Adaptive Minimum-BER Linear Multiuser Detection for CDMA Signals in Multipath Channels with 4-QAM Constellation

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Indexing terms: CDMA, Multiuser detection, mean sqaure error, bit error rate, quadrature amplitude modulation

We extend an adaptive minimum-BER linear multiuser detector called the least BER (LBER) algorithm, originally developed for the BPSK modulation, to the 4-QAM modulation.

Introduction

The design of linear multiuser detector is often based on the minimum mean square error (MMSE) principle, and adaptive MMSE detector can readily be implemented using the LMS algorithm [1]. However, it is well known that the MMSE solution can in certain cases be distinctly inferior to the optimal minimum bit error rate (MBER) solution. Adaptive MBER linear multiuser detectors have recently been developed [2],[3]. These two adaptive MBER multiuser detectors were inspired respectively by the two adaptive MBER linear equalisers called the approximate MBER (AMBER) algorithm [4] and the LBER algorithm [5], and they are both designed for the binary signalling scheme. Previous study [3],[5] has shown that the LBER algorithm has better performance than the AMBER algorithm, in terms of convergence speed and steady-state BER. We extend the LBER detector to complex-valued signalling schemes. The 4-QAM modulation scheme is used for this extension.

System model

The discrete-time model of the synchronous CDMA downlink system with N users and M chips per symbol is depicted in Fig. 1, where $b_i(k) = b_{R,i}(k) + j\, b_{I,i}(k) \in \{\pm 1 \pm j\}$ denotes the k-th symbol of user i, the unit-length spreading code for user i is $\bar{\mathbf{s}}_i = [\bar{s}_{i,1} \ \cdots \ \bar{s}_{i,M}]^T$, and the channel impulse response (CIR) is defined by $C(z) = c_0 + c_1 z^{-1} + \cdots + c_{n_c-1} z^{-n_c+1}$, with $\{c_i\}$ denoting the complex-valued channel taps. The received signal sampled at chip rate is given

$$\mathbf{r}(k) = \mathbf{P} \begin{bmatrix} \mathbf{b}(k) \\ \mathbf{b}(k-1) \\ \vdots \\ \mathbf{b}(k-L+1) \end{bmatrix} + \mathbf{n}(k) = \bar{\mathbf{r}}(k) + \mathbf{n}(k)\,, \tag{1}$$

where the complex Gaussian channel noise vector $\mathbf{n}(k) = [n_1(k) \cdots n_M(k)]^T$ with $E[\mathbf{n}(k)\mathbf{n}^H(k)] = 2\sigma_n^2 \mathbf{I}$, $\mathbf{b}(k) = [b_1(k) \cdots b_N(k)]^T$ is the user symbol vector, and the $M \times LN$ system matrix

$$\mathbf{P} = \mathbf{C} \begin{bmatrix} \bar{\mathbf{S}}\mathbf{A} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{S}}\mathbf{A} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \bar{\mathbf{S}}\mathbf{A} \end{bmatrix}, \tag{2}$$

with $\bar{\mathbf{S}} = [\bar{\mathbf{s}}_1 \cdots \bar{\mathbf{s}}_N]$, the user amplitude matrix $\mathbf{A} = \mathrm{diag}\{A_1 \cdots A_N\}$, and the $M \times LM$ CIR matrix

$$\mathbf{C} = \begin{bmatrix} c_0 & c_1 & \cdots & c_{n_c-1} & & & \\ & c_0 & c_1 & \cdots & c_{n_c-1} & & \\ & & \ddots & \ddots & \cdots & \ddots & \\ & & & c_0 & c_1 & \cdots & c_{n_c-1} \end{bmatrix} . \tag{3}$$

The intersymbol interference span L depends on n_c and M: $n_c = 1$, $L = 1$; $1 < n_c \le M$, $L = 2$; $M < n_c \le 2M$, $L = 3$; and so on. The linear detector for user i is given by

$$\hat{b}_i(k) = \mathrm{sgn}(y_R(k)) + j\,\mathrm{sgn}(y_I(k)) \quad \text{with} \quad y(k) = y_R(k) + j\,y_I(k) = \mathbf{w}^H \mathbf{r}(k) . \tag{4}$$

