Optimal Finite Element Modelling and Efficient Reconstruction in Non-Linear 3D Electrical Resistance Tomography

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Abstract

Electrical Impedance Tomography can provide images with well-defined characteristics using a fully non-linear reconstruction process when appropriate constraints are imposed on the solution to allow the ill-posed inverse problem to be solved. Using appropriate finite element discretizations for forward solution and inverse problem offers additional advantages in the image reconstruction process, such as (a) inclusion of prior knowledge, (b) generic model templating to adapt to, for example, individual head shapes, and (c) obtaining accurate results without unnecessary computational overhead.

We have developed an efficient 3D non-linear reconstruction algorithm based on a regularized inverse conjugate gradient solver which incorporates (a) local image smoothness constraints, and (b) a number of optimisations which reduce the computing power required to obtain an accurate solution. We show results from applying this to various problems which arise in medical resistivity reconstruction given only surface potential measurements and demonstrate the importance of the FE discretization.

Keywords

3D non-linear electrical impedance tomography, FE template modelling, optimal finite element meshes, 3D visualization, FE discretization.

1. Introduction

Recent medical interest is focused on accurately representing geometric objects (Gibson et al, 2000) and on near real-time reconstruction. We discuss how our algorithm, which is based on the finite element method, can be applied to these problems. In particular, the optimizations we have implemented, such as the use of adaptive meshing to improve the reconstruction resolution and parallel computing methods (Blott et al, 2000), are highly applicable.

Electrical Impedance Tomography algorithms in two dimensions are well established. Many different flavours of algorithms have been devised to enable fast and accurate 2D reconstructions. Several of these algorithms can be applied or at least extended three-dimensional problems. In three dimensional EIT the main problems to be overcome are

- accurate boundary shape representation
- suitable algorithms for general 3D imaging (non-circular, no application of dedicated interpolation functions such as Bessel functions)
- high reconstruction time caused by large dense matrices
- resolution inaccuracies caused by deficient element quality

Jain et al (1997) showed for 2D circular problems that inaccurate modelled object boundaries can cause large errors in the material reconstruction. In particular, a circular representation of an elliptic boundary caused 37% additional error in the reconstruction. We summarise the requirements for an efficient reconstruction algorithm in table 1.

In this paper, we demonstrate the importance of using a high-quality finite element object representation.

Speed	application of sparse matrix storage schemes and solver techniques problem-adapted mesh density parallelization of code
Accuracy	usage of high-quality domain discretization robustness with respect to noise minimal influence of constraints and regularization on solution accuracy suitable algorithm for the problem's non-linear nature
Flexibility	accurate modelling of complex 2D and 3D geometries allow for easy application of differing boundary conditions FE mesh "*templating*" and node relocation for dynamic imaging

Table 1 Requirements for an efficient reconstruction algorithm

2. Method

Our reconstruction of a conductivity distribution σ within a volume conductor Ω by means of Electrical Tomography is based on a least square minimization of a functional ϕ, which

(a) employs a λ-weighted χ^2-statistic incorporating the difference between computed and measured electrode voltages U_i^{comp} -U_i^{meas} and accounting for the measurement error δU_i^{meas}

(b) includes a term based on image properties, such as image smoothness, $\nabla \log(\sigma)$, to determine a well-defined reconstruction:

$$\phi = \frac{1}{2}\lambda \sum_{i=1}^{\text{electrodes}} \frac{\left(U_i^{comp} - U_i^{meas}\right)^2}{(\delta U_i^{meas})^2} + \frac{1}{2}\int_\Omega (\nabla \log \sigma)^2 dxdydz \qquad (1)$$

This system is equivalent for two- and three-dimensional problems. To solve the above equations for the unknown conductivity distribution σ, we have employed a conjugate gradient algorithm based on a modified Newton-Raphson scheme for which Yorkey et al (1987) showed fastest convergence of a number of EIT reconstruction processes. For more details about the algorithm please refer to the paper of Blott et al in these proceedings.

