The Probability of Multiple Correct Packet Reception in Coded Synchronous Frequency-Hopped Spread-Spectrum Networks

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***Abstract*— In this paper we present a computationally efficient method of evaluating the probability of multiple correct packet reception in coded synchronous frequency-hopped spread-spectrum (FHSS) networks. We show that the approximation using the independent receiver operation assumption (IROA), which has been frequently employed in the literature without rigorous validation, produces reasonable results in most network load conditions when compared to the exact computations derived from our proposed method. Specifically, the expected value of the absolute error was in the range of 0.0055%–18.21% in the investigated scenarios.**

I. Introduction

SPREAD-SPECTRUM networks employing slotted transmissions can be characterized by the probability of multiple correct packet reception, which is defined as the probability that exactly m out of k arbitrarily selected packets are received correctly, given that L packets are transmitted simultaneously in the network. This probability is denoted by $P(m, k-m|L)$. In the generic performance study of slotted multiple access spread-spectrum networks, this quantity has been evaluated approximately for various cases, for example, in the analysis by Polydoros and Silvester [1]. Later, exact and approximate methods of obtaining this quantity have been presented for frequency-hopped spread-spectrum (FHSS) networks in [2] and for direct-sequence spreading in [3]. Recently, Murali and Hughes [4] have investigated the tradeoffs in terms of coding, throughput, delay, and stability of FHSS networks, estimating the associated quantities by simulation due to the computationally intensive nature of the methods suggested in [2].

In this paper we present a computationally efficient method of evaluating the probability of multiple correct packet reception in coded FHSS networks. In Section II we describe and model the FHSS network under consideration. In Section III we detail the procedure of evaluating the probability of multiple correct packet reception, and in Section IV we present our numerical results. Finally, our conclusions are offered in Section V.

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II. Network Description

We consider a network in which L transmitters (or users) communicate with L distinct receivers. Each user is assumed to transmit packets according to a slotted channel access scheme. The packets are constructed and transmitted using Reed–Solomon (RS) error correction coding and frequency-hopped signaling, employing the (n_c, k_c) singly extended RS code, where each symbol in the codeword can assume a value from the alphabet of size n_c. Specifically, each transmitted packet consists of one RS-coded codeword and each RS-coded symbol in the codeword is transmitted on a frequency determined by the random frequency-hopping algorithm employed. We consider synchronous frequency hopping among users of the network, which is the case, when the propagation delay of the network is low compared to the hop interval. We assume that the frequency hopping pattern of each user is memoryless and uniformly distributed over q frequency slots and that those of different users are statistically identical and independent of each other. The specific modulation type is not considered. We take into account, however, the effect of the additive white Gaussian noise (AWGN) on the modulation and demodulation process by means of the symbol error rate P_N.

When more than two users occupy the same frequency slot in a hop interval, there is a high probability of symbol errors in this hop interval, a condition which is referred to as a frequency *hit* in [5]. Several methods have been devised to provide more accurate information concerning the occurrences of the hit [5], [6]. We consider two special cases, namely *no side information* (NSI) and *perfect side information* (PSI). No side information refers to the case when no information is available concerning the occurrence of hits, while perfect side information refers to the case when the occurrence of a hit is known exactly, i.e., with a probability of one. At the receiver, the decoding method applied can vary according to the availability of the hit-related side information. In case of no side information, we assume that every hit causes symbol errors in the corresponding hop interval with a probability of one and that the packet is decoded by a standard decoding method, such as the Berlekamp–Massey algorithm [7]. For the case of perfect side information, the symbols involved in the hit hop interval are marked as erasures and error-and-erasure decoding can be invoked where, due to the knowledge of the symbol error positions, the maximum number of correctable errors is doubled, since all the syndrome equations can be solved for a doubled number of error magnitudes, instead of having to compute both the error magnitudes and positions [7]. More specifically, the so-called bounded-distance decoding is assumed and the received packet is guaranteed to be correct if twice the number of errors t plus the number of erasures s remain within the minimum distance of the RS code, i.e., if [7]

