A Fast Converging Fractionally Spaced Equaliser

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Abstract

We present subband adaptive structures that can counteract the slow convergence which is imposed on the feed-forward part of a fractionally spaced equaliser by the spectral dynamics of the oversampled channel output. While the prewhitening of the subband approach is therefore beneficial to the feed-forward part, additionally operating the feedback part in subbands results in a more balanced and faster convergence, which is demonstrated in simulations.

1. Introduction

Fractionally spaced equalisers (FSE) are known to outperform their symbol spaced counterparts in reducing the effect of inter-symbol interference (ISI) at the communication receiver [1]. However, results in [2] indicate that due to the colouring imposed by the channel, the eigenvalue spread of the fractionally spaced equaliser input can significantly slow down the convergence speed of LMS-type algorithms [3]. To obtain faster convergence, the use of recursive least squares (RLS) type algorithm has to be bought at the expense of high computational cost [4]. Therefore, in this paper we proposed subband approach for adaptive equalisation using LMS algorithm in order to improve the convergence rate at low computational complexity.

In the context of related adaptive filter applications, subband decompositions are known to prewhiten the signal and hence increase the convergence speed of LMS-type algorithms [5]. In the past, subband equaliser structures have mainly targeted the reduction of complexity when operating under severely distorting channel conditions [6]. In this paper, we discuss two potential subband FSE architectures, whereby either the FF part only or the entire FSE is realised in subbands.

The paper is organised as follows. In Sec. 2, subband decompositions are introduced and reviewed. Based on the fullband FSE in Sec. 3, we introduce two different subband FSE structures in Secs. 4 and 5. These are compared to the fullband case in simulations as presented in Sec. 6. Finally, conclusions are drawn in Sec. 7.

2. Subband Decompositions

A general subband decomposition into K frequency bands decimated by N is shown in Fig. 1. The filters in both analysis and synthesis bank are bandpass filters, which, together with the decimation process, yield a prewhitening of the subband signals compared to the fullband input. Further, computational savings arise due to an N times lower update rate and lower filter orders when filtering in subbands compared to a fullband implementation. For adaptive signal processing applications, adaptive filters can be operated in each band independently, which lends itself to a parallel implementations. As a drawback, subband structures generally introduce aliasing in the subband signals due to decimation which limits the algorithm's performance [7]. Therefore, oversampled modulated filter banks (OSFB) with $K/N > 1$ are preferred here [5]. Convergence limitations due to aliasing can however be determined [7] and controlled by appropriate filter design [8].

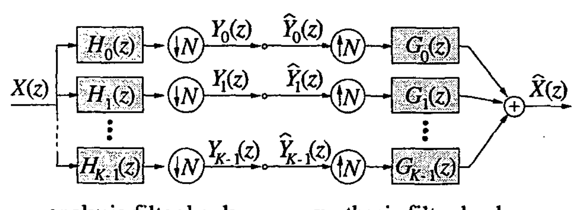

Fig. 1. K-channel filter bank decimated by N with analysis filters $H_k(z)$ and synthesis filters $G_k(z)$.

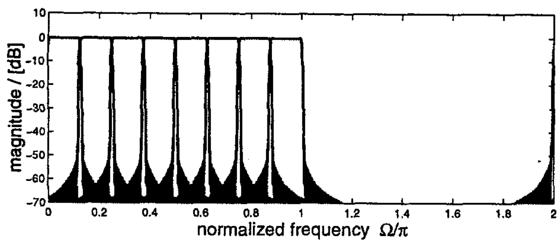

Fig. 2. Example of a generalised discrete Fourier transform (GDFT) modulated filter banks, with $K = 16$, showing only the lower $K/2$ bandpass filters.

The analysis and synthesis filter banks can be derived from a prototype low-pass filter by using, for example, a generalized discrete Fourier transform (GDFT) modulation. For modulated filter banks, both synthesis and analysis filters, $h_k(n)$ and $g_k(n)$, can be derived from a prototype filter by modulation. In the case of GDFT filter banks, the filter components of the filter banks are derived from a real valued finite impulse response (FIR) prototype low-pass filter $p(n)$ of length L_p by modulation with a GDFT

$$h_k(n) = p(n) \cdot e^{j\frac{2\pi}{K}(k+k_0)(n+n_0)} \tag{1}$$

for $n = 0(1)Lp - 1$ and $k = 0(1)K - 1$, where offset k_0 and n_0 for frequency and time indices are introduced [9]. For even K, the frequency interval $\Omega \in [0; \pi]$ will be covered by exactly $K/2$ subband signals; it is only these subbands that need to be processed if the inout signal is real valued, as the remaining subbands will be complex conjugate copies and therefore redundant. An example of GDFT modulated filter banks is illustrated in Fig. 2.

