Overlapping M-ary Frequency Shift Keying Spread-Spectrum Multiple-Access Systems Using Random Signature Sequences

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***Abstract*— In this paper, a multiple-access spread-spectrum communication system using binary frequency shift keying (BFSK) or M-ary frequency shift keying (MFSK) and noncoherent demodulation is considered. In contrast to previous work typically assuming that the frequency shift keying (FSK) tones are nonoverlapping after direct-sequence (DS) spreading, here we consider a spread-spectrum multiple-access (SSMA) system under the assumption that the DS spread signals of different FSK tones are only orthogonal over the information symbol duration. Consequently, the frequency band of a spread FSK tone may be fully or partially overlapping with the other spread signals. An estimate of the variance of the multiple-access interference is obtained by assuming that the phase angles and time delays of the received signals are mutually independent random variables, provided that random signature sequences are employed for spreading. On the basis of the above assumptions, the bit error rate (BER) of our DS spread-spectrum multiple-access (DS-SSMA) and that of our hybrid DS slow frequency-hopping spread-spectrum multiple-access (DS-SFH SSMA) systems using FSK modulation is analyzed, when the channel impairments are constituted by a combination of additive white Gaussian noise (AWGN) and multiple-access interference. From our analysis and the numerical results, we concluded that, for a given system bandwidth and for a certain value of M, the system's BER performance can be optimized by controlling the amount of overlapping and that the systems with optimized overlapping outperformed the systems using no overlapping.**

***Index Terms*— Frequency shift keying (FSK), slow frequency hopping, spread-spectrum multiple access.**

I. Introduction

SPREAD-SPECTRUM multiple-access (SSMA) schemes have received considerable attention and have been proposed for use in a wide variety of applications, in order to mitigate the different problems encountered over different communications media [1], [2]. In addition to its multiple-access potential, SSMA is also capable of combating various types of interferences. The most commonly used forms of SSMA are direct-sequence SSMA (DS-SSMA) [3], [4], frequency hopping SSMA (FH-SSMA) [5], [6], and hybrid DS slow frequency-hopping SSMA (DS-SFH SSMA) [7]–[11]. In multiuser communication environments, however, research has been focused on DS-SSMA and hybrid DS-SFH SSMA systems due to their multiple-access potential. In the context of DS-SSMA and hybrid DS-SFH SSMA subjected to multiple-access interference, [4] and [7]–[11] analyzed the performance of binary and M-ary frequency shift keying (FSK) using noncoherent demodulation. Their analysis was based on the assumption [4] that the frequency tones are at frequencies of $f_c + m\Delta$ ($m = +1, -1$ for BFSK, and $m = 1, 2, \cdots, M$ for MFSK) with $\Delta \gg T^{-1}$, where T is the signaling interval duration, and that the interference among the FSK tones of a given signal can be considered negligible. In other words, the frequency tones used for signaling were spaced far apart in comparison to the signaling rate or Baud rate. More specifically, in the above-mentioned references concerning BFSK and MFSK the approximations to the bit (symbol) error probabilities were derived by assuming that $\Delta \geq T_c^{-1}$ and that the effect of the sidelobes of the DS spread FSK tones for frequencies outside this T_c^{-1}-related range was negligible, where T_c was the chip duration of the spreading sequences, as noted for example in [4] and [8]. These assumptions indicate that the main lobes of the frequency tones remain nonoverlapping even after DS spreading.

However, due to the inherent properties of pseudonoise sequences, the orthogonality of the system is not affected by direct sequence spectrum spreading [12] and the spread signals are required to be orthogonal only over the frequency range related to the reciprocal of the symbol duration, i.e., over T^{-1}, rather than over the reciprocal of the DS chip duration, i.e., over T_c^{-1}. Hence, for BFSK and MFSK DS-SSMA systems, Δ is only required to be wider than T^{-1} after DS spreading, i.e., $\Delta \geq T^{-1}$. Although this implies that there is a strong spectral overlap of the different frequency tones after DS spreading, which increases the multiple-access interference, however, for a given overall system bandwidth of each frequency tone the DS spreading bandwidth can be increased, which potentially improves the system's performance.

In this paper, we consider a general DS-SSMA or DS-SFH SSMA system with BFSK or MFSK modulation, in which the FSK tones after DS spreading are required to be spaced wider than the reciprocal of the symbol duration, i.e., $\Delta \geq T^{-1}$ or T_s^{-1} for BFSK or MFSK, respectively, where T_s is the MFSK symbol duration. We assume furthermore that $\Delta = (i/T)$ for BFSK and $\Delta = (i/T_s)$ for MFSK, where the integer i has to obey $i \geq 1$, in order to optimize the spacing of the

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DS spread frequency tones and to minimize the effect of the multiuser interference. Although the overlapping main lobes and sidelobes of the FSK tones after DS spreading increase the interference inflicted upon the reference user, nonetheless, the numerical results in Section IV show that an enhanced bit error rate (BER) performance can be achieved by optimizing the amount of the spectral overlap of the spread FSK tones.

The remainder of this paper is organized as follows. In Sections II and III, we are concerned with the bit error probabilities of DS-SSMA and hybrid DS-SFH SSMA systems, under the assumption that the FSK tones after DS spreading are only spaced wider than the reciprocal of the information symbol duration, i.e., T^{-1} or T_s^{-1} for BFSK or MFSK, respectively. The performance of these systems is compared in Section IV by assuming a constant total system bandwidth. Similarly to [4] and [7]–[11], we assume that the channel impairments are a combination of additive white Gaussian noise (AWGN) and multiple-access interference. We also assume that all transmitted signals have the same power at the receiver, i.e., ideal power control is used. We note that in order to make the paper more readable we consolidated the more detailed mathematics in Appendixes I–IV, easing the readers' passage through Sections II and III to the most informative part, namely, Section IV.

II. SSMA System with DS Spread-Spectrum Signal

A. The Transmitted Signals

Our model for the asynchronous DS-SSMA system resembles that of [4], where BFSK and MFSK modulation schemes with noncoherent demodulation are employed. The kth user's $(1 \leq k \leq K)$ transmitted signal for the BFSK system can be expressed as

$$s_k(t) = \sqrt{2P}a_k(t)\cos\{2\pi[f_c + b_k(t)\Delta]t + \theta_k(t)\},\quad 1 \leq k \leq K \tag{1}$$

where f_c is the carrier frequency and P is the power of the transmitted signal. The signature waveform $a_k(t)$ consists of a sequence of rectangular pulses $a_l^{(k)}$ of duration T_c for $lT_c \leq t < (l+1)T_c$, and has amplitudes of $+1$ or -1 with equal probability for the random signature sequences considered in this paper. The kth user's data signal $b_k(t)$ is a sequence of mutually independent random variables with unit amplitude, positive and negative, corresponding to rectangular pulses of duration T. Again, we assume that $\Delta \geq T^{-1}$ so that the spacing between two FSK tones after DS spreading becomes wider than the reciprocal of the symbol duration. The waveform $\theta_k(t)$ is the phase introduced by the kth FSK modulator, that is, if $b_l^{(k)} = m$ for $m = +1$ or -1, then $\theta_k(t) = \theta_{k,m}$ for $lT \leq t < (l+1)T$, where $\theta_{k,m}$ is the phase corresponding to the frequency tone $f_c + m\Delta$.

For an MFSK rather than BFSK system, the transmitted signal is expressed also as in (1). We only need to modify the BFSK model in a straightforward fashion in order to account for the fact that the information sequence is M-ary, rather than binary and, hence, m takes values from the set $\{1-M, 3-M, \cdots, -1, 1, \cdots, M-3, M-1\}$ with equal

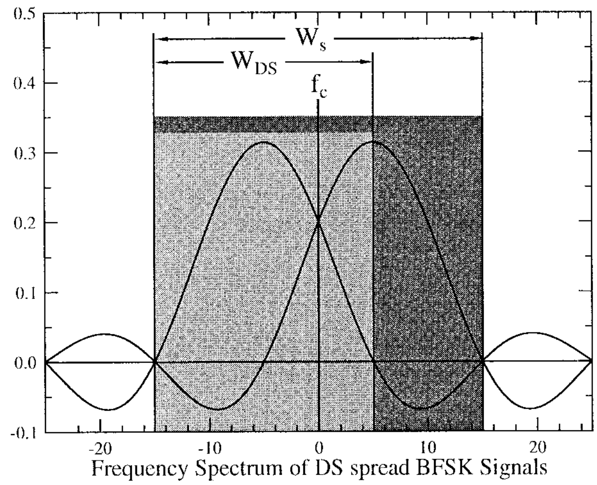

Fig. 1. Frequency spectrum of DS spread-spectrum signal with binary FSK modulation.

probability, instead of the set $\{+1, -1\}$. In addition, the bit duration T has to be replaced by the symbol duration $T_s = T\ \log_2\ M$ for the data sequence. Furthermore, the assumption of $\Delta \geq T^{-1}$ stipulated for the BFSK system should be replaced by $\Delta \geq T_s^{-1}$.

