PERFORMANCE UPPER BOUND OF ADAPTIVE QAM IN SLOW RAYLEIGH FADING ENVIRONMENTS

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ABSTRACT

The performance of adaptive modulation schemes that employ either no transmission, BPSK, QPSK, 16 Square QAM and 64 Square QAM is characterised for transmissions over Rayleigh channels. The optimum value for the switching levels between the fixed modulation schemes is found. This is achieved by optimising the BER and throughput for wireless computer data and speech systems. Using the optimised switching thresholds, the system performance is also evaluated over Rician channels.

1. INTRODUCTION

Following Shannon's fundamental work and Lee's seminal contribution on the channel capacity of fading channels [1] in this treatise we propose a scheme allowing us to approach the predicted channel capacity potential of fading wireless channels. The rapid fluctuations in received power result from multi-path propagation in a mobile radio environment. In the past systems operated at increased average transmit powers in order to account for these fluctuations and hence to obtain the desired Bit Error Rate (BER). However, in a mobile radio environment increasing the average transmit power has undesirable consequences in terms of co- and adjacent-channel interference and power consumption. A more attractive alternative to mitigate this fast fading is to employ a more robust modulation scheme, when a low instantaneous SNR is expected at the receiver. Moreover, when the instantaneous SNR increases again, a less rugged scheme exhibiting a higher throughput should be employed. This was initiated by Steele and Webb and presented in a keynote paper [2], which discussed adaptive differential modulation and considered the effect of SNR and co-channel interference. Morinaga [3] considers the employment of such adaptive modulation in terms of its future within mobile multimedia apparatus.

In order for a transmitting radio to select the appropriate modulation scheme there must be some information available to it upon the instantaneous quality of the channel. This can be achieved by using Time Division Duplex (TDD), which provides a convenient framework for exploiting the correlation between the up- and down-link complex envelope of the channel, given that the normalised Doppler frequency is sufficiently low. Therefore, adaptive modulation, within a TDD scenario relies upon the correlation

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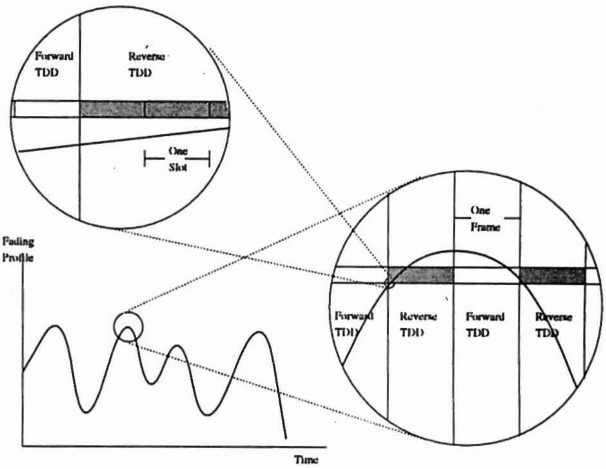

Figure 1: Received power fluctuation over the duration of one TDD slot, when compared with the deviation over the whole channel results in a single slot exhibiting near-Gaussian channel characteristics. The correlation between corresponding slots in adjacent TDD frames may be exploited.

between received and transmitted frames to estimate s, the instantaneous power that will be received for a given set of symbols. Figure 1 shows this principle, and demonstrates that it is reasonable to assume that the channel does not fluctuate rapidly over the duration of a received slot. The estimation of s is compared against a set of n switching levels, l_n, and the appropriate modulation scheme is selected accordingly. Following Kamio et al [4] Binary Phase Shift Keying (BPSK), Quaternary Phase Shift Keying (QPSK) and multilevel Quadrature Amplitude Modulation (QAM) can be invoked on the basis of the following thresholding operations:

$$\mathrm{MS} = \begin{cases} \text{No Transmission} & \text{if } l_1 < s \\ \text{BPSK} & \text{if } l_1 \le s < l_2 \\ \text{QPSK} & \text{if } l_2 \le s < l_3 \\ \text{Square 16 Point QAM} & \text{if } l_3 \le s < l_4 \\ \text{Square 64 Point QAM} & \text{if } s \le l_4 \end{cases} \quad (1)$$

where the threshold values of l_n will be discussed later.