$$2t + s < d_{\min} = n_c - k_c + 1. \tag{1}$$

III. Analysis

In order to determine whether a packet is correct or not, we have to count the number of errors and erasures which occur during the packet's reception at the corresponding receiver. Thus, it is necessary to keep track of every packet which is being received in a slot in order to evaluate the desired probability of multiple correct packet reception $P(m, k - m|L)$. We let $t_j^{(i)}$ be the actual number of errors encountered and $s_j^{(i)}$ the number of erasures counted at the receiver for the packet transmitted by the ith user during the period spanning the first to the jth hopping interval. Furthermore, we define the random variables $\{X_j^{(i)}, j = 0, 1, \cdots, n_c\}$, as

$$X_j^{(i)} = \begin{cases} \min\left(t_j^{(i)}, E\right), & \text{for NSI} \\ \min\left(2t_j^{(i)} + s_j^{(i)}, E\right), & \text{for PSI} \end{cases} \tag{2}$$

where

$$E = \begin{cases} \lfloor (n_c - k_c)/2 \rfloor + 1, & \text{for NSI} \\ n_c - k_c + 1, & \text{for PSI.} \end{cases} \tag{3}$$

We introduced the limit E into the definition of $X_j^{(i)}$ since there is no need to count the number of errors beyond E corresponding to the maximum error correction ability of the RS code plus one, beyond which the packet cannot be decoded correctly, regardless of the presence or absence of further hits.

Now we briefly state a theorem which will be used in our forthcoming analysis. This is referred to as *the inclusion and exclusion formula* in combinatorics, and the proof can be found in [8] and [9].

Theorem 1—The Realization of m Among K Events: For any integer m with $0 \le m \le K$, the probability $P_{[m]}$ that exactly m among the K events $A_1, A_2, \cdots, A_K$ occur simultaneously is given by

$$P_{[m]} = \sum_{i=0}^{K-m} (-1)^i \binom{m+i}{m} S_{m+i} \tag{4}$$

where

$$\begin{aligned} S_0 &= 1 \\ S_1 &= \sum_{i_1} P\{A_{i_1}\} \\ S_2 &= \sum_{i_1 < i_2} P\{A_{i_1} A_{i_2}\} \\ &\vdots \\ S_K &= \sum_{i_1 < i_2 < \cdots < i_K} P\{A_{i_1} A_{i_2} \cdots A_{i_K}\}. \end{aligned} \tag{5}$$

By applying the above theorem, we can determine $P(m, k - m|L)$ as

$$P(m, k - m|L) = \sum_{i=0}^{m} (-1)^i \binom{k-m+i}{k-m} S_{k-m+i} \tag{6}$$

where

$$\begin{aligned} S_r &= \sum_{i_1 < i_2 < \cdots < i_r} P\left\{X_{n_c}^{(i_1)} = E, X_{n_c}^{(i_2)} = E, \cdots, X_{n_c}^{(i_r)} = E\right\} \\ &= \binom{k}{r} P\left\{X_{n_c}^{(1)} = E, X_{n_c}^{(2)} = E, \cdots, X_{n_c}^{(r)} = E\right\}. \end{aligned} \tag{7}$$

Equation (7) accrues from the assumption that the packets transmitted in the same hop interval are, statistically speaking, exposed to identical conditions.

We now introduce vector notations where the vectors are r-dimensional, if not specified otherwise. We let $\mathbf{X}_j = (X_j^{(1)}, X_j^{(2)}, \cdots, X_j^{(r)})$ and define the initial state as $\mathbf{X}_0 = \mathbf{0}$. At the end of jth hop interval, $X_j^{(i)}$ must be in the ranges given below in order for the event $\{X_{n_c}^{(i)} = E\}$ to occur

$$\begin{aligned} \max(0, E + j - n_c) \le X_j^{(i)} \le \min(E, j), &\quad \text{for NSI} \\ \max(0, E + 2(j - n_c)) \le X_j^{(i)} \le \min(E, 2j), &\quad \text{for PSI.} \end{aligned} \tag{8}$$

We let Ω_j denote the set of $\mathbf{X}_j$ necessary for the event $\{\mathbf{X}_{n_c} = \mathbf{E}\}$ to be encountered, where $\mathbf{E} = (E, E, \cdots, E)$. Then the number of elements in Ω_j is given as

$$|\Omega_j| = \eta^r \tag{9}$$

where

$$\eta = \begin{cases} \min(E, j) - \max(0, E + j - n_c) + 1, & \text{for NSI} \\ \min(E, 2j) - \max(0, E + 2(j - n_c)) + 1, & \text{for PSI.} \end{cases} \tag{10}$$