B. Restrictions on the Signal Design and Analysis

In Section II-A, we made the assumptions that $\Delta \geq T^{-1}$ for the binary FSK DS-SSMA system and $\Delta \geq T_s^{-1}$ for the M-ary FSK DS-SSMA system. In order to optimize the spacing of the DS spread FSK tones and consequently to minimize the multiuser interference inflicted to the signal of the reference user, in the following analysis, we impose the restriction of $\Delta = (i/T)$ or $\Delta = (i/T_s)$, where the integer i obeys $i \geq 1$ following $\Delta \geq T^{-1}$ or T_s^{-1}. Using this restriction, the optimized spacing of the DS spread FSK tones can be found by optimizing the value of i, an issue which will be analyzed in Section IV. The frequency spectra of the DS spread BFSK and MFSK signals are shown in Figs. 1 and 2, respectively. We assume that the total system bandwidth W_s (Hz) is a constant and $(W_s/W_d) = N \gg 1$, where W_d (Hz) represents the binary baseband signal's bandwidth and N is an integer representing the total spreading gain of a DS-SSMA system using binary modulation. This assumption is readily applicable in practice, since the signal bandwidth after spreading is usually much higher than that of the original information signal.

Based on the above assumption, it is plausible that, for the BPSK DS-SSMA system of [3] or for the DPSK DS-SSMA system of [4], the bandwidth expansion factor, defined as the DS bandwidth W_{DS} (Hz) divided by the baseband information signal's bandwidth W_d (Hz) is N, since each DS spread BPSK signal or DPSK signal occupies the whole system bandwidth. However, for the BFSK and MFSK communication systems considered here, the DS spread signals activate different frequency tones according to the transmitted information. Hence, the bandwidth expansion factor of each BFSK and MFSK tone is typically lower than that of BPSK and DPSK, when using a constant total system bandwidth, which is a consequence

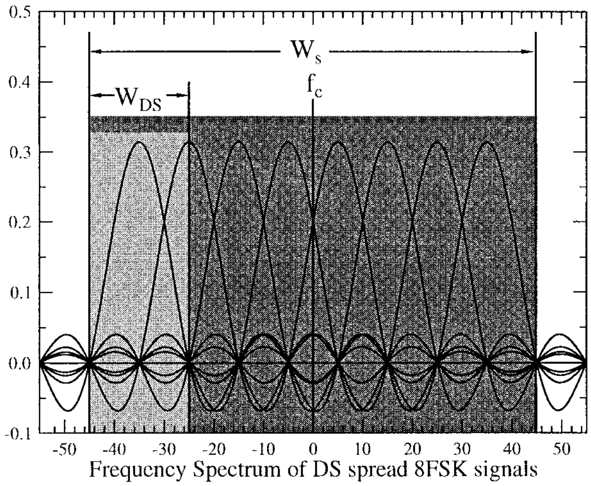

Fig. 2. Frequency spectrum of DS spread-spectrum signal with 8-ary FSK modulation.

of the orthogonality of different frequency tones at least over the reciprocal of the information symbol duration. From Fig. 1 we can infer that the bandwidth expansion factor of a DS spread BFSK tone is given by $N_B = (W_{DS}/W_d) = ((W_s - 2\Delta)/W_d)$. Substituting $\Delta = (i/T)$, $W_d = (2/T)$, and using $(W_s/W_d) = N$, we obtain

$$N_B = N - i. \tag{2}$$

For MFSK DS-SSMA, the bandwidth expansion factor of a DS spread MFSK tone can be similarly computed by $N_M = ((W_s - 2(M-1)\Delta)/W_d)$ with $\Delta = (i/T_s)$, $W_d = (2/T_s)$. Using $(W_s/W_d) = N(\log_2 M)$ for data sequences with a bit duration of T and with the chip rate fixed to $T_c^{-1} = NT^{-1}$, N_M can be simplified to

$$N_M = N \log_2 M - i(M-1). \tag{3}$$

As shown in Figs. 1 and 2, there is an overlap of the spectra associated with different FSK tones. However, since they are spaced wider than the reciprocal of the symbol duration, the signal associated with each FSK tone can be recovered, as long as the channel does not violate this condition. We will show that, although the spreading-induced overlapping implies that there is interference amongst the FSK tones, which aggravates the multiple-access interference, nonetheless the system benefits from the increased spreading gains. From (2) and (3) we can infer that the bandwidth expansion factor, N_B is larger than $N/2$, when $1 \leq i < N/2$, and N_M is larger than $(N/M)(\log_2 M)$, if $1 \leq i < (N/M)(\log_2 M)$. However, for a conventional system, where the main lobes of the FSK tones remain nonoverlapping even after DS spreading, the largest possible bandwidth expansion factor for each FSK tone is $N/2$ or $(N/M)(\log_2 M)$ for the BFSK and MFSK systems, respectively, since, in these systems the largest mainlobes' bandwidth occupied by the spread signals equals to W_s/M. Actually, the bandwidth expansion factor N_B or N_M of the spread-spectrum system using overlapping tones is $N/2$ or $(N/M)(\log_2 M)$ if $i = N/2$ or $(N/M)(\log_2 M)$ for BFSK or MFSK, respectively. Consequently, the spread-spectrum model with nonoverlapping frequency tones is included in our model, and an estimate of its average error probability can be obtained from the results, which will be derived in this paper, simply by replacing i with $N/2$ or $(N/M)(\log_2 M)$ and ignoring the effects of the interference from the spectral sidelobes of the interfering signals.

C. Receiver Description

Let us assume that we want to receive the signal transmitted by the first user which is treated as the reference user or reference signal in our analysis and that the receiver is capable of acquiring perfect time-domain synchronization. We use a receiver structure that is optimum for the AWGN channel. However, since in our system all users communicate over the same frequency band, there is multiple-access interference in addition to the AWGN, rendering this receiver suboptimum. Nevertheless, we evaluate its performance next.

For binary FSK modulation, the K users' signals in the form of (1) are transmitted asynchronously over the AWGN channel, where the received signal $r(t)$ is given by

$$r(t) = s_1(t) + \sum_{k=2}^{K} s_k(t-\tau_k) + n(t) \tag{4}$$

$$\begin{aligned} s_k(t-\tau_k) &= \sqrt{2P}a_k(t-\tau_k) \\ &\quad \cdot \cos\{2\pi[f_c + b_k(t-\tau_k)\Delta]t + \varphi_k(t)\}, \\ &\quad 1 \leq k \leq K \end{aligned} \tag{5}$$

where $\varphi_k(t) = \theta_k(t) - 2\pi[f_c + b_k(t-\tau_k)\Delta]\tau_k$, $n(t)$ is the channel noise, which is assumed to be a zero-mean stationary Gaussian process with double-sided spectral density of $N_0/2$; $\tau_k, 2 \leq k \leq K$ are the time delays relative to the reference signal, which are modeled as uniformly distributed independent random variables over $[0, T]$.

For noncoherent reception and a correlation receiver matched to the reference user's signal, during the reception of the data bit b_0 the in-phase component of the two quadrature branches is given by Fig. 2 of [4], which is redrawn here in Fig. 3 for convenience, yielding

$$Z_{c,m} = \int_0^T r(t)a_1(t) \cos[2\pi(f_c + m\Delta)t]\, dt \tag{6}$$

when assuming a rectangular chip waveform and $m = +1, -1$. In order to obtain the quadrature component $Z_{s,m}$, we simply replace $\cos(\cdot)$ by $\sin(\cdot)$ in (6). From (28) of Appendix I, the above expression of $Z_{c,m}$ can be simplified as

$$Z_{c,m} = \sqrt{\frac{P}{2}}\, T \left[D_{c,m} + \sum_{k=2}^{K} I_{c,m}(k,1) + N_{c,m}\right] \tag{7}$$

where

$$D_{c,m} = \delta(b_0, m) \cos\varphi_m \tag{8}$$

is the desired signal component, $\delta(b_0, m) = 1$ for $b_0 = m$, otherwise $\delta(b_0, m) = 0$ and φ_m is the phase angle of the reference signal when $b_0 = m$. Furthermore

$$N_{c,m} = \left(\sqrt{\frac{P}{2}}\, T\right)^{-1} \int_0^T n(t)a_1(t) \cos[2\pi(f_c + m\Delta)t]\, dt \tag{9}$$

