frac{\partial \lambda_i}{\partial p_j} \Big|_{X=X_0} \right| |\Delta p_j| \leq \mu(\Delta X) \sum_{j=1}^N \left| \frac{\partial \lambda_i}{\partial p_j} \Big|_{X=X_0} \right| \quad (14)$$

Thus $\forall i \in \{1, \dots, m+n\}$, if

$$\mu(\Delta X) < \frac{\frac{1}{h} - \left| \lambda_i(\bar{A}(X_0)) + \frac{1}{h} \right|}{\sum_{j=1}^N \left| \frac{\partial \lambda_i}{\partial p_j} \right|_{X=X_0}} \quad (15)$$

we have

$$\left| \lambda_i(\bar{A}(X_0 + \Delta X)) + \frac{1}{h} \right| \leq \frac{1}{h} \quad (16)$$

which means the sampled data system Σ_1 is stable. Let

$$\mu_1(X_0) \triangleq \min_{i \in \{1, \dots, m+n\}} \frac{\frac{1}{h} - \left| \lambda_i(\bar{A}(X_0)) + \frac{1}{h} \right|}{\sum_{j=1}^N \left| \frac{\partial \lambda_i}{\partial p_j} \right|_{X=X_0}} \quad (17)$$

From (16), one can reach to the following:

Theorem 1: System Σ_1 with FWL effect is stable when $\mu(\Delta X) < \mu_1(X_0)$.

One can compute $\mu_1(X_0)$ using the following lemma, which was proved by Li and Gevers (1996) and Istepanian *et al.* (1998a).

Lemma 3: Assume $\bar{A}(X)$ described in (7) be diagonalizable and have $\{\lambda_i\} = \{\lambda_i(\bar{A}(X))\}$ as its eigenvalues, let x_i be a right eigenvector of $\bar{A}(X)$ corresponding to the eigenvalue λ_i . Denote $M_x = [x_1 \ x_2 \ \dots \ x_{m+n}]$ and $M_y = [y_1 \ y_2 \ \dots \ y_{m+n}] = M_x^{-H}$, where y_i is called the reciprocal left eigenvector corresponding to λ_i . Then $\forall i \in \{1, \dots, m+n\}$

$$\frac{\partial \lambda_i}{\partial X} = M_1^T y_i x_i^T M_2^T \quad (18)$$

where y_i is conjugate to x_i , "T" denotes the transpose operation.

Based on $\mu_1(X_0)$, one can compute \hat{B}_{sl}^{\min} , the smallest integer which is not less than $-\log_2 \mu_1(X_0) - 1 + B_X$. From (11), (12) and theorem 1, we know that the sampled data system Σ_1 is stable when X_0 is implemented with a DCP of at least \hat{B}_{sl}^{\min} bits.

4. OPTIMAL DELTA REALIZATION OF CONTROLLER WITH FWL CONSIDERATION

In this section we will discuss the optimal realization

problem of digital SISO (Single Input Single Output) two order controllers because of the wider industrial use and simple application of such controllers. The usual PID controller is a kind of two order controller.

As a SISO two order controller, the initial realization $A_C^0 \in R^{2 \times 2}, B_C^0 \in R^{2 \times 1}, C_C^0 \in R^{1 \times 2}, D_C^0 \in R$. Then

$X_0 = \begin{bmatrix} D_C^0 & C_C^0 \\ B_C^0 & A_C^0 \end{bmatrix} \in R^{3 \times 3}$ and any realization of

$C(\delta)$ is $X_T \triangleq \begin{bmatrix} 1 & 0 \\ 0 & T^{-1} \end{bmatrix} X_0 \begin{bmatrix} 1 & 0 \\ 0 & T \end{bmatrix}$ where $T \in R^{2 \times 2}$

and $\det(T) \neq 0$. Then from (7),

$$\bar{A}(X_T) = \begin{bmatrix} I_m & 0 \\ 0 & T^{-1} \end{bmatrix} \bar{A}(X_0) \begin{bmatrix} I_m & 0 \\ 0 & T \end{bmatrix} \quad (19)$$

