Subband Adaptive Equalization of Time-Varying Channels

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Abstract

The potential presence of fractional delays, non-minimum phase parts, and a colouring of the channel output can require very large adaptive equalizers exhibiting slow convergence for LMS-type algorithms. In this paper, we address the equalization problem via a subband approach, which can significantly reduce computational complexity and improve convergence speed. Of particular interest in this paper is the tracking behaviour, which determines the speed with which the algorithm is capable of following time variations in the channel. We give reasons why tracking of LMS-type subband adaptive algorithms should be comparable to the fullband case. Simulation results with a time varying channel are presented.

1. Introduction

Minimum mean square error (MMSE) adaptive equalisers attempt to create an inverse of a transmission channel, such that the overall system of channel and inverse as shown in Fig. 1 allows the transmission of information at an acceptable error rate [12]. Sometimes the inverse system which the adaptive filter has to model can require rather large FIR filters, particularly if, for example, fractional delays have to be equalized [8]. Therefore, for a number of applications such as advanced digital subscriber lines (ADSL), adaptive solutions become very computationally complex and slow convergent when LMS-type algorithms are employed [13].

This has motivated the use of subband adaptive filtering (SAF) techniques [7, 3], which in the case of very long adaptive filters reduces the complexity and can considerably enhance the convergence rate for LMS-type algorithms. These benefits have been shown to also hold for SAF based equalisers [17, 21]. Similar to fractionally spaced equalizers [15, 6], SAF equalisation aims at reduction of parameters to be adapted.

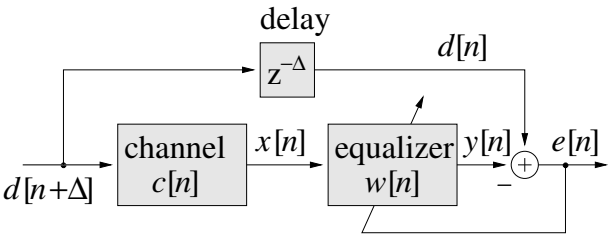


Fig. 1. Generic adaptive equaliser.

With regard to the adaptation behaviour, the convergence speed of SAF systems in general has been explored in, for example, [11]. However, to date very little is known about the tracking behaviour [10, 5] of SAFs. This is an important issue, as particularly in the context of communications, equalizers are often required to cope with the time varying behaviour of the transmission channel. Therefore, the aim of this paper is to focus on the tracking ability of subband adaptive equalizers.

In the following, Sec. 2 briefly reviews the background of subband adaptive filtering. The proposed subband adaptive equalizer structure will be introduced in Sec. 3. In Sec. 4 some positive and adverse influences of the SAF equaliser structure on the tracking performance are discussed. The validity of these theoretical considerations are illustrated by some simulation examples presented in Sec. 5.

2. Subband Adaptive Filtering

To perform SAF, the input $x[n]$ and desired signal $d[n]$ to the adaptive system [22, 5] as shown in Fig. 1 are split into K frequency bands. Due to their reduced bandwidth, these frequency bands can be decimated by a factor $N < K$. This signal decomposition is performed by analysis filter banks as given in Fig. 3 [1].

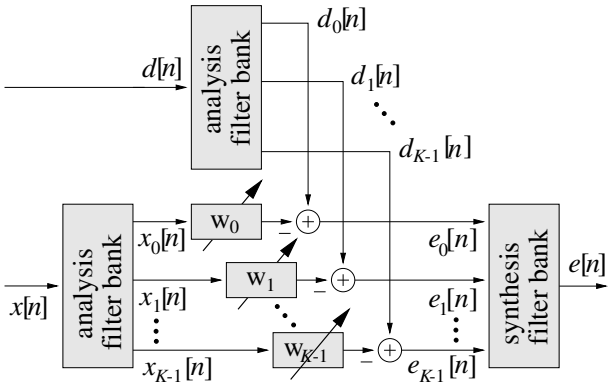


Fig. 2. Subband adaptive filter.