The thresholds l_1, l_2, l_3 and l_4 predetermine the performance of an adaptive modulation scheme under given channel conditions. The lower the threshold values, the higher the throughput, that is, on average more bits will be transmitted using a higher number of Bits Per Symbol (BPS). Conversely, the higher the values of l_1, l_2, l_3 and l_4, the lower the BER of the overall adaptive modulation scheme for given channel conditions. Clearly, BER can be traded with BPS performance and vice versa. The optimum trade-off will depend upon the type of information being transmitted and the source and channel coders used.

The trade-off is dependent upon the type of information that is to be transmitted, in order to satisfy the different network characteristics required by video, voice and computer data. Generally, interactive video and voice information cannot sustain as much latency across the link as computer data information. However, computer data information is less robust to channel errors. The BER performance of the adaptive modulation identifies how robust the information transmitted over the link needs to be. The latency introduced depends upon whether the information is buffered at the transmitter, when the instantaneous channel SNR is low. As the instantaneous channel SNR increases, the buffer may be emptied but latency has been incurred. The latency is therefore dependent upon the average BPS performance of the system.

The source coder and adaptive modulation scheme should be selected for optimum compatibility. Correct selection of the source coder [5] can allow improved quality of transmission in poor channel conditions by reducing the bit rate and consequently the data experiences a lower BER. Moreover, in good channel conditions, the overall source representation quality can be improved by increasing the bit rate, although the associated BER will typically also be increased.

2. PERFORMANCE OPTIMISATION

The upper-bound performance of the underlying adaptive modem scheme was characterised in Reference [6]. The BER performance of coherent modulation schemes with 1, 2, 4 and 6 Bits Per Symbol (BPS) assuming perfect clock and carrier recovery, in a Gaussian channel are known [7]. The corresponding expressions are given below:

$$P_b(\gamma) = Q\left(\sqrt{2\gamma}\right), \tag{2}$$

$$P_q(\gamma) = Q\left(\sqrt{\gamma}\right), \tag{3}$$

$$P_{16}(\gamma) = \frac{1}{4}\left[Q\left(\sqrt{\frac{\gamma}{5}}\right) + \left(3Q\sqrt{\frac{\gamma}{5}}\right)\right] + \frac{1}{2}Q\left(\sqrt{\frac{\gamma}{5}}\right), \tag{4}$$

$$\begin{aligned} P_{64}(\gamma) &= \frac{1}{12}\left[\begin{array}{c} Q\left(\sqrt{\frac{\gamma}{21}}\right) + Q\left(3\cdot\sqrt{\frac{\gamma}{21}}\right) \\ +Q\left(5\cdot\sqrt{\frac{\gamma}{21}}\right) + Q\left(7\cdot\sqrt{\frac{\gamma}{21}}\right) \end{array}\right] \\ &+ \frac{1}{6}Q\left(\sqrt{\frac{\gamma}{21}}\right) + \frac{1}{6}Q\left(3\cdot\sqrt{\frac{\gamma}{21}}\right) \\ &+ \frac{1}{12}Q\left(5\cdot\sqrt{\frac{\gamma}{21}}\right) + \frac{1}{12}Q\left(7\cdot\sqrt{\frac{\gamma}{21}}\right) \\ &+ \frac{1}{3}Q\left(\sqrt{\frac{\gamma}{21}}\right) + \frac{1}{4}Q\left(3\cdot\sqrt{\frac{\gamma}{21}}\right) \\ &- \frac{1}{4}Q\left(5\cdot\sqrt{\frac{\gamma}{21}}\right) - \frac{1}{6}Q\left(7\cdot\sqrt{\frac{\gamma}{21}}\right) \\ &+ \frac{1}{6}Q\left(9\cdot\sqrt{\frac{\gamma}{21}}\right) + \frac{1}{12}Q\left(11\cdot\sqrt{\frac{\gamma}{21}}\right) \\ &- \frac{1}{12}Q\left(13\cdot\sqrt{\frac{\gamma}{21}}\right), \end{aligned} \tag{5}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-x^2/2}dx$, γ is the signal-to-noise ratio (SNR), while $P_b(\gamma)$, $P_q(\gamma)$, $P_{16}(\gamma)$ and $P_{64}(\gamma)$, are the mean BERs of BPSK, QPSK, square 16 point square QAM, and square 64 point QAM, respectively. The PDF of the fluctuations in instantaneous received power, s, over a Rayleigh channel are given by:

$$F(s,S) = \frac{2s^{\frac{1}{2}}}{S}\cdot e^{-s/S}, \tag{6}$$

where S is the average signal power. Assuming a sufficiently low normalised Doppler frequency in order to maintain a near-constant fading envelope and hence Gaussian conditions for the duration of a modulation symbol and employing Pilot Symbol Assisted Modulation (PSAM) [8], upper bound BER performances can be obtained for the above four modulation schemes over a Rayleigh channel. For any of the modulation schemes, if $X_g(\gamma)$ is the Gaussian BER performance, as given in Equations (2), (3), (4) or (5), then $X_r(S/N)$ given below will be the upper bound for the BER performance in a Rayleigh channel:

$$X_r(S/N) = \int_0^\infty X_g(s/N)\cdot F(s,S)\,ds \tag{7}$$

Therefore, the narrow-band upper bound BER performance of an adaptive modulation scheme similar to that described in [4] may be computed from:

$$P_a(S/N) = B^{-1}\left[\begin{array}{ll} 1\int_{l_1}^{l_2} P_b(s/N)\cdot F(s,S) & ds \\ +2\int_{l_2}^{l_3} P_q(s/N)\cdot F(s,S) & ds \\ +4\int_{l_3}^{l_4} P_{16}(s/N)\cdot F(s,S) & ds \\ +6\int_{l_4}^{\infty} P_{64}(s/N)\cdot F(s,S) & ds \end{array}\right] \tag{8}$$

where l_1, l_2, l_3, l_4 and B are the thresholds between transmission off, BPSK, QPSK, square 16 point and square 64 point QAM, while B is the mean number of BPS, given by:

$$\begin{aligned} B &= 1\cdot\int_{l_1}^{l_2} F(s,S)\,ds + 2\cdot\int_{l_2}^{l_3} F(s,S)\,ds \\ &+ 4\cdot\int_{l_3}^{l_4} F(s,S)\,ds + 6\cdot\int_{l_4}^{\infty} F(s,S)\,ds \end{aligned} \tag{9}$$

Adaptive modulation schemes may select the appropriate modulation level for the transmitted frame on the basis of either the number of errors encountered in the received frame or the received signal strength [7]. The former is only possible in a system that includes some error detection. Here, the received signal strength relative to the switching values of l_1, l_2, l_3 and l_4 is used to select the appropriate modulation level for each frame. The difference between this upper bound performance and a certain desired performance can be used as a cost function for given modulation switching levels of l_1, l_2, l_3 and l_4. Minimisation of this cost function may be achieved iteratively by varying the switching levels [9] l_1, l_2, l_3 and l_4 using Powell's optimisation algorithm [10].