Let x_i^0 be a right eigenvector of $\bar{A}(X_0)$ corresponding to the eigenvalue $\lambda_i^0 = \lambda_i(\bar{A}(X_0))$, y_i^0 be the reciprocal left eigenvector corresponding to x_i^0 . Applying lemma 3, we have

$$\begin{aligned} & \frac{\partial \lambda_i}{\partial X} \Big|_{X=X_T} \\ &= \begin{bmatrix} B_C^T & 0 \\ 0 & I_2 \end{bmatrix} \begin{bmatrix} I_m & 0 \\ 0 & T^T \end{bmatrix} (y_i^0)^T (x_i^0)^T \begin{bmatrix} I_m & 0 \\ 0 & T^{-T} \end{bmatrix} \begin{bmatrix} C_C^T & 0 \\ 0 & I_2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & T^T \end{bmatrix} \frac{\partial \lambda_i}{\partial X} \Big|_{X=X_0} \begin{bmatrix} 1 & 0 \\ 0 & T^{-T} \end{bmatrix} \end{aligned} \quad (20)$$

For complex matrix

$$M = \begin{bmatrix} M_{11} & \dots & M_{1n} \\ \vdots & \dots & \vdots \\ M_{m1} & \dots & M_{mn} \end{bmatrix} \in C^{m \times n} \quad (21)$$

denote

$$\|M\|_s = \sum_{i=1}^m \sum_{j=1}^n |M_{ij}| \quad (22)$$

From (17), (18), (20) and (22), we can describe the optimal FWL realization problem of digital SISO two order controller as an optimization problem.

$$\nu = \frac{1}{\max_{X_T} \mu_1(X_T)}$$

$$\begin{aligned} &= \min_{\substack{T \in R^{2 \times 2} \\ \det(T) \neq 0}} \max_{i \in \{1, \dots, m+2\}} \left\| \begin{bmatrix} 1 & 0 \\ 0 & T^T \end{bmatrix} \frac{\partial \lambda_i}{\partial X} \Big|_{X=X_0} \begin{bmatrix} 1 & 0 \\ 0 & T^{-T} \end{bmatrix} \right\|_s \\ &\triangleq \min_{\substack{T \in R^{2 \times 2} \\ \det(T) \neq 0}} \max_{i \in \{1, \dots, m+2\}} \left\| \begin{bmatrix} 1 & 0 \\ 0 & T^T \end{bmatrix} \Phi_i \begin{bmatrix} 1 & 0 \\ 0 & T^{-T} \end{bmatrix} \right\|_s \end{aligned} \quad (23)$$

where Φ_i is a fixed complex matrix. The following lemma proven by Istepanian *et al.*, (1998b) show that optimization problem (23) can be expressed by solving the other two associated optimization

problems (24) and (26).

Lemma 4: Let

$$v_1 = \min_{\substack{x \in (0, +\infty) \\ y \in (-\infty, +\infty) \\ w \in (0, +\infty)}} f_1(x, y, w) \tag{24}$$

where

$$f_1(x, y, w) = \max_{i \in \{1, \dots, m+2\}} \left\| \begin{bmatrix} w & 0 & 0 \\ 0 & x & 0 \\ 0 & y & 1/x \end{bmatrix} \Phi_i \begin{bmatrix} 1/w & 0 & 0 \\ 0 & 1/x & 0 \\ 0 & -y & x \end{bmatrix} \right\|_s \tag{25}$$

Let

$$v_2 = \min_{\substack{x \in (-\infty, +\infty) \\ y \in (-\infty, +\infty) \\ z \in (0, +\infty) \\ w \in (0, +\infty)}} f_2(x, y, z, w) \tag{26}$$

where

$$f_2(x, y, z, w) = \max_{i \in \{1, \dots, m+2\}} \left\| \begin{bmatrix} w & 0 & 0 \\ 0 & x & z \\ 0 & \frac{xy-1}{z} & y \end{bmatrix} \Phi_i \begin{bmatrix} \frac{1}{w} & 0 & 0 \\ 0 & y & -z \\ 0 & \frac{1-xy}{z} & x \end{bmatrix} \right\|_s \tag{27}$$

Then $v = \min(v_1, v_2)$. If $v = v_1$ and $(x_{opt1}, y_{opt1}, w_{opt1})$ is the optimal solution of problem (24), the optimal solution of problem (23) can be given as:

$$T_{opt} = \frac{1}{w_{opt1}} \begin{bmatrix} x_{opt1} & y_{opt1} \\ 0 & 1/x_{opt1} \end{bmatrix} \tag{28}$$

If $v = v_2$ and $(x_{opt2}, y_{opt2}, z_{opt2}, w_{opt2})$ is the optimal solution of problem (26), the optimal solution of problem (23) can be given as:

$$T_{opt} = \frac{1}{w_{opt2}} \begin{bmatrix} x_{opt2} & (x_{opt2}y_{opt2} - 1)/z_{opt2} \\ z_{opt2} & y_{opt2} \end{bmatrix} \tag{29}$$

Evidently, the problems (24) and (26) are multivariable nonlinear programming problems which can be solved efficiently using the Rosenbrock algorithm (Dixon, 1972).