Let the $N_b = 4^{LN}$ possible combinations of $[\mathbf{b}^T(k)\ \mathbf{b}^T(k-1) \cdots \mathbf{b}^T(k-L+1)]^T$ be

$$\mathbf{b}^{(l)} = \begin{bmatrix} \mathbf{b}^{(l)}(k) \\ \mathbf{b}^{(l)}(k-1) \\ \vdots \\ \mathbf{b}^{(l)}(k-L+1) \end{bmatrix}, \ 1 \le l \le N_b , \tag{5}$$

and $b_i^{(l)} = b_{R,i}^{(l)} + j\,b_{I,i}^{(l)}$ the ith element of $\mathbf{b}^{(l)}(k)$. Define the N_b noise-free received signal states $\mathbf{r}_l = \mathbf{r}_{R,l} + j\,\mathbf{r}_{I,l} = \mathbf{P}\mathbf{b}^{(l)}$, $1 \le l \le N_b$, and the set of N_b scalars $y_l = y_{R,l} + j y_{I,l} = \mathbf{w}^H \mathbf{r}_l$, $1 \le l \le N_b$.

The MBER detector

The probability density functions of the signed decision variables $\mathrm{sgn}(b_{R,i}^{(l)}) y_R(k)$ and $\mathrm{sgn}(b_{I,i}^{(l)}) y_I(k)$ are

$$p_{R,s}(x) = \frac{1}{N_b \sqrt{2\pi}\sigma_n \sqrt{\mathbf{w}^H\mathbf{w}}} \sum_{l=1}^{N_b} \exp\left(-\frac{(x - \mathrm{sgn}(b_{R,i}^{(l)}) y_{R,l})^2}{2\sigma_n^2 \mathbf{w}^H \mathbf{w}} \right) \tag{6}$$

and

$$p_{I,s}(x) = \frac{1}{N_b \sqrt{2\pi}\sigma_n \sqrt{\mathbf{w}^H\mathbf{w}}} \sum_{l=1}^{N_b} \exp\left(-\frac{(x - \mathrm{sgn}(b_{I,i}^{(l)}) y_{I,l})^2}{2\sigma_n^2 \mathbf{w}^H \mathbf{w}} \right) , \tag{7}$$

respectively. The BER of the linear detector (4) is $P_E(\mathbf{w}) = \frac{1}{2}\left(P_{E,R}(\mathbf{w}) + P_{E,I}(\mathbf{w})\right)$, where

$$P_{E,R}(\mathbf{w}) = \int_{-\infty}^{0} p_{R,s}(x)dx \quad \text{and} \quad P_{E,I}(\mathbf{w}) = \int_{-\infty}^{0} p_{I,s}(x)dx . \tag{8}$$

After some manipulations and with the help of weight normalization $\mathbf{w}^H\mathbf{w} = 1$, it can be shown that

$$\nabla P_{E,R}(\mathbf{w}) = \frac{1}{N_b \sqrt{2\pi}\sigma_n} \sum_{l=1}^{N_b} \exp\left(-\frac{y_{R,l}^2}{2\sigma_n^2} \right) \mathrm{sgn}(b_{R,i}^{(l)}) (y_{R,l}\mathbf{w} - \mathbf{r}_l) \tag{9}$$

$$\nabla P_{E,I}(\mathbf{w}) = \frac{1}{N_b \sqrt{2\pi}\sigma_n} \sum_{l=1}^{N_b} \exp\left(-\frac{y_{I,l}^2}{2\sigma_n^2} \right) \mathrm{sgn}(b_{I,i}^{(l)}) (y_{I,l}\mathbf{w} + j\,\mathbf{r}_l) \tag{10}$$

The exact MBER solution can thus be obtained using the gradient descent algorithm

$$\mathbf{w}(t+1) = \mathbf{w}(t) - \frac{\mu}{2}\left(\nabla P_{E,R}(\mathbf{w}(t)) + \nabla P_{E,I}(\mathbf{w}(t)) \right) , \tag{11}$$

where t denotes the iteration number.

