SUBBAND ADAPTIVE GENERALIZED SIDELOBE CANCELLER FOR BROADBAND BEAMFORMING

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ABSTRACT

In this paper, we propose a novel subband adaptive broadband beamforming architecture based on the generalised sidelobe canceller (GSC), in which we decompose each of the tapped delay-line signals feeding the adaptive part of the GSC and the reference signal into subbands and perform adaptive minimisation of the mean squared error in each subband independently. Besides its lower computational complexity, this new subband adaptive GSC outperforms its fullband counterpart in terms of convergence speed because of its prewhitening effect. Simulations based on different kinds of blocking matrices with different orders of derivative constraints are presented to support these findings.

1. INTRODUCTION

Adaptive beamforming has found many applications in various areas ranging from sonar and radar to wireless communications. It is based on a technique where, by adjusting the weights of a sensor array with attached filters, a prescribed spatial and spectral selectivity is achieved. Fig. 1 shows a beamformer with M sensors receiving a signal of interest from the direction of arrival (DOA) angle ϑ.

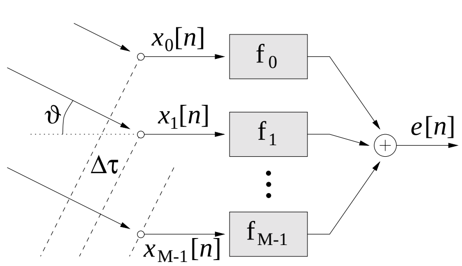

Fig. 1: A signal impinging from an angle ϑ onto a beamformer with M sensors.

To perform beamforming with high interference rejection and resolution, arrays with a large number of sensors and filter coefficients have to be employed. To facilitate real-time implementation, various methods are employed to reduce the computational complexity, such as the partially adaptive beamforming [1], wavelet-based beamforming [2] and subband beamforming [3]. In the latter, the received sensor signals are first split into decimated subbands, then an independent beamformer is applied to each subband. The advantage arises from the processing in decimated subbands, although at the expense of having to project constraints into the subband domain as well.

We here focus on a linearly constrained minimum variance (LCMV) beamformer, which can be efficiently implemented as a generalized sidelobe canceller (GSC) [4, 5]. Different from [3], instead of performing beamforming in subbands by decomposing the input sensor signals, we employ subband adaptive filtering techniques for the adaptive process of the GSC structure only. Specifically, noting that there are in total $M - S$ input tapped delay-lines for the adaptive part of the GSC, we decompose each of the tap-delay line signals and the reference signal $d[n]$ into K subbands by a K-channel filter banks as shown in Fig. 3 and perform adaptive minimisation in each subband. Simulation results with different blocking matrices and different order of derivative constraints show that this new method outperforms the fullband counterpart in addition to its very low computational complexity.

The rest of this paper is organised as follows: Section 2 is a brief review of GSC-based broadband beamforming based on a generalized sidelobe canceller with derivative constraints. In Section 3, we introduce the proposed subband-based GSC structure. Simulation and results will be given in Section 4 and conclusions are drawn in Section 5.

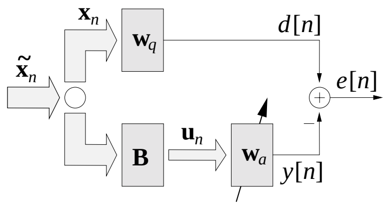

Fig. 2: Structure of a generalized sidelobe canceller.

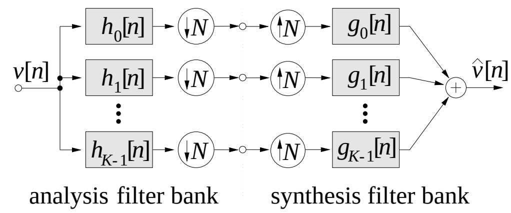

Fig. 3: K channel filter banks with decimation N.

2. GENERALIZED SIDELOBE CANCELLER

An LCMV beamformer performs the minimization of the variance or power of the output signal with respect to some given spatial and spectral constraints. For a beamformer with M sensors and J filter taps following each sensor as shown in Fig. 1, the output $e[n]$ can be expressed as:

$$e[n] = \mathbf{w}^{\mathrm{H}} \cdot \mathbf{x}_n \tag{1}$$

where coefficients and input sample values are defined as

$$\mathbf{w} = \left[\mathbf{w}_0^{\mathrm{T}} \;\; \mathbf{w}_1^{\mathrm{T}} \;\; \ldots \;\; \mathbf{w}_{J-1}^{\mathrm{T}}\right]^{\mathrm{H}} \tag{2}$$

