Adaptive Equalization in Oversampled Subbands

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Abstract

The potential presence of fractional delays, non-minimum phase parts, and a colouring of the channel output can require adaptive equalizers to adapt very long filters, which can have slow convergence for LMS-type adaptive algorithms. These problems can be addressed by a subband approach to reduce computational complexity and improve convergence speed. We discuss, why amongst other possibilities of subband processing the oversampled approach is particularly appealing to significantly reduce computational complexity and improve convergence speed. Simulation results for typical systems found in acoustics and communication channels are presented and highlight the benefit of our method.

1. Introduction

When data is transmitted through a channel, the characteristics of the channel generally create a signal distortion which may cause bit errors on the receiving side. Thus, in communication systems a filter is employed to equalize any linear distortions of the channel [13]. Similarly, audio systems may require equalization of the room acoustics for dereverberation or feedback suppression [11, 12]. To account for time-varying behaviour of the system to be equalized, usually an adaptive solution is preferred. The general set-up of an adaptive equalizer is shown in Fig. 1. There, the channel output $x[n]$ is fed into the equalizer $w[n]$. After adaptation, the overall system $s[n] * w[n]$ should ideally resemble a delay $\delta(n-\Delta)$ only. In an audio application, $s[n]$ would incorporate the room acoustics, amplifiers, and electro-acoustic transducers. For either application, we will here assume linearity of $s[n]$.

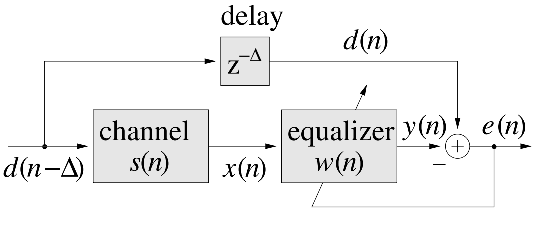

Fig. 1. Adaptive equalizer set-up.

Equalization is generally difficult due to a number of problems. Firstly, the system to be equalized may be very long, as for example the case in room acoustics, where the sampled impulse response can have several 1000's of coefficients [4]. Secondly, some types of characteristics are particularly hard to equalize, for example non-minimum phase systems [11, 21], and fractional delays [8], which involve the identification of very long and generally non-causal impulse responses. For adaptive equalization operating in the fullband, adaptive algorithms with high computational complexity such as the RLS cannot be used, while for computationally less complex LMS-type algorithms the convergence speed depends on the filter length. Finally, if the channel exhibits large spectral dynamics, the filter input signal $x[n]$ will have a large eigenvalue spread and the convergence speed of LMS-type adaptive algorithms is further slowed down [6].

Therefore, the application of subband adaptive filters (SAF) appears sensible. In an SAF system as shown in Fig. 2, both input and desired signal, $x[n]$ and $d[n]$, are decomposed into decimated frequency bands. By operating adaptive filters independetly in these subbands, both the update rate and the length of the adaptive filters can be greatly reduced leading to a lower computational complexity. Further, the subband decomposition performs a whitening of the input signal, resulting in improved convergence behaviour [7, 3].

In the following, we will give a brief introduction to subband adaptive filter structures, and concentrate on a particular SAF system using complex oversampled SAFs based on a signal decomposition using generalized DFT (GDFT) filter banks [1], which allows a very inexpensive implementation. Besides this, Sec. 2 also summarizes limiting influ-

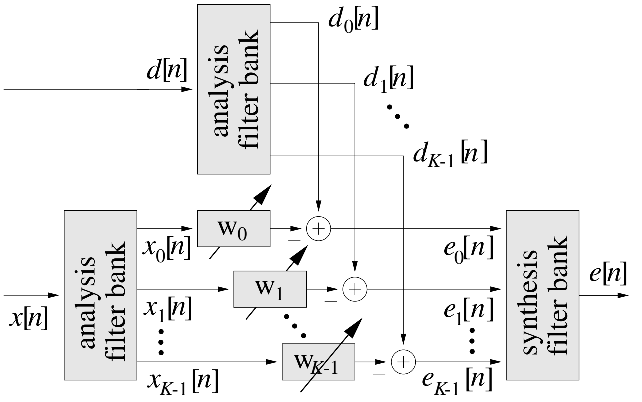

Fig. 2. SAF system with adaptive filters working independently in K decimated subbands; the subband splitting and fullband error reconstruction is performed by filter banks.