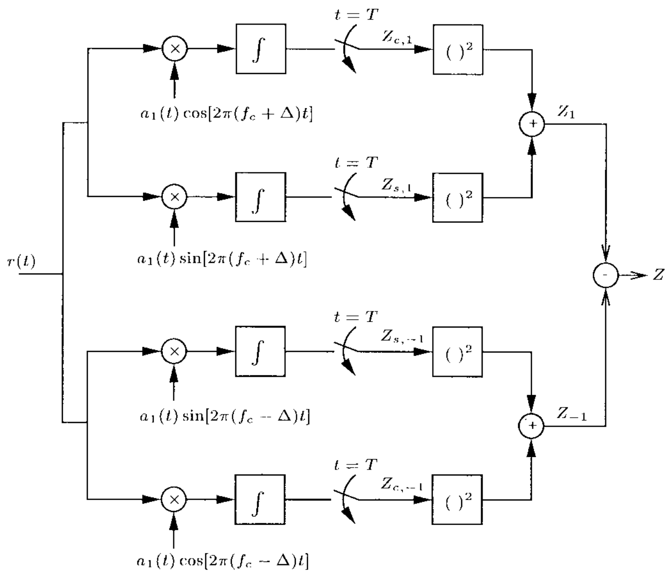

Fig. 3. Receiver for a DS-SSMA system employing binary FSK modulation.

is a normally distributed Gaussian random variable with zero mean and a variance of $(2E_b/N_0)^{-1}$ [4], where $E_b = PT$ is the transmitted signal's energy per bit. The interference term due to the kth user's signal is defined by

$$I_{c,m}(k,1) = \frac{1}{T}\,\{R_c[\tau_k, b_{-1}^{(k)}, \varphi_{-1}^{(k)}] + \hat{R}_c[\tau_k, b_0^{(k)}, \varphi_0^{(k)}]\} \quad (10)$$

where $\varphi_{-1}^{(k)}$ and $\varphi_0^{(k)}$ are the phase angles of the kth user relative to the reference signal for the transmitted data bits $b_{-1}^{(k)}$ and $b_0^{(k)}$. They are independent, identically distributed (i.i.d.) random variables uniformly distributed over $[0, 2\pi]$. In the spirit of [3], the terms $R_c[\tau_k, b_{-1}^{(k)}, \varphi_{-1}^{(k)}]$ and $\hat{R}_c[\tau_k, b_0^{(k)}, \varphi_0^{(k)}]$ are continuous-time partial cross-correlation functions defined by

$$R_c[\tau_k, b_{-1}^{(k)}, \varphi_{-1}^{(k)}] = \int_0^{\tau_k} a_1(t)a_k(t-\tau_k) \cdot \cos\{2\pi[b_{-1}^{(k)} - m]\Delta t + \varphi_{-1}^{(k)}\}\, dt \quad (11)$$

$$\hat{R}_c[\tau_k, b_0^{(k)}, \varphi_0^{(k)}] = \int_{\tau_k}^{T} a_1(t)a_k(t-\tau_k) \cdot \cos\{2\pi[b_0^{(k)} - m]\Delta t + \varphi_0^{(k)}\}\, dt \quad (12)$$

for $0 \le \tau_k \le T$. However, they are different from those defined in [3], hence, $I_{c,m}(k,1)$ in (10) cannot be evaluated by the conventional method of [3], [4], in which the interfering signal employs the same frequency tone as the desired signal. It follows namely from these references that the variance of the multiple-access interference from user k can be approximated by $1/3N$, where N is the bandwidth expansion factor, unless $b_{-1}^{(k)} = m$ and $b_0^{(k)} = m$ in (11) and (12).

The quadrature component $Z_{s,m}, (m = +1, -1)$ can be obtained from the in-phase component, if we replace $\cos(\cdot)$ by $-\sin(\cdot)$ in (7)–(12).

For MFSK modulation the output of the in-phase branch of the receiver matched to the reference signal is also given by (6), where m takes values of $(2j - M + 1)$ with $j = 0, 1, \cdots, M-1$ instead of $+1, -1$. Furthermore, in (7) and (9)–(12), T has to be replaced by $T_s = T(\log_2\ M)$ for an information sequence with constant bit duration of T, the variance of $N_{c,m}$ by $(2E_b\ \log_2\ M/N_0)^{-1}$ and b_0 by the M-ary symbol $b_{M,0}$. In (10)–(12), $b_{-1}^{(k)}$ and $b_0^{(k)}$ have to be replaced by the M-ary symbols $b_{M,-1}^{(k)}$ and $b_{M,0}^{(k)}$, which are assumed to take values in the range of $\{1-M, 3-M, \cdots, -1, 1, \cdots, M-3, M-1\}$. Last, $\varphi_{-1}^{(k)}$ and $\varphi_0^{(k)}$ must be replaced by $\varphi_{M,-1}^{(k)}$ and $\varphi_{M,0}^{(k)}$, which are the phase angles of the kth user relative to the reference signal for the transmitted data symbols $b_{M,-1}^{(k)}$ and $b_{M,0}^{(k)}$.

D. Average Bit Error Probability

In this section, the BER performance of the binary and M-ary FSK DS-SSMA system will be evaluated by assuming that all interferences are Gaussian distributed and treated as additional noise. The interferences from different users are treated as mutually independent random variables.

For BFSK modulation, the interference term due to the kth user's signal can be approximated as a Gaussian distributed variable with zero mean, while its variance conditioned on $x_k,\ \hat{x}_k$ is given by

$$\mathrm{Var}[I_{c,m}(k,1;x_k,\hat{x}_k)] = \frac{1}{T^2}\,[\gamma(x_k) + \hat{\gamma}(\hat{x}_k)] \quad (13)$$

where x_k and $\hat{x}_k$ are related to the data bits transmitted by user k and are defined as $x_k = b_{-1}^{(k)} - m$ and $\hat{x}_k = b_0^{(k)} - m$ according to (11) and (12) and (29) and (30), respectively, of Appendix II. The quantities $\gamma(x_k)$ and $\hat{\gamma}(\hat{x}_k)$ are the variance of $R_c[\tau_k, b_{-1}^{(k)}, \varphi_{-1}^{(k)}]$ and $\hat{R}_c[\tau_k, b_0^{(k)}, \varphi_0^{(k)}]$ for the given bits $b_{-1}^{(k)}$ and $b_0^{(k)}$, which are computed by considering τ_k, $\varphi_{-1}^{(k)}$ and $\varphi_0^{(k)}$ as mutually independent random variables over the appropriate interval. Substituting $\gamma(x_k)$ and $\hat{\gamma}(\hat{x}_k)$ from (35) and (36) of Appendix II into (13), and considering that $\Delta = (i/T)$, $G = (T/T_c) = N_B$, and $N_B = N - i$, we can simplify (13) as

$$\mathrm{Var}[I_{c,m}(k,1;x_k,\hat{x}_k)] = \frac{N-i}{4\pi^2 i^2}\left(\frac{1}{x_k^2}\left[1 - \mathrm{sinc}\left(\frac{2\pi x_k i}{N-i}\right)\right] + \frac{1}{\hat{x}_k^2}\left[1 - \mathrm{sinc}\left(\frac{2\pi \hat{x}_k i}{N-i}\right)\right]\right) \quad (14)$$

where $\mathrm{sinc}(x) = (\sin(x)/x)$.

Let x_k and $\hat{x}_k$ of (14) be replaced by x and y, and compute the limit $\lim_{x\to 0, y\to 0}\ \mathrm{Var}[I_{c,m}(k,1;x,y)]$. Then we find that the variance of the interference from the kth user equals to $1/3(N-i)$. This is the variance, when a reference signal is totally overlapped by the kth interfering signal, that is $b_{-1}^{(k)} = m$ and $b_0^{(k)} = m$. In addition, when $i = 0$, then all the frequency tones are fully overlapped, which only happens when BPSK or DPSK baseband modulation is employed. Hence, the spectrum of the modulation scheme occupies the whole system bandwidth all the time. Consequently, the variance of the interference from an interfering user's signal is reduced to $1/3N$, which is a typical approximated value used for performance evaluations of DS-SSMA systems.

The variance given by (14) is conditioned on x_k and $\hat{x}_k$, which are related to $b_{-1}^{(k)}$ and $b_0^{(k)}$. Below the variance will be averaged by considering $b_{-1}^{(k)}$ and $b_0^{(k)}$ as independent random variables, which take values from the set of $\{+1, -1\}$ for BFSK and from $\{2j-M+1, j=0,1,\cdots,M-1\}$ for MFSK with equal probability.

