Efficient Implementations of Complex and Real Valued Filter Banks for Comparative Subband Processing with an Application to Adaptive Filtering

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Abstract. In this paper, we discuss efficient methods to implement correlation type algorithms in subbands. Based on a polyphase representation, a modulated GDFT filter bank with arbitrary integer decimation ratio can be performed at low cost, yielding complex subband signals. Real valued subbands can be achieved by appropriate postprocessing of the complex filter bank. Together with consideration of the computational order of the algorithm applied for subband processing, complex or real implementation may be better suited, for which we present criteria. For the example of subband adaptive filtering as applied in acoustic echo cancellation, we compare a number of algorithms in terms of the most efficient implementation.

1. Introduction

Often the processing of signals is performed in frequency bands. This signal decomposition allows a lowering the sampling rate in the subbands, thus reducing the computational complexity of implemented systems, and leads to a convenient parallelization. For comparing the subbands of different signals as e.g. performed in adaptive filtering, critical decimation by a factor N (where $N = K$ the number of uniform subbands), requires either cross-terms between overlapping frequency bands [4], or the use of gap filter banks [11]. The cross-terms are necessary to compensate for information leaked into adjacent bands in the decimation stage, whereas in the case of gap filters there is a loss of spectral information which may not be acceptable. Oversampled filter banks can resolve this problem by introducing spectral redundancy. While complex subband signals can be decimated at any integer rate $N < K$, thus allowing a choice of a high decimation rate close to K while suppressing aliasing in the subband signals, for real valued subband signals oversampling is problematic and requires either non-uniform filter banks [5] or single side band (SSB) modulation into the baseband prior to decimation [2, 1, 7].

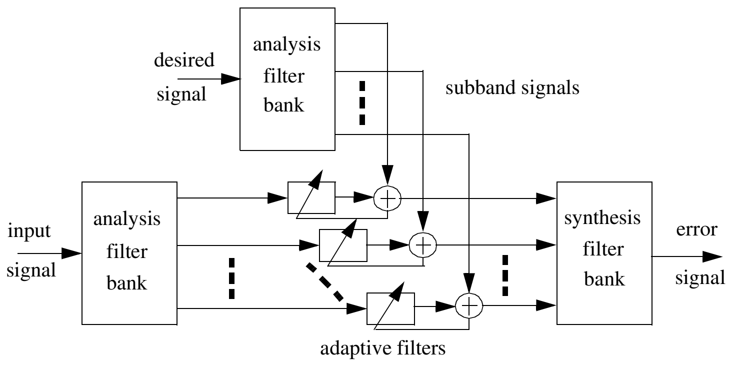

Fig. 1: Subband adaptive filter setup, where the adaptive filters try to produce output signals such, that when subtracted from the desired signals, the error signal is minimized (e.g. in a least mean square sense); for the comparison of filter outputs and desired signals, the subband signals should be alias free.

In this paper we endeavour to present an efficient polyphase implementation of a complex valued oversampled generalized DFT (GDFT) filter bank [2], extending it to an arbitrary integer decimation $N < K$. We use a polyphase factorization based on [3], which allow a filtering of the input signal by a polyphase network followed by a GDFT transformation, which can be efficiently implemented using an FFT [9]. This approach can be extended by a complex modulation and real part operation on the subband signals to yield a highly efficient SSB modulated, real valued filter bank. Although complex processing can be shown to be computationally more efficient in most cases, there exist adaptive filtering algorithms where non-complex arithmetic offers computational savings, like the affine projection algorithm [7] widely used in acoustic echo cancellation.

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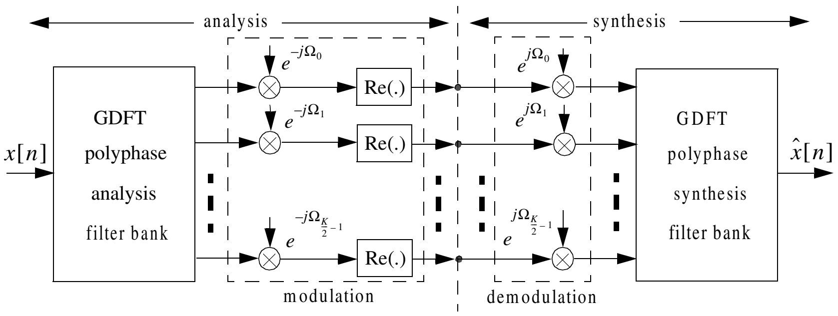


Fig. 2: GDFT analysis and synthesis filter bank with modulation (in dashed boxes) to achieve real valued subband signals.