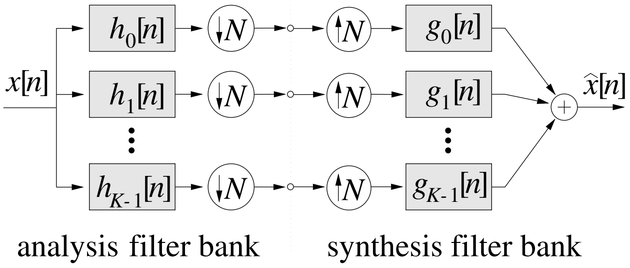

Fig. 3. Decomposition of a signal $x[n]$ by an analysis bank into K subbands decimated by $N \le K$; a fullband signal $\hat{x}[n]$ can be reconstructed by a synthesis bank.

ences in SAF systems. Sec. 3 discusses some fundamental problems of equalization and presents our approach to perform adaptive equalization in oversampled subbands. Finally, examples and results underlining the benefit of the proposed method are given in Sec. 4.

2. Adaptive Filtering in Subbands

2.1. Structural Approaches to SAF

Subband adaptive filter (SAF) systems, as shown in Fig. 2, are widely used for problems such as acoustic echo cancellation (AEC), where a large number of adaptive parameters has to be adjusted and the adaptive filter input is coloured. Therefore general fullband adaptive systems are very costly to implement and show a considerable decrease in convergence speed. The reduction in complexity by the subband approach becomes viable due to decreased complexity by processing in decimated subbands. Furthermore, the separation into frequency bands can reduce spectral dynamics.

However, the case of critical decimation, where the decimation ratio N equals the number of uniform subbands K, requires either cross-terms at least between adjacent frequency bands [3], which compensate for the information loss in the region of spectral overlap, or gap filter banks [23, 15], which introduce spectral loss to avoid any aliasing problems. The drawbacks are, that the inclusion of cross-terms requires multichannel adaptive algorithms with generally slower convergences and again increased computational cost, while the distortion produced by gap-filter banks may not be acceptable.

Oversampled SAF systems, with a decimation ratio $N < K$, are designed such that after decimation the alias level within the subbands is kept sufficiently low. Differences arise for the decimation of complex or real valued frequency bands. The decimation of real valued bandpass signals is generally complicated, and real valued signals have to be either modulated into the baseband prior to decimation by, for example, single sideband modulation (SSB,[1, 19]), or their bandwidth and decimation ratio has to be chosen in accordance with the sampling theorem, leading to non-uniform filter banks [14, 5]. In contrast, the decimation of complex valued bandpass signals with any integer factor $N < K$ is straightforward. Therefore, we focus on an SAF system which generalized DFT (GDFT) filter banks [1], performing a particular type of complex valued subband decomposition. In general, complex valued filter banks can be shown to be at least as efficient to implement as their real valued counterparts.