For BFSK, the information sequence is an independent random variable with unit amplitude, as noted in Section II-A. Hence, x_k and $\hat{x}_k$ in (14) take values of $\{-2, 0\}$ or $\{0, 2\}$ with equal probability for the values of $m=+1$ or $m=-1$. As a result, $\mathrm{Var}[I_{c,m}(k,1)]$ can be obtained by averaging $\mathrm{Var}[I_{c,m}(k,1;x_k,\hat{x}_k)]$ for the x_k and $\hat{x}_k$ values of $\{-2,0\}$ and $\{0,2\}$ for the values of $m=+1$ and -1

$$\mathrm{Var}[I_{c,m}(k,1)] = \tfrac{1}{8}\left\{\left[\sum_{x_k,\hat{x}_k\in\{-2,0\}} \mathrm{var}[I_{c,m}(k,1;x_k,\hat{x}_k)]\right]_{m=+1} + \left[\sum_{x_k,\hat{x}_k\in\{0,2\}} \mathrm{var}[I_{c,m}(k,1;x_k,\hat{x}_k)]\right]_{m=-1}\right\}. \quad (15)$$

Substituting $\mathrm{Var}[I_{c,m}(k,1;x_k,\hat{x}_k)]$ from (14) into (15) for all possible values of x_k and $\hat{x}_k$, the result is computed as

$$\mathrm{Var}[I_{c,m}(k,1)] = \frac{1}{6(N-i)} + \frac{N-i}{16\pi^2 i^2} \cdot \left[1-\mathrm{sinc}\left(\frac{4\pi i}{N-i}\right)\right]. \quad (16)$$

As noted in Section II-C, $N_{c,m}$ is a normally distributed Gaussian variable with variance of $(2E_b/N_0)^{-1}$, and all interferences are treated as additional noise. Hence, according to (7) the normalized variance of $Z_{c,m}$ can be computed by $\sigma_{c,m}^2 = (2E_b/N_0)^{-1} + (K-1)\,\mathrm{Var}[I_{c,m}(k,1)]$ or alternatively as

$$\sigma_{c,m}^2 = \left(\frac{2E_b}{N_0}\right)^{-1} + (K-1) \cdot \left(\frac{1}{6(N-i)} + \frac{N-i}{16\pi^2 i^2}\left[1-\mathrm{sinc}\left(\frac{4\pi i}{N-i}\right)\right]\right). \quad (17)$$

The normalized variance of the quadrature-phase component $Z_{s,m}$ can also be obtained according to the method above, resulting in $\sigma_{s,m}^2 = \sigma_{c,m}^2$.

Finally, the bit error probability of the BFSK DS-SSMA system with $(K-1)$ interfering users is approximated as (41) of [4]

$$P_{\mathrm{BFSK}}(K-1) = \frac{1}{2}\exp\left(-\frac{1}{4\sigma_{c,m}^2}\right) \quad (18)$$

for a receiver employing square-law detection.

For MFSK modulation, the variance of the interference inflicted upon the in-phase branch of the receiver conditioned on x_k and $\hat{x}_k$ is also given by (14), where x_k and $\hat{x}_k$ may take values from the set $\{0, \pm 2, \pm 4, \cdots, \pm 2(M-1)\}$ and $N-i$ should be replaced by $N(\log_2 M) - i(M-1)$. When independent information sequences are concerned, then $\mathrm{Var}[I_{c,m}(k,1;x_k,\hat{x}_k)]$ can be averaged by considering all possible values of x_k and $\hat{x}_k$ for M different frequency tones of the reference signal. However, it is cumbersome to do this, when M is a high integer. Hence, below we only aim for computing the upper bound and the lower bound of the average value of $\mathrm{Var}[I_{c,m}(k,1)]$.

From Fig. 2 we infer that $\mathrm{Var}[I_{c,m}(k,1)]$ is upper bounded, when the reference user activates frequency tones $f_c \pm \Delta$, since these two frequency tones are overlapped by the M number of legitimate frequency tones with a maximum overlapping area from both left and right because the frequency tones' sidelobes close to the main lobe have not decayed to a low value. Hence, the upper bound can be expressed as [see Appendix III]

$$\mathrm{Var}[I_{c,m}(k,1)]_U = \frac{1}{M}\left\{\frac{1}{3(N\log_2 M - i(M-1))} + \frac{N\log_2 M - i(M-1)}{2\pi^2 M^2 i^2} \cdot \left[1-\mathrm{sinc}\left(\frac{2\pi M i}{N\log_2 M - i(M-1)}\right)\right] + \sum_{\lambda=1}^{(M/2)-1} \frac{N\log_2 M - i(M-1)}{4\pi^2\lambda^2 i^2} \cdot \left[1-\mathrm{sinc}\left(\frac{4\pi\lambda i}{N\log_2 M - i(M-1)}\right)\right]\right\}. \quad (19)$$

By contrast, $\mathrm{Var}[I_{c,m}(k,1)]$ is lower bounded, when the reference user activates frequency tones $f_c \pm (M-1)\Delta$ since these two frequency tones are overlapped by the M number of legitimate frequency tones of the interfering signals with a minimum overlapping area because in this case the frequency tones' sidelobes have decayed to a lower value due to their higher frequency-domain separation. Therefore, the lower bound can be expressed as [see Appendix IV]

$$\mathrm{Var}[I_{c,m}(k,1)]_L = \frac{1}{M}\left\{\frac{1}{3(N\log_2 M - i(M-1))} + \sum_{\lambda=1}^{M-1} \frac{N\log_2 M - i(M-1)}{8\pi^2\lambda^2 i^2} \cdot \left[1-\mathrm{sinc}\left(\frac{4\pi\lambda i}{N\log_2 M - i(M-1)}\right)\right]\right\}. \quad (20)$$

Hence, the normalized variance of $Z_{c,m}$ for MFSK systems can be obtained according to the method employed above for BFSK with $(2E_b/N_0)^{-1}$ replaced by $(2E_b \log_2 M/N_0)^{-1}$. From (19) and (20), we can infer that the variance of $Z_{c,m}$ is upper bounded by

$$\sigma_{c,m}^2(U) = \left(\frac{2E_b\log_2 M}{N_0}\right)^{-1} + (K-1) \cdot \mathrm{Var}[I_{c,m}(k,1)]_U \quad (21)$$

and lower bounded by

$$\sigma_{c,m}^2(L) = \left(\frac{2E_b \log_2 M}{N_0}\right)^{-1} + (K-1) \cdot \mathrm{Var}[I_{c,m}(k,1)]_L. \tag{22}$$

Similarly, the variance of $Z_{s,m}$ can be computed, whose upper bound and lower bound become identical to (21) and (22). Finally, the upper bound and the lower bound of the bit error probabilities of the DS-SSMA system using MFSK modulation can be expressed as [4]

$$P_{\mathrm{MFSK}}(K-1)|_{\Re} = \frac{2^{h-1}}{M-1} \sum_{n=1}^{M-1} (-1)^{n+1} \binom{M-1}{n} \frac{1}{n+1} \cdot \exp\left(-\frac{n}{2(n+1)\sigma_{c,m}^2(\Re)}\right) \tag{23}$$

where $\Re$ represents U or L for the upper bound and the lower bound, respectively, and $h = \log_2 M$ is the number of bits per symbol.

III. SSMA System with DS-SFH Spread-Spectrum Signaling

DS systems exhibit high-antimultipath resistance [2], [13], [14], while frequency-hopping (FH) schemes are robust against partial-band jamming and the near–far problem [5], [6], [8], [9]. Hence, the hybrid form of DS-SFH spread-spectrum systems have received considerable interest in recent years [7]–[11], [13]–[15]. The performance of hybrid DS-SFH systems has been widely studied when multiple-access is concerned and when the channels are modeled as Gaussian or multipath fading with coherent or noncoherent receivers using different modulation schemes [7]–[11], [13]–[15]. Typically, in a hybrid system, the information symbols to be transmitted are first DS modulated, and then frequency hopped according to the frequency hopping pattern in order to form the transmitted signal. At the receiver the signal is demodulated similarly to receiving pure DS signals, except that the received signals need to be firstly dehopped to perform the appropriate frequency translation. For a hybrid DS-SFH SSMA system with BFSK or MFSK modulation, the transmitted signal can be expressed as

$$s_k(t) = \sqrt{2P} a_k(t) \cos\{2\pi[f_c + b_k(t)\Delta + f_k(t)]t + \theta_k(t) + \alpha_k(t)\}, \qquad 1 \le k \le K \tag{24}$$

where $\{f_k\}$ represents the frequency hopping pattern of user k, which is derived from a sequence $(f_j^{(k)})$ of frequencies, while $\alpha_k(t)$ represents the phase waveform introduced by the kth frequency hopper, which takes on the constant value $\alpha_j^{(k)}$ during the jth frequency hopping dwell time. The other parameters in (24) are the same as in (1).

In DS-SFH systems the DS bandwidth may be designed wider than the FH carrier spacing, in order to overlay the frequency hopping "slots," where the term slots was used to indicate the frequencies for frequency hopping, in order to distinguish them from the DS spread FSK tones. Laxpati and Gluck [15] have shown that by designing the DS-SFH system with overlapping frequency hopping slots, the system can improve its antimultitone jamming capability in comparison to those with nonoverlapping slots. Moreover, the results of [16] showed that, for a given total bandwidth and for a given modulation scheme, a hybrid DS-SFH system with optimized overlapping frequency slots outperformed the systems with nonoverlapping frequency slots, when they were compared in a multiple-access environment. Furthermore, in DS-SFH SSMA systems with BFSK or MFSK modulation, both the frequency hopping slots and the DS spread FSK tones can be designed with overlap and hence enhance the system performance by optimizing both overlaps. However, in this paper, we consider that the frequency hopping slots are nonoverlapping, while the DS spread FSK tones can overlap with each other.

