COMPEL - The International Journal for Computation and Mathematics in Electrical and Electronic Engineering
Vol 11, No 1, 73-76 (c) James & James Science Publishers Ltd

APPLICATION OF FINITE ELEMENT MODELLING IN LVDT DESIGN

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Abstract

Application of *finite-element* modelling of magnetic fields in a *linear variable differential transformer* (LVDT) is demonstrated and electromagnetics of such devices briefly discussed. A *finite-element* aided design of a particular LVDT, built and tested at USITT, Southampton, is used to illustrate practical aspects of such modelling.

1. Introduction

The *linear variable differential transformer* (LVDT) is the most common mutual inductance passive transducer element. It produces an electrical output proportional to the displacement of a separate movable core. An AC carrier excitation is applied to the primary. Two identical secondaries, symmetrically spaced from the primary, are connected externally in a series-opposing circuit. Motion of the non-contacting magnetic core varies the mutual inductance of each secondary to the primary, which determines the voltage induced from the primary to each secondary. If the core is centred between the secondary windings, the voltage induced in each secondary is identical and 180^{o} out-of-phase, so there is no output. If the core is moved off centre, the mutual inductance of the primary with one secondary will be greater than with the other, and a differential voltage will appear across the secondaries in series. For off-centre displacement within the range of operation, this voltage is essentially a linear function of displacement.

The LVDT has many commendable features that make it useful for a wide variety of applications. Some of these features are unique to the LVDT and are not available in any other transducer. These features are: frictionless measurement, virtually infinite mechanical life, core and coil separation, infinite resolution, null repeatability, input/output isolation.

2. LVDT design

The analysis and design of LVDT's is usually based on a semi-empirical approach. In particular, the electromagnetics of these devices tends to be over-simplified. New models and geometries are usually interpolated from existing designs. The impedance of the primary winding is then calculated using standard formulae and the voltage ratio is estimated. A series of models are built and tested before the final design is achieved.

Amongst various LVDT ratings and characteristics, the most important are: the *nominal linear range* - which is usually stated as plus or minus (±) full-scale displacement; the *sensitivity* - perhaps best stated in terms of output voltage per unit displacement per input voltage; the *linearity* - maximum deviation of the output from a best-fit straight line; *resolution* and *repeatability* - which are theoretically infinite but in practice will depend on the associated electronic equipment. Another important characteristic of an LVDT is the *null voltage*. Ideally, the output of an LVDT at null core position should be zero. However, quadrature voltages and harmonic components of the excitation source do not cancel out. Quadrature voltages result from the difference in winding capacitance between the primary and each secondary. The secondary outputs do not have exactly the same phase angle; therefore, they will not completely cancel out at any core position. The above LVDT characteristics are usually established experimentally.

Simplified mathematical analysis of an LVDT assumes an idealized distribution of the leakage flux, which ignores any non-uniformity of the field strength along the axis of the coil and neglects end effects. Appropriate equations are derived for example in Reference [1]. For a sinusoidal primary excitation current (rms) of I at a frequency f, the differential output voltage may be expressed as

$$\Delta v = k_1 \, x \, (1 - x^2/k_2) \tag{1}$$

where x is the core displacement. The coefficient of non-linearity, $(1-x^2/k_2)$, in equation (1) can be evaluated from:

$$k_2 = \tfrac{1}{4}\,(l^2-b^2) \tag{2}$$

where l is the core length and b is the axial separation between secondary windings. The sensitivity of the LVDT is given by:

$$k_1 = \frac{4\pi\mu_0 f l N_p N_s k_2}{dc\,\ln(r_2/r_1)} \tag{3}$$

where N_p and N_s are the number of turns on the primary and secondary, respectively, c is the length of the core, d is the length of the secondary, r_1 is the core radius and r_2 is the inner radius of the outer case. The accuracy of the above equations is rather poor as is demonstrated in Fig. 3.

3. Finite-element modelling

The benefits of using *finite-element modelling* as an aid to the LVDT design are three-fold. First, it provides an in-depth understanding of the magnetic processes. Secondly, accurate predictions can be made in terms of the value of reflected impedance of the windings, degree of non-linearity of the response, sensitivity, etc. Lastly, any changes to the geometry, dimensions and material properties can be incorporated quickly and efficiently. To that extent the FE model must be truly interactive and computing times short and to achieve this some simplifications may have to be made. The effects of such simplifications on the overall accuracy of the model may, however, be studied and assessed independently in advance of the design stage.