2. Filter Bank Implementation

2.1 Complex Valued GDFT Filter Banks

Generalized DFT (GDFT, [2]) filter banks are characterized by a DFT-like modulation of a real valued prototype lowpass filter, $p[n]$,

$$h_k[n] = e^{j\frac{2\pi}{K}(k+1/2)(n-(L_p-1)/2)} \cdot p[n], \qquad k = 0(1)K,\;\; n = 0(1)L_p - 1, \tag{1}$$

where K denotes the number of complex subbands and L_p the length of $p[n]$. The offset values $k_0 = \frac{1}{2}$ and $n_0 = -(L_p - 1)/2$ are chosen such as to obtain linear phase filters $h_k[n]$ and to achieve a uniform coveraged of the normalized angular frequency interval $\Omega = [0; \pi]$ by $K/2$ analytic bandpass filters. For real input, the remaining subbands will be complex conjugate and their processing therefore redundant.
The synthesis filters $g_k[n]$ can be obtained by time reversion of the analysis filter, i.e. $g_k[n] = \tilde{h}_k[n] = h_k^*[L_p - n + 1]$. Thus, all filters can be derived from one single prototype $p[n]$, which has to be designed appropriately.

Polyphase Representation. To describe the analysis bank decimated by a factor N, a matrix $\mathbf{H}(z)$ can be created,

$$\mathbf{H}(z) = \begin{bmatrix} H_{0|0}(z) & H_{0|1}(z) & \cdots & H_{0|N-1}(z) \\ H_{1|0}(z) & H_{1|1}(z) & & H_{1|N-1}(z) \\ \vdots & & \ddots & \vdots \\ H_{K-1|0}(z) & H_{K-1|1}(z) & \cdots & H_{K-1|N-1}(z) \end{bmatrix} \qquad \text{where} \quad H_k(z) = \sum_{j=0}^{N-1} z^{-j} H_{k|j}(z^N), \tag{2}$$

holding the $H_{k|j}(z)$, $j = 0(1)N-1$ polyphase components of the kth analysis filter. With a similar polyphase decomposition of the input signal $x[n]$, $X(z) = \sum_{j=0}^{N-1} z^{-j} X_j(z^N)$

$$\underline{X}(z) = [X_0(z), X_1(z), \ldots X_{N-1}(z)]^T \tag{3}$$

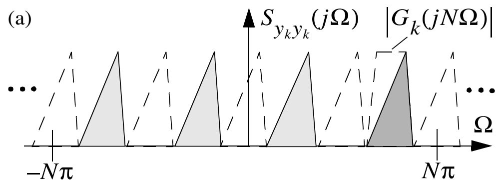
the analysis bank operation is denoted as $\underline{Y}(z) = \mathbf{H}(z) \cdot \underline{X}(z)$, where $\underline{Y}(z)$ contains the K subband signals. With careful filter design, the polyphase matrix $\mathbf{H}(z)$ is paraunitary, i.e. the input signal $x[n]$ can be perfectly reconstructed in the synthesis bank with the above choice of synthesis filter $g_k[n]$ [3].

Polyphase Factorization. Let M be the least common multiple (lcm) of the periodicity of the GDFT transform in (1), $2K$, and the decimation ratio N, $M = \mathrm{lcm}(2K, N)$, with $M = J \cdot 2K = L \cdot N$, $\quad J, L \in \mathbb{Z}$. To exploit common calculations between filters $H_k(z)$, the polyphase matrix $\mathbf{H}(z)$ can be written in terms of the M polyphase components of the prototype filter $P(z) = \sum_{m=0}^{M-1} z^{-m} P_m(z^M)$,

$$H_{k|n}(z) = \sum_{l=0}^{L-1} z^{-l} \cdot t_{k,lN+n} \cdot P_{lN+n}(z^L). \tag{4}$$

If the periodicity $2K$ of the transform coefficients $t_{k,n}$ is considered, it is possible to formulate a dense matrix notation

$$\mathbf{H}(z) = \mathbf{T}_{\text{GDFT}} \cdot \mathbf{P}(z) \tag{5}$$



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