Parallel Numerical Modelling of Ice Flow in Antarctica

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Abstract *The Antarctic Ice Sheet is made up of the West Antarctic Ice Sheet (WAIS) and the much larger East Antarctic Ice Sheet (EAIS). Previous numerical models have focussed on ice flow in WAIS, since the size of the EAIS has precluded studies at a resolution adequate to identify complex flow features. The equations describing ice flow are highly non-linear, making this a computationally intensive problem. We use a staggered grid to overcome numerical instability and a sparse packing scheme to take account of the irregular boundary of Antarctica. We have developed a highly efficient parallel thermally-coupled ice flow model of the entire Antarctic Ice Sheet at a resolution of 20km and present performance results obtained on a commodity cluster of workstations. Our initial results show areas of East Antarctica at pressure melting point that may be characterised by fast ice flow.*

Keywords: Antarctica, parallel computing, non-linear partial differential equations, commodity supercomputing

1 Introduction

Antarctica is the largest ice mass on Earth, and has a strong influence over global climate and ocean circulation. The ice sheet consists of the West Antarctic Ice Sheet (WAIS) and the much larger eastern part (EAIS). The complex feedback between temperature and ice flow is an important factor in the dynamics of an ice sheet. Melting of the WAIS and EAIS would contribute ~6m and 60m to global sea-level, respectively. It is therefore essential to model both of these parts together.

Numerical models have been developed to study ice streams in WAIS at a resolution of 20km or less [1]. The ice streams are regions of fast flow, and are thought to be a key feature affecting the stability (or instability) of the whole of the WAIS. Until recently, the EAIS was thought to be a region of stable ice flow, with topographically controlled outlet glaciers, but no WAIS-type ice streams. However, recent remote sensing images have uncovered ice-stream-like features stretching for many kilometres inland in the EAIS [2, 3].

The system is governed by a pair of highly non-linear coupled partial differential equations. This complexity, and the vast size of the EAIS, has restricted previous modelling to the study of WAIS at a high resolution, or the whole of Antarctica at a resolution too low to resolve complex flow features. To overcome these limitations, we have developed an efficient

parallel thermally-coupled ice flow model of the whole of Antarctica, at a resolution of 20km.
The layout of the paper is as follows. In Section 2 we discuss the non-linear equations governing the flow of an ice sheet and evolution of ice temperature. Section 3 presents our parallel computational method and some performance results. The model is tested against benchmarks, and used to investigate ice flow in Antarctica. Results are shown in Section 4, and conclusions given in Section 5.

2 Numerical modelling of ice flow

The evolution of an ice sheet through time is analogous to the behaviour of sand poured onto a flat surface. It builds up uniformly until reaching a critical threshold, when it begins to flow, producing a dome shape. In an ice sheet, ice thickness H increases by the mass balance b (rate of snow accumulation - rate of melting) over the bedrock until it reaches this threshold. Although topographic variation will complicate the early stages of growth, ice will begin to form the characteristic domed shape when it becomes thick enough to cover the landscape uniformly. Figure 1 summarises the variables acting on a hypothetical column of ice.