5. APPLICATION TO ELECTROHYDRAULIC ACTUATOR SYSTEM

In order to illustrate how the optimization approach presented earlier can be used efficiently for the delta-based parameterization issues of optimal FWL SISO two order controller structures with improved stability bounds and minimum word-length requirements. We consider the following industrial electrohydraulic actuator PI control system to confirm our theoretical results.

The continuous time linearized model $P(s)$ of electrohydraulic actuator system is given by :

$$A_P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2.4667 \times 10^6 & -62800 & 0 & 0 \\ -2.5485 \times 10^2 & 0 & -551.13 & 0 \\ 1.8595 \times 10^2 & 0 & 0 & -29359 \end{bmatrix}$$

$$B_P = \begin{bmatrix} 0 \\ 18933 \\ 0 \\ 0 \end{bmatrix}, C_P = \begin{bmatrix} 0 & 0 & -2.2500 \times 10^{-4} & 4.0200 \times 10^{-4} \end{bmatrix}$$

To provide a high bandwidth force control loop, a stabilized force feedback PI controller with a pre-filter is designed:

$$\frac{-1}{0.0001s + 1} \left(0.005 + \frac{0.05}{s} \right)$$

The sampling period chosen as $h = 2^{-12}$. Using bilinear (Tustin) method we obtain the initial realization of $C(\delta)$ as:

$$A_C^0 = \begin{bmatrix} 0 & 0 \\ 1 & -4.5031 \times 10^3 \end{bmatrix}, B_C^0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C_C^0 = [-10.179 \quad 45610], D_C^0 = [-2.7518 \times 10^{-3}]$$

For problem (24), we set initial point $x_{01} = 1, y_{01} = 0, w_{01} = 1$. The realization corresponding to (x_{01}, y_{01}, w_{01}) is X_0 . By using Rosenbrock iterative algorithm, we get the solution $x_{opt1} = 17.688, y_{opt1} = 253.86, w_{opt1} = 56.399$ and $v_1 = 5353.7158$. The new realization corresponding to $(x_{opt1}, y_{opt1}, w_{opt1})$ is X_{opt1} . For problem (26), we set initial point $x_{02} = 1, y_{02} = 1, z_{02} = 1, w_{02} = 1$. The resultant realization corresponding to $(x_{02}, y_{02}, z_{02}, w_{02})$ is X_2 . By using Rosenbrock iterative algorithm, we get the solution $x_{opt2} = 2.5207 \times 10^{-2}, y_{opt2} = 2.3182 \times 10^{-3}, z_{opt2} = 5.6092 \times 10^{-2}, w_{opt2} = 5.6776 \times 10^{-2}$ and $v_2 = 5353.7027$. The realization corresponding to $(x_{opt2}, y_{opt2}, z_{opt2}, w_{opt2})$ is given as:

$$X_{opt2} = \begin{bmatrix} -2.7518 \times 10^{-3} & -1.2974 \times 10^{-1} & 3.1908 \\ 1.3162 \times 10^{-1} & -3.8316 & -132.27 \\ -3.1847 & -1.3034 \times 10^2 & -4499.2 \end{bmatrix}$$

Hence $v = \min(v_1, v_2) = v_2 = 5353.7027$ and X_{opt2} is the relevant optimal realization.

Table 1 Stability Measures and word lengths

Realization	μ_1	\hat{B}_{s1}^{\min}
X_0	1.6647×10^{-5}	31
X_2	8.7076×10^{-6}	32
X_{opt1}	1.867862×10^{-4}	25
X_{opt2}	1.867866×10^{-4}	25

Table 1 summarizes the results for the different finite realizations and show their relevant stability robustness measure μ_1 and the stabilized word length $\hat{b}_{s,1}^{\min}$ for the electrohydraulic actuator sampled data system.

6. CONCLUSION

In this paper a new delta-based FWL stability measure is introduced. It has been shown that the relevant optimal FWL realizations for two order controller structures can be interpreted as a constrained nonlinear optimization problem. A numerical example using a PI electrohydraulic actuator sampled data control system is presented to verify the improved parametrization characteristics at high sampling rates. Ongoing work is currently underway to generalize the methodology to higher order controller structures with integrated robust formulations.

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