PARALLEL CODE-ACQUISITION FOR MULTICARRIER DS-CDMA SYSTEMS COMMUNICATING OVER MULTIPATH NAKAGAMI FADING CHANNELS

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ABSTRACT
A parallel pseudo-noise (PN) code acquisition scheme is investigated for multicarrier CDMA (MC-CDMA) systems. The channel is modeled by a frequency-selective Nakagami distribution. The parallel acquisition scheme's performance is evaluated, when multiple synchronous states (H_1 cells) are assumed in the uncertainty region of the PN code, and when the correlator outputs of the subcarriers associated with the same phase of the local PN code replica are noncoherently combined with the aid of equal gain combining (EGC). It is demonstrated that when EGC is employed, the acquisition performance of the MC-CDMA scheme is better than that of the equivalent single-carrier CDMA benchmarker scheme over the investigated multipath Nakagami fading channels.

1. INTRODUCTION
Pseudo-noise (PN) code acquisition is the first action of any direct-sequence code division multiple-access (DS-CDMA) system. Acquisition refers to the coarse synchronization of the received PN sequence and the locally generated PN sequence within a fraction of the chip duration of the code sequence. The performance of code acquisition systems has been widely investigated, when the communication channels were modeled as non-fading Additive White Gaussian Noise (AWGN) channels [1, 2, 3], frequency-nonselective or frequency-selective Rayleigh or Rician fading channels [4, 5, 2, 6], respectively.

A more general fading channel model, which is often used in the literature for characterizing the fading statistics in a digital mobile radio channel, is the Nakagami-\( m \) distribution [8]. This model is versatile, and often better fits experimental data generated in a variety of fading environments - including urban as well as indoor radio propagation channel - than the Rayleigh and Rice distributions [9]. Moreover, the Nakagami-\( m \) distribution function models a continuous transition from a Rayleigh fading channel to a non-fading Gaussian channel by varying a single parameter, namely \( m \), from one to infinity [8]. Despite the above-mentioned advantages provided by the Nakagami-\( m \) distribution, however, the Nakagami fading channel model so far has not been invoked for the analysis of the initial synchronization performance of CDMA systems.

Our goal in this paper, therefore, is to quantify the performance of a parallel acquisition scheme over Nakagami-\( m \) fading channels, under the hypothesis that there are multiple synchronous states (H_1 cells) in the uncertainty region of the PN code [6]. Specifically, the acquisition performance of a pure parallel acquisition scheme is evaluated for a high-data rate multicarrier CDMA (MC-CDMA) system, since MC-CDMA transmission schemes have been proposed, in order to achieve further advantages in terms of bandwidth efficiency, frequency diversity, reduced complexity parallel signal processing and interference rejection capability in high data-rate transmissions [10]. The acquisition performance of the MC-CDMA system has received comparatively little attention in the literature, except for the contribution of [2]. In a high data rate MC-CDMA system considered in [11], the transmitted data bits are serial-to-parallel converted to a number of parallel streams and each stream is transmitted by a subcarrier as portrayed in Fig.1. In contrast to the MC-CDMA scheme considered in [2], where each subcarrier's signal was subjected to frequency-nonselective fading, in the MC-CDMA system considered here, frequency-selective fading may be encountered by the signals transmitted over a given subcarrier, since the bandwidth of the direct-sequence (DS) spread signal on each subcarrier may be wider than the coherence bandwidth of the channel [11]. However, since in many CDMA systems the transmitter aids the initial acquisition by transmitting the phase-coded carrier signal without data modulation, consequently, in a MC-CDMA scenario, the outputs of the noncoherent subcarrier correlators associated with the same phase of the local PN code replica can be combined, as shown in [2]. Hence, in our analysis a noncoherent equal gain combining (EGC) scheme is investigated and its performance is compared to that of the equivalent single-carrier CDMA scheme over frequency-selective Nakagami-\( m \) fading channels.