For the above-mentioned system, which uses nonoverlapping frequency hopping slots and overlapping DS spread FSK tones, as in [8], we can compute the upper bound of the average error probability by first evaluating the conditional error probability given the number of "hits," where a "hit" implies that an interfering signal activates the same frequency hopping slots as the desired signal. Otherwise, when the reference user and the interfering user activate different frequency slots, there will be no multiple-access interference to the reference signal due to their nonoverlapping frequency hopping slots and due to using matched bandpass filters in the receiver. After obtaining the conditional BER, the unconditional error probability can be computed by averaging the conditional BER with respect to the distribution of hits.

When independent random signature sequences as well as random memoryless frequency hopping patterns are considered and the powers of the transmitted signals are assumed to be equal, we can write the upper bound of the bit error probability of the hybrid SSMA system with binary and M-ary FSK modulation as

$$P_H(\Theta FSK) \le \sum_{k=0}^{K-1} \binom{K-1}{k} P_h^k (1-P_h)^{K-1-k} \cdot P_{\Theta FSK}(k) \tag{25}$$

where $0 \le k \le K-1$ and P_h is the probability of a hit from an interfering signal. In asynchronous systems, the upper bound of P_h is given by [9]

$$P_h = \frac{2}{q} \tag{26}$$

where q is the number of frequency slots of the frequency hopping pattern. $P_{\Theta FSK}(k)$ in (25) denotes the conditional bit error probability, given that k hits occurred from the other $K-1$ interfering users, i.e., k out of $K-1$ users in the system activated the same frequency slot as the reference user, which is given by (18) or (23) with the related $K-1$ represented by k, while Θ represents "B" or "M," corresponding to Binary or M-ary FSK.

IV. Numerical Results and Concluding Remarks

In this section, the previously derived analytical performance formulas are numerically evaluated and compared with the conventional systems using nonoverlapping frequency hopping slots and nonoverlapping DS spread FSK tones, under the assumption that the systems employ the same total bandwidth.

In Figs. 4 and 5, the variance of the multiple-access interference term $I_{c,m}(k,1)$ from one of the interfering users was evaluated for a constant total system bandwidth of $W_s = 256 * 2T^{-1}$. The curves were plotted as a function of i, which controlled the amount of overlap, with parameters $N = (W_s/W_d) = 256$, and $M = 2$ (Fig. 4) as well as 8 (Fig. 5), where increasing i corresponded to higher spectral spacing and less overlap between frequency tones for a constant value of M and a constant total system bandwidth. For $M = 2$, the exact value of the variance was computed by (16). For $M = 8$, the upper bound and the lower bound of the variance were computed using (19) and (20). In addition, when $i \geq (N/M)(\log_2 M)$, the DS spread MFSK discussed in this paper is reduced to the conventional nonoverlapping scenario, if the spectral sidelobes are ignored, and the approximation to the multiple-access interference term of the MFSK DS-SSMA system given by $I \geq (1/3N \log_2 M)$ was plotted as a horizontal line in both Figs. 4 and 5, so that the performance of the system with overlapping frequency tones can be conveniently compared with that of the system having nonoverlapping frequency tones. From these results, we can conclude that the multiple-access interference can be minimized by controlling the overlap. Thus, the advantage of the increased spreading gain of the frequency tones outweighs the disadvantage of the increased multiple-access interference, provided that the optimum i values are used. For example, for the binary FSK DS-SSMA system with a total spreading gain of 256, the variance of the multiple-access interference inflicted upon the reference user is minimized by letting $\Delta \approx (85/T)$, as seen in Fig. 3. Apparently, two frequency tones are partially overlapped for $\Delta \approx (85/T) < (128/T)$, where $128/T$ is the minimum frequency spacing of the conventional MFSK DS system using nonoverlapping frequency tones. Furthermore, we observe, also from the analysis that the optimum value of i for DS-SSMA using BFSK, where $\mathrm{Var}[I_{c,m}(k,1)]$ achieves its minimum value is $N/3$, but the optimum value of i for DS-SSMA using MFSK achieving the minimum value of $\mathrm{Var}[I_{c,m}(k,1)]$ is not analytically tractable. These optimum values of i for different MFSK systems can be obtained by numerical computations.

In Fig. 6, we estimated the performance of various M-ary FSK DS-SSMA systems for a constant total system bandwidth. The upper bounds of $\mathrm{Var}[I_{c,m}(k,1)]$ were computed with $N = 512$ and $M = 2, 4, 8, 16, 32$. In the computations, the available maximum value of i for different values of M was obtained from $i \leq (N/M)(\log_2 M)$ for systems with overlapping frequency tones. From (19) to (22), we can infer that, for a constant system bandwidth, constant bit rate and bit energy, increasing M means that not only the ratio of $2E_b \log_2 M/N_0$ is increased, but also the spreading gain $[N \log_2 M - i(M-1)]$, which results in the reduction of the multiple-access interference. Hence, the figure illustrates that the variance of the multiple-access interference is gracefully improved, as the value of M increases.

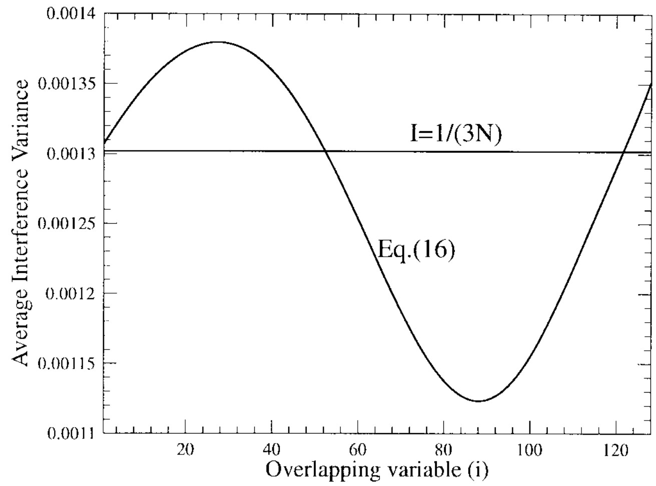

Fig. 4. Variance of $I_{c,m}(k,1)$ from (16) with a bandwidth expression of $N = 256$ and the binary case of $M = 2$.

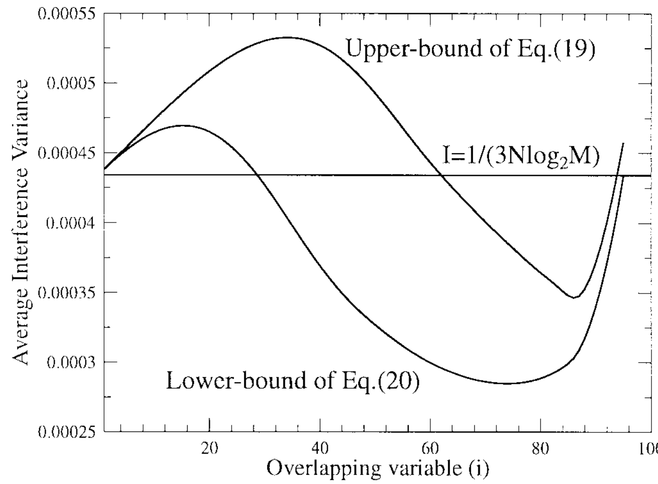

Fig. 5. The upper and lower bounds of the variance of $I_{c,m}(k,1)$ from (19) and (20) with $N = 256$ and $M = 8$.

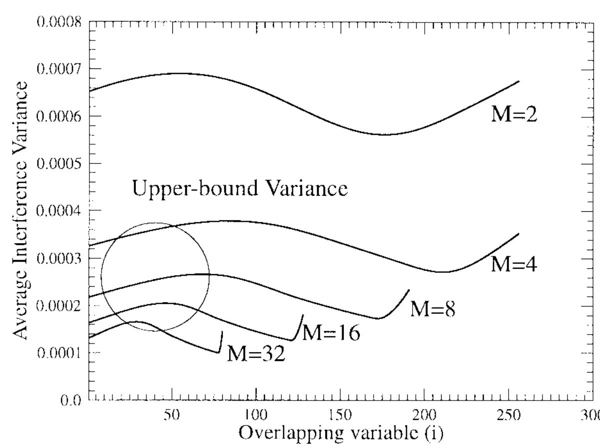

Fig. 6. Comparison of the variance of $I_{c,m}(k,1)$ with a total spreading gain of $N = 512$ and $M = 2, 4, 8, 16, 32$ computed from (16) for $M = 2$ and (19) otherwise.

