EFFICIENT SUBBAND ADAPTIVE FILTERING WITH OVERSAMPLED GDFT FILTER BANKS

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Abstract. This paper addresses the numerical efficiency of adaptive filtering implemented in subbands. Our approach first focuses on oversampled GDFT filter banks and their potential benefits over other possible subband decompositions. Although the subband filters presented use complex arithmetic, the discussed method allows factorization into a real valued polyphase network, followed by a complex GDFT modulation, which can be mostly implemented via an FFT. Secondly we discuss the advantages and potential savings that can be gained by processing complex subband signals, with particular reference to adaptive system identification problems, for which we give demonstrations of the potential benefits of our GDFT approach compared to adaptive identification in both fullband and critically sampled DCT-IV based pseudo-QMF subbands.

1. INTRODUCTION

Adaptive identification of long impulse responses, as required for acoustic echo cancellation, is unlikely to be implemented as a fullband FIR system due to computational limitations [4]. Strategies to lower the computational complexity of adaptive DSP algorithms include the application of decimated subband structures [7, 3, 13], where both input and desired signal are split into a number of frequency bands, as depicted in Fig. 1. The reduced spectra allow a downsampling of the subband signals, which enables computational savings when applying adaptive filtering to the subbands. Via a synthesis bank operation, a fullband error signal can then subsequently be reconstructed.

The downsampling operation in filter banks causes aliasing, which usually is suppressed in the synthesis stage by careful design [14]. For processing that involves correlating between different subband signals, like adaptive filtering, aliasing is a disturbing non-linear distortion, which decreases performance. There exist several subband approaches to overcome this problem. Critically sampled systems require the use of cross-terms

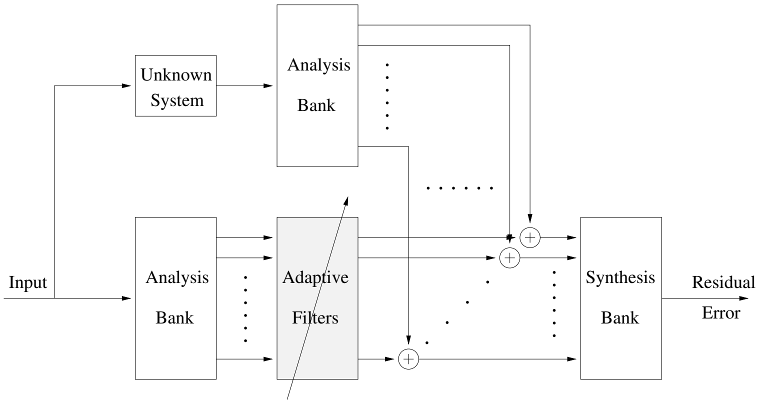

Fig. 1: Subband adaptive filter architecture to identify an unknown system.

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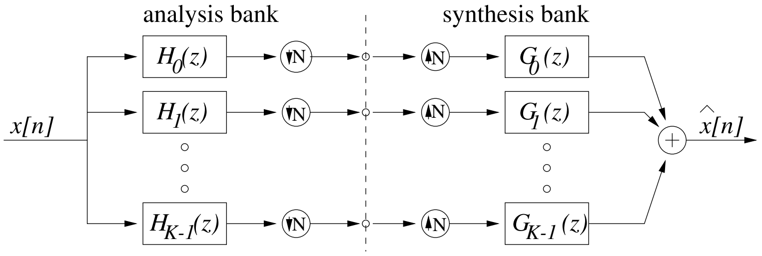


Fig. 2: Analysis and synthesis branch of a K-channel filter bank with subbands decimated by N.

between adjacent bands [3] to cover for lost information, or gap filter banks [17] which are non-perfectly reconstructing by definition. For real-valued bandpass signals, non-critical subsampling may violate the sampling theorem and requires single sideband (SSB) modulation into the baseband prior to decimation [1] or the use of non-uniform filter banks with unequal decimation ratios [5], which require the handling of different sampling rates. On the contrary, uniform complex valued subbands are unproblematic to decimate [1, 2] and can usually be implemented at least as efficiently as real valued subband adaptive filter systems, as we will show in the sequel.