2. SYSTEM DESCRIPTION
2.1. Transmitted Signal
The \( k \)th user's transmitter is shown in Fig.1 in the framework of the MC-CDMA system considered, which is similar to that considered in [11], except that no interleaving across the different subcarriers is assumed in our analysis. However, the analysis in this contribution can be readily extended to the PN code acquisition study of MC-CDMA.
systems using interleaving across the different subcarriers. As shown in Fig. 1, the bit stream having a bit duration of $T_b$ is serial-to-parallel converted to $U$ parallel streams at the transmitter. The bit duration of each stream modulated on to the subcarrier hence equals $T_b = U T_c$. All data streams are spread by the same signature sequence, which is given by $c_k(t) = \sum_{n=-\infty}^{\infty} c_{kn}(t) P_{U T_c} (t-n U T_c)$, where $\{c_{kn}(t)\}$ is a random sequence with $c_{kn}(t)$ taking values of $\pm 1$ with equal probability, $P_{U T_c}(t) = 1$ for $0 \leq t < U T_c$, otherwise, $P_{U T_c}(t) = 0$. We assume that the system supports $K$ users and that all users have $U$ number of subcarriers. Moreover, we assume that the first user is the user-of-interest, while the other users are synchronized and reached the data transmission stage. Furthermore, we assume that ideal power control is employed for all the communicating users. Consequently, after modulating the corresponding subcarrier, the transmitted signal for user $k$ can be expressed as:

$$s_k(t) = \frac{1}{U} \sum_{u=1}^{U} \sqrt{2} F_u b_u(t) c_k(t) \cos(2 \pi f_d t + \phi_u),$$

where the subcarrier data stream $b_u(t) = \sum_{k=-\infty}^{\infty} s_{ku}(t) P_{U T_c} (t-u U T_c)$ consists of a sequence of mutually independent rectangular pulses of duration $T_b = U T_c$ and of amplitude $+1$ or $-1$ having an equal probability. Furthermore, the associated subcarrier transmitted powers are $P_k = P_1$, while $P_k = P_k$ for $k \neq 1$. Lastly, $f_d$ is the $u$th subcarrier frequency and $\phi_u$ is the random phase of each subcarrier. In our forthcoming analysis for the sake of simplicity, we assume that there exists no spectral overlap between the spectral main-lobes of two adjacent subcarriers [2], and there is no adjacent-channel interference between subcarriers. More explicitly, interference is inflicted only when an interfering user activates the same subcarrier, as the reference user. Let $N = U T_c / U T_c = T_b / T_c$ be the length of the spreading sequence $\{c_{kn}(t)\}$, where $T_b$ and $T_c$ are the corresponding bit duration and chip duration, respectively, associated with the equivalent single-carrier CDMA system ($U = 1$). Consequently, both the single-carrier CDMA and MC-CDMA systems considered have the same information rate, the same system bandwidth as well as the same PN spreading sequence length, which allows their direct comparison by comparing their detection probabilities.

### 2.2. Nakagami-$m$ Fading Channel

The channel model considered is the frequency selective Nakagami-$m$ fading channel having a complex low-pass channel impulse response (CIR) matched to user $k$ and subcarrier $u$ expressed as [12]:

$$h_{u}(t) = \sum_{l=0}^{L-1} a_{u,l} \delta(t-l U T_c) e^{-j \phi_{u,l}},$$

where $l U T_c$ is the relative delay of the $l$-th path of user $k$ with respect to the main path, the phases $\{\phi_{u,l}\}$ are independent identically distributed (iid) random variables uniformly distributed in the interval $[0, 2\pi)$. Furthermore, the $L$ tap weights $\{a_{u,l}\}$ are independent Nakagami-$m$ random variables with a PDF of [12]:

$$f_{a_{u,l}}(R) = \frac{2 m^m R^{m-1}}{T(m) \Gamma(m)} e^{-m/m R^2},$$

where $T(.)$ represents the gamma function, and $m$ is the Nakagami-$m$ fading parameter, which is defined as $m = E^2((a_{u,l})^2)/Var((a_{u,l})^2)$, while $\Gamma(n) = E((a_{u,l})^2)$.

When users’ signals in the form of Eq. (1) are transmitted asynchronously over the multipath fading channel characterized by Eq. (2), the received signal can be expressed as:

$$r(t) = \sum_{u=1}^{U} \sum_{l=0}^{L-1} \sqrt{2} F_u a_{u,l} c_k(t-l U T_c) \cos(2 \pi f_d t + \phi_u) + \sum_{u=1}^{U} \sum_{v=1}^{U} \sum_{l=0}^{L-1} \sqrt{2} F_u a_{u,l} b_v(t-l U T_c) \cos(2 \pi f_d t + \phi_v) + n(t),$$

where $\tau_{u,l}$ is the relative time delay between the user’s transmitted signal associated with an asynchronous transmission scheme, $\phi_{u,l} = \phi_u - \phi_{u,l} - 2 \pi f_d \tau_{u,l} + U T_c$ is a phase term, which is modeled with the aid of iid random variables uniformly distributed in the interval $[0, 2\pi)$ for different values of $u$, $k$ and $l$, while $n(t)$ represents the AWGN with a double-sided power spectral density of $N_0/2$.