The BER and BPS performances were evaluated for average channel SNRs in the range of 0 dB to 50 dB in 1dB intervals. The optimisation cost function was defined as:

$$\text{Total Cost} = \sum_{i=0}^{50} \text{BER Cost}(i) + \text{BPS Cost}(i) \tag{10}$$

where

$$\text{BER Cost}(i) = \begin{cases} 10log_{10}\left(\frac{\text{BER}_m(i)}{\text{BER}_d(i)}\right) & \text{if BER}_m(i) > \text{BER}_d(i) \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

$$\text{BPS Cost}(i) = \begin{cases} \text{BPS}_d(i) - \text{BPS}_m(i) & \text{if BPS}_d(i) > \text{BPS}_m(i) \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

and $\text{BER}_m(i)$, $\text{BER}_d(i)$, $\text{BPS}_m(i)$ and $\text{BPS}_d(i)$ are respectively the measured and desired BER and BPS at an average channel SNR of i. It can be seen from Equations 10, 11 and 12 that the cost function can only be positive and increases when either the BER or the BPS performance become inferior to their desired performance at an average channel SNR of i. The cost function cannot be negative, therefore at high average channel SNRs, where both the BER and the BPS outperform their respective desired performance targets, the combined performance will be sub-optimum. The advantage of this approach is that it reduces the minimum average channel SNR above which both the desired BER and BPS performance criteria is achieved. However, the performance does improve at higher average SNRs because of the favourable prevailing channel conditions. Equation 11 utilises the logarithm function to increase the significance of small BERs. A weighting factor of 10 is employed in order to bias the optimisation towards achieving the desired BER performance in preference to the BPS performance.

3. RESULTS AND DISCUSSION

Two desired system performance profiles were considered. One of them was optimised for a speech codec with a target BER and BPS of 0.01 and 4.5 respectively. The other performance profile was intended for computer data transfer with target BER and BPS performances of 0.0001 and 3 respectively. Although the important speech interactivity aspects of buffer size, delay and latency issues were left for future study, it is convenient to refer to these systems as the speech and data schemes, which are here synonymous to the higher and lower BER systems. Naturally, the portrayed techniques can be invoked in the context of arbitrary desired BERs. The optimisation was performed for Rayleigh channel conditions and the initial condition for both minimisations was l_1 = 5dB, l_2 = 8dB, l_3 = 14dB and l_4 = 20dB. After optimisation the values of l_1 = 3.31, l_2 = 6.48, l_3 = 11.61 and l_4 = 17.64 were registered for the speech system. For the computer data system the values of l_1 = 7.98, l_2 = 10.42, l_3 = 16.76 and l_4 = 26.33 were recorded.

Considering Figure 2, the desired BER is achieved between 0 and 50 dBs for both the speech and computer data switching levels schemes. The targeted BPS performance is achieved at about 18 dB and 19 dB average channel SNRs for the speech and computer data schemes, respectively. Observe in the Figure that both the speech and data BER profiles outperform the BER requirements for average channel SNRs greater than these values. The system was capable of maintaining the target BER performances at extremely low average SNR values. This robust performance was achieved

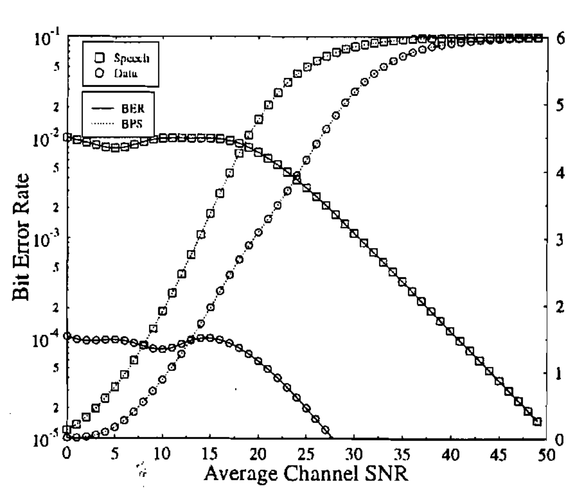

Figure 2: Upper bound BER and BPS performance of adaptive QAM in Rayleigh Channel optimised separately for Speech and Data transfer

at the cost of reducing the BPS channel capacity below that of BPSK, which was possible due to disabling transmissions for low instantaneous SNR values. This can be further studied by considering Figures 3-6. Figures 3 and 5 show the distribution of employment of each of the fixed modulation schemes, as the average channel SNR varies, for the computer data optimised and voice systems, respectively. For both optimised schemes the relative use of no transmission and the lower order modulation schemes reduces, as the average channel SNR increases. For the speech-optimised system the maximum probability for BPSK, QPSK and Square 16 QAM is lower than the corresponding values for the computer data optimised system. The explanation for this is that the switching levels in the speech system are further apart.