3. Fractionally Spaced Equaliser

The schematic of an FSE is depicted in Fig. 4, whereby a feed-forward (FF) filter is operated at twice the symbol rate, in addition to a symbol spaced feedback (FB) filter. The realisation of the FF part in Fig. 4 is such that the filter is split into two polyphase components, $a_0[n]$ and $a_1[n]$, which have here been exchanged with the twofold decimator. This has no influence of the functionality of the FSE, but will simplify the later derivation of FSE subband structures.

However, the input to the FF part generally exhibit a large eigenvalue spread, due to often severe spectral dynamics in the channel in combination with the increased bandwidth and the influence of the pulse shaping filters. An example for the channel dynamics is

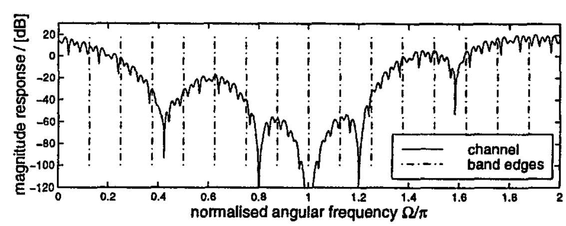

Fig. 3. Channel spectral dynamics characteristic with transmit- and receive filter.

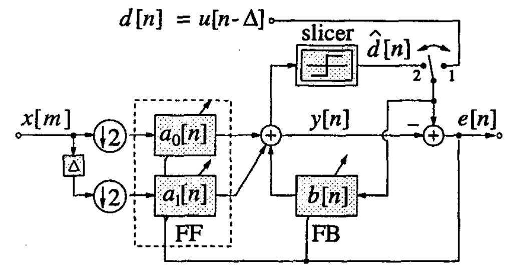

Fig. 4. Fractionally spaced equaliser.

given in Fig. 3. For the popularly applied least mean squares (LMS) type adaptive algorithms in FSEs, this results in slow convergence [2].

To minimize the influence of noise on the FB part, an equation error structure is selected [10], feeding the FB filter with the desired symbol sequence $d[n]$. The FB part $b[n]$ can be excited by either a training signal (switch position 1) — a copy of the transmitted symbol sequence $u[n]$ delayed by Δ periods — or in decision feedback mode (switch position 2) is shown in Figure 4. As this fedback data is generally scrambled, the inout to the FB filter $b[n]$ is white in good approximation and its convergence generally poses no problem.

4. Subband Equaliser Structure I

Based on the FSE structure in Fig. 4, the polyphase components of the FF part, $a_0[n]$ and $a_1[n]$ are projected into subbands. The resulting architecture is shown in Fig. 5, whereby $\mathbf{H}$ and $\mathbf{G}$ denote analysis and filter bank blocks including decimation and expansion as given in Fig. 1, respectively, and $\mathbf{A}_0$ and $\mathbf{A}_1$ represent independent adaptive filters within each of the K subbands.

The subband decomposition, as indicated by the filter characteristics in Fig. 2, devides the input spectrum into narrower bands with reduced spectral dynamics. For LMS-type algorithm, this prewhitening is expected to improve the convergence speed for the FF part of the algorithm. As the FB has to be performed at symbol

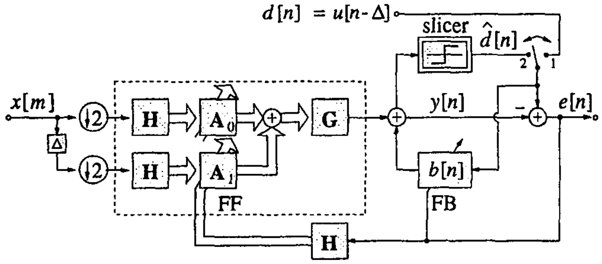

Fig. 5. Subband adaptive equaliser structure I.

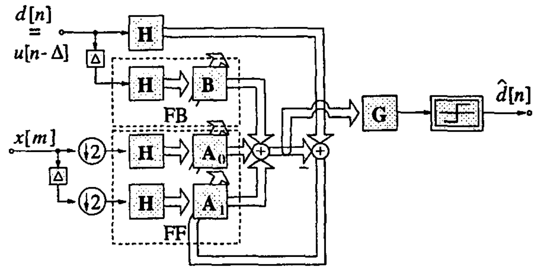

Fig. 6. Subband adaptive equaliser structure II.

rate, the error is evaluated based on the FF outputs reconstructed by **G**, and is projected back into the subband domain to update the filters in $\mathbf{A}_0$ and $\mathbf{A}_1$.