### 2.3. Code-Acquisition Process

We investigate a parallel implementation of the maximum-likelihood acquisition scheme, as in [5], where all possible PN code phases of the received signal are tested simultaneously. We assume that the acquisition search step size for the MC-CDMA system concerned is $U T_c/2$, and that there are $L$ resolvable paths for each subcarrier signal. Hence, for a PN sequence having an uncertainty region of $q$ cells in the AWGN channel, the total number of cells to be searched in the $L$-tap dispersive fading channel is $(q U T_c / U T_c + (L-1) U T_c) / U T_c = q + 2(L-1)$ and the number of $H_1$ cells is $U T_c / U T_c / 2 = 2L$, where $(L-1) U T_c$ represents the maximum delay-spread of the $L$-tap fading channel. Let $\{z_0, z_1, \ldots, z_{q+2L-1}\}$ denote the decision variables corresponding to all possible PN code phases, where each element $z_i$ of the set of $q + 2(L-1)$ number of decision variables corresponds to one of the possible $q + 2(L-1)$
Figure 2: Schematic of generating the decision variable $Z_i$ for the multi-carrier DS-CDMA code-acquisition system.

phases. The block diagram of the proposed scheme used to generate the decision variable $Z_i$ is shown in Fig. 2. The philosophy of the parallel search scheme considered is that, the largest decision variable of $\{Z_0, Z_1, \ldots, Z_q+2L-3\}$ corresponds to $q + 2(L - 1)$ cells is selected, which is then forwarded to the associated verification mode. If this cell is confirmed during the verification mode, acquisition is declared. Otherwise, if the largest decision variable $Z_i$ of the set $\{Z_0, Z_1, \ldots, Z_q+2L-3\}$ cannot be confirmed by the verification mode, then a set of $q + 2(L - 1)$ new decision variables is collected, and the above process is repeated.

In the MC-CDMA system of Fig. 1, since all subcarriers employ the same PN spreading sequence, and since no data modulation is imposed during the acquisition stage, the outputs of the $U$ noncoherent subcarrier correlators seen in Fig. 2, which are associated with the same specific phase of the local PN code replica can be combined. In this treatment, noncoherent EGC combining scheme is investigated, since its acquisition performance is better than that of the associated selection combining (SC) scheme [2]. Furthermore, since the PN code chip rate of the MC-CDMA system using $U$ subcarriers is a factor of $U$ lower, than that of the corresponding single-carrier CDMA system, the number of chips collected during the integration period is also reduced by a factor of $U$. Hence, the average energy captured by the correlator of Fig. 2 of the MC-CDMA system during the associated constant integration dwell-time is also a factor of $U$ lower, than that of the corresponding single-carrier CDMA system. However, since the chip rate of the single-carrier CDMA system is $U$ times higher, than that of the MC-CDMA system, the number of resolvable paths for the single-carrier CDMA system will be $U$ times higher, than that of the MC-CDMA system. Bearing these relations between the single-carrier and MC-CDMA systems in mind, let us now derive the detection probability for the EGC scheme. We proceed by deriving the statistics of the decision variables.

3. DECISION VARIABLE STATISTICS

Since we have assumed that there exists no spectral main-lobes of two adjacent subcarriers, hence the statistics associated with subcarrier $f_u$ are identical to those of a single-carrier CDMA system over the multipath fading channel. The output variable of Fig. 2 matched to the subcarrier $f_u$ for the user-of-interest can be expressed as:

$$Z_{uv} = Z_{uv}^2 + Z_{uv}^2$$

$$= \left[ \int_{0}^{UT_L} r(t)c(t-iUT_L/2)\sqrt{2}\cos(2\pi f_u t)dt \right]^2 + \left[ \int_{0}^{UT_L} r(t)c(t-iUT_L/2)\sqrt{2}\sin(2\pi f_u t)dt \right]^2,$$

where $i = 0, 1, 2, \ldots, q + 2L - 3$, $u = 1, 2, \ldots, U$. It can be observed that both $Z_{uv}$ and $Z_{uv}^2$ can be approximated by Gaussian random variables having normalized means given by $\frac{3}{4}c_{uv}^2 \cos \theta_{uv}$ and $\frac{3}{4}c_{uv}^2 (-\sin \theta_{uv})$ [6, 7], if an $H_1$ cell is being tested, and zero, if an $H_0$ cell being tested. The normalized variance for both $Z_{uv}$ and $Z_{uv}^2$ is given by $\sigma^2_{Z_{uv}} = \frac{(L-1)^2}{3M} + \frac{(K-1)\rho^2}{3M} + \frac{1}{5MC}$ [6], where $\gamma = E_c/N_0$ is the signal-to-noise ratio (SNR) per chip (SNR/chip) and $E_c$ is the chip energy, $\rho = E_f/P_f$ is the ratio of subcarrier transmitted power of the interfering user to that of the user-of-interest.