The correct presentation of parameters involved in the reconstruction process is essential. This includes the correct modelling of the object's boundary, the correct location of electrodes and the application of a suitable discretization of the object for the numerical procedure.

2.1 Finite Element Discretization

Since there exist no analytic solutions to the generalized problem, the object under investigation needs to be discretized to enable the application of numerical techniques. We employ the well-known and most suitable Finite Element (FE) Method which provides many advantages over other methods in the reconstruction process.

Finite element techniques are well studied and understood (Burnett, 1987) and in comparison with Finite Difference methods produce piecewise solutions on the domain (compared to only pointwise in FD). Boundary Elements are applicable only in certain cases where the volume contains only few differing materials (Webster, 1990).

The main advantages of Finite Element discretizations are flexibility in terms of geometry and in the application of different types of boundary conditions. However, the speed and accuracy of the imaging process – the reconstruction of conductivities given only surface potential measurements – highly depends on the underlying FE discretization.

A finite element discretization appropriate to a certain problem, allows for additional advantages in the imaging procedure:

1) *Inclusion of prior image knowledge.* Prior knowledge about the problem or object can be included in the reconstruction process, for example image smoothness (Blott et al, 1998)
2) *Inclusion of prior material and geometric information*. Features extracted from MRI or CT scans can be used to improve the image iteration process by constraining it to respect certain boundaries (skull, bone, gray matter) or by applying an approximate extracted conductivity distribution as algorithm starting vector or prior (Vauhkonen, 1997).
3) *Problem adapted element density*. Pre-defined element density at locations where there are expected to be high current density gradients (eg around electrodes or the optical nerve)
4) *Generic model templating*. We will call the preparation of generic models for application to individually differing shapes/patients Finite Element mesh *templating*. This does not form part of the actual reconstruction process, however, if these templates are applied, it must be assured that elements are not distorted. Templates simplify the incorporation of the above-mentioned features.
5) *Adaptive meshing techniques*. Accurate results can be obtained without computational overhead when the elements are only added where necessary during image reconstruction (Molinari et al, 2000)

3. 3D non-linear solver optimisations

We have employed the following optimisations to our 3D reconstruction algorithm to reduce the required computing power and obtain an accurate solution:

High quality mesh generation. An initial high quality discretization of the object is carried out with a suitable Finite Element mesh generator (eg. Netgen, Geompack, Bubble meshing method).

Three-dimensional adaptive mesh refinement methods. Adaptive Meshing method in three dimensions are much more complex than in two dimensions and require the application of appropriate strategies such as, for example, Bubble Meshing (Cingoski et al, 1997).

Conjugate gradient solver. The conjugate gradient solver is one of the most suitable solvers for ill-posed large scale problems. It scales with $O(n^{3/2})$ in 2D and $O(n^{4/3})$ in 3D where n is the number of elements per row in a matrix used in the reconstruction (Jacobian or sensitivity matrix). An additional advantage is that the non-linear conjugate gradient algorithm can easily be parallelized onto several processors.

Local image smoothness constraints. The ill-conditioning of the problem makes it important to regularize the algorithm to stabilize it and to improve convergence. Regularization can either be carried out by standard Tikhonov regularization or, preferably, by applying the above presented objective function which uses the smoothness of image as a property constraining the reconstruction process.

Parallel computing methods. Parallel computing methods are highly applicable to our reconstruction algorithm. We have demonstrated that a solution of the forward problem can be obtained using a cluster of computers working in parallel (Blott et al, 2000). In particular, the conjugate gradient solver is very efficient in a parallelised version (Hake, 1992). Current work involves the implementation of these techniques in Object oriented C++ code using MPI (Message Passing Interface) programming.

Visualization. The visual representation of values such as potentials, current densities, reconstructed material is much more demanding in three dimensions compared to two-dimensional imaging. Visualization using software such as Matlab or graphics libraries, for example OpenGL implementations, enables us to image isosurfaces, isopotentials, 3D material distributions, etc. based on the used or derived finite element mesh.