Here, we discuss a complex valued filter bank which is derived from a real prototype lowpass filter by generalized DFT (GDFT) modulation [1], such that the frequency range $\Omega = [0, \pi]$ is covered by $K/2$ subbands, which can be decimated by a factor $N \leq K$. In Sec. 2, we introduce the modulation procedure, specify design criteria, and propose an efficient polyphase structure for this filter bank. Polyphase structures are often believed to be restricted to critical decimation and integer oversampling ratios [1, 10]. Our formulation uses a factorization similar to [2] and allows any integer decimation $N \leq K$, thus including non-integer oversampling ratios. Based on our polyphase filter bank realization, we derive some results for the complexity of subband filtering in Sec. 3, depending on the order of the algorithm. Finally, Sec. 4 compares the GDFT method to a critically decimated subband adaptive filter approach [3] for the normalized least mean squares (NLMS) algorithm.

2. OVERSAMPLED GDFT FILTER BANK

2.1. GDFT Filter Banks

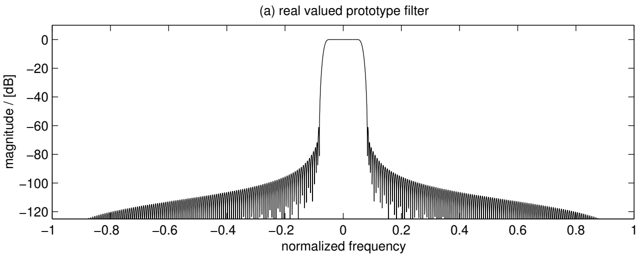
A general structure of a filter bank is shown in Fig. 2. The analysis bank decomposes a signal $x[n]$ into K subbands, each produced by a branch $H_k(z)$ of the analysis bank. After decimation and expansion by a factor N, the fullband signal is reconstructed from the subbands in the synthesis bank by filtering with filters $G_k(z)$ followed by summation. The analysis filters $h_k[n]$ are derived from a real valued lowpass prototype FIR filter $p[n]$ of even length L_p by a generalized discrete Fourier transform (GDFT),

$$h_k[n] = t_{k,n} \cdot p[n] \quad , \qquad t_{k,n} = e^{j\frac{2\pi}{K}(k+k_0)(n+n_0)}, \qquad k, n \in \mathbb{N}. \tag{1}$$

The term generalized DFT [1] stems from offsets k_0 and n_0 introduced into the frequency and time indices, which serve two purposes. Firstly, a frequency offset $k_0 = \frac{1}{2}$ shifts the bandpass characteristics of the filters $h_k[n]$ and yields the frequency range $[0; \pi]$ to be covered by exactly $K/2$ subbands for an even K. For real input signals $x[n]$, it is sufficient to keep only these $K/2$ subbands, from which the complete spectrum can be reconstructed by a real operation Re$\{\cdot\}$. An example for the described GDFT modulation of an $\frac{1}{8}$-band lowpass filter with $k_0 = \frac{1}{2}$ in (1) and the spectral coverage can be seen in Fig. 3. This *reduced* filter bank with only $K/2$ subbands has serious implications on the computational efficiency of processing complex subbands, as will be further explained in Sec. 3. Secondly, we can ensure the linear phase property of the filters $h_k[n]$ by choosing a linear phase prototype filter $h_p[n]$ and a symmetric transform with respect to $(L_p - 1)/2$ by appropriately setting n_0.

For the synthesis filters, we employ time reversed copies of the analysis filters,

$$g_k[n] = \tilde{h}_k[n] = h_k[L_p - n + 1]. \tag{2}$$


(a) real valued prototype filter