Below we define a specific operation which the vector $\mathbf{x}$ is exposed to. We let $O_{\text{NI}}(\mathbf{x})$ denote the operation that sorts the elements of $\mathbf{x}$ in nonincreasing order. If we apply this operation to every element in Ω_j and discard the resulting duplicates, we can obtain a new set $\vec{\Omega}_j$. The number of the elements in $\vec{\Omega}_j$ is equal to the number of selections with repetition of r objects chosen from the η types of objects. Thus, according to [9] we have

$$|\vec{\Omega}_j| = \binom{\eta + r - 1}{r}. \tag{11}$$

Noting that $\{\mathbf{X}_j, j = 0, 1, \cdots, n_c\}$ is a Markov chain, we can express the probability in (7) as

$$\begin{aligned} P\{\mathbf{X}_{n_c} = \mathbf{E}\} &= \sum_{\mathbf{y}\in\Omega_j} P\{\mathbf{X}_j = \mathbf{y}\} \cdot P\{\mathbf{X}_{n_c} = \mathbf{E} | \mathbf{X}_j = \mathbf{y}\} \\ &= \sum_{\mathbf{y}\in\vec{\Omega}_j} P\{\mathbf{X}_j = \mathbf{y}\} \cdot N_j(\mathbf{y}) \cdot P\{\mathbf{X}_{n_c} = \mathbf{E} | \mathbf{X}_j = \mathbf{y}\} \\ &= \sum_{\mathbf{y}\in\vec{\Omega}_j} P\{\mathbf{X}_j = \mathbf{y}\} \cdot f_j(\mathbf{y}) \end{aligned} \tag{12}$$

where the above functions are defined as

$$N_j(\mathbf{y}) = \text{number of elements in the set } \{\mathbf{x} | \mathbf{x} \in \Omega_j, O_{\mathrm{NI}}(\mathbf{x}) = \mathbf{y}\} \tag{13}$$

$$f_j(\mathbf{y}) = N_j(\mathbf{y}) \cdot P\{\mathbf{X}_{n_c} = \mathbf{E} | \mathbf{X}_j = \mathbf{y}\}. \tag{14}$$

Upon taking into account the one-step transition probability of the Markov chain, we have

$$\begin{aligned} &\text{RHS of (12)} \\ &= \sum_{\mathbf{y}\in\vec{\Omega}_j} \sum_{\mathbf{x}\in\Omega_{j-1}} P\{\mathbf{X}_{j-1} = \mathbf{x}\} \\ &\quad \cdot P\{\mathbf{X}_j = \mathbf{y} | \mathbf{X}_{j-1} = \mathbf{x}\} \cdot f_j(\mathbf{y}) \\ &= \sum_{\mathbf{x}\in\Omega_{j-1}} P\{\mathbf{X}_{j-1} = \mathbf{x}\} \cdot \left[\sum_{\mathbf{y}\in\vec{\Omega}_j} P_{\mathbf{xy}} \cdot f_j(\mathbf{y}) \right] \\ &\quad \cdot \sum_{\mathbf{x}\in\vec{\Omega}_{j-1}} P\{\mathbf{X}_{j-1} = \mathbf{x}\} \\ &\quad \cdot \left[\sum_{\mathbf{z}\in\{\mathbf{z}|O_{\mathrm{NI}}(\mathbf{z})=\mathbf{x}\}} \sum_{\mathbf{y}\in\vec{\Omega}_j} P_{\mathbf{zy}} \cdot f_j(\mathbf{y}) \right] \end{aligned} \tag{15}$$

where the one-step transition probability is denoted by

$$P_{\mathbf{xy}} = P\{\mathbf{X}_j = \mathbf{y} | \mathbf{X}_{j-1} = \mathbf{x}\}. \tag{16}$$

The one-step transition probabilities for the NSI and PSI cases are derived in the Appendix. Comparing (12) and (15), we arrive at a recursive equation for $f_j(\cdot)$

$$f_{n_c}(\mathbf{E}) = 1 \tag{17}$$

$$f_{j-1}(\mathbf{x}) = \sum_{\mathbf{y}\in\vec{\Omega}_j} \sum_{\mathbf{z}\in\{\mathbf{z}\,|\,O_{\mathrm{NI}}(\mathbf{z})=\mathbf{x}\}} P_{\mathbf{zy}} \cdot f_j(\mathbf{y}). \tag{18}$$