The LBER detector

We will follow the approach [3],[5] to derive a stochastic gradient adaptive MBER algorithm. One-sample estimates of $p_{R,s}(x)$ and $p_{I,s}(x)$ are

$$\hat{p}_{R,s}(x;k) = \frac{1}{\sqrt{2\pi}\rho_n\sqrt{\mathbf{w}^H(k)\mathbf{w}(k)}} \exp\left(-\frac{(x - \mathrm{sgn}(b_{R,i}(k))y_R(k))^2}{2\rho_n^2\mathbf{w}^H(k)\mathbf{w}(k)}\right) \tag{12}$$

and

$$\hat{p}_{I,s}(x;k) = \frac{1}{\sqrt{2\pi}\rho_n\sqrt{\mathbf{w}^H(k)\mathbf{w}(k)}} \exp\left(-\frac{(x - \mathrm{sgn}(b_{I,i}(k))y_I(k))^2}{2\rho_n^2\mathbf{w}^H(k)\mathbf{w}(k)}\right), \tag{13}$$

respectively, where ρ_n is the width for a kernel density estimation. With a re-scaling after each sample updating to ensure $\mathbf{w}^H(k)\mathbf{w}(k) = 1$ and using instantaneous gradients

$$\nabla\hat{P}_{E,R}(\mathbf{w}(k);k) = \frac{1}{\sqrt{2\pi}\rho_n} \exp\left(-\frac{y_R^2(k)}{2\rho_n^2}\right) \mathrm{sgn}(b_{R,i}(k))(y_R(k)\mathbf{w}(k) - \mathbf{r}(k)) \tag{14}$$

$$\nabla\hat{P}_{E,I}(\mathbf{w}(k);k) = \frac{1}{\sqrt{2\pi}\rho_n} \exp\left(-\frac{y_I^2(k)}{2\rho_n^2}\right) \mathrm{sgn}(b_{I,i}(k))(y_I(k)\mathbf{w}(k) + j\,\mathbf{r}(k)) \tag{15}$$

leads to the LBER algorithm

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \frac{\mu}{2}\left(\nabla\hat{P}_{E,R}(\mathbf{w}(k);k) + \nabla\hat{P}_{E,I}(\mathbf{w}(k);k)\right), \tag{16}$$

$$\mathbf{w}(k+1) = \frac{\mathbf{w}(k+1)}{\sqrt{\mathbf{w}^H(k+1)\mathbf{w}(k+1)}}, \tag{17}$$

where the adaptive gain μ and the width parameter ρ_n are to be chosen appropriately to ensure a best combined performance of convergence rate and steady-state BER misadjustment.

A simulation example

A four-equal-power-user system with the 4-QAM modulation was used in simulation. The four user spreading codes were $(+1,+1,+1,+1,-1,-1,-1,-1)$, $(+1,-1,+1,-1,-1,+1,-1,+1)$, $(+1,+1,-1,-1,-1,-1,+1,+1)$ and $(+1,-1,-1,+1,-1,+1,+1,-1)$, respectively. The CIR was $C(z) = (0.4 + 0.4j) + (0.7+0.7j)z^{-1} + (0.4+0.4j)z^{-2}$. For user 1 with an SNR=14 dB, and $\mu = 5/k$ and $\rho_n = \sigma_n$, the convergence performance of the LBER algorithm was shown in Fig. 2, where the results were averaged on 10 runs. The AMBER algorithm [2] was also modified to work for the 4-QAM, and its performance is also depicted in Fig. 2. It can be seen that the LBER algorithm has a faster convergence rate and a smaller steady-state BER. Fig. 3 compares the BER of the LBER detector with those of the MMSE and MBER detectors for user 1 over a range of SNR values.

Conclusion

A stochastic gradient adaptive MBER linear multiuser detector, originally developed for binary signalling schemes, has been extended to the complex-valued 4-QAM signalling scheme. Initial simulation results show that this proposed LBER detector has better performance, in terms of faster convergence rate and smaller steady-state BER, than an existing AMBER detector.