As only the FF input suffers from a high eigenvalue spread due to channel distortions and receive and transmit filters, the FB part has remained in the fullband domain. Similar to the original fullband FSE, this subband FSE structure can be operated in either training mode (switch position 1 in Fig. 5) or in decision directed mode (switch position 2). In both cases the FB input is scrambled and therefore approximately uncorrelated.

However note, that the updating of the FF subband part is delayed w.r.t. FB, such that the convergence of the overall system is potentially unbalanced. To overcome this problem, a modification of the architecture in Fig. 5 (from now on referred to as structure I) will be introduced by integrating the FB part into subbands next.

5. Subband Equaliser Structure II

To overcome the imbalance in updating the FF and FB parts of the FSE architecture of Fig. 5, we also project the FB part into subbands as shown in Fig. 6. The error signal is now formed in the subband domain and can be used to delaylessly update both the FF and FB parts. Similarly to Sec. 3, **B** is of diagonal polynomial form holding the adaptive FB filters running independently within each subband.

Note that different from Structure I, in the new architecture (referred to as Structure II) all adaptive filters are updated by the immediately formed subband errors *at the same time*. Hence any drawbacks such as reduced convergence speed due to delayed updating [11] or the earlier noted updating with unmatched delays is avoided.

However, as the error is now formed in the subband domain, this structure can only be used in training mode. The decision directed learning mode — switch position 2 in the fullband structure in Fig. 4 and the subband structure I in Fig. 5 — requires a non-linearity that cannot be transferred into the subband domain. Therefore, if decision directed learning was to be performed, Structure I would have to be selected. By appropriate subband projections, the FB filter $b[n]$ in Fig. 5 can be reconstructed from **B** in Fig. 6.

Equaliser structure	L_{FF}	L_{FB}
Fullband	500	100
Structure I	70	100
Structure II	70	40

Tab. 1. Number of coefficients in the FF and FB parts of the different simulated equaliser structures.

6. Simulation Results

A severely dispersive Gaussian channel having a delay spread of approximately 100 symbol periods and spectral zeros was employed for testing the performance of the different FSE structures in conjunction with 64-QAM modulation for signal transmission. The channel SNR was variable. The delay Δ was selected such that the FF-part of the FSE mainly acted on the pre-cursor ISI. The filter banks used for structures I and II employed $K = 16$ subbands decimated by a factor of $N = 14$, with prototype filter length of $Lp = 448$.

The equalisers in all three structures are updated in training mode by normalised LMS (NLMS) algorithms with a normalised step size $\tilde{\mu} = 0.4$ [11]. The only exception is the FF section in Structure I, where a delay NLMS [11] was employed. Due to the increased sampling period in the subbands, the order of subband-based FSE sections can be reduced by approximate a factor of N without impairing the steady-state mean square error (MSE) performance [7]. The filter lengths for our simulations are based on the parameters in Tab. 1.

In a first set of simulations, the convergence of the subband FSE structures is compared to the fullband

FSE for various multilevel QAM schemes and variable channel SNR. Both structure I and II exhibit very similar steady-state characteristics; it is for this reason that Fig. 7 shows the it error rate (BER) in the steady state for structure II only. It is interesting to note, that the proposed subband scheme has advantages over the fullband FSE particularly for high SNR. There, the modelling capabilities of appear to be enhanced due to the fullband structure levelling out at less favourable BERs. For 64-QAM modulation and no noise noise, the convergence curve is shown in Fig. 8. The subband structures converge significantly faster, whereby structure II has an initially high convergence speed than structure I due to the balanced updating.

To further investigate the leaning behaviour of the different FSE structures, the simulations in Figs. 9 and 10 assess the convergence speed of the adaptive equalisers. The MSE curves for an SNR of 20dB of the three different FSE structures are shown in Fig. 9, after averaging over 20 ensemble runs. The graphs show a general advantage for the subband FSE structures in adaptation speed, whereby structure II exhibits the best initial convergence speed amongst the three. An overall faster convergence of subband FSE structures is observed in comparison to the fullband realisation, which can be attributed to the prewhitening effect of the subband decomposition of the FF section.

To explore the adaptation behaviour of the different FSE sections, the transient behaviour of the largest coefficients of both FF sections and that of the FB section has been recorded. The moduli of these coefficients are displayed in Fig. 10, normalised w.r.t. their steady-state value. Since the two FF sections only insignificantly varied from each other, they are only given as a single averaged curve. We observe in Fig. 10 that for structure I the FF and FB sections show a considerable difference in terms of their convergence, whereby the slower FB part limits the overall adaptation speed. For structure II the convergence speed of the FB part was increased. Since in this case the eigenvalue spread of the FB input was not improved over structure I, the faster convergence is likely to be due to the improved updating procedure based on the same error subband signals.