4. Results

We show results from simulated reconstructions applying an algorithm which incorporates some of the above optimisations. We have computed several reconstructions of a simple head model with three conductivity parameters. We assumed a head filled with cranio-spinal fluid (CSF, 1.55 S/m) and a mouth cave region (0.01 S/m) as well as a blood clot (6.67 S/m) in a central region behind the left eye of the patient.

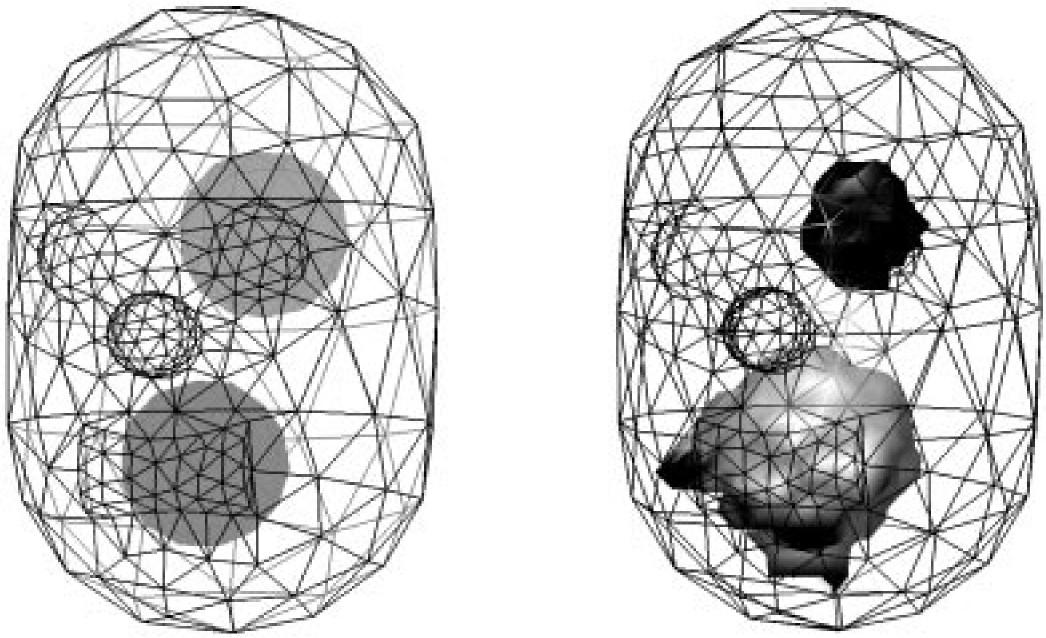
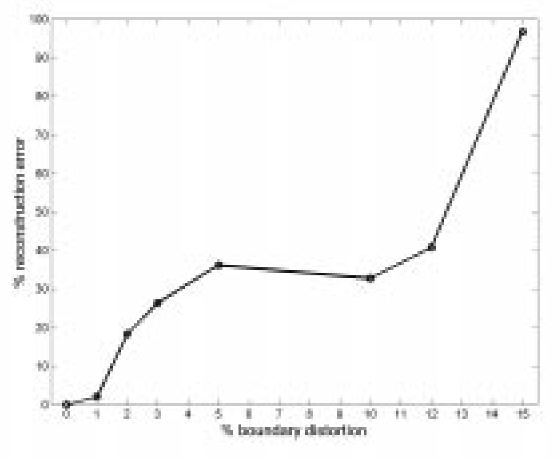


Figure 1: (Left) Our 3D 'dummy' head to investigate the effects of boundary distortion on the reconstruction. (Right) interpolated isosurface of the reconstructed conductivity distribution.