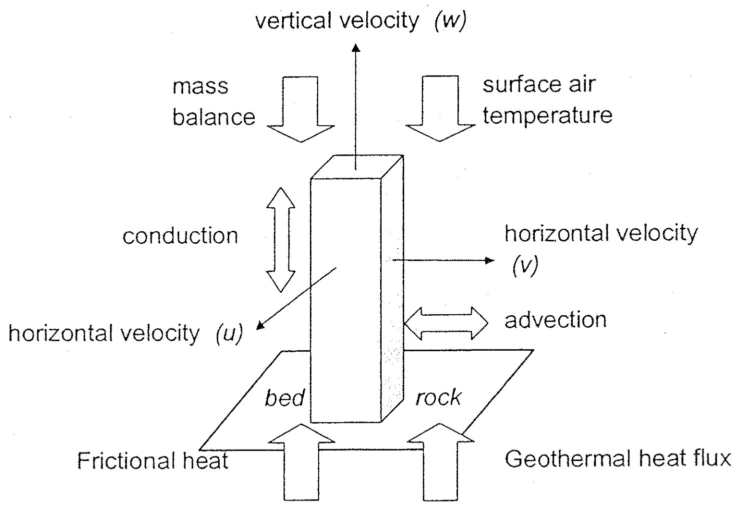

Figure 1 Summary of variables acting on a hypothetical column of ice

The continuity equation describing the evolution of ice thickness with time is highly non-linear. The velocity of ice flow depends on ice thickness to the fifth power and on surface slope to the third power, making the solution of these equations extremely computationally intensive. Ice viscosity depends on temperature, and inclusion of thermal coupling improves the model, but adds a third dimension to the computation. Thermomechanical models that allow feedback between ice temperature and the flow parameter have successfully been used to explain many of the characteristics of ice streams in WAIS [1].

The continuity equation (1) expresses conservation of mass within a hypothetical ice block, where t is time, and x and y the two horizontal directions.

$$\frac{\partial H}{\partial t} = b - \frac{\partial}{\partial x}\left(\bar{u}\, H\right) - \frac{\partial}{\partial y}\left(\bar{v} H\right) \qquad (1)$$

This equation requires the depth-averaged velocities $\bar{u}$ and $\bar{v}$. We replace depth invariant constants by ϕ:

$$\phi = \left(\left(\frac{\partial s}{\partial x} \right)^2 + \left(\frac{\partial s}{\partial y} \right)^2 \right)^{\frac{n-1}{2}} (\rho g)^n \frac{\partial s}{\partial x} \qquad (2)$$

and compute $\bar{u}$ (and analogously $\bar{v}$) by averaging the velocity over each column of ice:

$$\bar{u} = \int_h^s u_z \, dz' = -2\phi \int_h^s \int_h^z A(z')(s-z')^n \, dz' \, dz \qquad (3)$$

$$= -2\phi H^{n+2} \tilde{A}$$

where

$$\tilde{A} = \int_h^s \int_h^z A(z') \sigma^n \, dz' \, dz \qquad (4)$$

and A is the temperature dependent flow parameter, ρ is the density of ice (910 kgm^{-3}), g is acceleration due to gravity (9.81ms^{-2}), n is a parameter from Glen's flow law [4], generally taken to be 3, and s is surface elevation. These can be substituted into the continuity equation (1) to give:

$$\frac{\partial H}{\partial t} = b + \frac{\partial}{\partial x}\left(D \frac{\partial s}{\partial x} \right) + \frac{\partial}{\partial y}\left(D \frac{\partial s}{\partial y} \right) \qquad (5)$$

where D describes diffusivity:

$$D = 2\tilde{A}(\rho g)^n H^{n+2} \left(\left(\frac{\partial s}{\partial x} \right)^2 + \left(\frac{\partial s}{\partial y} \right)^2 \right)^{\frac{n-1}{2}} . \qquad (6)$$

The flow factor is determined using an Arrhenius relation:

$$A(T^*) = a \exp\left\{ \frac{-Q}{RT^*} \right\} , \qquad (7)$$

where a is a multiplier with the value 1.14 x 10^5 Pa^{-3} a^{-1} if $T^* < 263$K, and 5.47 x 10^{10} Pa^{-3} a^{-1} if $T^* \geq 263$K . Q is the activation energy for creep, with a value of 60 kJ mol^{-1} for $T^* < 263$K and 139kJ mol^{-1} for $T^* > 263$K. R is the gas constant, and has a value of 8.31 J mol^{-1} K^{-1}. The melting point of ice is dependent on pressure, which is in turn affected by depth. This temperature dependence is corrected for using:

$$T^* = T - T_{pmp} + T_0 \qquad (8)$$

with

$$T_{pmp} = T_0 - \Phi(s - z) \qquad (9)$$

where T_0 is the triple point temperature of ice (273.15K) and Φ is the dependence of melting point on pressure (8.71 x 10^{-4}K m^{-1}). s is the surface, and z the vertical coordinate.

Temperature must be calculated to allow thermal coupling between the temperature of ice and its viscosity via the flow parameter A. Ice temperature evolves with time according to Equation (10)

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c} \nabla^2 T - \vec{U} \nabla T + \frac{F}{\rho c} , \qquad (10)$$

where the first term is temperature diffusion, the second advection and the third dissipation, or strain heating. k is the thermal conductivity of ice (6.62 x 10^7 Jm^{-1}K^{-1} yr^{-1}), c the specific heat capacity of ice (2009 J kg^{-1} K^{-1}) and ∇ the two-dimensional horizontal divergence operator. $\vec{U}$ is the three dimensional ice velocity vector, and F is frictional heat generated internally by ice deformation.