This section shows a finite-element aided design of a particular LVDT which has been built and tested at USITT, Southampton. Fig. 1. shows a typical finite-element mesh used in calculations. This mesh is a result of several preliminary tests performed to minimise local solution errors. Both the core and the casing are magnetically non-linear and electrically conducting, whereas the core tube is non-magnetic but conducting.

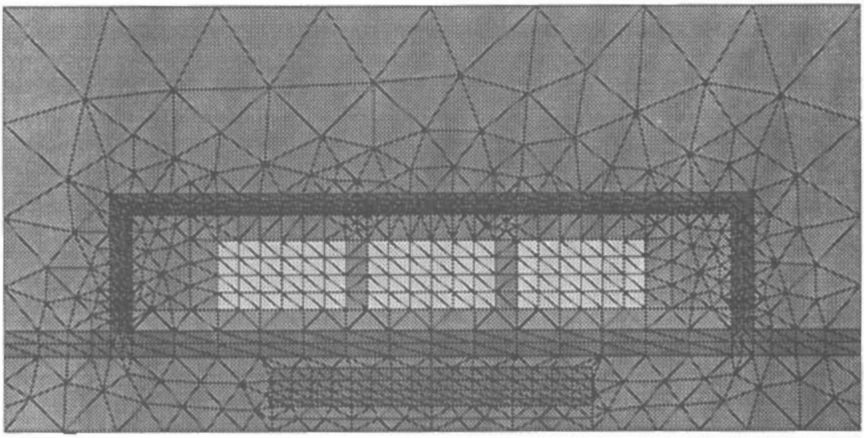

Fig. 1. *FE mesh used in calculations.*

Magnetic field distributions for different core displacements are shown in Fig. 2. From these field solutions global parameters such as coil impedances and induced voltages can easily be found and thus the most important LVDT ratings established. One parameter which cannot be evaluated, as it is not included in the above model, is the *null voltage* (see section 2). The magnitude of the combined quadrature and harmonic voltages is usually insignificant; in other cases a simple corrective circuit will eliminate them from the output signal. Nevertheless, these unaccounted voltages have been identified as the main source of error in predicting the LVDT parameters from the finite element calculations.