Finally, we obtain the desired probability of multiple correct packet reception $P\{\mathbf{X}_{n_c} = \mathbf{E}\}$ by solving the above equation recursively, since

$$P\{\mathbf{X}_{n_c} = \mathbf{E}\} = P\{\mathbf{X}_0 = \mathbf{0}\} \cdot f_0(\mathbf{0}) = f_0(\mathbf{0}). \tag{19}$$

In Fig. 1, we provided the pseudocode of the algorithm, $REQN$, proposed for solving the recursive equation. This algorithm sequentially produces $f_{n_c}(\cdot)$, $f_{n_c-1}(\cdot)$, $\cdots$, $f_0(\cdot)$. Noting that the set involved in the algorithm is now shifted from Ω_j to $\vec{\Omega}_j$ and comparing the size of the two sets, we can see that the computational complexity of the algorithm is substantially reduced. In order to quantify the reduction in computational complexity achieved by our proposed method

```
procedure REQN(r, n_c, k_c, P_xy)
// This algorithm finds f_0(0) recursively
   from the initial condition, f_{n_c}(E) = 1. //
f_{n_c}(E) ← 1
for j ← n_c to 1 do
    for each x ∈ Ω⃗_{j−1} do
        f_{j−1}(x) ← 0
    repeat
    for each y ∈ Ω⃗_j do
        for each z ∈ {z | z ∈ Ω_{j−1}, P_zy > 0} do
            x ← O_NI(z)
            f_{j−1}(x) ← f_{j−1}(x) + P_zy · f_j(y)
        repeat
    repeat
repeat
return f_0(0)
end REQN
```

Fig. 1. Algorithm $REQN$ for solving the recursive equation.

in comparison to the technique advocated in [2], we compare the number of nested iterations involved in both methods of evaluating the probability of multiple correct packet reception. From (6) and (7) and Fig. 1, the number of iterations involved in our method, N_{prop}, can be shown to be

$$N_{\mathrm{prop}} = \sum_{r=1}^{k} \sum_{j=1}^{n_c} \binom{\eta + r - 1}{r} \cdot M^r \tag{20}$$

where η is defined in (10) and $M = 2$ and 3 for the NSI and the PSI scenarios, respectively. On the other hand, from [2, eqs. (3) and (17)], we can see that the number of iterations involved in the previous proposed method, namely N_{prev}, is given by

$$N_{\mathrm{prev}} = \binom{M^k + n_c - 1}{n_c}. \tag{21}$$

In Fig. 2, we compare the number of iterations involved in both methods with increasing number of users k for both the NSI and PSI scenarios using (32,16) RS coding. Note that the parameters L, q, and P_N are irrelevant to the computational complexity. From the curves, we can see that the reduction of computational complexity, when using our proposed method, becomes very substantial as the number of users increases.

IV. Numerical Results

From our simulations, we found that approximately 1×10^{10} iterations can be carried out at the time of writing within the CPU time of 24 h using state-of-the-art Pentium personal computers. Hence, in the following numerical analysis, we set the computational limit of 24 h on the CPU time. With this limitation imposed, the numerical evaluation of the correct packet reception probability is confined to $k \leq 10$ users for the NSI case and to $k \leq 6$ for the PSI case, as evidenced by

