Fast implementation of oversampled modulated filter banks

S. Weiss and R.W. Stewart

The authors present an efficient implementation of oversampled filter banks derived from a prototype filter by modulation. Via a polyphase analysis, redundancies in the filter operations are removed. With some modifications, a very simple and efficient implementation is found, which is briefly compared to existing realisations.

Introduction: Oversampled filter banks are widely used for reducing the computational complexity of resource-demanding signal processing algorithms, such as subband adaptive filtering applied to acoustic echo control [1, 2]. Therefore, low complexity realisations of such filter banks are desirable. However, despite this motivation and in contrast to their critically decimated counterparts [3], numerically efficient implementations of noncritically (or 'oversampled') filter banks have received little attention.

The sparse literature on the implementation of oversampled filter banks includes the work of Wackersreuther [4], where a time domain approach leads to a factorisation of the analysis filter bank operation into a filtering operation linked to the prototype filter coefficients, a cyclical shift, and the applications of the appropriate modulating transform (e.g. a DFT). In [5], the analysis filter bank is in the time-domain divided into a filter operation with time-varying components of the prototype filter, followed by the modulating transform. More recently, polyphase factorisations in the z-domain have been presented [6, 7]. For all cases [4 – 7], a dual implementation can be found for the synthesis filter bank operation.

Here, the polyphase approach [6, 7] is utilised as a starting point to yield, with some modifications and rearrangements, novel, simple and efficient filter bank implementations.

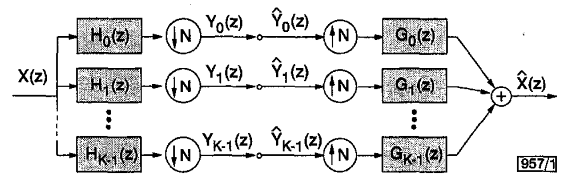

Fig. 1 *Analysis and synthesis filter bank for subband decomposition of a signal X(z)*

Analysis filter bank: Consider the analysis filter bank of Fig. 1 producing K subband signals. To remove computational redundancies due to the decimation by N, a type-1 polyphase notation [3] is introduced for the analysis filters

$$H_k(z) = \sum_{n=0}^{N-1} z^{-n} H_{k|n}(z) \tag{1}$$

The input signal is decomposed into N type-2 polyphase components

$$X(z) = \sum_{n=0}^{N-1} z^{-N+n-1} X_n(z) \tag{2}$$

Organising the polyphase components in vector form

$$\underline{H}_k(z) = [\,H_{k|0}(z) \quad H_{k|1}(z) \quad \cdots \quad H_{k|N-1}(z)\,]^T \tag{3}$$

$$\underline{X}(z) = [\,X_0(z) \quad X_1(z) \quad \cdots \quad X_{N-1}(z)\,]^T \tag{4}$$

the K subband signals $\underline{Y}(z) = [Y_0(z)\ Y_1(z)\ \ldots\ Y_{K-1}(z)]^T$ can be denoted by

$$\begin{aligned} \underline{Y}(z) &= [\,\underline{H}_0(z) \quad \underline{H}_1(z) \quad \cdots \quad \underline{H}_{K-1}(z)\,]^T \cdot \underline{X}(z) \\ &= \mathbf{H}(z) \cdot \underline{X}(z) \end{aligned} \tag{5}$$

where $\mathbf{H}(z)$ is the polyphase analysis matrix [6] and describes a linear periodically time-varying system of period N.

We assume that the analysis filters are FIR with L_p coefficients, $h_{k,0} \ldots h_{k,L_p-1}$, which are derived from a prototype lowpass filter by a modulation sequence $t_{k,0} \ldots t_{k,L_p-1}$, with period K. For simplicity without loss of generality we assume that L_p is a common multiple of both N and K. With $\mathbf{I}_N$ being an $N \times N$ identity matrix and the filter coefficients of $H_k(z)$ organised in a vector $\mathbf{h}_k$, the polyphase components in eqn. 3 can be written as

$$\underline{H}_k(z) = \underbrace{[\,\mathbf{I}_N \quad z^{-1}\cdot\mathbf{I}_N \quad \cdots \quad z^{-L_p/N+1}\cdot\mathbf{I}_N\,]}_{\mathbf{L}_1^T(z)} \cdot \mathbf{h}_k \tag{6}$$

$$= \mathbf{L}_1^T(z) \cdot \underbrace{\begin{bmatrix} p_0 & & & \mathbf{0} \\ & p_1 & & \\ & & \ddots & \\ \mathbf{0} & & & p_{L_p-1} \end{bmatrix}}_{\mathbf{P}} \cdot \underbrace{\begin{bmatrix} t_{k,0} \\ t_{k,1} \\ \vdots \\ t_{k,L_p-1} \end{bmatrix}}_{\mathbf{t}_k} \tag{7}$$