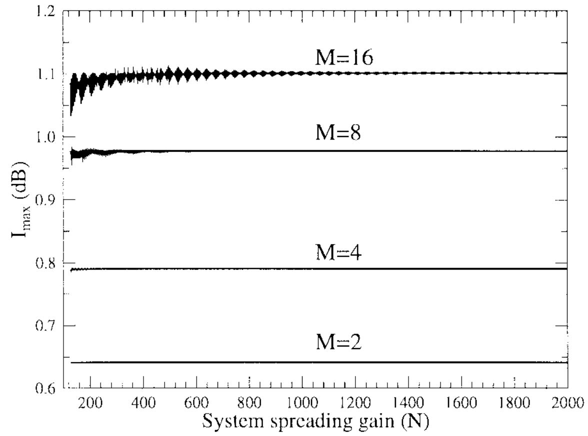

Fig. 7. Achievable multiple-access interference variance reduction of the proposed overlapping scheme over the conventional orthogonal scheme with $M = 2, 4, 8, 16$.

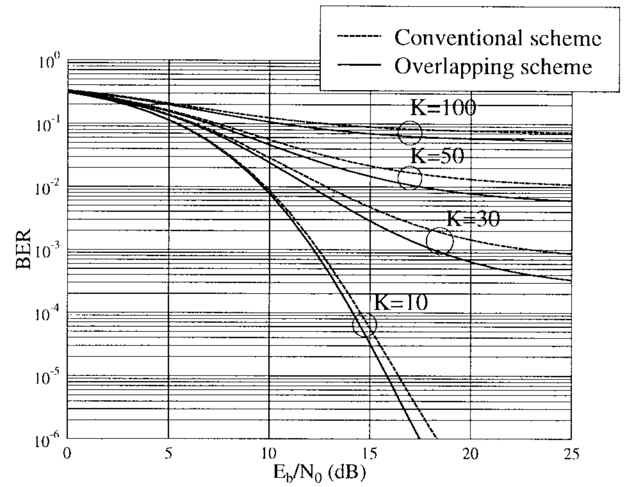

Fig. 8. Bit error probability of DS-SSMA using binary FSK and a total spreading gain of $N = 256$ for $K = 10, 30, 50$, and 100 users computed from (18).

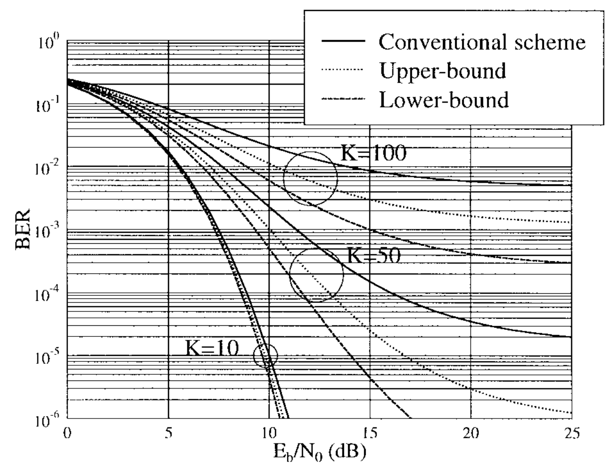

Fig. 9. Bit error probability of DS-SSMA using 8-ary FSK and a total spreading gain of $N = 256$ for $K = 10, 50$, and 100 users computed from (23).

Note that, since the sidelobes of the DS spread FSK frequency tones were included in our computations, from the results of Figs. 4–6 we observed that the multiuser interference power from the kth user at the point $i = (N/2)$ for BFSK, or at an overlap of $i = (N/M)(\log_2 M)$ for MFSK was higher than $1/3N$ for BFSK or $1/3N \log_2 M$ for MFSK, which was the approximated interference power inflicted by an interfering user, an estimate, which is frequently invoked in conventional DS-SSMA systems. Hence, we can argue that $1/3N$ and $1/3N \log_2 M$ are the approximated interference lower bounds of the conventional DS-SSMA systems using BFSK and MFSK with nonoverlapping DS spread FSK tones, respectively.

In Fig. 7, $I_{\max} = (1/3N \log_2 M)/I_U$ expressed in decibels was plotted versus N, the total system spreading gain, when different values of M were concerned. The term $1/3N \log_2 M$ was the approximated lower bound interference variance from an interfering signal of the conventional MFSK DS-SSMA system with nonoverlapping tones, as noted previously, while I_U was the optimum (minimum) upper bound of $\text{Var}[I_{c,m}(k,1)]$ for a given M and N, which was quantified by (19). It can be observed from the results of Fig. 7, and also from the results of Figs. 4–6 that the achievable multiple-access interference reduction due to using overlapping tones is on the order of 0.5–1.5 dB for a single interfering signal, depending on the value of M, rather than on the system's total bandwidth. As observed in the figure, there is no improvement, as the total system spreading gain, N increases. This is particularly pronounced for the systems employing a sufficiently high DS spreading bandwidth, or in other words, when the value of N_B or N_M computed from N was high enough. In addition, we can conclude that the BER performance improvement due to increasing the total system bandwidth (or N) is the same, whether overlapping frequency tones or conventional nonoverlapping frequency tones are employed by the MFSK DS-SSMA systems.

In Fig. 8, the BER versus the bit energy-to-noise ratio of E_b/N_0 was plotted for the binary FSK DS-SSMA system with a bandwidth expansion factor of $N = 256$ and for $K = 10, 30, 50, 100$ users. In Fig. 9, the upper bound and the lower bound bit error probabilities of the M-ary FSK DS-SSMA system using overlapping frequency tones, and that of the conventional system using nonoverlapping tones were plotted versus E_b/N_0 with the above parameters of $N = 256, K = 10, 50, 100$ for an 8-ary FSK system. Note that the upper bound and the lower bound of the bit error probabilities were computed by assuming that the frequency tones were optimally overlapped and hence the variance of multiple-access interference was minimized. As expected, the results show that the bit error performances degrade, as the number of active users K increases and the proposed system with overlapping frequency tones achieves a lower BER, than the conventional system using nonoverlapping tones, provided that the frequency tones are optimally overlapped. Taking an 8-ary FSK system with $K = 50$ users as an example, the DS-SSMA system with MFSK tones optimally overlapped needs 3–4 dB less SNR per bit, than the conventional system in order to achieve the same BER of 1×10^{-4}.

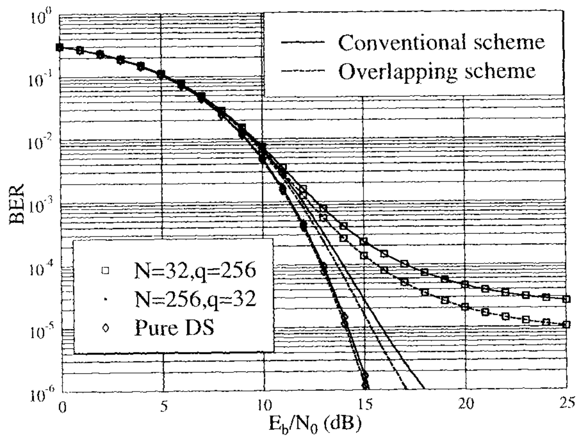

Fig. 10. Bit error probability of DS-SFH SSMA using binary FSK, supporting $K = 100$ users, and for $qN = 8192$ in various combinations of the number of frequency slots q and spreading gain N computed from (25) for the hybrid DS-SFH case and (18) for the pure DS case.

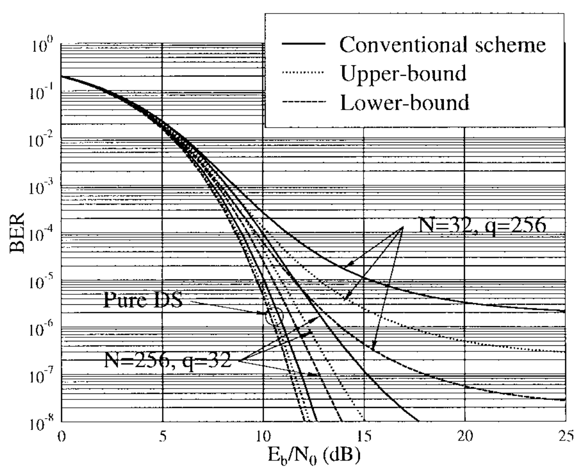

Fig. 11. Bit error probability of DS-SFH SSMA using 8-ary FSK, supporting $K = 300$ users, and for $qN = 8192$ in various combinations of the number of frequency slots q and spreading gain N computed from (25) for the hybrid DS-SFH case and (23) for the pure DS case.

In Fig. 10, we plotted the BER of a binary FSK scheme, while in Fig. 11 the upper bound and lower bound BER of an 8-ary FSK hybrid DS-SFH system. The figures were plotted versus bit energy-to-noise ratio E_b/N_0, assuming $K = 100$ (Fig. 10), $K = 300$ (Fig. 11), and a constant product of $qN = 8192$, where q was the number of frequency slots of the frequency hopping pattern, while N was the frequency band expansion factor of each frequency slot. Note that, $N = 8192, q = 1$ indicated a pure DS-SSMA system, which marked as pure DS in the figures. From the results we can infer that, as shown in the pure DS-SSMA case of Figs. 8 and 9, the DS-SFH SSMA system with the MFSK tones optimally overlapped achieves a lower BER than the conventional DS-SFH SSMA system using nonoverlapping tones. Moreover, the bit error performance improvement depends mainly on the DS spreading. This conclusion was shown in numerous papers [8]–[11] related to the analysis of hybrid DS-SFH SSMA systems, under the assumption of perfect power control.