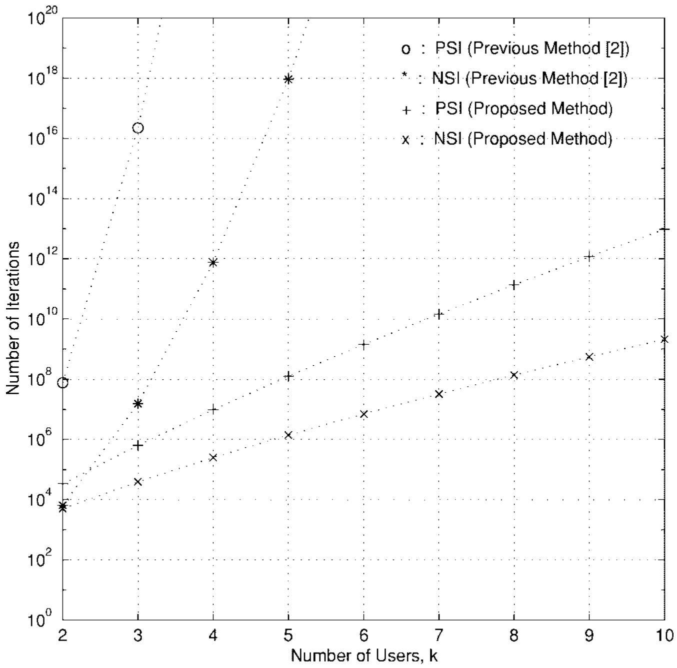

Fig. 2. Comparison of the computational complexity between our proposed method and the previous method [2] in terms of the number of nested iterations with increasing number of users k for both NSI and PSI cases with (32,16) RS coding.

Fig. 2. In this context, we note that the numerical evaluations in [2] were limited to $k \leq 3$ users for the NSI case and to $k \leq 2$ for the PSI case for reasons which become explicit in Fig. 2, and that the numerical results according to [2], as expected, coincided with our results.

In Figs. 3 and 4 we present numerical results of our analysis for the NSI and the PSI scenarios, respectively. Upon varying the available number of frequency slots q, the curves in Figs. 3 and 4 represent situations where the network load is light ($q = 80$ for the NSI case and $q = 30$ for the PSI case), medium ($q = 29$ for the NSI case and $q = 13$ for the PSI case), and heavy ($q = 14$ for the NSI case and $q = 7$ for the PSI case). Due to numerical precision limitations and due to the resultant buildup of round-off errors, numerical results below 1×10^{-12} became less reliable and thus they were dropped from the figures. The dotted curves in the figures show the results obtained according to the independent receiver operation assumption (IROA) [2], [3]. We can see that the relative error expressed in percent tends to increase as the probability reduces.

As a measure of estimating the accuracy of the IROA approximation, we introduced the expected absolute error (EAE), defined as

$$\begin{aligned} \text{EAE} &= \sum \left[\frac{|P(\text{IROA}) - P(\text{Exact})|}{P(\text{Exact})} \right] \cdot P(\text{Exact}) \\ &= \sum |P(\text{IROA}) - P(\text{Exact})|. \end{aligned} \tag{22}$$

The EAE values expressed in terms of percent for the results shown in Fig. 3 were 0.70, 18.21, and 0.12 for the cases of $q = 80, 29$, and 14, respectively, and those for Fig. 4 were 0.011, 4.1, and 0.0055 for the cases of $q = 30, 13$, and 7, respectively. Judging from the EAE values, we can see that the IROA approximation gives reasonable results under most network load conditions, although it becomes less accurate under the medium load condition.

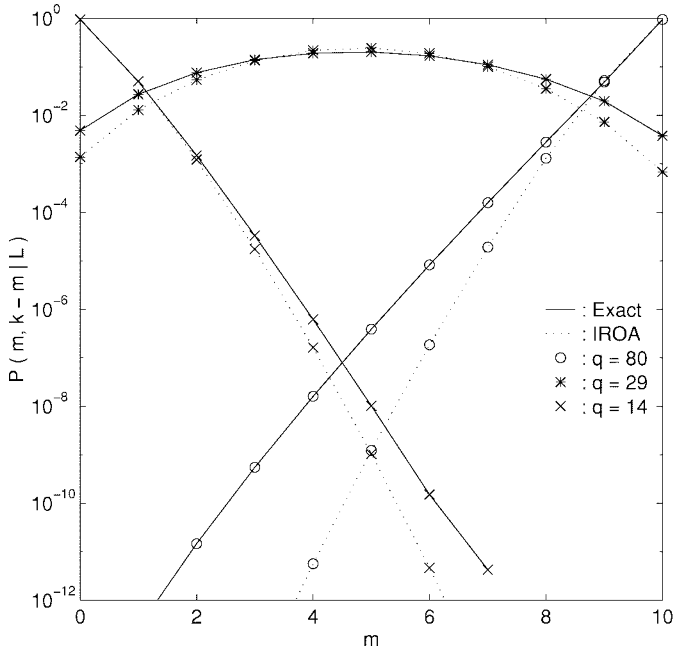

Fig. 3. The probability $P(m, k - m \mid L)$ of correctly receiving m out of k arbitrarily selected packets when transmitting L simultaneous packets for the NSI case with $L = 10$, $k = 10$, for $q = 80, 29$, and 14 frequency slots and for a modem symbol error rate of $P_N = 1 \times 10^{-3}$, and $(32, 16)$RS coding.