Adaptive filters $w_k[n]$ are then applied in each subband, attempting to minimize the subband error signals $e_k[n]$ in some sense. From the outputs of these subband adaptive filters (SAF), a fullband error signal $e[k]$ can be re-established by means of a synthesis filter bank as depicted in Fig. 3, which upsamples and appropriately interpolates the subband signals.

Amongst a large variety of subband structures, complex oversampled SAFs [7] with a decimation rate $N < K$ are particularly advantageous, as independent adaptive filters can be applied to each subband. Otherwise, critically sampled SAF systems require — different from Fig. 2 — adaptive cross-filters between adjacent subbands [3], or alternatively spectral loss has to be accepted [24]. For oversampled systems, oversampling ratio K/N is chosen such that the aliasing caused in the subband signals is small.

The advantages of SAF are due to the decimation, since the N times longer sampling period allows the use of a shorter filter, and updating occurs at the reduced rate. In general, this allows a reduction in computational complexity of K/N^2 for $\mathcal{O}(N)$ adaptive filters such as the LMS or K/N^3 for $\mathcal{O}(N^2)$ such as RLS type algorithms [5]. On top of that, computationally efficient and memory-saving implementations for the subband splitting can be achieved through modulated filter banks [19]. Other advantages of SAFs include spectral whitening, as the subbands have smaller spectral dynamics than the fullband signal, and the division of a single fullband problem into several parallel tasks in the subbands, which can be independently solved on separate processors [3].

In the absence of other error sources such as observation noise or model truncation, the limitations of SAF lie in the aliasing produced in the subbands, which limits the achievable final MSE. Another limitation is the modelling capability, i.e. how accurately an SAF

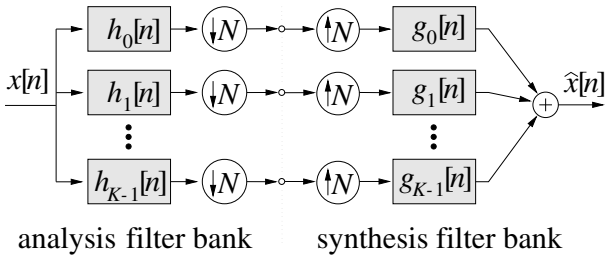


Fig. 3. Analysis and synthesis filter bank.

can represent a fullband system. The mismatch in this model is given by the reconstruction error of the filter bank system in Fig. 3, which ideally should only impose a delay on the signal sent through analysis and synthesis filter bank. In the case of modulated filter banks, both errors can be stated in terms of the prototype filter [18] and controlled by application specific design [4].

3. Subband Adaptive Channel Equalisation

For standard channel equalization (or inverse system identification [22]), the channel output $x(n)$ is passed to the input of the adaptive filter. The output of the latter, $y(n)$, is compared to a delayed version of the channel input, $d(n)$. This delay is necessary to cater for non-causalities arising from the inversion of non-minimum phase components in the channel [23, 17].

For subband adaptive equalization, an SAF structure as discussed in Sec. 2 is performed on $x(n)$ and $d(n)$ [17, 20]. Besides the advantages and limitations discussed in Sec. 1, the potential increase of convergence speed is a key feature of the SAF equalizer. Since the channel and possibly also the transmitted signal may exhibit large spectral dynamics (spectral nulls are not considered here but can be address by zero-forcing equalizers [14]), the input to the equalizer has a large eigenvalue spread which is inversely proportional to the convergence speed of LMS-type adaptive filters [5]. However, the separation of the input signal's spectrum into frequency bands ensures that all subbands have smaller spectral dynamics than the fullband signal, hence yielding a faster convergence than possible in the fullband.