At the surface, the upper boundary value is surface air temperature. Basal boundary conditions depend on whether or not the ice is at pressure melting point. If the ice has not yet reached melting point geothermal heat flux continues to warm the base:

$$\left. \frac{\partial T}{\partial t} \right|_1 = -\frac{GH}{k} . \qquad (11)$$

Once pressure melting point has been reached, the energy from the geothermal heat flux goes into melting more ice, so temperatures can never rise above T_{pmp}.

3 Parallel Computational Method

The ice flow equation (1) is solved on a two dimensional grid using a parallel sparse conjugate gradient method. The ice temperature evolution equation is then solved in the third dimension, as an individual column of ice for each grid cell using an ADI tridiagonal solver. Although temperature at the surrounding gridpoints is used to calculate horizontal advection, this necessitates nearest-neighbour communication only.

The data-set used in this project consists of a square grid 281x281 cells in size, and is taken from work by Budd *et al* [5]. The grid contains the Antarctic Ice Sheet, but surrounding ocean is also included, covering approximately one third of the grid. To overcome the problem of including irrelevant sea areas in expensive but unnecessary computations, land values are indexed, so every grid point that falls on the ice mass rather than the surrounding ocean has its own identity number. A lookup subroutine then allows the original co-ordinates of a grid point to be determined, given only its identity number.

The map of the Antarctic Ice Sheet is split into strips of approximately equal size by dividing the total number of land cells by the number of processors. Each processor's computational domain is then extended to the next coastal point if necessary to create a complete strip. This is shown in Figure 2.

We found that, whilst not optimal, a strip decomposition was more efficient on the modest number of processors employed than a block decomposition.

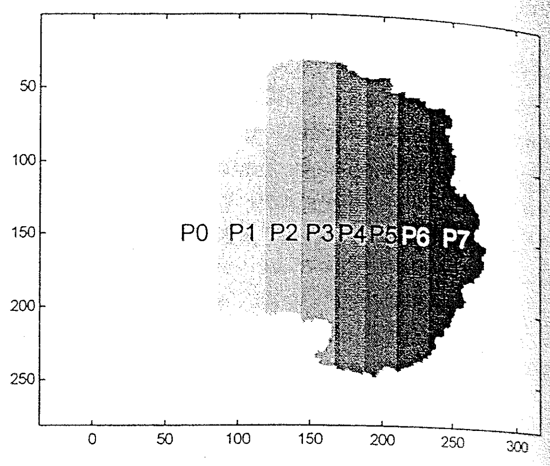

Figure 2 Antarctica is divided into n strips of approximately equal size to run on n processors.

All computations on the sparse matrix representing the discretized version of Equation (1) proceed in parallel and the array is globally reduced to a single processor at the end for output to disk. If the resolution of the grid was further enhanced we would consider using the parallel I/O facilities of MPI 2 [6].