2.2. Oversampled Modulated Filter Banks

A general structure of a K band filter bank with decimation by a factor $N \le K$ is shown in Fig. 3. The analysis filters $h_k[n]$ are derived from a real valued lowpass prototype FIR filter $p[n]$ of length L_p by a generalized discrete Fourier transform (GDFT),

$$h_k[n] = e^{j\frac{2\pi}{K}(k+k_0)(n+n_0)} \cdot p[n], \qquad k,n \in \mathbb{N}. \qquad (1)$$

The term generalized DFT [1] stems from offsets k_0 and n_0 introduced into the frequency and time indices. With $k_0 = 1/2$, for a real valued input signal $x[n]$ it is sufficient to process the first $K/2$ subbands covering the frequency interval $[0;\pi]$ as shown in Fig. 4, while the remaining subbands are redundant. Together with conditions on $p[n]$, the time offset n_0 can be set appropriately to ensure useful properties such as linear phase. The synthesis filters $g_k[n]$ can be obtained by time reversion and complex conjugation of the analysis filters,

$$g_k[n] = h_k^*[L_p - n + 1] \quad . \qquad (2)$$

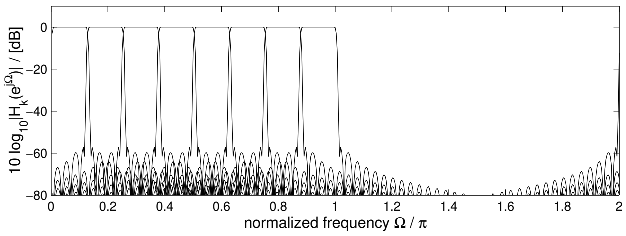

Fig. 4. Example of a $K = 16$ modulated filter bank.

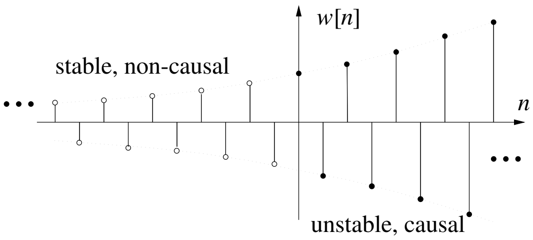

Fig. 5. Two possibilities of representing an unstable pole by all-zero models.

The modulation approach allows for both low memory consumption for storing filter coefficients and an efficient polyphase implementation. The latter even works for non-integer oversampling ratios K/N, and allows a factorization of the filter bank into a real valued polyphase network only depending on the prototype filter [2, 17]. The output of this network is rotated by a GDFT transform, which can be mainly implemented using an FFT.

Through the above modulation, the filter bank design reduces to an appropriate choice of the prototype filter, which has to fulfill two criteria. Firstly, the filter's attenuation in the stopband, $\Omega \in [\pi/N; \pi]$, has to be sufficiently large. Every frequency of the input signal $x[n]$ lying within the interval $[\pi/N; \pi]$ will be aliased into the baseband after filtering and decimation, and cause a distortion of the subband signal. A second constraint on the design is the perfect reconstruction condition. If stopband attenuation of the prototype filter is high enough to sufficiently suppress aliasing, this condition reduces to the consideration of inaccuracies in power complementarity [16]:

$$\sum_{k=0}^{K-1} |H_k(e^{j\Omega})|^2 \overset{!}{=} 1. \tag{3}$$

A prototype filter approximating these constraints can be constructed by an iterative least-squares method [19].

2.3. Subband Adaptive Filtering

When performing adaptive equalization in oversampled complex valued subbands, the adaptive filter lengths can be chosen shorter compare to a fullband adaptive filter in accordance with the sampling rate reduction by a factor $N < K$. Further, updating now occurs at the lower rate. Following the approach in [19, 20], for real valued signals $x[n]$ and $d[n]$ only $K/2$ complex subbands decimated by $N < K$ need to be processed, since the remaining subbands are complex conjugate and therefore redundant. This yields a reduction in computational complexity by a factor $r = \frac{K}{2N^2}$ for LMS-type algorithms (omitting any additional costs introduced by the filter banks). Therefore, it is advantageous in terms of computational cost to keep the oversampling ratio K/N close to one, i.e. choose non-integer oversampling. For an extensive discussion of computational costs, please refer to [17].