where $\mathbf{t}_k \in \mathbb{C}^{L_p}$ is the modulation sequence for the kth analysis filter. The periodicity of this sequence can be exploited by

$$\mathbf{t}_k = [\underbrace{\mathbf{I}_K \quad \mathbf{I}_K \quad \cdots \quad \mathbf{I}_K}_{\mathbf{L}_2}]^T \cdot \tilde{\mathbf{t}}_k \tag{8}$$

where $\tilde{\mathbf{t}}_k \in \mathbb{C}^K$. The K modulation sequences are collected in a matrix

$$\mathbf{T} = [\,\tilde{\mathbf{t}}_0 \quad \cdots \quad \tilde{\mathbf{t}}_{L_p-1}\,]^T \tag{9}$$

which for example for a DFT modulated filter bank would be a $K \times K$ DFT matrix. The polyphase analysis matrix is now given by

$$\mathbf{H}(z) = \mathbf{T} \cdot \mathbf{L}_2 \cdot \mathbf{P} \cdot \mathbf{L}_1(z) \tag{10}$$

Hence, a factorisation into prototype filter components and a rotation by a transform matrix $\mathbf{T}$ has been established similar to [6, 7]. The difference is that the diagonal matrix $\mathbf{P}$ contains no sparse filters but only the prototype filter coefficients.

Synthesis filter bank: The synthesis filter bank with expansion by N followed by interpolation filters $G_k(z)$ as shown in Fig. 1 can be performed in an analogous fashion to the analysis filter bank operation. The condition that all filters $G_k(z)$ and $H_k(z)$ are derived from the same prototype lowpass filter and that the filter bank is perfectly reconstructing is guaranteed by $\mathbf{H}(z)$ being paraunitary [6]. Reconstruction is then given by the polyphase synthesis matrix

$$\mathbf{G}(z) = \mathbf{H}^H(z^{-1}) = \mathbf{L}_1^T(z^{-1}) \cdot \mathbf{P} \cdot \mathbf{L}_2^T \cdot \mathbf{T}^H \tag{11}$$

This polyphase synthesis matrix relates the subband samples back to the polyphase components of the fullband signal, $\underline{\hat{X}}(z) = \mathbf{G}(z) \cdot \underline{\hat{Y}}(z)$.

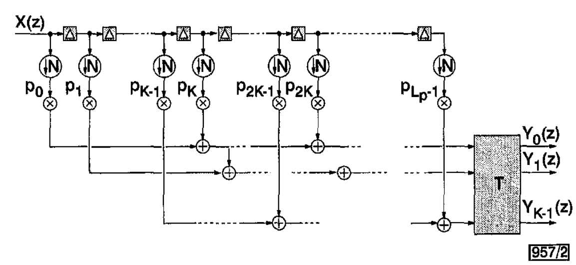

Fig. 2 *Analysis filter bank signal flow graph*

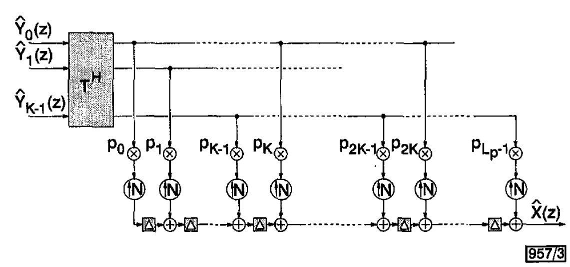

Fig. 3 *Synthesis filter bank signal flow graph*

Implementation: The analysis filter bank operation in eqn. 10 can be executed in two steps. First, the memory-requiring multiplication $\mathbf{L}_N(z) \cdot \underline{X}(z)$ can be brought into the form of a tapped delay line (TDL) block updated with N new samples for every operation. This is shown in Fig. 2. The second part of the operation is memoryless, and consists of the multiplication with the L_p prototype filter coefficients, forming K polyphase components, which are then rotated by the modulation matrix $\mathbf{T}$ to yield the subband samples.

For the synthesis filter bank operations, we split the equation $\underline{X}(z) = \mathbf{G}(z) \cdot \underline{Y}(z)$ into two parts by introducing an intermediate variable $\underline{V}(z) \in \mathbb{C}^{L_p}(z)$:

$$\underline{V}(z) = \begin{bmatrix} \mathbf{0}_N & & & \mathbf{0} \\ \mathbf{I}_N & \mathbf{0}_N & & \\ & \ddots & \ddots & \\ \mathbf{0} & & \mathbf{I}_N & \mathbf{0}_N \end{bmatrix} \cdot \underline{V}(z)z^{-1} + \mathbf{P} \cdot \mathbf{L}_2^T \cdot \mathbf{T}^H \cdot \underline{Y}(z) \quad (12)$$