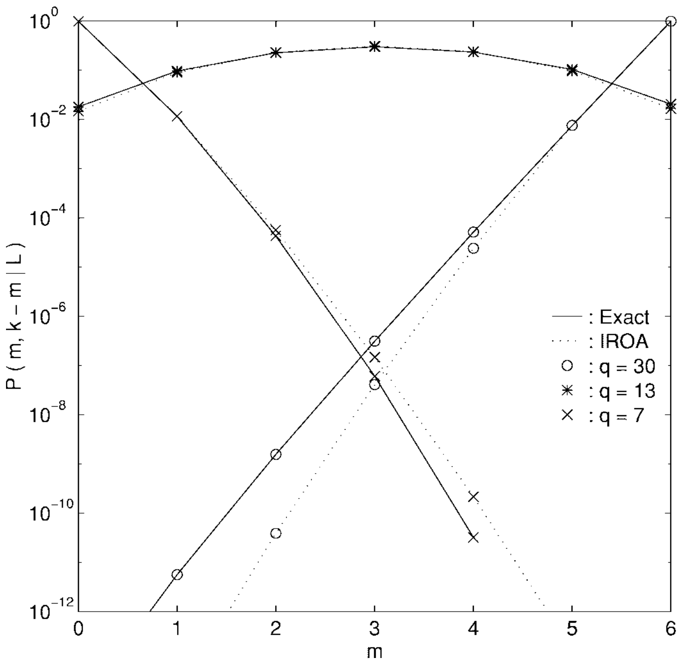

Fig. 4. The probability $P(m, k - m|L)$ of correctly receiving m out of k arbitrarily selected packets, when transmitting L simultaneous packets for the PSI case with $L = 10$, $k = 6$, for $q = 30, 13$, and 7 frequency slots and for a modem symbol error rate of $P_N = 1 \times 10^{-3}$, and $(32, 16)$RS coding.

V. Conclusions

In this paper, we presented a computationally efficient method of evaluating the probability of multiple correct packet receptions in coded synchronous FHSS networks. We used the inclusion and exclusion formula of combinatorics in order to determine the probability systematically, and we could greatly reduce the computational complexity involved in solving the resultant recursive equation upon exploiting that the packets transmitted in the same hop interval are, statistically speaking, exposed to identical conditions. Nevertheless, the presented method becomes computationally too intensive as the number of users in the network increases beyond single figures. The approximation method with IROA, which has been frequently used in the literature, was shown to produce reasonable results under various network load conditions compared to the exact results derived from our proposed method. The expected value of the absolute error was in the range of 0.0055%–18.21% in the investigated scenarios. A future challenge which remains to be solved is applying our proposed technique also to the asynchronous hopping scenario.

APPENDIX

In this appendix we derive the one-step transition probabilities $P_{\mathbf{xy}}$ for the NSI and PSI cases. We first find the probability distribution of the number of symbol errors, erasures, and correct symbols among the k symbols in a hop interval, given that L packets are transmitted simultaneously in the network. Based on this probability distribution, we derive the one-step transition probabilities. As before, all vectors are r-dimensional, unless specified otherwise.

A. NSI Case

We denote the number of symbol errors and correct symbols among the k symbols in a hop interval by N_E and N_C, respectively. We let C_i be the event that the symbol contained in the packet transmitted by ith user in a hop interval is received correctly. By applying the inclusion and exclusion formula, we have

$$P\{N_E = e, N_C = k-e|L\} = \sum_{i=0}^{e} (-1)^i \binom{k-e+i}{k-e} S_{k-e+i} \tag{23}$$

where

$$\begin{aligned} S_r &= \sum_{i_1<i_2<\cdots<i_r} P\{C_{i_1} C_{i_2} \cdots C_{i_r}\} \\ &= \binom{k}{r} P\{C_1 C_2 \cdots C_r\} \\ &= \binom{k}{r}\left(\frac{1}{q^r}\right)\left[\binom{q}{r} r!(q-r)^{L-r}\right](1-P_N)^r. \end{aligned} \tag{24}$$