In conclusion, we provided explicit formulas for the performance of BFSK and MFSK DS-SSMA as well as for DS-SFH SSMA with nonoverlapped and partially overlapped frequency tones. The optimum frequency overlap was also determined. The approach can be used for direct sequence spread-spectrum systems with MFSK modulation to minimize the partial band jamming by optimizing the frequency tones' overlap, and the work can be extended to the analysis of multicarrier CDMA systems, which use overlapping frequency bands.

APPENDIX I
SIMPLIFICATION OF THE IN-PHASE COMPONENT $Z_{c,m}$

This Appendix shows, how to obtain and simplify the in-phase component of (6). We assume that the data bit b_0 is sent by the reference user in the first bit period, and the phase angle is φ_m when $b_0 = m$. Then substituting $r(t)$ from (4) into (6), and simplifying it, we obtain

$$\begin{aligned} Z_{c,m} &= \sqrt{\frac{P}{2}}\,T\Bigg\{\delta(b_0,m)\,\cos\varphi_m + \left(\sqrt{\frac{P}{2}}\,T\right)^{-1} \\ &\quad\cdot \int_0^T n(t)a_1(t)\,\cos[2\pi(f_c+m\Delta)t]\,dt \\ &\quad+ \sum_{k=2}^{K}\frac{1}{T}\int_0^T a_1(t)a_k(t-\tau_k) \\ &\quad\cdot \cos\{2\pi[b_k(t-\tau_k)-m]\Delta t+\varphi_k(t)\}\,dt\Bigg\}. \end{aligned} \tag{27}$$

Let us assume that $b_{-1}^{(k)}$ and $b_0^{(k)}$ are the data bits transmitted by the kth user in the time interval $[0,\tau_k]$ and $[\tau_k, T]$ relative to the reference signal, and $\varphi_{-1}^{(k)}$ and $\varphi_0^{(k)}$ are the phase angles of the kth signal for transmitting the data bits $b_{-1}^{(k)}$ and $b_0^{(k)}$. The third term of (27) can be expressed by the partial correlation method [3]. Hence, (27) can be written as

$$\begin{aligned} Z_{c,m} &= \sqrt{\frac{P}{2}}\,T\Bigg\{\delta(b_0,m)\,\cos\varphi_m + \left(\sqrt{\frac{P}{2}}\,T\right)^{-1} \\ &\quad\cdot \int_0^T n(t)a_1(t)\,\cos[2\pi(f_c+m\Delta)t]\,dt \\ &\quad+ \sum_{k=2}^{K}\Bigg[\frac{1}{T}\int_0^{\tau_k} a_1(t)a_k(t-\tau_k) \\ &\quad\cdot \cos[2\pi(b_{-1}^{(k)}-m)\Delta t+\varphi_{-1}^{(k)}]\,dt \\ &\quad+ \frac{1}{T}\int_{\tau_k}^{T} a_1(t)a_k(t-\tau_k) \\ &\quad\cdot\,\cos[2\pi(b_0^{(k)}-m)\Delta t+\varphi_0^{(k)}]\,dt\Bigg]\Bigg\}. \end{aligned} \tag{28}$$

APPENDIX II
VARIANCES OF THE PARTIAL CROSS-CORRELATION FUNCTIONS

In this Appendix, we will analyze the continuous partial cross-correlation functions of (11) and (12) and show how to compute their expectations by assuming that the phase angles and time delays are modeled as mutually independent random variables, each of which is uniformly distributed over the appropriate interval. We can find the variance of the multiple-access interference term of (11) and (12) by computing the expectations of the square of

$$R(\tau, x, \varphi) = \int_0^{\tau} a_i(t)a_k(t-\tau)\cos(2\pi x\Delta t + \varphi)\, dt \quad (29)$$

and

$$\hat{R}(\tau, \hat{x}, \varphi) = \int_{\tau}^{T} a_i(t)a_k(t-\tau)\cos(2\pi \hat{x}\Delta t + \varphi)\, dt. \quad (30)$$

Below we only analyze the expectation of $R(\tau, x, \varphi)$, the expectation of $\hat{R}(\tau, \hat{x}, \varphi)$ can be obtained following the same approach.

From (29) we have

$$\begin{aligned} R(\tau, x, \varphi) = &\sum_{j=0}^{l} a_i(j)a_k(G-l-1+j) \\ &\cdot \int_{jT_c}^{jT_c+\tau-lT_c} \cos(2\pi x\Delta t + \varphi)\, dt \\ &+ \sum_{j=0}^{l-1} a_i(j)a_k(G-l+j) \\ &\cdot \int_{jT_c+\tau-lT_c}^{(j+1)T_c} \cos(2\pi x\Delta t + \varphi)\, dt \end{aligned} \quad (31)$$

where $lT_c \le \tau < (l+1)T_c$ and $G = N_B$ or N_M for BFSK or MFSK, respectively, representing the number of chips per data symbol period or the bandwidth-expansion factor per spread FSK tone. Upon evaluating the integrals in (31), we find that

$$\begin{aligned} R(\tau, x, \varphi) = &(\tau - lT_c)\,\mathrm{sinc}[\pi x\Delta(\tau - lT_c)] \\ &\cdot \sum_{j=0}^{l} a_i(j)a_k(G-l-1+j)\cos\beta_{ka}(j,\tau) \\ &+ [(l+1)T_c - \tau]\,\mathrm{sinc}[\pi x\Delta((l+1)T_c - \tau)] \\ &\cdot \sum_{j=0}^{l-1} a_i(j)a_k(G-l+j)\cos\beta_{kb}(j,\tau) \end{aligned} \quad (32)$$

where

$$\begin{aligned} \beta_{ka}(j,\tau) &= \pi x\Delta(2jT_c + \tau - lT_c) + \varphi \\ \beta_{kb}(j,\tau) &= \pi x\Delta((2j+1)T_c + \tau - lT_c) + \varphi. \end{aligned}$$

The expression of $R^2(\tau, x, \varphi)$ is computed from (32) as

$$\begin{aligned} &R^2(\tau, x, \varphi) \\ &= (\tau - lT_c)^2\,\mathrm{sinc}^2[\pi x\Delta(\tau - lT_c)] \\ &\quad\cdot \Bigg[\sum_{j=0}^{l} a_i^2(j)a_k^2(G-l-1+j)\cos^2\beta_{ka}(j,\tau) \\ &\quad + \sum_{r=0}^{l}\sum_{\substack{s=0\\ s\neq r}}^{l} a_i(r)a_k(G-l-1+r) \\ &\quad \cdot a_i(s)a_k(G-l-1+s) \\ &\quad \cdot \cos\beta_{ka}(r,\tau)\cos\beta_{ka}(s,\tau)\Bigg] \\ &\quad + (\tau - lT_c)[(l+1)T_c - \tau]\,\mathrm{sinc}[\pi x\Delta(\tau - lT_c)] \\ &\quad \cdot \mathrm{sinc}[\pi x\Delta((l+1)T_c - \tau)] \\ &\quad \cdot \Bigg[\sum_{r=0}^{l}\sum_{s=0}^{l-1} a_i(r)a_k(G-l-1+r) \\ &\quad \cdot a_i(s)a_k(G-l+s)\cos\beta_{ka}(r,\tau) \\ &\quad \cdot \cos\beta_{kb}(s,\tau) + \sum_{r=0}^{l-1}\sum_{s=0}^{l} a_i(r)a_k(G-l+r)a_i(s) \\ &\quad \cdot a_k(G-l-1+s)\cos\beta_{kb}(r,\tau)\cos\beta_{ka}(s,\tau)\Bigg] \\ &\quad + [(l+1)T_c - \tau]^2\,\mathrm{sinc}^2[\pi x\Delta((l+1)T_c - \tau)] \\ &\quad \cdot \Bigg[\sum_{r=0}^{l-1} a_i^2(r)a_k^2(G-l+r)\cos^2\beta_{kb}(r,\tau) \\ &\quad + \sum_{r=0}^{l-1}\sum_{\substack{s=0\\ s\neq r}}^{l-1} a_i(r)a_k(G-l+r)a_i(s)a_k(G-l+s) \\ &\quad \cdot \cos\beta_{kb}(r,\tau)\cos\beta_{kb}(s,\tau)\Bigg]. \end{aligned} \quad (33)$$