The second summand on the right-hand side of eqn. 12 is memoryless, and the system matrix in eqn. 12 performs a shift operation by N samples. The desired output can be derived from the intermediate variable $\underline{V}(z)$ as

$$\underline{X}(z) = [\mathbf{0} \ \cdots \ \mathbf{0} \ \ \mathbf{I}_N] \cdot \underline{V}(z) \quad (13)$$

Thus, the only memory-exhibiting operation in the synthesis filter bank is the shift operation in eqn. 12, and the circuit given in Fig. 3 results. This circuit shows the derotation of the subband samples by $\mathbf{T}^H$, the copying by $\mathbf{L}_2$ such that the L_p multipliers are excited, and accumulation of the products into a TDL. This TDL results from rearranging the multiplexing of the N polyphase outputs in $\underline{X}(z)$, and only requires a shift operation every N sampling periods.

Computational complexity: From the signal flow graphs for analysis and synthesis in Figs. 2 and 3, the computational complexities for both operations in terms of multiply-accumulates (MACs) evaluations is

$$C = \frac{1}{N} \cdot (2L_p + 4K \log_2 K) \qquad [\text{MACs}] \qquad (14)$$

for a complex input $x[n]$. Multiplication of the complex samples with the real valued prototype filter coefficients accrues to $2L_p$ MACs. The modulation matrix $\mathbf{T}$ is assumed to be implemented by a K-point FFT [7] requiring $4K\log_2 K$ real valued MACs. If the fullband signal $X(z)$ is real valued, the complexity C in eqn. 14 is halved since all MACs outwith the transform are entirely real valued and half of the subband signals $Y_k(z)$ are complex conjugate copies of others subbands, and therefore do not need to be generated nor processed.

Although other methods reported in the literature give identical complexities in terms of MACs, the realisations in Figs. 2 and 3 do not require any additional circular shifts [4], or the indexing of time-varying filters [5] or filters with sparse coefficients [6, 7].

Conclusion: A filter bank analysis has been presented based on the polyphase approach. With some modifications, filter bank realisations have been derived which are efficient, very simple, and avoid some of the disadvantages of previous fast implementations.

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Feedback phase-shift compensation for adaptive interference cancellers

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An algorithm typically used for adaptive co-site interference cancellers (for frequency-hopping spread-spectrum communications systems) is the least-mean-square error (LMS) algorithm. As adaptive cancellation moves into L-band and S-band, feedback system stability becomes a critical issue for co-site interference cancellers using the LMS algorithm. The authors present a technique for compensating the high-frequency feedback path phase shifts leading to instability with an additional complex weight multiplication at baseband. Simulation and experimental results confirm the effectiveness of the technique.

The co-site interference problem impacts a number of frequency-hopping spread-spectrum communications platforms, and is now being addressed for next-generation systems in L-band and S-band. Co-site interference occurs owing to the physical proximity of transmit and receive antennas (e.g. on a ship or rotorcraft), and because transmission and reception occur simultaneously. Adaptive co-site interference cancellers reduce the interfering signal (due to the transmit antenna) from the signal at the receive antenna, to prevent the receiver from being overdriven. Using a reference signal from the transmitter, adaptive co-site interference cancellers at VHF and UHF can typically achieve cancellation of the interferer of 30 dB or more [1 – 3].

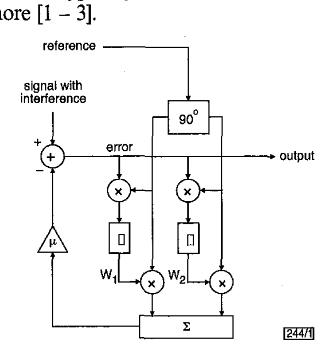

Fig. 1 *Adaptive canceller with single complex weight*

A standard algorithm for co-site interference cancellers is the least-mean-square error (LMS) algorithm [4, 5]. For narrowband interferers and relatively slow frequency hopping at VHF or UHF, a single complex weight circuit, as shown in Fig. 1, can provide adequate cancellation. The adaptive canceller of Fig. 1 works by generating a copy of the interfering signal from the reference signal, and then subtracting the copy of the interfering signal from the receiver signal (which contains both the desired receive signal and the co-site interference signal). The copy of the interfering signal is produced by appropriately weighting the in-phase and quadrature-phase copies of the reference signal, using the weights w_1 and w_2 in Fig. 1. The weights are determined adaptively by integrating the correlation between the error signal and the in-phase and quadrature-phase versions of the reference signal (i.e. using the LMS algorithm). The feedback loop consists of the low-frequency path through the integrators, and the high-frequency path through the summer, feedback amplifier, and subtractor (to which the multipliers also contribute). The delay and phase-shift through