Now we consider the one-step transition probability. Noting the definition of $X_j^{(i)}$ for NSI, we can see that $X_j^{(i)}$ can increase at most by one. Letting $\mathbf{x} = (x_1, x_2, \cdots, x_r)$ and $\mathbf{y} = (y_1, y_2, \cdots, y_r)$, the one-step transition probability is given by

$$\begin{aligned} P_{\mathbf{xy}} &= P\{\mathbf{X}_j = \mathbf{y}|\mathbf{X}_{j-1} = \mathbf{x}\} \\ &= \sum_{i=\alpha}^{k-\beta} \left[\frac{\binom{\gamma+k-r}{i-\alpha}}{\binom{k}{i}} \right] \\ &\quad \cdot P\{N_E = i, N_C = k-i \,|\, L\} \end{aligned} \tag{25}$$

TABLE I
EVENT DESCRIPTION FOR THE NSI CASE

Event	Description
$\mathcal{A}$	$y_i = x_i + 1$
$\mathcal{B}$	$y_i = x_i,\ y_i < E$
$\mathcal{C}$	$y_i = x_i,\ y_i = E$

TABLE II
EVENT DESCRIPTION FOR THE PSI CASE

Event	Description
$\mathcal{A}$	$y_i = x_i + 2$
$\mathcal{B}$	$y_i = x_i + 1,\ y_i < E$
$\mathcal{C}$	$y_i = x_i,\ y_i < E$
$\mathcal{D}$	$y_i = x_i + 1,\ y_i = E$
$\mathcal{E}$	$y_i = x_i,\ y_i = E$

where α, β, and γ ($\alpha+\beta+\gamma = k$) represent the number of events $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$, respectively, defined in Table I.

B. PSI Case

When we consider the PSI case, we have to include the occurrences of symbol erasures. We denote the number of symbol errors, erasures, and correct symbols among the k symbols in a hop interval by N_E, N_S, and N_C, respectively, and in addition by N_H and $N_{\overline{H}}$ the number of symbols, which are hit and hit-free, respectively. We let $\overline{H}_i$ be the event that the symbol contained in the packet transmitted by the ith user in a hop interval is not hit. By applying the inclusion and exclusion formula [8], [9], we have

$$\begin{aligned} P\{N_E &= e, N_S = s, N_C = k-e-s|L\} \\ &= P\{N_H = s, N_{\overline{H}} = k-s \,|\, L\} \\ &\quad \cdot P\{N_E = e, N_C = k-s-e|L, N_{\overline{H}} = k-s\} \\ &= \sum_{i=0}^{s} (-1)^i \binom{k-s+i}{k-s} S_{k-s+i} \\ &\quad \cdot \binom{k-s}{e} P_N^e (1-P_N)^{k-s-e} \end{aligned} \tag{26}$$

where

$$\begin{aligned} S_r &= \sum_{i_1<i_2<\cdots<i_r} P\{\overline{H}_{i_1}\overline{H}_{i_2}\cdots\overline{H}_{i_r}\} \\ &= \binom{k}{r} P\{\overline{H}_1\overline{H}_2\cdots\overline{H}_r\} \\ &= \binom{k}{r}\left(\frac{1}{q^r}\right)\left[\binom{q}{r} r!(q-r)^{L-r}\right]. \end{aligned} \tag{27}$$

The procedure to derive the one-step transition probability for the PSI case is similar to that for the NSI case. Noting the definition of $X_j^{(i)}$ for the PSI case, we can see that $X_j^{(i)}$ can increase at most by two. Letting $\mathbf{x} = (x_1, x_2, \cdots, x_r)$

and $\mathbf{y} = (y_1, y_2, \cdots, y_r)$, the one-step transition probability is given by

$$P_{\mathbf{xy}} = P\{\mathbf{X}_j = \mathbf{y} | \mathbf{X}_{j-1} = \mathbf{x}\} = \sum_{i=\gamma}^{k-\alpha-\beta-\delta} \sum_{j=\alpha}^{k-i-\beta} \left[\frac{\binom{\epsilon+k-r}{i-\gamma}\binom{\delta+\epsilon+k-r-i+\gamma}{j-\alpha}}{\binom{k}{i}\binom{k-i}{j}} \right] \cdot P\{N_E = j, N_S = k-i-j, N_C = i | L\} \tag{28}$$

where α, β, γ, δ, and ϵ $(\alpha+\beta+\gamma+\delta+\epsilon = k)$ represent the number of events $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, and $\mathcal{E}$, respectively, defined in Table II.

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