$$\mathbf{w}_l = [w_0[l] \;\; w_1[l] \;\; \ldots \;\; w_{M-1}[l]]^{\mathrm{T}} \tag{3}$$

$$\mathbf{x}_n = \left[\tilde{\mathbf{x}}_n^{\mathrm{T}} \;\; \tilde{\mathbf{x}}_{n-1}^{\mathrm{T}} \;\; \ldots \;\; \tilde{\mathbf{x}}_{n-J+1}^{\mathrm{T}}\right]^{\mathrm{T}} \tag{4}$$

$$\tilde{\mathbf{x}}_n = [x_0[n] \;\; x_1[n] \;\; \ldots \;\; x_{M-1}[n]]^{\mathrm{T}} \quad . \tag{5}$$

The data vector $\tilde{\mathbf{x}}_n$ is a time slice as given in Fig. 1. A coefficient $w_m[l]$ is defined to sit at the tap position l of the mth filter f_m. The LCMV problem can now be formulated as [6]

$$\min_{\mathbf{w}} \mathbf{w}^{\mathrm{H}} \mathbf{R}_{xx} \mathbf{w} \qquad \text{subject to} \qquad \mathbf{C}^{\mathrm{H}} \mathbf{w} = \mathbf{f} \quad , \tag{6}$$

where $\mathbf{R}_{xx}$ is the covariance matrix of observed array data in $\mathbf{x}_n$, $\mathbf{C} \in \mathbb{C}^{MJ \times SJ}$ is a constraint matrix and $\mathbf{f} \in \mathbb{C}^{SJ}$ is the constraining vector. The constraint matrix here imposes derivative constraints of order $S-1$ [7],

$$\mathbf{C} = \left[\hat{\mathbf{C}}_0 \ldots \hat{\mathbf{C}}_{S-1}\right] \qquad \text{with} \quad \hat{\mathbf{C}}_i = \begin{bmatrix} \mathbf{c}_i & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{c}_i \end{bmatrix} \tag{7}$$

with $\mathbf{c}_i = \left[(-m_0)^i \;\; (1-m_0)^i \;\; \cdots \;\; (M-1-m_0)^i\right]^{\mathrm{T}}$ and a phase origin point m_0.

The constrained optimisation of the LCMV problem in (6) can be conveniently solved using a GSC. The GSC performs a projection of the data onto an unconstrained subspace by means of a blocking matrix $\mathbf{B}$ and a quiescent vector $\mathbf{w}_{\mathrm{q}}$. Thereafter, standard unconstrained optimisation algorithms such as least mean square (LMS) or recursive least squares (RLS) algorithms can be invoked [8]. Fig. 2 shows the principle of a GSC, where the desired signal $d[n]$ is obtained via $\mathbf{w}_{\mathrm{q}}$,

$$d[n] = \mathbf{w}_{\mathrm{q}}^{\mathrm{H}} \cdot \mathbf{x}_n \qquad \text{with} \quad \mathbf{w}_{\mathrm{q}} = \mathbf{C}(\mathbf{C}^{\mathrm{H}}\mathbf{C})^{-1}\mathbf{f} \quad . \tag{8}$$

The input signal $\mathbf{u}_n = [u_0[n] \;\; u_1[n] \;\; \ldots \;\; u_{M-S-1}[n]]^{\mathrm{T}}$ to the following multichannel adaptive filter (MCAF) is obtained by $\mathbf{u}_n = \mathbf{B}^{\mathrm{H}} \tilde{\mathbf{x}}_n$, whereby the $M \times (M-S)$ blocking matrix $\mathbf{B}$ must satisfy

$$\tilde{\mathbf{C}}^{\mathrm{H}} \mathbf{B} = \mathbf{0} \qquad \text{where} \quad \tilde{\mathbf{C}} = [\mathbf{c}_0 \;\; \cdots \mathbf{c}_{S-1}] \quad . \tag{9}$$

In the next section, we will focus on the multiple-input optimisation process and introduce our subband adaptive GSC structure by employing the subband adaptive filtering techniques.