In terms of computational complexity, we evaluate the computational cost comparison for the subband equalisers implementations compared to the fullband realisation according derivation in reference [12]. In addition to their increased convergence speed, the subband structures I and II only require 37% and 29% of the fullband FSE's computational complexity, respectively, due to their proportionately reduced filter orders.

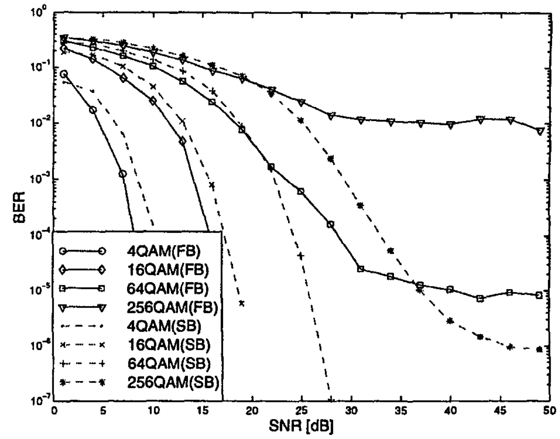

Fig. 7. BER performance of fullband (FB) and subband (SB) structure II over variable channel SNR for various modulation levels.

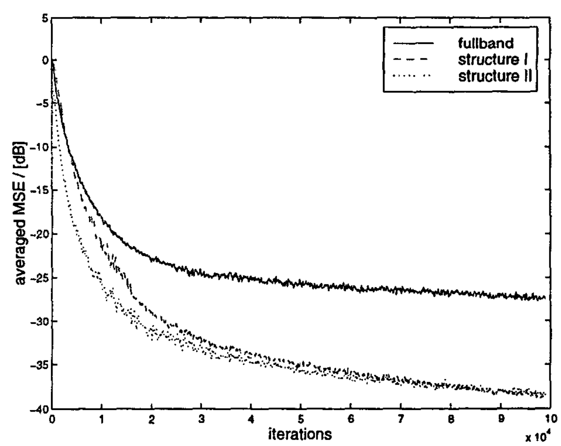

Fig. 8. MSE performance for fullband and subband (structure I and II) equalisers for a noise free channel.

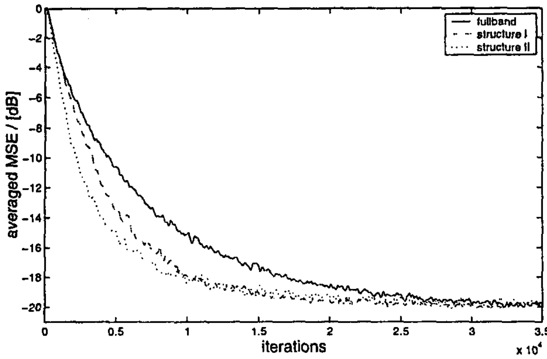

Fig. 9. Ensemble MSE of fullband and subband FSEs.

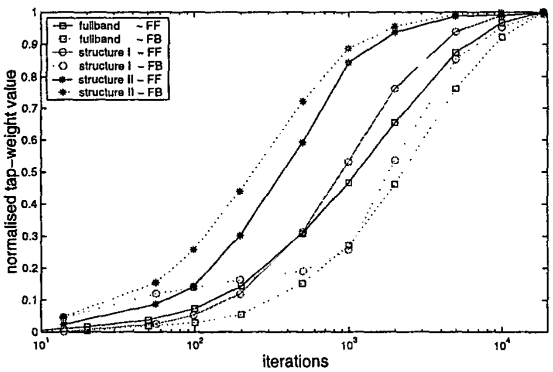

Fig. 10. Trajectories of normalised maximum coefficients within each of the FF and FB parts.

7. Conclusions

Subband decompositions have been considered to overcome the slow convergence behaviour of FSEs, which has led to the definition and evaluation of two different subband FSE structures that benefit from the prewhitening effect of subband decompositions. The structures differ in the extend to which FSE parts are operated in the subband domain. While structure I can yield improvements over the fullband case, structure II, which is entirely operated in the subband domain, shows a further enhanced performance in learning with a known training sequence, as demonstrated in simulations. Although not explicitly derived, an additional appeal of these structures is their reduced computational cost.

Although not explicitly simulated here, the subband FSE can also operated in decision directed mode, whereby only structure I can be employed. If training occurs with structure II with its superior convergence speed, and then the subband FB section needs to be projected into the fullband domain, and structure I be used for decision directed updating.

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