For EGC scheme investigated, the decision variable $Z_i$ is given by the sum of the synchronous outputs $\{Z_{uv}\}$ for $u = 1, 2, \ldots, U$ seen in Fig. 2, i.e.

$$Z_i = \sum_{u=1}^{U} Z_{uv}.$$

Assuming that $\{Z_{uv}\}$ are iid variables, and provided that $Z_i$ in Fig. 2 constitutes an $H_1$ sample, $Z_i$ obeys the chi-square distribution with 2L degrees of freedom [13]. After the normalization of $Z_i$ by $\sigma^2_{Z_{uv}}$, the normalized PDF of $Z_i$ can be expressed as:

$$f_{Z_i}(y|S^2, H_1) = \frac{1}{2} \left( \frac{y}{S^2} \right)^{(U-1)/2} e^{-\left(y^2 + y\right)/2},$$

$$\gamma_{Z_{uv}} = \frac{3}{4} \left( \frac{(L-1)^2}{3M} + \frac{(K-1)\rho^2}{3M} + \frac{1}{5MC} \right)^{-1/2}.$$

We assume throughout that all the path gains are iid random variables. Consequently, with the aid of Eq. (3), it can be shown that the PDF of $S^2$ can be expressed as:

$$f_{S^2}(\gamma) = \left( \frac{m}{n} \right)^{\gamma_{S^2}} \Gamma(\gamma_{S^2}^{(U-1)}) \Gamma(U) e^{-m\gamma/2}, \quad \gamma \geq 0.$$

By contrast, when $Z_i$ constitutes an $H_0$ sample, $Z_i$ in Fig. 2 is the central chi-square distribution and its normalized PDF can be expressed as [13]:

$$f_{Z_i}(y|H_0) = \frac{1}{2^{(U-1)/2}} \Gamma(U) e^{-y/2}, \quad y \geq 0.$$

Let us assume that the decision variables $\{Z_0, Z_1, \ldots, Z_{q+2L-3}\}$ are independent random variables. Then the conditioning upon $S^2$ in Eq. (7) may be removed by averaging.

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\( f_{x_2}(y; S^2, H_1) \) over the valid range of \( S^2 \), which can be expressed as:

\[
f_{x_2}(y; H_1) = \int_0^{\infty} f_{x_2}(y; H_1)f_{S^2}(\gamma)d\gamma.
\]  \hspace{1cm} (11)

Upon substituting Eq. (7) and Eq. (9) into Eq. (11), it can be shown that:

\[
f_{x_2}(y; H_1) = \frac{\gamma^{U-1}}{2^U(U-1)!} \exp \left( -\frac{\gamma y}{\sigma_e^2 + 2m} \right)
\times \frac{1}{\Gamma(1-m,\gamma)} \frac{\gamma y}{(1 + \sigma_e^2/2m)^{U+m}}
\times \frac{\gamma^2}{2\sigma_e^2 + 4m}, \hspace{1cm} (12)
\]

where \( \Gamma(1-m,\gamma) = \int_0^{\infty} \frac{\gamma^x e^{-\gamma}}{x^m} dx \) is the confluent hypergeometric function, which is defined as [13]:

\[
\Gamma(a + k) = \sum_{k=0}^{\infty} \frac{\Gamma(a)\Gamma(b + k)}{\Gamma(b + 1)k!}, \hspace{1cm} b \neq 0, -1, -2, \ldots \hspace{1cm} (13)
\]

4. DETECTION PROBABILITY

The detection probability is defined as the chance that the largest of the decision variables \( \{Z_0, Z_1, \ldots, Z_{2L-1}\} \) which are associated with the \( q + 2(L-1) \) number of cells in the uncertainty region - corresponds to one of the \( 2L \) number of \( H_1 \) cells. Hence, the detection probability can be expressed as:

\[
P_D = 2L \int_0^{\infty} f_{x_2}(y; H_1) \left( \int_0^{L} f_{S^2}(\gamma; H_1)d\gamma \right)^{2L-1}
\times \left( \int_0^{\infty} f_{S^2}(\gamma; H_0)d\gamma \right)^{2L-1} dy.
\]