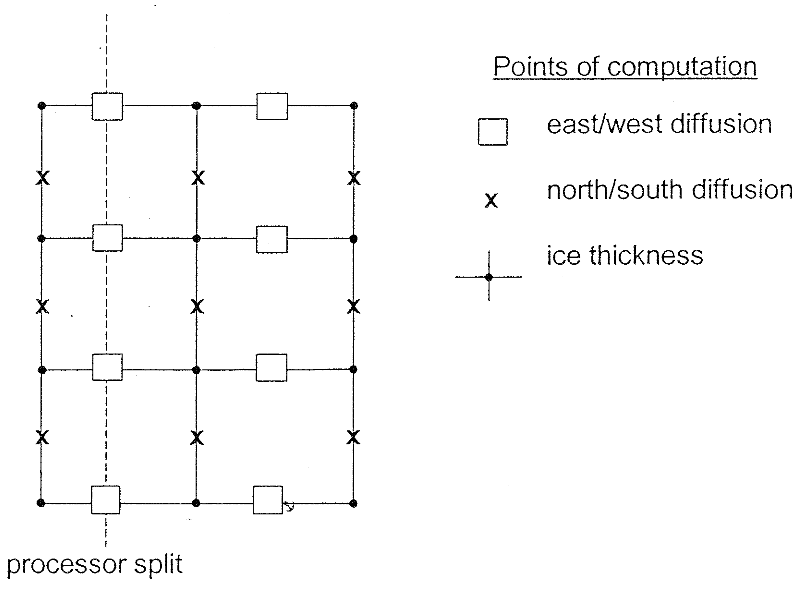

Figure 3 Points of computation for diffusion and ice thickness

For numerical stability, calculation of ice diffusivity takes place on a staggered grid system, shown in Figure 3. Along with the irregular boundaries of our grid, this prevented us from using the standard parallel template libraries for our solvers, e.g. KELP [7]. The staggered diffusion grid means that the split between processors dissects the east-west diffusion term for ice thickness values on the processor border. For this reason there is an overlap, with diffusivity at the border being calculated by both processors. Diffusivity in the north-south direction is calculated on the same rows as the ice thickness points. It is therefore not affected by the processor split, so does not need to be duplicated.

Information regarding the north, south, east and west neighbours of individual grid points, and of the six staggered grid diffusion neighbours is stored in two arrays. Using these, it is possible to identify the neighbours of the grid points located on the processor boundaries, and to work out the limits of each processor's computational domain. Packing of the diffusivity terms in north-south and east-west directions can then be run in parallel using these limits for each processor. These arrays are used to pack the information from the sparse irregular grid into arrays to be sent between processors. This helps to increase latency tolerance of the solver, which is important for running on clusters of PCs.

A test program has been run to assess parallel performance. A reduced grid was used, with only 31 grid points in each of the horizontal directions, but the full 11 layers in the vertical. Datafiles were read in containing the state of the ice sheet at 10,000 years of simulation time, and the model run for a further 1,000 years. Speedup and timing results are presented in Figure 4.

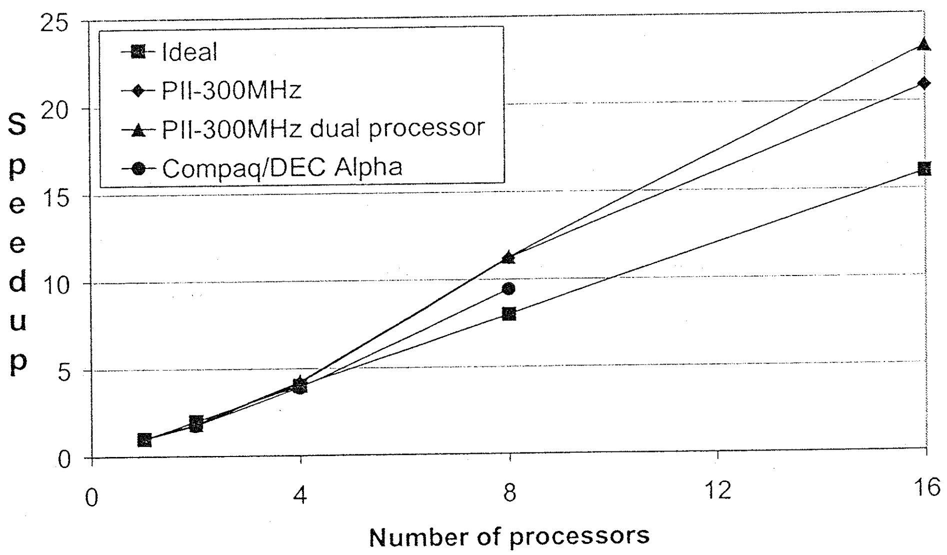

Figure 4 Speedup of code

The code was run on a cluster of eight 500MHz, 21164 Compaq/DEC Alphas running Windows NT and MPI/Pro [8]. Microsoft Research provided additional timings, running on a PII-300MHz system with 16 dual processors under Windows NT 4 SP3 using MPI/Pro v1.5.

The larger scale model performs slightly better than the small test case presented here, particularly when run for the full 150,000 simulation years. The superlinear speedups are processor memory cache effects.