For an SAF system to perform satisfactorily, a delay in the path of the desired signal is required to account for various decimated transients of the analysis filter banks. If the SAF length is sufficient, no model truncation occurs, and if observation noise is absent, the minimum mean square error (MMSE) is limited only by the aliasing level produced by the decimation in the analysis filter banks, and the maximally achievable accuracy of the equalizer bound by the distortion function of the filter bank. In the case of modulated filter banks, both errors can be stated in terms of the prototype filter [20].

3. Subband Adaptive Equalization

For successful equalization, an adaptive algorithm is expected to perform an inverse system identification of the channel [21]. This becomes awkward as many real world channels are non-minimum phase systems, i.e. posses zeros outside the unit circle, e.g. in room acoustics [11] or communication systems [8]. Considering a maximum phase zero location a, $|a| > 1$, the inverse system can be stated by two representations for FIR equalizers, obtained by polynomial division:

$$\frac{1}{1 - az^{-1}} = \begin{cases} \sum_{i=0}^{\infty} a^i z^{-i} & (4) \\ \frac{-a^{-1}z}{1 - a^{-1}z} = -\sum_{i=1}^{\infty} a^{-i} z^i & (5) \end{cases}$$

Although derived from the same left hand side, the expression (4) and (5) are fundamentally different: while (4) is causal but unstable, 5 is stable but non-causal. The two solutions are illustrated in Fig. 5. Only solution (5) is viable, and can be made partially causal by introducing a delay. If the zero position a is close to the unit circle, the impulse response of the inverse system can become very long. Furthermore, the channel $s[n]$ generally colours the signal $x[n]$.

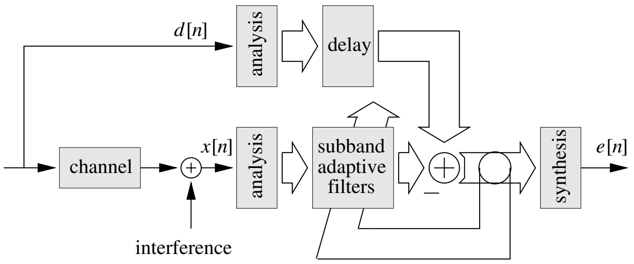

Fig. 6. Adaptive equalization in subbands.

A simple structure for an equalizer was introduced in Fig. 1. When incorporating the subband approach, the structure shown in Fig. 6 arises, where the SAF system is applied in series to the distorting channel. The delay inserted in the desired path should be approximately half the length of the SAFs [21], which can be justified from the assumption, that maximum- and minimum-phase zeros of the channel are located at similar distances from the unit circle, and causal and non-causal part of the impulse response to be identified should therefore have approximately the same length.

4. Application Example

To demonstrate the benefit of adaptive equalization in subbands, a bandpass channel with magnitude and a nonlinear phase response as shown Fig. 7(b) is used for simulation. Fig. 7(a) shows its pole-zero plot. Noise interference with spectral characteristics similar to the channel has been added at 40dB SNR. To invert this system, we apply an NLMS adaptive algorithm to both a 1500 tap fullband filter and an SAF system with $K/2 = 16$ complex subbands (with bandedges indicated by vertical dashed lines in Fig. 7(b)), decimated by $N = 28$ and having $(1500 + 896)/N \approx 86$ tap filters in each channel. The increase in length by $L_p = 896$ is to approximately compensate for transients caused by the filter banks, which are modulated from a lowpass prototype with L_p coefficients. Both fullband and subband adaptive algorithms operate with the same normalized LMS step size [21]. From the ensemble mean squared error curves in Fig. 7(c), a clear advantage in adaptation speed for the SAF equalization system is evident, while also requiring only 22% of the computational resources of the fullband equalizer.

Since SAF exhibit slower convergence at band edges [10], the choice of K forms a trade-off between the prewhitening effect of the subband approach and the number of band egdes introduced. Simulations in [17] indicate that the decimation ratio N has hardly any influence on the convergence speed, and should therefore be choosen as close as possible to K to allow an inexpensive implementation.