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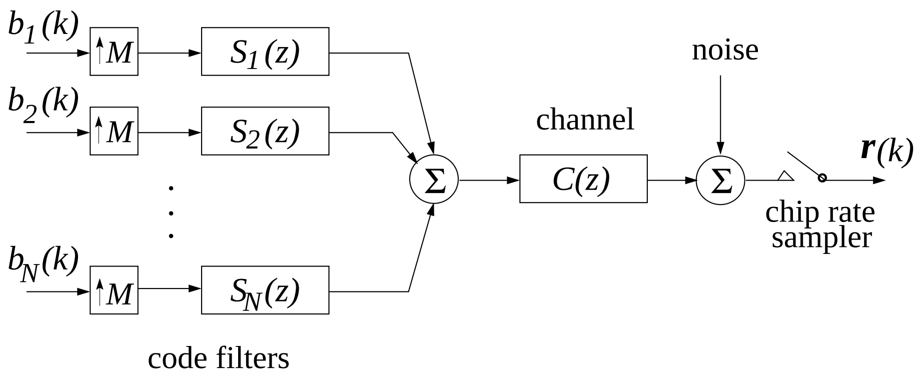

Figure 1: Discrete-time model of synchronous CDMA downlink.

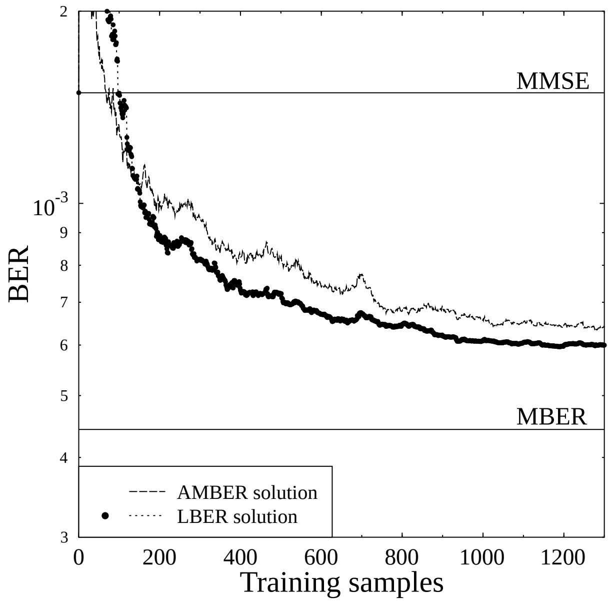

Figure 2: Learning curves of the LBER and AMBER algorithms for user 1 with SNR=14 dB.

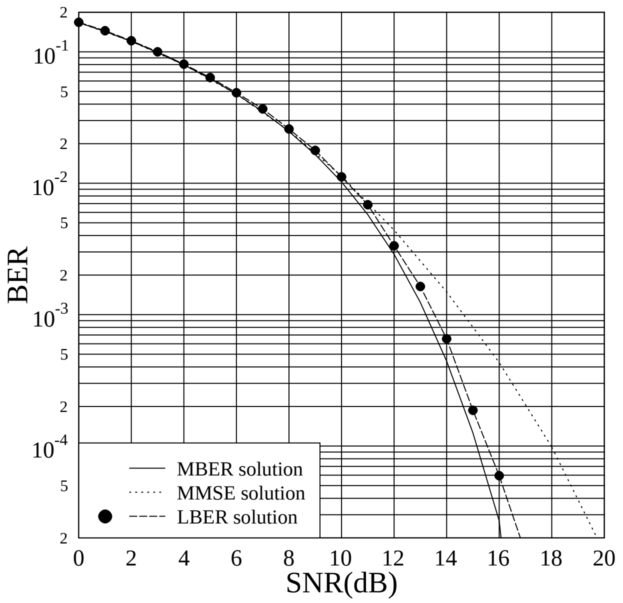

Figure 3: Performance comparison of the MMSE, MBER and LBER detectors for user 1.