Figures 4 and 6 show the BER performance of the individual fixed modulation schemes at a range of average channel SNRs for both the computer data and speech optimised systems. It is interesting to note for both optimisation scenarios that the particular fixed scheme that has the highest BER varies with the average channel SNR. Furthermore, the most rugged schemes do not necessarily have the lowest BER. Both these phenomena are as a result of the shape of the fading Probability Distribution Function (PDF) and because the rugged schemes are only employed when the channel is poor. To illustrate this, consider the BER performance of QPSK in Figure 6, which (within the range of average SNRs plotted) starts at a maximum of approximately 2.8×10^{-4} and drops smoothly to an asymptotic value of approximately 2.2×10^{-5}. The BER is extremely close to the asymptote at 20 dB average channel SNR, so when the average channel SNR is below this value, the instantaneous SNRs at the lower range of the QPSK window ($s \rightarrow l_2$) are significantly more likely than instantaneous SNRs at the higher range of the QPSK window ($s \rightarrow l_3$). Confirmation of this can be obtained from considering Figure 5, where it can be seen that the occurance of a QPSK symbol is fairly unlikely, when the average channel SNR reaches 20 dB.

In order to ensure that the optimisation for Rayleigh

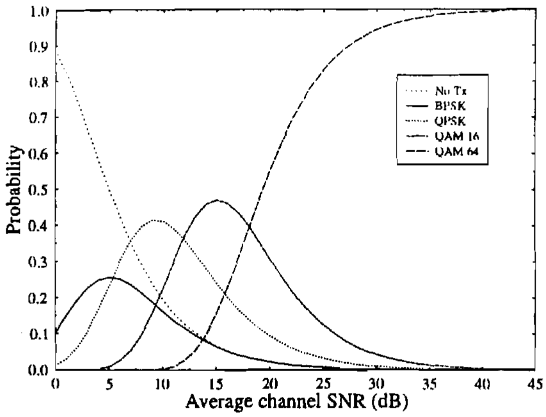

Figure 3: Upper bound probability of the individual modulation schemes for adaptive modulation optimised separately for speech plotted against average channel SNR for a Rayleigh channel

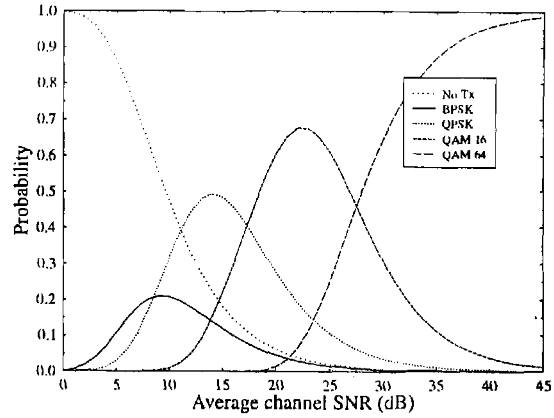

Figure 5: Upper bound probability of the individual modulation schemes for adaptive modulation optimised separately for data transfer plotted against average channel SNR for a Rayleigh channel

