APPLICATION OF FINITE ELEMENT MODELLING IN LVDT DESIGN

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Abstract

Application of finite-element modelling of magnetic fields in a linear variable
differential transformer (LVDT) is demonstrated and electromagnetics of such devices
briefly discussed. A finite-element aided design of a particular LVDT, built and tested at
IWT, Southampton, is used to illustrate practical aspects of such modelling.

1. Introduction

The linear variable differential transformer (LVDT) is the most common magnetic
sensor used in a wide range of applications, and its design and characteristics are
very well known. In this paper, we focus on the design and analysis of LVDTs using
finite-element methods.

2. LVDT design

The analysis and design of LVDTs is usually based on a semi-empirical approach. In
particular, the electromagnetics of these devices tends to be over-simplified. New models
and approaches are usually interpreted from existing designs. The impedances of the
primary winding is then calculated using standard formulae and the voltage ratio is
estimated. A series of models are built and tested before the final design is achieved.

Amongst various LVDT ratings and characteristics, the most important are:
 the linear range - which is usually stated as plus or minus (x) full-scale
 displacement; the sensitivity - which is usually stated as output voltage per unit
 displacement per input voltage; the linearity - maximum deviation from the output of a
 linear straight line, resolution and repeatability - which are theoretically infinite but in
 practice will depend on the associated electronic equipment. Another important
 characteristic of an LVDT is the zero. Ideally, the output of an LVDT at zero
 position should be zero. However, quadrature voltages and harmonic components of
 the excitation source do not cancel out. Quadrature voltages result from the difference
 in winding capacitance between the primary and secondary. The secondary outputs do
 not have exactly the same phase angle; therefore, they will not completely cancel out at
 any zero position. The above LVDT characteristics are usually emulated experimentally.

Simplified mathematical analysis of an LVDT assumes an idealized distribution of
 the linkage flux, which ignores any non-uniformity of the field strength along the axis of
 the coil and neglects end effects. Approximate equations are derived for example in Reference
 [1]. For a sinusoidal primary excitation current (mu) of I at a frequency f, the
 differential output voltage may be expressed as

$$v_{oc} = k, \sin(\omega t + \phi)$$

(1)
where \( x \) is the core displacement. The coefficient of non-linearity, \( J - x \gamma k \), in equation (1) can be evaluated from:
\[
\gamma = \frac{1}{J - x^2} \frac{b}{a}
\]
where \( J \) is the core length and \( b \) is the axial separation between secondary windings. The sensitivity of the LVDT is given by:
\[
\frac{\Delta x}{L_{x}} \approx \frac{\Delta x}{L_{y}} = \frac{1}{L_{y}} \frac{1}{L_{x}} \frac{1}{L_{z}} \frac{b}{a}
\]
where \( L_{y} \) and \( L_{y} \) are the number of turns on the primary and secondary, respectively, \( a \) is the length of the core, \( d \) is the length of the secondary, \( r_{1} \) is the core radius and \( r_{2} \) is the inner radius of the outer case. The accuracy of the above equations is rather poor as is demonstrated in Fig. 3.

3. Finite-element modelling

The benefit of using finite-element modeling as an aid to the LVDT design are three-fold. First, it provides an in-depth understanding of the magnetic processes. Secondly, accuracy predictions can be made in terms of the value of reflected impedance of the windings, degree of non-linearity of the response, sensitivity, etc. Lastly, any changes to the geometry, dimensions of the material properties can be incorporated quickly and efficiently. To that extent the FE model must be truly interactive and computing time short and to achieve this some simplifications may have to be made. The effects of such simplifications on the overall accuracy of the model may, however, be studied and assessed independently in advance of the design stage.

This section shows a finite-element aided design of a particular LVDT which has been built and tested at LSTF, Southampton. Fig. 1 shows a typical finite-element mesh used in calculation. This mesh is a result of overall preliminary tests performed to minimize local skin losses. Both the core and the casing are magnetically non-linear and electrically conducting, whereas the core tube is non-magnetic but conducting.

Fig. 1. FE mesh used in calculations.

Magnetic field distributions for different core displacements are shown in Fig. 2. From these field solutions global parameters such as coil impedances and induced voltages can easily be found and thus the most important LVDT ratings established. One parameter which cannot be evaluated, as it is not included in the above model, is the null voltage (see section 2). The magnitude of the combined quadrature and harmonic voltages is usually insignificant; in other cases a simple corrective circuit will eliminate them from the output signal. Nevertheless, these unaccounted voltages have been identified as the main source of error in predicting the LVDT parameters from the finite element calculations.
Fig. 3: Magnetic field distribution for different core displacements (with and without the core tube).
4. Discussion of results and conclusions

The induced voltages in both secondary windings may be found from the finite-element solution by integrating the solution pointwise:

\[ V = \int_{-\text{length}}^{\text{length}} \frac{d\phi}{d\theta} \, d\theta \]

where \( V \) is the core of the winding and integration is carried over this area. Other quantities, such as the inductance, have also been calculated using appropriate volume integration. Comparison with experiment has shown excellent agreement. In particular, Fig. 3 shows the predicted and measured variation of the output signal as a function of the core displacement. Two different core lengths have been used (15.5 mm and 8.2 mm) and calculations and measurements performed with and without the core tube. Values obtained using eqn. 1 are also shown. Notice, however, that eqn. 1 fails completely for the short core length, and that it takes no account of the presence of the core tube.

![Graphs showing core displacement vs. output voltage](image)

**Fig. 3.** Output voltage as a function of core displacement

- a = long core, simulation
- b = long core, FE calculation
- c = long core, linear
- d = short core, formula (3)
- e = short core, measurement
- f = short core, FE calculation
- g = short core, linear

The results show a slight (about 3%) but consistent overestimate of the output voltage using the finite-element modelling, whereas the shape of the currents and hence the degree of nonlinearity of the LVDT response are predicted almost perfectly. The effect of the core tube is relatively small in this case (a small reduction of output voltage); nevertheless, in practice, it may be modelled very accurately. Thus, the finite-element modelling has been found extremely helpful in its aid in the design and development of LVDTs. Most of the important outputs and characteristics can be predicted accurately and changes in the geometry and material properties can be easily accommodated.

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**References**