3. SUBBAND ADAPTIVE GENERALIZED SIDELOBE CANCELLER

Subband decompositions for adaptive filtering applications are commonly based on oversampled modulated filter banks (OSFB) as shown in Fig. 3 , where the input signal is divided into K frequency bands by analysis filters and then decimated by a factor N. Due to oversampling, i.e. $N < K$, a low alias level in the subband signals can be achieved. This is important since aliasing will limit the performance of an subband adaptive filtering (SAF) system [9]. Due to its lower update rate and fewer coefficients to represent an impulse response of a given length, the subband implementation only necessitates K/N^2 (K/N^3) of the operations required for a fullband adaptive algorithm with a complexity of $\mathcal{O}(L_{\mathrm{a}})$ ($\mathcal{O}(L_{\mathrm{a}}^2)$), where L_{a} is the total number of coefficients in the fullband realisation [3].

When applying SAF techniques to the MCAF in the GSC structure in Fig. 2, the subband setup as shown in Fig. 4 arises. There, the blocks labelled A perform an OSFB analysis operations, splitting the signal into K frequency bands each running at an N times lower sampling rate compared to the fullband input to the

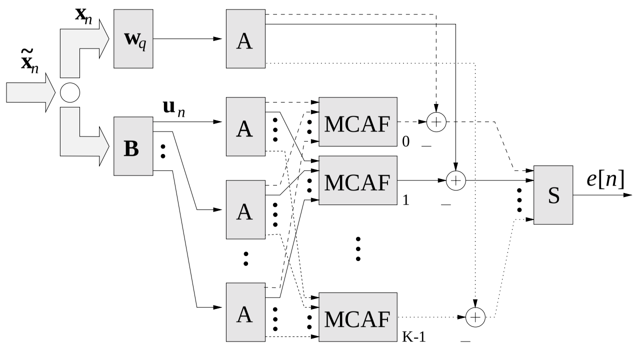

Fig. 4: Subband adaptive GSC; an independent MCAF is applied to each subband.

block. Within each subband, an independent MCAF is operated, and a synthesis filter bank, labelled S, recombines the different subsystem outputs to a fullband beamformer output $e[n]$.

In addition to the lower computational complexity of this subband adaptive GSC, it promises faster convergence speed for LMS-type adaptive algorithms because of the pre-whitening effect of the input signal. Next, we will give some simulation results to demonstrate the performance of our subband adaptive GSC.

4. SIMULATIONS AND RESULTS

In our simulation, we use a beamformer with $M = 15$ sensors and $J = 60$ coefficients for each attached filter. Each of the input signals $u_i[n]$ ($i = 0, 2, \cdots, M-S-1$) and the reference signal $d[n]$ are divided into $K = 8$ subbands by an oversampled GDFT filter bank [10] with decimation factor $N = 6$ as characterised in Fig. 5. This subband adaptive GSC is constrained to received a signal of interest from broadside, which is white Gaussian with unit variance. The beamformer should adaptively suppress a broadband interference signal covering the frequency interval $\Omega = [0.25\pi; 0.75\pi]$ from $\vartheta = 30°$ and with a signal-to-interference ratio (SIR) of -24 dB. The sensor signals are corrupted by additive Gaussian noise at an SNR of 20 dB.

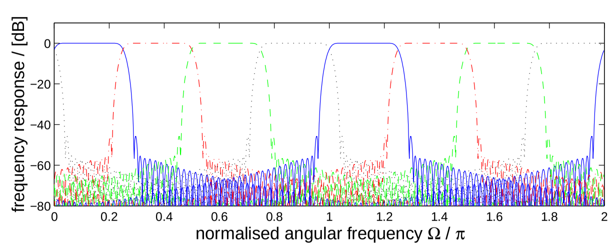

Fig. 5: Magnitude response of $K = 8$ channel filter bank decimated by $N = 6$.

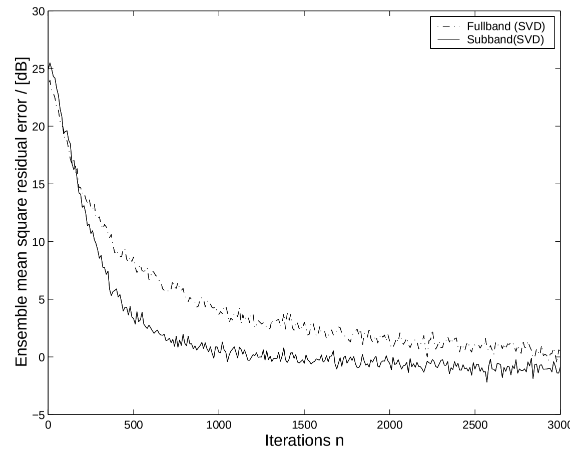

Fig. 6: Learning curves for simulation I ($S = 2$).