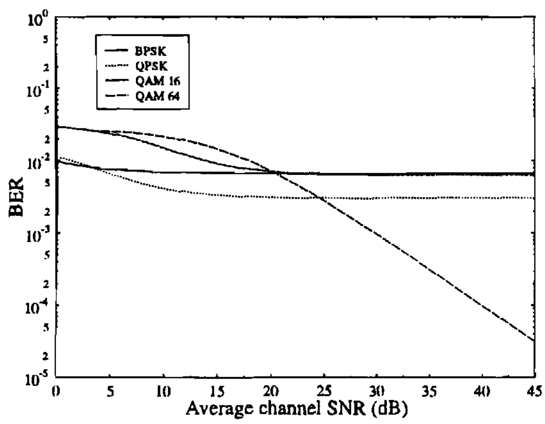

Figure 4: Upper bound BER of the individual modulation schemes for adaptive modulation optimised separately for speech plotted against average channel SNR for a Rayleigh channel

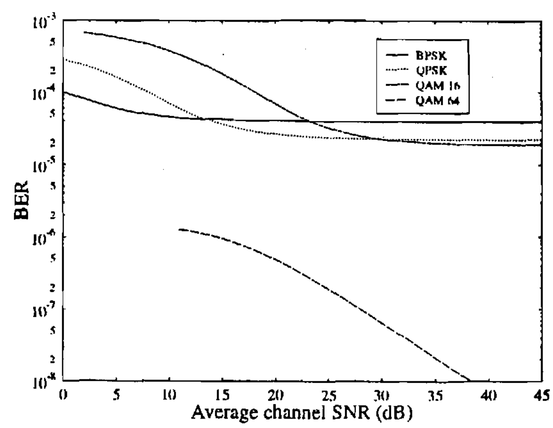

Figure 6: Upper bound BER of the individual modulation schemes for adaptive modulation optimised separately for data transfer plotted against average channel SNR for a Rayleigh channel

channel conditions did not yield switching values that were too specific, the same switching values were evaluated through Rician channels with K factors of 4 and 16. Here the K factor is defined as the ratio of received power in the dominant path to the power in the other paths and the results are given in Figure 7. The performance through Rician channels was quite similar to that for Rayleigh channels for average channel SNRs below about 20 dB. However, as the K factor increased, both the BER and BPS performance was more undulating. This was because the variance of the fading amplitude PDF reduces as the K factor is increased. Consequently, for larger values of K there becomes more of a discrete switch from one modulation scheme to another, as the average channel SNR increases. When using the Rayleigh-optimised switching levels over Rician channels exhibiting K factors of both 4 and 16, the BER never exceeded twice the desired BER and was often below it in the average channel SNR range from 0 to 20 dB. Above average SNRs of 20dB the BER performance over Rician channels improved dramatically, as the K factor was increased. This was the result of a significant reduction in the probability of deep fades.

4. CONCLUSIONS

The optimisation of switching levels within adaptive modulation schemes was discussed and two schemes optimised for computer data transfer and speech were presented. The optimisation was performed with the assumption of Rayleigh channel conditions and it was shown that the same levels are suitable for other Rician channels.

5. ACKNOWLEDGEMENT

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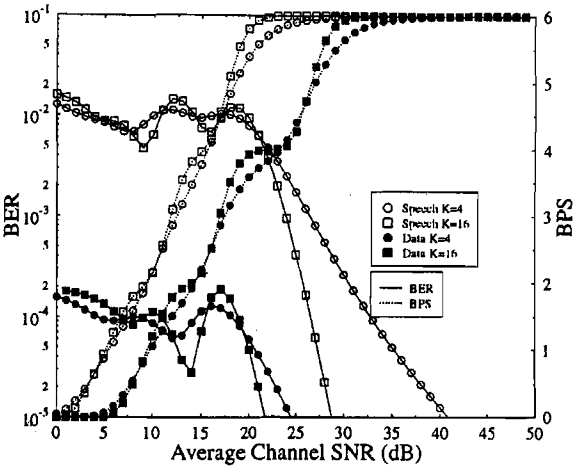

Figure 7: Upper bound BER and BPS performance of adaptive QAM in Rician Channels having been optimised separately for Speech and Data transfer and found by numerical solution

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