To exploit the subband splitting for spectral dynamics, the convergence rate of LMS-type algorithms should be adjusted individually in each subband. This can be achieved by employing an normalised LMS (NLMS) algorithm [5], whereby the step size in each band is normalised with respect to the power of the

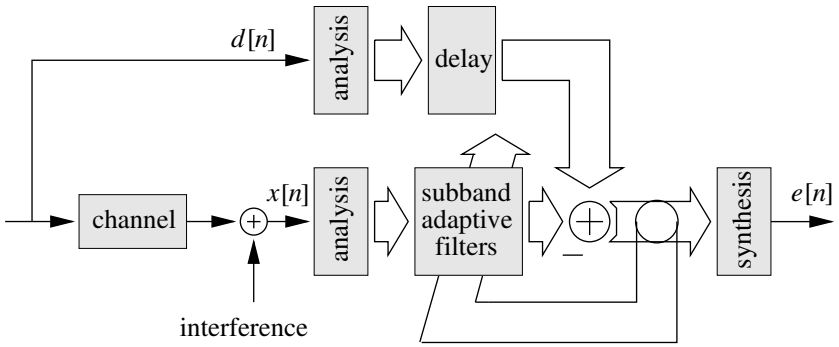


Fig. 4. Subband adaptive equaliser structure.

kth subband input signal, $x_k[n]$.

4. Tracking Performance

Tracking characterizes the ability of an adaptive algorithm to follow parameter changes in the transmission channel, which is a behaviour generally different from the convergence rate. In the communications application targeted here, time variations are due to, for example, movement of a cellular telephone user resulting in changing multipath propagation and Doppler shift, modelled by a Rayleigh fading channel [16].

Analytically, the optimum solution is now time-varying. As a result, the final MSE ξ, given by the contributions from the MMSE and excess MSE ξ_{ex}. The excess MSE can be separated into a gradient noise term and an additional term which is due to the adaptive solution lagging behind in pursuing the time-varying optimum [9]:

$$\xi_{\text{ex}} \simeq \mu\sigma_s^2 \text{tr}\{\mathbf{R}\} + \frac{1}{4\mu}\text{tr}\{\mathbf{O}\} \quad , \tag{1}$$

where, $\text{tr}\{\cdot\}$ denotes trace of a matrix, $\mathbf{R}$ is the covariance matrix of the input signal, $\mathbf{O}$ the correlation matrix of the optimum weight changes, and σ_s^2 the power of any observation noise on the system. Obviously, through $\text{tr}\{\mathbf{O}\}$ the lag error is proportional to the length of the adaptive filter.

Considering the subband adaptive equalizer in relation to a fullband equalizer of equivalent modelling capabilities, the filter length L_{sb} of the SAFs compared to the fullband filter length L has to be chosen as [19]

$$L_{sb} = \frac{L + L_p}{N} \quad , \tag{2}$$

where L_p is the length of the anaylsis filters. For $L \gg L_p$, the subband filter length is reduced a factor N. Thus, from theory the effect on the tracking performance is twofold:

- shortening the SAF filter length to approximately a factor of N with respect to a fullband adaptive equalizer decreases the lag error by about a factor of N;
- the update rate of the SAF is reduced by N compared to the fullband, which slows tracking down.

Hence, from a rather crude argumentative point of view, the subband approach should be neither beneficial nor harmful for the tracking ability of an adaptive filter.

5. Simulations and Results

A channel of 20μs (400 taps) length modelling a hilly terrain [16] is used to transmit a signal which is sampled at a rate of 20MHz at the receiver. The steady state characteristics of the sampled channel are shown in Fig. 5. Instead of statistical fading to model the time variation of the channel, for simplicity a Doppler shift of 40Hz is imposed by applying a complex modulation to the coefficients of the channel model.