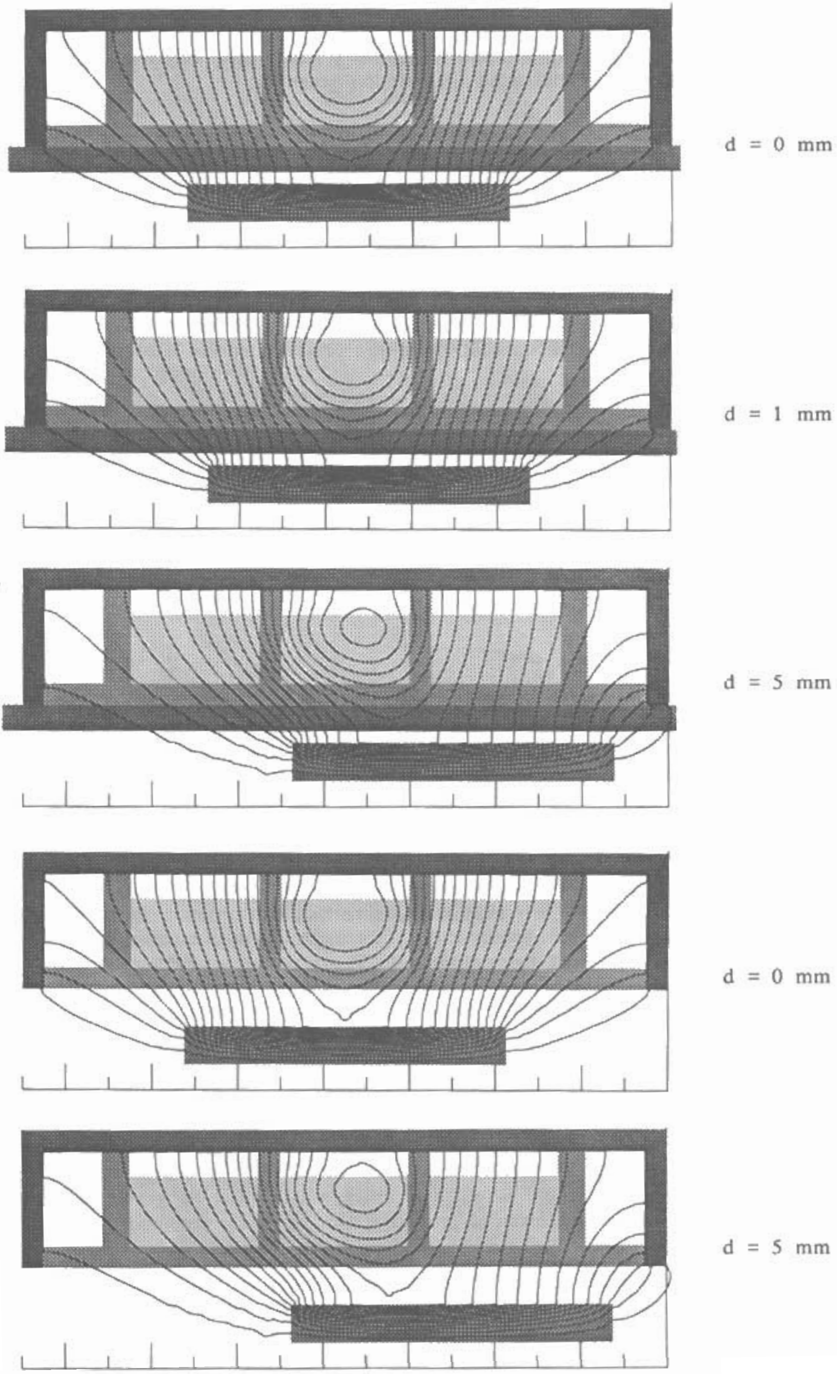

Fig. 2. *Magnetic field distribution for different core displacements (with and without the core tube)*

4. Discussion of results and conclusions

The induced voltages in both secondary windings may be found from the finite-element solution by integrating the solution potential:

$$V = 4\pi^2 \times f \times \frac{N}{S} \int_S r \, A \, ds$$

where S is the area of the winding and integration is carried over this area. Other quantities, such as the inductance, have also been calculated using appropriate volume integration. Comparison with experiment has shown excellent agreement. In particular, Fig. 3 shows the predicted and measured variation of the output signal as a function of the core displacement. Two different core lengths have been used (15.5 mm and 8.2 mm) and calculations and measurements performed with and without the core tube. Values obtained using eqn. 1 are also shown. Notice, however, that eqn. 1 fails completely for the short core length, and that it takes no account of the presence of the core tube.

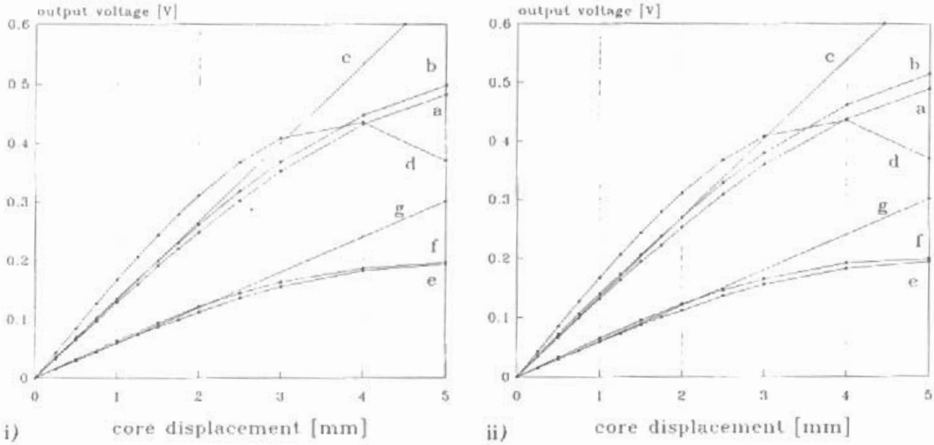

Fig. 3. *Output voltage as a function of core displacement*
i) with the core tube
ii) without the core tube

a – long core, measurement *e – short core, measurement*
b – long core, FE calculation *f – short core, FE calculation*
c – long core, linear *g – short core, linear*
d – long core, formula (1)

The results show a slight (about 3%) but consistent over-estimate of the output voltage using the finite-element modelling, whereas the shape of the curves and hence the degree of nonlinearity of the LVDT response are predicted almost perfectly. The effect of the core tube is relatively small in this case (a small reduction of output voltage); nevertheless its presence may be modelled very accurately. Thus the finite-element modelling has been found extremelly helpful as an aid in the design and development of LVDTs. Most of the important ratings and characteristics can be predicted accurately and changes to the geometry and material properties can be easily accomodated.

Acknowledgement

The collaboration with Ashridge Engineering Ltd, for whom the LVDT was originally designed, is gratefully acknowledged. Numerical results have been produced using the finite element package PE2D from Vector Fields Ltd.

References

[1] E.E. Herceg: *Handbook of Measurement and Control*, Schaevitz Engineering, 1983, Pennsauken, N.J.