4 Results

The code has been tested in detail against the EISMINT benchmarks for a regular grid, and compares within acceptable error bounds of the published results [9]. The benchmark tests include calculation of ice temperature, but do not allow feedback between temperature and ice flow parameter A. Following testing, the model has been developed to include thermal coupling, so that ice flow is dependent on temperature. It has been applied to Antarctica, allowing the basal thermal regime to be studied. The model is run for 150,000 years of simulation time, using a timestep of 1 year. The current topography of Antarctica is used, and observed snow accumulation rates are used for the mass balance. The model does not include basal sliding or isostatic bedrock loading at this stage.

Figure 5 shows areas of the modelled ice sheet at pressure melting point from preliminary results. Areas at pressure melting point are of interest as they may be associated with fast flow features. One of the underlying assumptions of the equations shown in Section 2 is that flow is dominated by longitudinal stresses. This assumption is not true of ice streams, where latitudinal stresses become important. For this reason, the model cannot model the flow of these fast features, but mapping areas of the base at pressure melting point gives a strong indication of their likely location.

The modelled ice sheet contains a good representation of parts of the WAIS where meltwater is known to be present. The areas o the EAIS where the basal ice is at pressur melting point are also potential regions of fas flow. Recent remote sensing work [2,3] ha identified areas of fast ice flow that correspon well with the regions of basal melting in EAI which our model identifies.

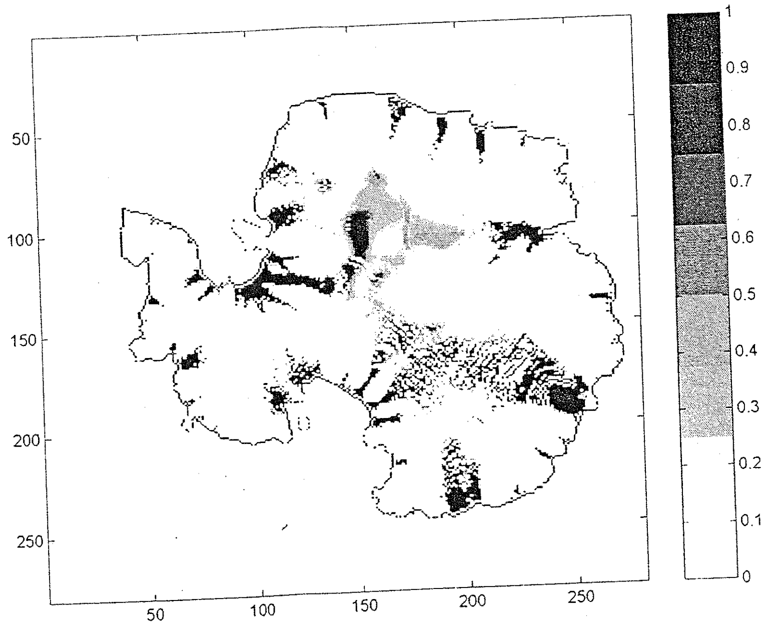

Figure 5 Basal melt rate of the modelled ice sheet. The scale shows the basal melt rate in mm, ar restricted so that any areas with a basal melt rate of over 1mm/yr are shown in black.

5 Conclusions

Fieldwork in Antarctica is restricted by high costs and poor accessibility, particularly in the inhospitable interior of the EAIS. Computational modelling and remote sensing are therefore vital tools in the study of ice flow in this region. Our model requires the coupled solution of highly non-linear partial differential equations on a staggered grid over a domain with irregular boundary. The scheme is semi-implicit, and thermal coupling requires a reduction in timestep to ensure numerical stability. The coupled model therefore takes much longer to run than the uncoupled version, due to the necessary timestep reduction in addition to the extra computation required. Parallelization of the code has therefore been essential in making a high-resolution thermally coupled model of the entire Antarctic ice sheet feasible.

Parallelization has allowed features of the EAIS to be seen at a higher resolution than previously possible. It will now be possible to study the modelled basal thermal regime beneath the EAIS, which is up to 4km thick in places. The parallel model also makes it feasible to run a greater range of sensitivity tests than previously possible, such as variations in geothermal heat flux. Possible further work includes implementing a fully implicit parallel solver to overcome the problem of a small timestep. Other multigrid-type solvers may also be investigated.

6 Acknowledgements

We acknowledge support from MPI Software Technology Inc. and Jonathan Hardwick at Microsoft Research. ALT is grateful to EPSRC for a studentship.

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