In a first simulation, a fullband adaptive equaliser with $L = 1000$ coefficients is employed. It is compared to an SAF equaliser working in $K = 32$ subbands decimated by $N = 28$ with same modelling capabilites, yielding a filter length of $L_{sb} = 70$. Both equalizers employ an NLMS algorithms [5] with normalized step size $\mu = 0.4$. The adaptation curves are given in Fig. 6. The time varying channel causes a cyclic behaviour of the MSE at the Doppler frequency, apart from a short initial transition, where the adaptive filters converge. Obviously, both equalisers perform very similar, with a worst case MSE of around -14 dB. However, the performance of the SAF equaliser is achieved at a complexity of around 6% of the fullband equaliser.

If both fullband and subband equalisers should have identical computational complexity, a fullband

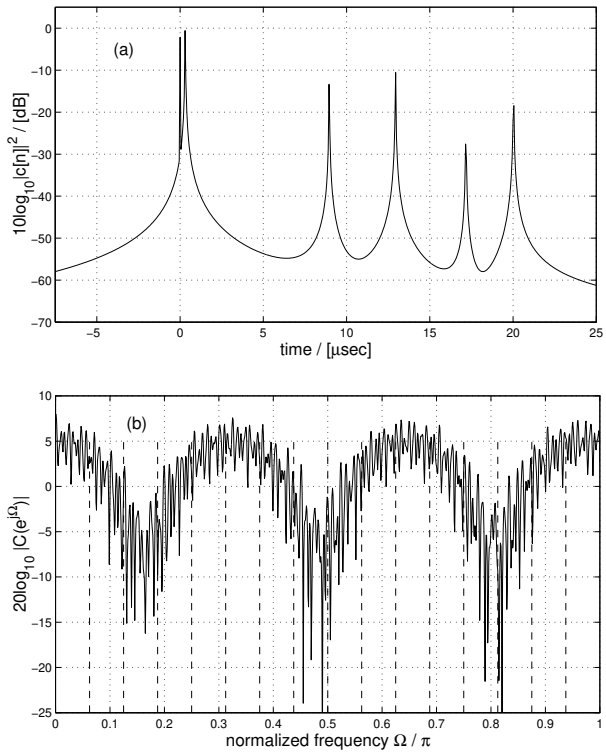
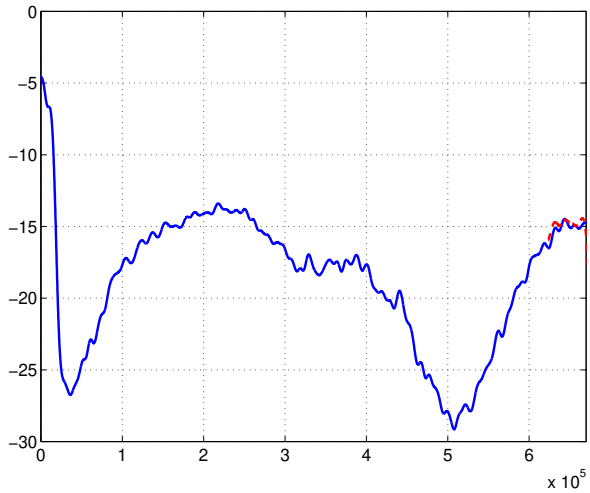


Fig. 5. Characteristics of stationary channel: (a) log-modulus of impulse response; (b) magnitude response



equaliser would only be allowed to have $L = 70$ coefficients. Although this should result in considerably faster tracking, the dash-dotted adaptation curve of this equaliser in Fig. 6 indicates, that the filter length is insufficient to properly model the channel inverse.

A second simulation compares a fullband equaliser of length $L = 2000$ with an SAF system of identical modelling capabilities with $L_{sb} = 140$. The adaptation curves are given in Fig. 7, indicating no degradation of the SAF's MSE over the MSE of the fullband equaliser. However, again the computational complexity of the SAF equaliser is only about 6% as compared to the complexity required for the fullband equalizer.

6. Conclusions

We have motivated the application of an SAF structure to the channel equalization problem. The structure has been introduced and its general advantages and limitations have been highlighted. Concerning the suitability of the SAF equalizer for the tracking of time-varying channels, we have presented some reasoning, underlined by some simulation results, that the track-