We have investigated the effect of inaccurate volume discretization on the reconstruction. For this, we used the mesh for the forward simulation and distorted it for the reconstruction by a small fraction. The resulting error in conductivities was obtained using $E = |\sigma^{\text{undistorted mesh}} - \sigma^{\text{distorted mesh}}| / |\sigma^{\text{undistorted mesh}}|$. Table 2 shows the results in comparison to an undistorted mesh, figure 2 shows the result graphically. Q is the – almost constant – average of the elemental quality distribution $q_i = D\ R_i/R_o$ where D denotes the dimension of the problem, R_i the inscribed radius and R_o the circumscribed radius of element *i* (Golias and Dutton, 1997). As only few elements were used to ensure a good reconstruction, the quality was quite low as the complex boundary causes high variations in mesh density.

Geometric distortion	Quality Q	Reconstruction error *E* relative to 'no distortion'
0	0.67	0
1 %	0.67	2.0 %
2 %	0.67	18.4 %
3 %	0.67	26.4 %
5 %	0.67	36.2 %
10 %	0.67	32.8 %
12 %	0.66	40.8 %
15 %	0.66	96.7 %*



(Left) Table 2: Effects of geometric modelling inaccuracies of the head boundary on the reconstructed conductivity distribution using a mesh with 409 nodes and 1495 elements.
(Right) Figure 2: Error of reconstruction on meshes with distorted geometry ('electrode dislocation')*

Mesh templating can cause elements and object boundaries to deform. In order to preserve the 'built-in' features such as the element density and quality, we need to utilize specific techniques (for example Bubble Meshing) when applying the template to a different shape/patient. As shown above, a mesh template used without these corrections will cause large errors.

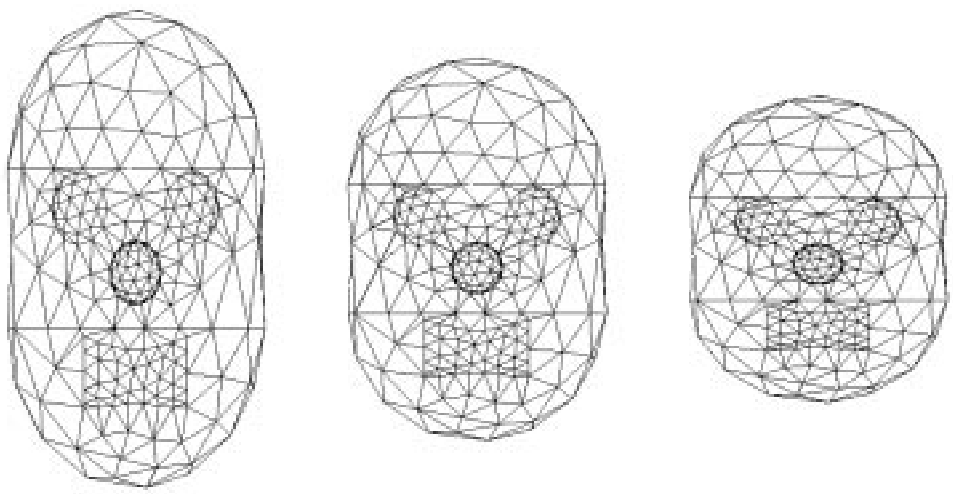


Figure 3: Mesh templates: From one mesh to many patients

5. Conclusions

We have shown that accurate presentation of the boundary of objects is essential for good reconstructions. It is also very important to employ quality finite elements in order not to deteriorate the possible reconstruction accuracy. To obtain these, a quality improving bubble meshing technique can be applied. This technique is also useful for reshaping meshes as used in Mesh Templating. We have discussed optimisations of the non-linear solver to obtain an efficient reconstruction and accurate results based on optimised finite element meshes. Adaptive finite element techniques can save a large amount of computation time by refining the mesh density only where necessary for a model accurate for the numerical process.

Further work will apply parallel computing methods which will then enable closer real-time imaging in the context of patient monitoring.

6. Acknowledgements

MM is grateful to EPSRC for funding.

7. References

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