Since we assumed that random signature sequences are concerned in our system, $a_i(r)$ is statistically independent of $a_k(s)$ when $i \neq k$ or $r \neq s$, and takes values of $+1$ or -1 with equal probability, hence, taking the expectation of $R^2(\tau, x, \varphi)$ with respect to the $a_i(r)$, $a_k(s)$, φ and τ we obtain

$$\begin{aligned} \gamma(x) &= \mathrm{E}[R^2(\tau, x, \varphi)] \\ &= \frac{1}{2T}\sum_{l=0}^{G-1}\int_{lT_c}^{(l+1)T_c} \{(\tau - lT_c)^2 \\ &\quad \cdot \mathrm{sinc}^2[\pi x\Delta(\tau - lT_c)]\cdot(l+1) + [(l+1)T_c - \tau]^2 \\ &\quad \cdot \mathrm{sinc}^2[\pi x\Delta((l+1)T_c - \tau)]\cdot l\}\, d\tau. \end{aligned} \quad (34)$$

Upon evaluating the resulting integral we find that

$$\gamma(x) = \frac{G}{4\pi x^2\Delta^2}\,[1 - \mathrm{sinc}(2\pi x\Delta T_c)]. \quad (35)$$

The corresponding expression for $\hat{\gamma}(\hat{x})$ can be obtained by evaluating the expectation of $\hat{R}^2(\tau, \hat{x}, \varphi)$ in the same way, as for $R^2(\tau, x, \varphi)$, and the result is expressed as

$$\hat{\gamma}(\hat{x}) = \frac{G}{4\pi \hat{x}^2\Delta^2}[1 - \mathrm{sinc}(2\pi \hat{x}\Delta T_c)]. \quad (36)$$

APPENDIX III
UPPER BOUND VARIANCE OF AN INTERFERENCE SIGNAL

This Appendix shows how to compute the upper bound variance of an MFSK DS-modulated interference signal inflicted upon the reference signal. For an MFSK DS-SSMA overlapping system, the maximum interference is experienced by the frequency tones $f_c \pm \Delta$ since they are overlapped by the other frequency tones with the maximum overlapping area for a given value of i and consequently experience maximum interference from the interfering signals.

When an interfering signal activates the specific frequency tones, which will inflict interference upon the desired signal, x_k and $\hat{x}_k$ can take values corresponding to the set $m = \{-M, -(M-2), \cdots, -2, 0, 2, \cdots, (M-2), M\}$ according to $x_k = b_{M,-1}^{(k)} - m$ and $\hat{x}_k = b_{M,0}^{(k)} - m$ for the given frequency tones of $f_c \pm \Delta$ of the reference user. Hence, when an independent information sequence is considered, the upper bound of $I_{c,m}(k,1)$ can be expressed as

$$\begin{aligned}\mathrm{Var}[I_{c,m}(k,1)]_U &= \frac{1}{M}\Bigg\{\mathrm{Var}[I_{c,m}(k,1;0,0)] \\ &\quad + \mathrm{Var}[I_{c,m}(k,1;M,M)] \\ &\quad + 2\cdot\sum_{\lambda=1}^{(M/2)-1}\mathrm{Var}[I_{c,m}(k,1;2\lambda,2\lambda)]\Bigg\}.\end{aligned} \tag{37}$$

Upon simplification using (14), we finally obtain

$$\begin{aligned}\mathrm{Var}[I_{c,m}(k,1)]_U &= \frac{1}{M}\Bigg\{\frac{1}{3(N\log_2 M - i(M-1))} \\ &\quad + \frac{N\log_2 M - i(M-1)}{2\pi^2M^2i^2} \\ &\quad \cdot\left[1-\mathrm{sinc}\left(\frac{2\pi Mi}{N\log_2 M - i(M-1)}\right)\right] \\ &\quad + \sum_{\lambda=1}^{(M/2)-1}\frac{N\log_2 M - i(M-1)}{4\pi^2\lambda^2i^2} \\ &\quad \cdot\left[1-\mathrm{sinc}\left(\frac{4\pi\lambda i}{N\log_2 M - i(M-1)}\right)\right]\Bigg\}.\end{aligned} \tag{38}$$

APPENDIX IV
LOWER BOUND VARIANCE OF AN INTERFERENCE SIGNAL

Here, the lower bound of the interference variance from an interfering signal is computed. Apparently, the lower bound is achieved when the desired signal activates frequency tones $f_c \pm (M-1)\Delta$, as argued in Section II-D. Now, from Appendix III, we can deduce that interference is inflicted upon the reference signal, where x_k and $\hat{x}_k$ take values corresponding to the set $m = \{0, 2, 4, \cdots, 2(M-1)\}$ and where $x_k = b_{M,-1}^{(k)} - m$ and $\hat{x}_k = b_{M,0}^{(k)} - m$. Hence, when an independent information sequence is considered, the lower bound can be expressed as

$$\mathrm{Var}[I_{c,m}(k,1)]_L = \frac{1}{M}\cdot\sum_{\lambda=0}^{M-1}\mathrm{Var}[I_{c,m}(k,1;2\lambda,2\lambda)]. \tag{39}$$

After simplification, we obtain

$$\begin{aligned}\mathrm{Var}[I_{c,m}(k,1)]_L &= \frac{1}{M}\Bigg\{\frac{1}{3(N\log_2 M - i(M-1))} \\ &\quad + \sum_{\lambda=1}^{M-1}\frac{N\log_2 M - i(M-1)}{8\pi^2\lambda^2i^2} \\ &\quad \cdot\left[1-\mathrm{sinc}\left(\frac{4\pi\lambda i}{N\log_2 M - i(M-1)}\right)\right]\Bigg\}.\end{aligned} \tag{40}$$

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REFERENCES

[1] M. K. Simon *et al.*, *Spread Spectrum Communications*. Baltimore, MD: Computer Science, 1985.
[2] A. J. Viterbi, *CDMA: Principles of Spread Spectrum Communication*. New York: Art & Licensing, 1995.
[3] M. B. Pursley, "Performance evaluation for phase-coded spread-spectrum multiple-access communication—Part I: System analysis," *IEEE Trans. Commun.*, vol. COM-25, pp. 795–799, Aug. 1977.
[4] E. Geraniotis, "Performance of noncoherent direct-sequence spread-spectrum multiple-access communications," *IEEE J. Select. Areas Commun.*, vol. SAC-3, pp. 687–694, Sept. 1985.
[5] M. B. Pursely and W. E. Stark, "Performance of Reed-Solomon coded FH SS communications in partial-band interference," *IEEE Trans. Commun.*, vol. COM-33, pp. 767–774, Aug. 1985.
[6] J. S. Lee and L. E. Miller, "Error performance analyses of differential phase-shift-keyed/frequency-hopping spread-spectrum communication systems in the partial-band-jamming environment," *IEEE Trans. Commun.*, vol. COM-30, pp. 943–952, May 1982.
[7] E. Geraniotis, "Multiple-access capability of frequency-hopped spread-spectrum revisited: An analysis of the effect of unequal power levels," *IEEE Trans. Commun.*, vol. 38, pp. 1066–1077, July 1990.
[8] ——, "Noncoherent hybrid DS-SFH spread-spectrum multiple-access communications," *IEEE Trans. Commun.*, vol. COM-34, pp. 862–872, Sept. 1986.
[9] T. Vlachos and E. Geraniotis, "Performance study of hybrid spread-spectrum random-access communications," *IEEE Trans. Commun.*, vol. 39, pp. 975–985, June 1991.
[10] M. Soroushnejad and E. Geraniotis, "Performance comparison of different spread-spectrum signaling schemes for cellular mobile radio networks," *IEEE Trans. Commun.*, vol. 40, pp. 947–956, May 1992.
[11] Z. Tan and I. F. Blake, "Performance analysis of noncoherent DS-SFH spread spectrum multiple access for indoor wireless communications," in *Proc. IEEE Military Commun. Conf.*, 1992, pp. 851–855.
[12] L. Vandendorpe, "Multitone spread spectrum multiple access communications system in a multipath Rician fading channel," *IEEE Trans. Veh. Technol.*, vol. 44, pp. 327–337, May 1995.
[13] J. Wang and M. Moeneclaey, "Hybrid DS/SFH spread-spectrum multiple access with predetection diversity and coding for indoor radio," *IEEE Trans. J. Select. Areas Commun.*, vol. 10, pp. 705–713, May 1992.
[14] ——, "Hybrid DS/SFH-SSMA with predetection diversity and coding over indoor radio multipath Rician-fading channels," *IEEE Trans. Commun.*, vol. 40, pp. 1654–1662, Oct. 1992.
[15] M. A. Laxpati and W. Gluck, "Optimization of a hybrid SFH/DS MFSK link in the presence of worst case multitone jamming," *IEEE Trans. Commun.*, vol. 43, pp. 2118–2125, June 1995.
[16] L. L. Yang, "On the study of hybrid DS-SFH spread-spectrum communication systems with bandwidth overlapping," Ph.D. dissertation, Northern Jiaotong Univ., Beijing, China, Apr. 1997.

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