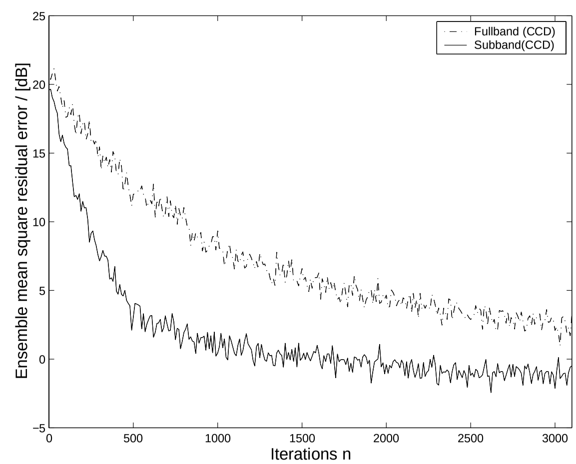

Fig. 7: Learning curves for simulation II ($S = 2$).

In order to compare the performance of our subband method with its fullband counterpart, we give four examples based on two commonly used approaches for building the blocking matrix, each with two different orders of constraints. The first approach is based on the cascaded columns of difference (CCD) method [11], the second on a singular value decomposition (SVD) [5]. The four examples are: (I) SVD method with first order derivative constraints ($S = 2$), (II) CCD method with $S = 2$, (III) SVD method with zero order derivative constraints ($S = 1$), (IV) CCD method with $S = 1$.

The step size in the NLMS adaptation for the first two examples is set to $\tilde{\mu} = 0.30$, and to $\tilde{\mu} = 0.20$ for examples (III) and (IV). Simulation results for these four cases are shown in Fig. 6 to Fig. 9, respectively. As a performance criterion, these figures display the ensemble mean square value of the residual error, which is defined as the difference between the beamformer output $e[n]$ and the appropriately delayed desired signal received from broadside.

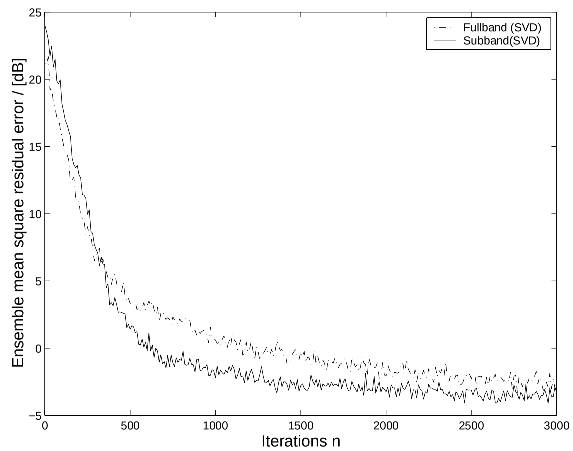

Fig. 8: Learning curves for simulation III ($S = 1$).

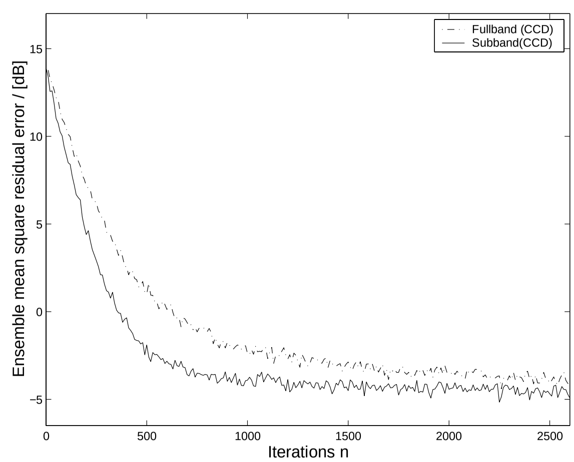

Fig. 9: Learning curves for simulation IV ($S = 1$).

From these results we can see that the subband adaptive method always has a faster convergence speed because of its pre-whitening effect. Comparing Fig. 6 with Fig. 7 and Fig. 8 with Fig. 9, we see the fullband performance changes according to different building of the blocking matrix, whereas the subband method has a relatively uniform performance independent of settings. With the added benefit of its low computational complexity due to processing in decimated subbands, the presented subband method outperforms the traditional fullband implementation.

5. CONCLUSIONS

A novel subband adaptive Generalized Sidelobe Canceller for broadband beamforming has been proposed. By employing subband adaptive filtering techniques, the computational complexity is greatly reduced. Moreover, the new method can also achieve a faster convergence speed because of its pre-whitening effect. Superiority of this new method to fullband implementation has been demonstrated by four examples based on different approaches for the blocking matrix and different orders of derivative constraints.

6. REFERENCES

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