Consequently, substituting \( f_{x_2}(y; H_1) \) and \( f_{S^2}(\gamma; H_1) \) from Eq. (12) as well as \( f_{S^2}(\gamma; H_0) \) from Eq. (10) into Eq. (14), finally, the detection probability using EGC can be simplified to:

\[
P_D = 2L \int_0^{\infty} \frac{\gamma^{U-1}}{2^U(U-1)!} \exp \left( -\frac{\gamma y}{\sigma_e^2 + 2m} \right)
\times \frac{1}{\Gamma(1-m,\gamma)} \frac{\gamma y}{(1 + \sigma_e^2/2m)^{U+m}}
\times \frac{\gamma^2}{2\sigma_e^2 + 4m}
\times \frac{1}{\Gamma(1-m,\gamma)} \frac{\gamma y}{(1 + \sigma_e^2/2m)^{U+m}}
\times \left( \int_0^{\infty} \frac{(y/2)^{n}}{(2m + \sigma_e^2)^{n}} (U m + n) ! B(U m + n + 1)! \right)^{2L-1} \int_0^{\infty} \exp \left( -\frac{y}{2} \right) \left( \sum_{k=0}^{U+n} \frac{(y/2)^k}{k!} \right)^{2L-1} dy.
\]

where \( B(\cdot) \) is the Beta function, which is defined as:

\[
B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}, \hspace{1cm} a > 0, b > 0.
\]

The associated erroneous detection probability is given by \( 1 - P_D \).

5. NUMERICAL RESULTS

The probability of erroneous detection for EGC over the frequency-selective Rayleigh fading \( (m = 1) \) channel and frequency-selective Nakagami-\( m \) fading \( (m = 5) \) channel is shown in Fig.3 and Fig.4, where \( L_p \) and \( M_p \) represent the number of resolvable paths and the number of chips in the integral dwell-time corresponding to a single-carrier CDMA system \( (U = 1) \), respectively. For MC-CDMA, the number of resolvable paths for each subcarrier signal is \( L = L_p / U \) and the number of chips collected during the integral dwell-time is \( M = M_p / U \). From the results we observe that over the multipath fading channels considered, the acquisition performance of the MC-CDMA system using EGC is better, than that of the single-carrier CDMA system.

In Fig.5, we estimated the influence of the fading parameter \( m \) on the detection performance of both the single-carrier CDMA and the eight-subcarrier MC-CDMA system using the parameters of \( N = 128, \quad q = 256, \quad K = 1, \quad \rho = \).
1. \( L_0 = 16 \) and \( M_p = 128 \). From the results, we observed that, for the eight-subcarrier MC-CDMA system, the detection performance improved, when the value of \( m \) increased, which implies that the channel quality became better. However, for the single-carrier CDMA system of \( U = 1 \), the detection performance degraded upon increasing the value of the parameter \( m \). This phenomenon can be explained with the aid of Eq. (14). According to Eq. (14), on the one hand, the detection probability will be increased, when the value of \( m \) increases, since the peak of \( f_2(q; H_1) \) will move away from the peak of \( f_2(x; H_0) \). However, on the other hand, the integral of \( \int f_2(z; H_1)dz \) will be decreased, when \( m \) increases, and the composite effect also depended on the exponent \( 2L-1 \) in Eq. (14). Consequently, for a high number of resolvable paths, such as \( L = L_0 = 16 \) in the single-carrier CDMA system, the resultant erroneous detection probability increases, as the value of \( m \) increases. However, for a low number of resolvable paths, such as \( L = 16/8 = 2 \) in the eight-subcarrier MC-CDMA system, the resultant erroneous detection probability decreases, as the value of \( m \) increases. Note that during the acquisition stage of the MC-CDMA system, the transmitted energy dispersed over the multipath components cannot be combined, but the transmitted energy assigned to the subcarrier signals can be efficiently combined, as discussed in this paper.

Finally, in Fig. 6 we estimated the erroneous detection probability versus the number of active users. The parameters employed were shown on the top of the figure. As expected, the detection performance degrades, when the number of active users increases. Furthermore, from the results of Figs. 3, 4 and 6, we can conclude that the detection performance of the MC-CDMA scheme is better than that of the single-carrier CDMA scheme, and the detection performance is enhanced, when increasing the number of subcarriers.

6. REFERENCES


